MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Martingale methods in statistics

11.12. bis 17.12.1988

The title "Martingale methods in statistics" can be interpreted in many ways, since the basic probabilistic tool of martingale theory can be used throughout mathematical statistics in a multitude of ways. In the present meeting there was a clear concentration on one particular area of mathematical statistics in which martingale theory (in its modern "French" continuous-time form) plays a dominating role, namely the area of *survival analysis:* the analysis of life-times or survival times in medical (biostatistical) and engineering applications. This has been an area of vigorous development in the last ten years. Other topics covered were: semimartingale theory and the theory of filtered statistical experiments, time-series analysis, sequential statistics, and miscellaneous applications and "pure" probabilistic subjects.

The meeting was felt by all participants to be a great success. Clear signs for the future were the growing interest in multi-dimensional time scales, and related to that the interaction between different time scales (e. g. age, calendar time, and duration of stay in a particular state in the modelling of one individual's life history).

It was especially nice at the meeting to have four participants from the Soviet Union. Two others were also invited but did not come for private reasons. There were also two participants from Australia, one from Israel, and eight from the U.S.A., and two from Finland, as well as participants from closer by: England, France, Belgium, Holland, Denmark and W. Germany.

The organisers (R. Gill, NL-Utrecht and H. Strasser, D-Bayreuth) especially appreciated the possibility of inviting several extra participants at very short notice, in particular one from the U.S.A. two weeks before the meeting. The new computing facilities were also a great success.

Abstracts

P. K. Andersen

THE COX REGRESSION MODEL FOR NON-HOMOGENEOUS MARKOV PROCESSES

We consider non-homogeneous Markov processes ($X_i(t)$, $i = 1, \ldots, n$) with finite state space S and assume that for every $h, j \in S, h \neq j$, the transition intensity is given by a Cox-type regression model $\alpha^i_{hj}(t) = \alpha^0_{hj}(t)\exp(\beta' Z_i)$ Then the regression coefficients β and the integrated underlying transition intensities $A^0_{hj}(t)$ can be estimated much like in the case of survival data and it is shown how these estimates can be converted into estimates for the transition probabilities $Pr(X(t) = j/X(s) = h, Z_0)$, $t > s$ for given covariates Z_0 using the product integral representation. Finally large sample properties of the transition probability estimates are derived.

E. Arjas

UNOBSERVABLES IN OBSERVATIONAL STUDIES AND FILTERING OF MARKED POINT PROCESSES

The term "unobservable variables" has come up quite often in the recent discussion concerning the methodologies of applied sciences such as econometrics and demography. In some other areas unobservables, of latent variables, have a long standing tradition. In this talk we review some situations where the natural approach to modelling seems to call for the introduction of unobservables. We discuss how such situations are conveniently described in terms of filtering from a marked point process, with respect to a range of possible histories. We also discuss briefly the consequent techniques of estimation, and the use of Bayesian ideas in this context.

N. Becker

ESTIMATING POPULATION SIZE FROM MULTIPLE RECAPTURE EXPERIMENTS IN CONTINUOUS TIME

The size of a closed population is to be estimated using data from a multiple recapture study in continuous time. A method of moments for martingales is used to define a class of estimators. The asymptotic relative efficiency of estimators contained in this class is defined, and thereby estimators which are both convenient and efficient are identified.

E. Bolthausen

A MARTINGALE APPROACH TO DIRECTED POLYMERS IN A RANDOM ENVIRONMENT

A short introduction to some open and one solved problem concerning random walks in random environments was given. The solved one is a central limit theorem in the following model:
Let $\xi(0) = 0, \xi(1), \dots$ be an ordinary symmetric random walk on $\mathbb{Z}^d . X(t,i), t \in \mathbb{N}, i \in \mathbb{Z}^d$ are i. i. d. random variables which are strictly positive, satisfy $EX = 1$ and are independent of ξ. The law of ξ_T is transformed by the Kernel $\kappa_T = \Pi_{s=1}^{T} X\left(s, \xi(s)\right)$, i. e. one defines the probability measure $\mu_{T,x}$ on $\mathbb{Z}^d$ depending on the realization of X by

$$\mu_{T,X}(A) = E(\kappa_T 1_A(\xi_T)|X.)/E(\kappa_T|X.)$$

A simple proof is given that for $d \geq 3$ and small variance of X $\mu_{T,X}\left(\sqrt{T}.\right)$ converges for almost all realizations of X to a normal distribution. This simplifies and extends results of I. Imbrie and T. Spencer (J. Stat. Phys., to appear) who evaluated the variance of $\mu_{T,X}$ in a special case by complicated cluster expansion methods.

R. Dahlhaus

PARAMETER ESTIMATION FOR PROCESSES WITH LONG RANGE DEPENDENCE

Stationary processes with a long range dependence (nonsummability of the autocorrelations) occur for example as increments of self-similar processes. They are of importance in data analysis as an alternative to the assumption of independence. If the process is Gaussian, the classical estimates, mean and sample variance, are no longer efficient. We consider weighted means and weighted M-estimates and prove for a certain (adaptive) choice of the weight function that the resulting estimates are Fisher-efficient. Furthermore, we prove efficiency of the maximum likelihood and a quasi-maximum likelihood estimator for σ^2 and the dependence parameters (e. g. the self-similarity parameter).

P. L. Davies

THE AVERAGE GERMAN STUDENT

The standard method of calculating the average length of study of German graduates is to average over the graduates of a particular year. It is pointed out that this is only equal to the average length of study if the system is in equilibrium. Given the paucity of information it would seem to be a non-trivial problem to detect trends in the average length of study. The talk by a non-expert, was an attempt to interest the experts in the problem and to get advice on how to tackle the problem

K. Dzhaparidze

CONDITIONS FOR ASYMPTOTIC EFFICIENCY OF MLE

In the joint paper with E. Valkeila, Helsinki, we study the Hellinger type distances between probability measures on a filtered probability space. We give upper (and lower) bounds for these distances, expressed in "predictable" terms – in terms of the Hellinger process and a certain process of a similar form, which naturally enter into considerations.
The upper bounds discussed are useful for checking in particular, one concrete condition for asymptotic efficiency of MLE.

E. Eberlein

ON APPROXIMATION OF SEMIMARTINGALES

We study the almost sure approximation of semimartingales by continuous processes with independent increments. The results are motivated by the functional central limit theorems of Liptser and Shiryayev and their applications to statistical invariance principles due to Greenwood and Shiryayev (discrete time case) and Jacod and Shiryayev (continuous time case).

P. Embrechts

PIECEWISE-DETERMINISTIC MARKOV PROCESSES: A MARTINGALE CALCULUS IN STOCHASTIC MODELLING

In this talk I shall:

i) Discuss the basic theory of piecewise-deterministic Markov processes as treated in (2). These processes provide an interesting class of non-diffusion type models for which a martingale stochastic calculus is available.

ii) Provide applications to Insurance Risk models where economic factors such as investment strategies, borrowing, inflation, dividend payment, ... are incorporated. See (1) for more details.

1. Dassios, A. and P. Embrechts: *Martingales and Insurance Risk, Stochastic Models (1989)*, to appear.

2. Davis , M. H. A.:*Piecewise-deterministic Markov Processes: A general Class of Nondiffusion Stochastic Models.* JRSS(B), 46 (1984), 353-388.

P. D. Feigin

MARTINGALE ESTIMATING EQUATIONS ON SEMIGROUPS (and Examples)

After considering the simple example $\{X_n = \theta X_{n-1} + E_n,\ n \geq 1\}$ where $\{E_n\}$ are i. i. d. exponentially distributed with mean 1, we readily see that the optimum estimator of θ cannot be obtained as a solution of a martingale estimating equation. However we may embed the process in the Abelian semigroup $(\mathbb{R}, \wedge)$ instead of $(\mathbb{R}, +)$. We develop a calculus involving generalized expectations and martingales for Abelian-semigroup-valued processes. These generalizations are derived from work due to S. Lauritzen on generalized exponential families in just such a setting. We are then able to derive generalized martingale estimating equations for models like the one above and which do lead to optimum estimators. A max-ARMA process example is also used to illustrate the theory.

F. Götze

STEIN'S METHOD FOR MULTIVARIATE STATISTICS

Using an inductive multivariate extension of Stein's method of differential equations we prove rates of convergence for multivariate statistics $T = t_n(X_1, \ldots, X_n)$ converging to a standard multivariate normal distribution. We use instead of Stein's equation an Ornstein-Uhlenbeck type diffusion equation $\Delta\Psi(x) - x \cdot \nabla\Psi(x) = h(x)$. In this way Berry-Esseen type results in $\mathbb{R}^k$ are proved extending one dimensional results of van Zwet and Friedrich.
More generally this method allows to treat statistics $T = t(\Pi)$, where Π is a random permutation on the integers $1, \ldots, N$ with uniform distribution. Berry-Esseen type results for these general permutation, sampling and independent situations are obtained by elementary Stein-type arguments jointly with E. Bolthausen (Berlin).

P. E. Greenwood

EFFICIENCY OF ESTIMATORS FOR PARTIALLY SPECIFIED FILTERED MODELS

This is joint work with W. Wefelmeyer. Consider triangular arrays of counting processes $X_{n1}, \ldots, X_{nn}$ and of vector-valued (predictable) covariate processes $Y_{n1}, \ldots, Y_{nn}$. Suppose we are given functions $a_\theta, \theta \in \Theta$ such that $a_\theta(Y_{ni}(t), t)$ is the

intensity of X_{ni} with respect to the filtration generated by both arrays and some unspecified P_θ. We formulate a version of the Hajek-LeCam convolution theorem for estimating real-valued functionals $K(\theta)$ in this context. It implies that an estimator sequence $\hat{K}_n$ is efficient if $n^{1/2}\left(\hat{K}_n - K(\theta)\right)$ is approximated by $Z_n(\theta)$, the partial log-likelihood in the least favourable direction.
We prove efficiency, within a certain class, of the Huffer-McKeague estimator in Aalen's additive risk model, of the Cox estimator in the proportional hazards model, and of the McKeague-Utikal estimator in Beran's general nonparametric survival regression model.

P. Groeneboom

INTERVAL CENSORING AND DECONVOLUTION

The situation is studied where the (real-valued) random variable of interest cannot be directly observed, but only information is available about an interval to which the random variable belongs or about the convolution of the unknown distribution with a known kernel. A famous example of the latter situation is the Wicksell problem. We determine nonparametric maximum likelihood estimators (NPMLE's) for the unknown distribution functions and discuss the close connection between the behaviour of these NPMLE's and certain local minimax results.

E. Haeusler

RATES OF CONVERGENCE IN THE MARTINGALE CENTRAL LIMIT THEOREM

Let the real-valued random variables $X_1, \ldots, X_n$ form a martingale difference sequence w.r.t. the σ-fields $\mathcal{F}_0 \subset \mathcal{F}1 \subset \ldots \subset \mathcal{F}_n$ and set $S_n = \sum_{i=1}^{n} X_i$. We present asymptotically optimal uniform and nonuniform bounds on $|P(S_n \leq x) - \Phi(x)|, x \in \mathbb{R}$, where Φ denotes the distribution function of the standard normal distribution. The bounds are expressed in terms of the Ljapunov sum $L_{n,2\delta} = \sum_{i=1}^{n} E(|X_i|^{2+2\delta})$ and the variance term $N_{n,2\delta} = E\{|\sum_{i=1}^{n} E(X_i^2|\mathcal{F}_{i-1}) - 1|^{1+\delta}\}$, where $0 < \delta < \infty$. The result in the nonuniform case is joint with K. Joas, Munich.

C. C. Heyde

ASYMPTOTIC INFERENCE FOR STOCHASTIC PROCESSES

Various competing approaches to what can be termed quasi-likelihood and asymptotic quasi-likelihood are available. These are outlined together with methods for comparison and combination of estimating functions. Temporal process can be conveniently discussed using semimartingale ideas but random fields are not so quite amenable to a comprehensive treatment.

R. Höpfner

ASYMPTOTIC INFERENCE FOR RECURRENT (BUT NON-ERGODIC) MARKOV CHAINS IN CONTINUOUS TIME

Consider a Markov chain Z (continuous time, countable state space) whose infinitesimal generator depends on some unknown parameter $\theta \in \Theta$. Assume that Z is irreducible recurrent for all $\theta \in \Theta$, $\Theta \subset \mathbb{R}$ open.

Let p_θ^t denote the restriction of P_θ (a probability on the canonical path space) to $\sigma(Z_s : s \leq t)$, which corresponds to observing a path of Z up to time t. A key step in asymptotic inference is an expansion (valid under some assumptions)

$$\ln\left(dP_{\theta_n}^{\cdot n}/dP_\theta^{\cdot n}\right) = (\theta_n - \theta)\, M_\theta(\cdot n) - \frac{1}{2}(\theta_n - \theta)^2\, [M_\theta]\,(\cdot n) + \ldots$$

where M_θ is a P_θ-martingale with quadratic variation $[M_\theta]$. After suitable choice of $\theta_n = \theta + h/\sqrt{a(n)}$, the local limit experiment at θ is determined by weak convergence under P_θ of the pair

(*)

$$\left(M_\theta(\cdot n)/\sqrt{a(n)}, [M_\theta](\cdot n)/a(n)\right)$$

as $n \to \infty$. Assuming that the Fisher information associated to observation of one full lifecycle of Z is finite for all $\theta \in \Theta$, only limits of type

(I) (B, id), with $a(t) \sim tL(t)$

(II) $(B(W^\alpha), W^\alpha)$, $0 < \alpha < 1$, with $a(t) \sim t^\alpha \tilde{L}(t)$

can arise in $(*)$. Here $L, \tilde{L}$ are slowly varying functions, B denotes standard Brownian motion, W^α is the process invers to the stable subordinator V^α of index α, $0 < \alpha < 1$, and $B(W^\alpha)$ ist time-changed Brownian motion, the time-change being given by the level-crossing times of a stable subordinator V^α independent of B. Note that α, $a(\cdot)$ depend on θ.

Case I corresponds to LAN, case II to LAMN of the statistical model at $\theta \in \Theta$. We present a simple example where Z is null recurrent and exhibits all case II limits, $\alpha = \alpha(\theta) \in (0, 1)$.

M. Jacobsen

THE MULTIPLICATIVE INTENSITY MODEL: RIGHT CENSORING AND LEFT TRUNCATION

Statistical models are considered for partial observation of iid failure times where the intensities for the occurrence of observed failures are as they would be under complete observation. For models with right censoring a minimal class of censoring patterns is introduced what permits a 1-1 correspondence between the joint distribution of all failure times and observations (censorings an observed failures) and the marginal distribution of the observations with failure intensities as above. For models with left truncation little is known, and the problem of even finding a wide class of truncation supplements with the derived intensity structure is left open.

J. Jacod

INFORMATION PROCESSES

Starting with a filtered statistical model $(\Omega, F., (F_\nu)_{\nu\geq 0}, (P_\theta)_{\theta\in\Theta})$, with Θ an open set of $\mathbb{R}^d$ containing 0, we define the notion of "local differentiability" of the model at point $\theta - 0$. For such models we have a derivative of the likelihood processes at $\theta = 0)$ which is a locally-square integrable martingale $V_t = (V_t^i)_{i\leq d}$ under P_0, and the Fisher information process is the matrix-valued process $\Lambda_t^{ij} =< V^i, V^j >_t$.
Then we give a necessary and sufficient condition for local differentiability in terms of a sort of second-order differentiability of the corresponding Hellinger processes; this allows to obtain a simple expression of the information process based on Hellinger processes. Finally we exhibit some examples when the information process can be explicitly computed.

A. Janssen

LOCAL ASYMPTOTIC NORMALITY OF RANDOMLY CENSORED MODELS AND CONDITIONAL TESTS

The topic of the talk is a new approach for test problems with randomly censored data. It is based on the local asymptotic normality of the underlying model, the classical results of Hájek and Sidák (1967) and a recent paper of Neuhaus (1988). Consider a two sample problem for randomly censored survival times. It is the purpose to introduce tests which are able to find differences between the two underlying groups. Since common rank tests with estimated variances do no attain the level at finite sample size, we introduce conditional tests which are a natural extension of the rank tests of Hájek and Sidák (1967). The following results are obtained: The conditional rank tests are asymptotically equivalent to unconditional tests. Prior information concerning the censoring procedure can be used to give a refinement of the model and to improve the tests. We compute the asymptotic power under local alternatives. Adaptation is possible and we get efficient tests for certain classes of hazard rates alternatives.

I. Johnstone

FISHER'S INFORMATION IN TERMS OF THE HAZARD RATE

If $\{g_\theta(t)\}$ is a regular family of probability densities on the real line, with corresponding hazard rates $\{h_\theta(t)\}$, then the Fisher information for θ can be expressed in terms of the hazard rate as follows,

$$I_\theta \equiv \int (\dot{g}_\theta/g_\theta)^2 g_\theta = \int \left(\dot{h}_\theta/h_\theta\right)^2 g_\theta, \ \theta \in \mathbb{R}$$

where the dot denotes $\partial/\partial\theta$. This identity shows that the hazard rate transform of a probability density has an unexpected length-preserving property. We explore this property in continuous and discrete settings, some geometric consequences and curvature formulas, its connection with martingale theory, and its relation to statistical issues in the theory of life-time distributions and censored data.

A. F. Karr

NONPARAMETRIC SURVIVAL ANALYSIS WITH TIME-DEPENDENT COVARIATE EFFECTS: A PENALIZED PARTIAL LIKELIHOOD APPROACH

Techniques are presented for nonparametric analysis of data under a Cox-regression-like model permitting time-dependent covariate effects determined by a regression function $\beta_0(t)$. Estimators resulting from maximization of an appropriate penalized partial likelihood are shown to exist and a computational approach is outlined. Weak uniform consistency (with a rate of convergence) and pointwise asymptotic normality of the estimators are established under regularity conditions. A consistent estimator of a common baseline hazard function is presented, and used to construct a consistent estimator of the asymptotic variance of the estimator of the regression function.

N. Keiding

NONPARAMETRIC ESTIMATION IN THE LEXIS DIAGRAM

The Lexis diagram is the (calender time $\times$ age) plane, each individual being represented by a line starting at (birth time t, birth age 0) and ending at (death time

$t + x$, death age x). (Calendar time, age) - specific death intensities (mortality rates) $\mu(t, x)$ and disease incidences (morbidity rates) $\alpha(t, x)$ are standard tools in demography and epidemiology, but so far no nonparametric statistical estimation techniques are available in continuous time. After a brief survey of the difficulties compared to the calendar time – homogeneous case the choice of kernel smoothing is motivated.
The main part of the paper is a presentation of a case study (joint work with C. Holst & A. Green, tent. acc., Amer. J. Epid). From a survey of all 1499 insulin-treated diabetics in Fyn county, Denmark, on 1 July, 1973, it is desired to estimate diabetic incidence 1933 - 1973 among 0 - 30 year old males and females. The (time, age) of onset is known retrospectively for all patients. Due to the truncated nature of the sampling (patients only sampled conditionally on survival until 1 July, 1973) each should be weighted $1/p$, p = this patient's survival probability from onset to sampling. The value of p is estimated from an independent sample. The results are shown to fit nicely with direct estimates of diabetes incidence for the 1953 cohort.

Alternative calculations done on the same data by Y. Ogata (Tokyo) using techniques by Ogata & Katsura (Ann. Inst. Statist. Math. (1988) are also presented (they give similar results).

E. V. Khamaladze

INNOVATION MARTINGALES FOR MULTIVARIATE TIME

This talk is mainly devoted to the following statistical problem: In case of random variables of any finite dimension and both simple or parametric hypotheses how to construct convenient "empirical" processes which could provide the basis for goodness of fit tests - more or less in the same way as the uniform empirical process does in the case of simple hypothesis and scalar random variables?
The solution of this problem is connected here with the theory of multiparameter martingales and the theory of function-parametric processes. Namely, for the limiting Gaussian processes some kind of filtration is introduced and so-called scanning innovation processes are constructed - the adapted standard Wiener processes in one-to-one correspondence with initial Gaussian processes. This is done for the function-parametric versions of the processes.

A. Kutoyantz

PARAMETER ESTIMATION IN AN INCORRECT DIFFUSION MODEL

Some nonstandard problems for diffusion processes with a small noise. Let the observations $X_\varepsilon = \{X_\varepsilon(t), 0 \leq t \leq T\}$ satisfy the SDE $dX_\varepsilon(t) = S(\theta, X_\varepsilon(t))\,dt +$

εdW_t, $X_\varepsilon(0) = x_0, 0 \leq t \leq T, \varepsilon \downarrow 0$ and the unknown parameter $\theta \in \Theta \subset \mathbb{R}$ is necessary to estimate. It is known, that if *a)* the function $S(\cdot)$ is smooth enough, *b)* x_0-nonrandom, *c)* the trend is really $S(\theta, x)$, *d)* the function $F(\theta_1, \theta) = \int_0^T [S(\theta_1, X_0(t)) - S\theta, X_0(t)]^2 dt$ has a unique minimum at $\theta_1 = \theta$, then the MLE is
1. consistent
2. asymptotically normal $(\hat{\theta}_\varepsilon - \theta)\varepsilon^{-1} \Longrightarrow \mathcal{N}(0, I(\theta)^{-1}), I(\theta) = \int_0^T S_\theta(\theta, X_0(t))^2\, dt$.
3. asymptotically efficient.

This report is devoted to the MLE investigation in the situations when a) - d) conditions are not fulfilled. For example, if the function $F(\cdot)$ has two zeros at $\theta_1 = \theta$ and $\theta_1 = \tilde{\theta}$ then MLE $\hat{\theta}_2 \to \theta$ with prob. p and $\hat{\theta}_\varepsilon \to \tilde{\theta}$ with prob. $T - p$ where $p = P\{\xi_1^2 > \xi_2^2\}$ and $\mathcal{L}\{\xi_1, \xi_2\} = \mathcal{N}(0, 0; 1.1, \rho)$.

S. Leurgans

AN INCONSISTENT GENERALIZED MAXIMUM LIKELIHOOD ESTIMATOR IN A MULTISTATE MODEL

The simplest nontrivial multistate model has instantaneous transitions from state 1 to state 2 at time S and from state 2 to state 3 at time T. Generalized maximum likelihood estimators (GMLE's) for two nonparametric models are known to be well-behaved.
The Markov model assumes that the conditional hazard rate of T given S depends only on t, for $t > S$. The semi-Markov model assumes that the conditional hazard rate of T given S depends only on $t - S$, for $t > S$. The GMLE for the combined model in which the conditional hazard rate is the sum of one function of t and another function of $t - s$ is derived. The estimators of integrals of the two functions are Fisher-inconsistent functionals of two counting processes.

J. Mau

PARTITIONED COUNTING PROCESSES IN CLINICAL STATISTICS

Consider an underlying counting process and a random partitioning of the time interval of observation. Counting points in each stochastic interval separately, produces a counting process on the total time interval for each stochastic interval of the partitioning. The partitioned counting process is the multivariate counting process constructed from the interval-specific counting processes. If the underlying counting process admits a multiplicative representation of its stochastic intensity as considered by Aalen (Ann. Stat., 1978), then the same is true for the components of the partitioned counting process. Though the deterministic part of the stochastic intensity, the hazard function, is still that of the underlying counting process probabilistically, separate inference about that hazard function based on data from observation of only the particular stochastic interval will practically lead to a statistical model with separate hazard functions with respect to each stochastic interval. In clinical applications, each patient is assumed to generate such an underlying counting process and also an individual sequence of random stopping times. The applications of this simple concept are, firstly, to simplify the stochastic modelling of multistate survival if one is only interested in the transition intensities associated with absorbing states, and secondly, to construct a statistical model for the monitoring of follow-up studies with staggered entry where the random stopping times are introduced via calendar time intervals.

I. McKeague

INFERENCE FOR SEMIMARTINGALE REGRESSION MODELS

Let X_t be a semimartingale which is either continuous or of counting process type and which satisfies the stochastic differential equation $dX_t = Y_t\alpha(t, Zt)dt + dM_t$, where Y and Z are predictable covariate processes, M is a martingale and α is an unknown, nonrandom function. We study inference for α by introducing an estimator for $\mathcal{A}(t,z) = \int_0^z \int_0^t \alpha(s,x)ds\ dx$ and deriving a functional central limit theorem for the estimator. The asymptotic distribution turns out to be given by a Gaussian random field that admits a representation as a stochastic integral with respect to a multiparameter Wiener process. This result is used to develop a test for independence of X from the covariate Z, a test for time-homogeneity of α, and goodness-of-fit tests for Cox's proportional hazards model and Aalen's additive risk model used in survival analysis.

A. A. Novikov

A CLASS OF MARTINGALES ASSOCIATED WITH AUTOREGRESSIVE PROCESSES

Let Y_t be a first order autoregressive process with discrete or continuous time parameter. A class of martingales which are functions of Y_t and t is presented for the case when $E\exp(nT_t) < \infty$ for all $u > 0$. These martingales are analogous to an exponential class of martingales for processes with independent increments. As an application of these martingales sufficient and necessary conditions for instance of moments of stopping time $\tau = \mathrm{int}\{t : Y_t \geq A\}$ are obtained. An application to a quickest detection problem is also considered.

K. Oelschläger

LIMIT THEOREMS FOR AGE- AND AGESTRUCTURE-DEPENDENT BRANCHING PROCESSES

Wir betrachten ein Modell für die zeitliche Entwicklung einer Population mit Altersstruktur, wobei die Vermehrungs- und Todesraten vom Alter des betroffenen Individuums und der Altersverteilung der Gesamtpopulation abhängig sind. Es zeigt sich, daß die zeitliche Entwicklung der Altersverteilung im Grenzwert großer Populationen deterministisch wird und durch die Lösung einer Integro-Differentialgleichung beschrieben werden kann. Außerdem konvergieren die Fluktuationen um den deterministischen Grenzwert gegen einen unendlich – dimensionalen Ornstein – Uhlenbeck Prozess.
Die Beweismethoden basieren auf der Stroock-Varadhan'schen Martingalcharakterisierung von Diffusionsprozessen und deren Weiterentwicklung von Holley-Stroock zur Untersuchung von Vielteilchensystemen.

P. Protter

WEAK CONVERGENCE OF STOCHASTIC INTEGRALS AND SOME APPLICATIONS

We give conditions which are simple to verify and that ensure the weak convergence of stochastic integrals, or of solutions of stochastic differential equations. These conditions are then illustrated with the Black-Scholes model in Finance Theory. This is joint work with Darrell Duffie.

H. Pruscha

STATISTICAL ANALYSIS OF TREND-AFFECTED AND OF DETRENDED MULTIVARIATE POINT PROCESSES

In a multivariate point process possessing an intensity process a trend can be defined by multiplying a factor to the intensity. The familiar asymptotic properties of ML-estimators of trend- as well as model-parameters can be derived by using the martingale theory of point processes and a transformation result of point process distributions. The trend is removed by transforming the occurrence times by means of the estimated integrated trend function. The usual statistics for stationary point processes (as, e. g., mean rate, periodogram, conditional (Palm) intensity function) are then defined on the basis of the detrended process. It is investigated how the asymptotic distribution results carry over from stationary model to the detrended model.

Y. Ritov

MARTINGALES IN CENSORED REGRESSION

Suppose $X, X_1, \ldots, X_n$ are iid where $X = (Y \wedge c, Z, 1_{\{Y \leq c\}}$, $Y = \beta^T Z + \varepsilon$ and ε and (Z, c) are independent. The distributions of ε and (Z, c) are assumed to be unknown. We investigate the connections between two types of estimators of β. The first is a class of M-estimators which are a generalization of the Buckley-James estimator. The second type is the class of estimators which are based on linear rank test and was suggested by Tsiatis (1988). It is proved that these two families are asymptotically equivalent. The proof is an extension of a basic connection between "conditional expectation martingales" and "counting process martingales" explored in Ritov and Wellner (1988) and Efron and Johnstone (1987). Some non standard martingales are used in the proof.

N. J. Schmitz

OPTIONAL SAMPLING THEOREMS FOR SEQUENTIAL SAMPLING PLANS

Doob's famous optional sampling theorem (1953) turned out to be a powerful tool for many statistical applications in particular in sequential analysis. But the implication of that theorem fails, in general, to hold for partially ordered index sets as arising e. g. in problems of sequential design of experiments or of sequential sampling plans. Under additional assumptions optional sampling theorems for partially ordered index sets have been proven e. g. by Haggstrom (1966), Washbun/Willsky (1981), Mandelbaum/Vanderbei (1981), Kurtz (1980) and Hüzeler (1985). We present a result, essentially due to Harenbrock (1988), which is general enough to cover all these theorems. For the index set a "scanning" structure is required and for the stopping time a reachability condition is assumed. This theorem allows applications for sequential sampling plans for continuous time processes.

T. Selke

SEQUENTIAL BEHAVIOUR OF THE KAPLAN-MEIER ESTIMATE OF THE PROBABILITY OF SURVIVAL TO A PARTICULAR AGE

Consider a staggered-entry clinical trial in which survival times are independent and identically distributed with distribution F. Suppose also that entry times and censoring times are independent of the survival times. I will state and discuss a theorem according to which, conditional on the entry times and censoring times, the Kaplan-Meier estimate of the probability of survival to a particular age behaves over time approximately like a backward Brownian motion when the estimated variance of the Kaplan-Meier estimator is used as a clock time.

E. V. Slud

RELATIVE EFFICIENCY OF TWO-SAMPLE RANK TESTS WITHIN A MULTIPLICATIVE INTENSITY MODEL

In the context of large sample clinical trials with independent individuals randomly allocated to two treatment groups, for which survival times follow a log-linear multiplicative intensity model with treatment group as covariate, we calculate the asymptotic relative efficiency of the logrank test for treatment effect as compared with the optimal score test. The main idea is to express explicitly the failure hazard intensity obtained by ignoring all covariates other than treatment group: this is done by a Bayes formula conditional expectation of the multiplicative intensity with respect to the predictable filtration generated by the failure counting processes, the at-risk indicators, and the treatment indicators. The relative efficiency formulas apply to situations with multiple failures and time-dependent covariates, and can be estimated from data with complete cover rates but mastered treatment-group indicators. Two theoretical and one data-analytic examples are given.

M. Sørensen

QUASI-LIKELIHOOD FOR SEMIMARTINGALES

A martingale estimating function intended for estimating a parameter determining the distribution of a semimartingale is presented. This estimating function is particularly intended for situations where the likelihood function does not exist or is intractable. The proposed estimating function is optimal within the class of all martingale estimating functions carrying information on the parameter and satisfying certain modest regularity conditions. The characterisation of this class follows from martingale representation theory. The optimality is according to the fixed sample as well as to the asymptotic criterion in the general Godambe-Heyde framework for quasi-likelihood inference. Hence the estimating function is a quasi-score function and the derived estimator a quasi-likelihood estimator. Conditions are given under which asymptotically the quasi-likelihood estimator exists, is consistent, and, when properly normalized, is asymptotically normal.

A. F. Taraskin

ASYMPTOTIC BEHAVIOUR OF THE LIKELIHOOD RATIO OF PROCESSES

For the sequence of processes $(X_t^n)_{t\geq 0}$ defined on some filtered space $(\Omega^n, \mathcal{F}^n, (\mathcal{F}_t^n)_{t\geq 0})$ which are semimartingales with respect to two measures the limit behaviour of the likelihood ratio process is studied. The limit process is the process with the independent increments. The conditions of convergence are formulated in terms of

triplets of predictable characteristics.
If the measure of the process X^n is parametrizied then the conditions of local asymptotic infinitely divisibility are proposed. This is natural generalization of the well known LAN-property.

E. Valkeila

LÉVY-PROHOROV DISTANCES FOR COUNTING PROCESSES

We derive an upper bound for the Lévi-Prohorov distance between an arbitrary counting process and a poisson process. We give some applications of our bounds and (re)prove a related weak convergence result.

W. Wefelmeyer

EFFICIENT NONPARAMETRIC MAXIMUM PARTIAL LIKELIHOOD ESTIMATORS FOR PARTIALLY SPECIFIED MODELS

This is joint work with P. E. Greenwood. Consider a triangular array of counting processes $X_{n1}, \ldots, X_{nn}$ and of vector-valued (predictable) covariate processes $Y_{n1}, \ldots, Y_{nn}$. Assume that the intensity process of X_{ni} is of the form $a_\theta(Y_{ni}(t), t)$, with a_θ known up to (possibly infinite-dimensional) parameter, but with unspecified joint distribution of (X_{ni}, Y_{ni}).
An efficiency concept for estimating real-valued functionals $k(\theta)$ in this set-up was introduced by Greenwood and Wefelmeyer (1988). It implies that an estimator $\hat{k}_n$ is efficient if its standardized error $n^{1/2}\left(\hat{k}_n - k(\theta)\right)$ is approximated by the derivative $Z_n(\theta)$ of the partial log-likelihood ratio in the least favourable direction. (For the definition of partial likelihoods used here see Gill, 1985, and Jacod, 1987.)
We show that any approximate zero $\hat{\theta}_n$ of Z_n leads to an efficient estimator $k(\hat{\theta}_n)$ of $k(\theta)$. The main part of the proof consists in checking that the partial log-likelihood ratio admits a quadratic approximation in the sense of LeCam (1986). For $k(\theta) = \theta(t)$ the result implies efficiency of nonparametric maximum partial estimators.

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