

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 4/1989

Mengenlehre

22.1. bis 28.1.1989

Die Tagung fand unter der Leitung der Herren M. Magidor (Jerusalem) und E.J. Thiele (Berlin) statt. Die Tagung konzentrierte sich auf drei Hauptthemen:

Große Kardinalzahlen und mengentheoretische Konsistenzbeweise,  
Verallgemeinerung des Jensehens Kernmodells,  
Kombinatorische Mengenlehre.

Besonders bei den beiden ersten Themen wurde der Einfluß der in den letzten Jahren entdeckten neuen großen Kardinalzahlen untersucht.

Außer diesen genannten Gebieten wurde in einer größeren Zahl von Vorträgen über andere Forschungsrichtungen der Mengenlehre referiert, von allgemeinen Grundlagenfragen der Mathematik über andere Axiomensysteme der Mengenlehre hin bis zu Anwendungen der Mengenlehre in der Analysis, so daß insgesamt die Tagung, außer den wichtigen Resultaten in den genannten Hauptgebieten, auch einen ziemlich vollständigen Überblick über die derzeitigen mengentheoretischen Aktivitäten gegeben hat.

Derartig umfassende Tagungen über Mengenlehre scheinen im Abstand von vier Jahren zweckmäßig und erfolgreich zu sein.

## J.E. BAUMGARTNER

## CONSISTENCY PROBLEMS IN THE PARTITION CALCULUS

Assume GCH. Call  $g: [\omega_2]^2 \rightarrow \omega_1$  an Erdős-Rado function if

$\forall f: [\omega_2]^2 \rightarrow \omega \exists X \subseteq \omega_2 |X| = 3$  and  $X$  is homogeneous for  $f$  and  $X$  is antihomogeneous for  $g$ . It is consistent that CH holds,  $2^{\omega_1} \geq \omega_3$  and there are no ER-functions, and a recent result of Shelah seems to show it consistent that GCH holds and there are ER-functions. A related problem: let

$K = \{(\alpha, \beta), (\gamma, \delta)\} \in [\omega_2 \times \omega_2]^2; \alpha < \gamma < \delta < \beta\}$ . Does  $K \rightarrow (3)_{\omega}^2$ ? What if  $\omega_2$  is replaced by  $\omega_3$ ? It is consistent that CH,  $2^{\omega_1} \geq \omega_3$  and  $\omega_3 \rightarrow (\alpha)_{\omega}^2$  for all  $\alpha < \omega_2$ . Does  $c^{++} \rightarrow (\alpha)_{\omega}^2$  all  $\alpha < \omega_2$ ?

Assume MA + non CH. It is known that  $\omega_1 \cdot \alpha \rightarrow (\omega_1 \cdot \alpha, n)^2$ , all  $n < \omega$ , if  $\alpha = \omega$  or  $\omega^2$  and Larson has shown this for  $\alpha = \omega^{\omega}$  and  $n = 3$ . What about other countable  $\alpha$ ? What about  $\omega_1^2 \rightarrow (\omega_1^2, 3)^2$ ? In some ways this is similar to the famous question  $\omega_1 \rightarrow (\omega_1, \omega_1)_2^2$ .

## A. BLASS

## CARDINAL INVARIANTS -- OLD AND MIDDLE-AGED

The following results were presented, mostly without proof. (The notation for the cardinal invariants is as in "Applications of superperfect forcing and its relatives", to appear in the STACY proceedings.)

1.  $\min(b, s)$  is the smallest cardinality of a family of partitions of  $[\omega]^2$  having no common almost homogeneous set (i.e. homogeneous after removal of a finite subset).
2. (joint with A. Taylor and P. Erdős)  $\aleph_1$  is the smallest cardinality of a family of partitions of  $\omega$  such that there is no single partition of  $[\omega]^2$  with the property that all homogeneous sets for the latter are also almost homogeneous for all of the former.
3. The intersection of the Turing cone filters relative to arbitrary reals is closed under intersections of fewer than  $g$  sets.
4. It is not possible to have  $b < u < g$  although either inequality alone is consistent.
5. If  $u < g$  then  $d = c$ .

**M. BOFFA**

**Z F J**

ZFJ is the system obtained by adding to ZF the axiom expressing that  $J$  is an automorphism of the universe. This system is interesting in connection with the consistency problem for Quine's system NF. If  $c$  is a cardinal such that  $J(c) = 2^c$ , then there is a "model" of NF of power  $c$ .

**N. BRUNNER**

**AUSWAHLAXIOM UND  $p$ -ADISCHE ZAHLEN**

Sei  $G$  eine monothetische  $T_2$ -Gruppe deren offene Untergruppen eine Umgebungsbasis der 1 bilden. Dann sind äquivalent

- (i) es gibt ein endliches Produkt  $P = \prod_{p \in A} J_p$ ,  $J_p$  die  $p$ -adischen ganzen Zahlen mit der  $p$ -adischen Topologie, und eine Gruppe  $H$ , die sowohl in  $G$  als auch in  $P$  als nicht nirgends dichte Gruppe einbettbar ist.
- (ii) das von  $G$  induzierte Fraenkel-Mostowski-Modell von ZFA erfüllt das wohlordbare Selektionsprinzip.
- (iii) die von  $G$  bzw.  $P$  induzierten Fraenkel-Mostowski-Modelle erfüllen dieselben Jech-Sochor beschränkten Aussagen.

**L. BUKOWSKÝ**

**MINIMAL COLLAPS**

Survey of minimal collapsing generic extensions of models of ZFC was presented. Some new results were presented. E.g.

Theorem (E. Hartová) 1 : If  $\kappa$  is a measurable cardinal then the forcing with  $\kappa$ -perfect trees collapses  $\kappa^+$  to  $\aleph_1$ .

Theorem (L.B. and E.H.) 2: If  $\kappa$  is regular,  $\aleph_1 < \kappa < \aleph_{\omega_1}$ , GCH holds true, then there exists a generic extension  $N$  of the ground model  $M$  preserving  $\aleph_{1,\kappa^{++}}$ , collapsing  $\kappa^+$  to  $\aleph_1$ , and such that: if  $K$  is a model,  $M \subseteq K \subseteq N$ ,  $K$  collapses some  $\lambda > \aleph_1$  to  $\aleph_1$ , then  $K = N$ .

Theorem (L.B.) 3: If  $\aleph_{\omega+1}$  of the ground model is  $\aleph_2$  of an extension then CH does not hold true in the extension.

Problem: Construct a generic extension in which  $\aleph_2$  is the  $\aleph_{\omega+1}$  of the ground model.

### J. CICHON

#### DECOMPOSITION OF BAIRE FUNCTIONS INTO CONTINUOUS FUNCTIONS

For any  $\sigma$ -class  $A \subseteq P(X)$  let

$$\underline{M}A = \{f \in {}^X[0,1]; (\forall a) f'((a,1)) \in A\},$$

$$\overline{M}A = 1 - \underline{M}A, MA = \underline{M}A \cap \overline{M}A.$$

For any classes  $F, G \subseteq {}^X[0,1]$  let

$$\text{dec}(F, G) = \min \{\kappa; (\forall f \in F)(\exists g_\alpha \in G, \alpha < \kappa)(\exists X_\alpha \subseteq X, \alpha < \kappa) f = \bigcup g_\alpha \upharpoonright X_\alpha\}.$$

Theorem 1: If  $A$  is a  $\sigma$ -class with universal sets then  $\text{dec}(\overline{M}A, \underline{M}A) > \omega$ .

Theorem 2: If  $\{A_n\}_{n < \omega}$  are  $\sigma$ -classes with universal sets,  $A \supseteq \bigcup_n (A_n \cup \neg A_n)$  has reduction property, then  $\text{dec}(MA, \bigcup_n (\underline{M}A_n \cup \overline{M}A_n)) > \omega$ .

Many classical results can be deduced from these theorems.

**P. DEHORNOY**

## STARTING FROM POLYNOMIALS OF ELEMENTARY EMBEDDINGS

Say that a set endowed by a binary operation  $*$  satisfying

$x * (y * z) = (x * y) * (x * z)$  is a clump. If  $j$  is an elementary embedding of  $R_\lambda$  into itself then the set  $A(j)$  of all elementary embeddings constructed from  $j$  using the operation  $k * l = \bigcup_{\alpha < \lambda} k(1 \downarrow R_\alpha)$  is a clumb.

**Conjecture:**  $A(j)$  is a free monogenic clump.

Some consequences of the conjecture were presented.

**C.A. DI PRISCO**

## THE NORMAL FILTER GENERATED BY A FAMILY OF SETS

This talk reported on work done jointly with James Henle. For any filter  $F$  on a regular uncountable cardinal  $\Delta^2 F = \Delta^3 F$ , where  $\Delta$  is the diagonal intersection operator. On the other hand, if  $\lambda > \kappa$  and there is  $\lambda', \kappa \leq \lambda' \leq \lambda$  satisfying the free set existence property  $(\lambda', n, \omega) \rightarrow n + 1$ , then there is a filter  $F_n$  on  $[\lambda]^{<\kappa}$  such that  $\Delta^{n-1} F \neq \Delta^n F = \text{CLUB}_{\kappa, \lambda} (n \geq 1)$ . If  $\lambda \geq \aleph_\omega$  then  $F \neq \Delta F \neq \Delta^2 F \neq \dots \neq \Delta^n F \neq \dots$  for some filter  $F$  on  $[\lambda]^{<\kappa}$ .

**F.R. DRAKE**

## FOUNDATIONS OF MATHEMATICS

Three theses were advanced to explain why I do not understand certain claims about "lack of certainty in mathematics".

- 1) Completed mathematics can always be regarded as fully formalised in some first-order system.
- 2) There can be no ontological distinction between very large finite numbers,  $\omega$ ,  $\omega_1$ , large cardinals, ....
- 3) The only possible explanations of mathematical objects are as a) concrete, feasible numbers or operations; b) elements of (models of) some first-order axiom system; c) elements of some heuristic, intuitive picture of some higher-order

system ("real" integers etc.); and the third thesis is that the only way we can explain elements of type c) is via type b).

Doubts about certainty and usefulness of mathematics can arise only from ignoring first, the correlation between theory as derived in b) and practice as observed in a); and second, the large agreement about what should be derived in b) from intuitions in c).

## M. FOREMAN

### THE CONSTRUCTIVE CONTINUUM HYPOTHESIS

Let  $\Theta = \sup \{ \alpha : (\exists f \in L(\mathbb{R})) f : \mathbb{R} \rightarrow \alpha \text{ onto} \}$ . The constructive continuum hypothesis is the inequality  $\Theta < \omega_2$ .

**Theorem** (M.F. - M. Magidor): If  $NS(\omega_2) \downarrow \text{cof}(\omega_1)$  is saturated and there is a supercompact cardinal, then  $\Theta < \omega_2$ .

## R. FRANKIEWICZ

### EMBEDDING AND REPRESENTATION OF BOOLEAN ALGEBRAS

Let  $F$  be a maximal almost disjoint family in  $\omega$ ; a set  $A \subseteq \omega$  is called a partitioner of  $F$  if each element is either almost contained in  $A$  or almost disjoint with  $A$ . The set of all partitioners forms a Boolean algebra  $B_F$  with an ideal generated by  $F$ . We say that a Boolean algebra  $B^*$  is representable if  $B^* = B_F/F$  for some  $F$ .

**Theorem 1:** Cons(ZFC) implies

Cons(ZFC + MA + there is a not representable algebra of cardinality  $2^{\aleph_0}$ ).

**Theorem 2:** Cons(ZFC) implies

Cons(ZFC + a free algebra of  $\omega_1$  generators is not representable).

## S. FRIEDMAN CODING OVER CORE MODELS

The author described what progress has been made in extending Jensen's Coding theorem into the context of large cardinals. An outline of the proof of the following result was given: If  $[V, \mathcal{A}]$  is a model of  $\text{ZFC} + "$   $\mu$  is a measure" then there is a  $[V, \mathcal{A}]$ -definable forcing for producing a real  $R$  such that  $V[R] = L[\mu^*, R]$  and  $\mu^*$  is a measure extending  $\mu$ . The proof can be carried out as well for an extender sequence  $\underline{E}$  provided  $\text{cof}(\kappa) < \kappa^{++}$  for all  $\kappa$ , subject to some fine-structural facts.

## M. GITIK RECONSTRUCTING EXTENDERS AND $\text{nonGCH}$ OVER MEASURABLES

Assuming there is no inner model with a strong cardinal and the  $K(\underline{F})$  has some weak covering properties it was shown that

- a) if  $j : V \rightarrow M$ ,  ${}^{\omega_1}M \subseteq M$  and the same extender was used  $\omega_1$ -times in the iterated ultrapower  $j \downarrow K(\underline{F})$  then its  $\omega_1$ -th image is in  $M$ .
- b) if  $j : V \rightarrow M$ ,  ${}^{\omega}M \subseteq M$ , all extenders in iteration  $j \downarrow K(\underline{F})$  satisfy  $\lambda < \kappa^{+\omega}$  then we can replace  $\omega_1$  by  $\omega$  in a).

## A. HAJNAL HOMOGENEITY OF INFINITE PERMUTATION GROUPS

A permutation group acting on  $\kappa > \lambda$  is  $\lambda$ -homogeneous if for all  $X, Y \in [\kappa]^\lambda$  there is a  $g \in G$  with  $g''X = Y$ .

**Theorem.**  $\square_{\omega_1}$  implies that there is a group  $G$  acting on  $\omega_2$  which is  $\omega_1$ -homogeneous but not  $\omega$ -homogeneous.

P.M. Neumann asked the problem if  $\mu < \lambda$  and  $\lambda$ -homogeneity implies  $\mu$ -homogeneity and proved that the answer is yes for  $\lambda \geq \omega$  and  $\mu < \omega$ . Independently P. Nyikos and S. Shelah with S. Thomas showed that  $\omega_1$ -homogeneity does not

imply  $\omega$ -homogeneity under the condition that  $MA + 2^{\aleph_0} > \aleph_1$  and  $MA + 2^{\aleph_0} > \aleph_2$  hold respectively.

**P. HINNION**

**THE PROBLEM OF THE CONSISTENCY OF FREGE'S NAIVE SET THEORY, WITH EXTENSIONALITY, FOR 3-VALUED LOGIC**

It is known that Frege's comprehension principle is consistent in a weak logic. The consistency of it with the axiom of extensionality is still an open question. However, there seem to be two ways of attacking the problem:

- 1) try to find a set theory in which one can give an interpretation of the 3-valued Frege's theory.
- 2) try to adapt the recent methods which permitted to prove the consistency of the generalized positive comprehension scheme.

**T. JECH**

**COFINALITIES IN MAGIDOR'S MODEL**

It was shown that in Magidor's Model

$$\text{cof}(\prod \aleph_{3n}) = \aleph_{\omega+1} \quad \text{and}$$

$$\text{cof}(\prod \aleph_{3n+1}) = \aleph_{\omega+2}.$$



**P. KOEPKE**  
**CORE MODELS**

Strong cardinals and extenders were introduced. Strong cardinals are a natural generalization of measurability in terms of elementary embedding; extenders allow to code such "strong" embeddings. Using the notion of coherency one can get inner models for strong cardinals. To obtain a more "L-like" hierarchization of such models, one has to admit partial extenders in the constructive predicate.:  $F_\nu$  is required to be an extender on  $J_\nu[F]$  only. This is a way of obtaining condensation for the  $J_\nu[F]$ -hierarchy. All the  $J_\nu[F]$  considered and  $L[F]$  satisfy the axioms for mice.

**P. KOMJATH**  
**COUNTABLE DECOMPOSITIONS OF EUCLIDEAN SPACES**

It was shown that

- 1)  $\mathbb{R}^n$  can be colored by countably many colors so that no two monochromatic points are of rational distance from each other.
- 2)  $\mathbb{R}^3$  can be colored by countably many colors so that the four nodes of a regular tetrahedron do not get the same color.

This can even be true for equilateral triangle instead of tetrahedron.

The proofs consist of a mixture of set theory, geometry and finite combinatorics.

**A. KRAWCZYK**  
**EXTENSIONS OF INVARIANT MEASURES**

The talk was based on a joint paper by P. Zakrewski and A.K. It was shown e.g. that there is no maximal  $\mathbb{Q}$ -invariant extension of the Lebesgue measure on  $\mathbb{R}$ . On the other hand if  $2^{\aleph_0}$  is real-valued measurable, then there is a maximal finite  $\mathbb{Q}$ -invariant measure on a proper  $\sigma$ -field of subsets of  $\mathbb{R}$ .

## J. LARSON

### CANONICAL PARTITIONS

Canonical partitions were developed by Hajnal and Galvin for proving partition relations  $\alpha \rightarrow (\beta, m)^2$  for ordinals  $\alpha, \beta$  and  $m < \omega$  in the case where  $\alpha, \beta$  are finite powers of  $\omega$ . The concept generalizes to partitions of the  $\omega^{\text{th}}$  power of a cardinal  $\kappa$ , and if  $\kappa$  is Ramsey, then any partition of  $\kappa^\omega$  can be reduced to a canonical one. The reduction is the first step in the proofs that

- 1) if  $\kappa$  is Ramsey, then for all  $m < \omega$ ,  $\kappa^\omega \rightarrow (\kappa^\omega, m)^2$ .
- 2) if  $\lambda < \kappa$  are both Ramsey, then for all  $m < \omega$ ,  $\kappa^\omega \cdot \lambda^\omega \rightarrow (\kappa^\omega \cdot \lambda^\omega, m)^2$ .

## J.P. LEVINSKI

### SOME FILTERS

For a filter  $F$  on  $\kappa$ , set  $B(F) = P(\kappa)/F$ . We look for properties  $E(\kappa)$ , which are very near to the measurability of  $\kappa$ , and are compatible with " $\kappa$  is the critical cardinal", i.e.  $(\forall \alpha)(\alpha < \kappa \rightarrow 2^\alpha = \alpha^+) \ \& \ 2^\kappa > \kappa^+$ . The author had former theorems where  $E(\kappa)$  is " $\kappa$  bears a normal precipitous filter". However, in these models,  $\kappa$ , if ineffable, is not  $\Pi_3^1$ -indiscernible. So the following two properties were considered:

- $E_1(\kappa)$  -  $\kappa$  bears a normal filter  $F$ , such that  $B(F)$  is  $\kappa^+$ -distributive.
- $E_2(\kappa)$  -  $\kappa$  bears an  $F$  such that  $B(F)$  admits a dense  $\kappa^+$ -closed subset.

Several consistency results for  $E$ 's were presented.

## A. LOUVEAU

### BOREL ORDERS AND BQO THEORY

The main known results about Borel orders, quasi-ordered under the embeddability relation were presented:

- (i) (Harrington-Shelah) the orders  $s(2^\xi, \text{lexicographic ordering})$  for  $\xi < \omega_1$  are cofinal;
- (ii) (Marker; Louveau) for any Borel order  $X$  and any  $\xi < \omega_1$  either  $X$  is embeddable in  $2^{<\omega^\xi}$ , or equiembeddable with  $2^{\omega^\xi}$ , or embeds  $2^{\omega^\xi+1}$ ;
- (iii) (Louveau-Saint Raymond) if  $\text{BOR}_\xi$  denotes the set of Borel orders embeddable in  $2^{\omega^\xi}$  then
  - (a)  $\text{ZFC} \vdash \text{BOR}_2$  is well quasiordered,
  - (b)  $\text{Proj.Det.} \vdash \forall n \text{ BOR}_n$  is well quasiordered,
  - (c)  $\text{Hyper Proj.Det.} \vdash \text{BOR}_\omega$  is well quasiordered

The author conjectures that "BOR is well quasi-ordered" is a theorem of ZFC, and in fact of  $2^{\text{nd}}$ -order arithmetics.

**P. MATET**

#### YET ANOTHER VARIANT OF DIAMOND

The diamond principle depending on three parameters  $\kappa, \lambda, p$  is defined just as the usual one, except that one is handling less than  $p$  many subsets of  $\lambda$  at a time, instead of just one. Assuming GCH it was shown that this principle holds whenever  $\lambda > p \geq \kappa$ . Moreover if  $\lambda$  has cofinality less than  $\kappa$  and  $S$  is a stationary subset of  $[\lambda]^{<\kappa}$  then the diamond principle for  $S$  holds true.

**A.R.D. MATHIAS**

#### THE IGNORANCE OF BOURBAKI

This paper criticizes the views of the Bourbaki group on logic, set theory and the foundations of mathematics, as set out in papers and books published in the years 1939-1948, on two grounds:

- 1) they declare themselves disciples of Hilbert yet make no explicit reference to Gödel's Incompleteness Theorems, which were published in 1931;

- 2) their chosen set theory, a version of set theory with AC is not adequate for all of mathematics, not even all the mathematics known in the 1930's.

It is suggested 1) that the Bourbakistes were psychologically unable to face the implications of Gödel's work for Hilbert's programme and therefore sought to pretend that this work did not exist; and 2) that their choice of a restricted set theory reflects, as does the even more restricted choice advocated by Saunders MacLane, the confinement of their mathematical interests to an area of mathematics that may be broadly described as geometry as opposed to arithmetic.

**W.J. MITCHELL**

#### **QUESTIONS ON SINGULAR CARDINALS**

The author started with questions "what is the consistency strength of the failure of the singular cardinal hypothesis". Five different possible reasons for witnessing singularity of a cardinal were discussed.

**L.J. STANLEY**

#### **A COMBINATORIAL METHOD OF CODING BY A REAL**

An approach to coding by a real was sketched which uses some combinatorial consequences of Fine Structure rather than Fine Structure directly. While the method is not yet worked out to models where zero sharp exists, it is still much simpler than Jensen's original method. Also, the forcing has nice properties related to properness, which allow it to be iterated. This is joint work with S. Shelah.

## K. STEFFENS

### SOME FACTS IN MATCHING THEORY

Some basic facts in matching theory of infinite graphs including Aharoni's Duality Theorem, an extension of Dilworth's Theorem for finite p.o. sets and an extension of Menger's Theorem were presented. After discussing a generalization of Tutte's Theorem a necessary and sufficient criterion for the existence of a perfect  $f$ -matching of a countable graph was given.

## C. SURESON

### COVERING PROPERTIES OF CHANG'S MODEL

Chang's Model  $C$  is the analog for the infinitary language  $L_{\omega_1\omega_1}$  of Gödel's Constructible Universe  $L$ . Following Jensen, we say that a model  $M$  satisfies the covering property if any set of ordinals in the universe can be included in a set in  $M$  of the same cardinality. It was shown that the existence of  $\omega_1$  measurable cardinals and the violation of the covering property by  $C$  are equiconsistent.

## F. TALL

### TOPOLOGICAL APPLICATIONS OF GENERIC HUGE EMBEDDINGS

The Foreman-Laver collapse of a huge cardinal to  $\aleph_1$  and target to  $\aleph_2$  has been previously used to transfer paracompactness properties from  $\aleph_1$  to  $\aleph_2$ . Now we instead collapse to  $\aleph_2$  and  $\aleph_3$  in order to take advantage of the countable closure of the partial ordering.

**Theorem 1.** If  $X$  is first countable  $T_2$  space and  $j(P)/P$  is countably closed then  $j''X$  is a closed subspace of  $j(X)$ ,

**Theorem 2.**  $\text{Cons}(\exists \text{huge})$  implies  $\text{Cons}$  (every first countable  $T_2$   $\aleph_2$ -paracompact space of size  $\leq \aleph_3$  is paracompact).

**P. VOJTÁŠ**

## SOME APPLICATIONS OF SET THEORY TO ANALYSIS

Let  $\underline{r}$  denote the minimal cardinality of such a set  $R \subseteq [\omega]^\omega$  that for any  $A \in [\omega]^\omega$  either  $A$  or  $\omega - A$  almost contains an element of  $R$ .

Then

1.  $\text{Cov}(\mathbb{K}) \cdot \text{Cov}(\mathbb{L}) \leq \underline{r} \leq \min(u, i)$ .
2.  $\min(\underline{r}, \underline{d}) \leq$  "the minimal size of a family of regular (Toeplitz) matrices such that every 0-1 sequence is summable by one of them"  $\leq \underline{r}$ .
3. Results comparing the strenght of different convergence tests for series were presented.

**W.A.R. WEISS**

## PARTITIONING THE QUADRUPLES OF TOPOLOGICAL SPACES

**Theorem.** Assume GCH and  $\square_\kappa$  for each singular cardinal  $\kappa$ . For each Hausdorff space  $X$  with cardinality not greater than the least weakly compact cardinal,  $X \rightarrow (Y)_\omega^4$  implies that  $Y$  is discrete. In fact, for each such  $X$  there is  $f: [X]^4 \rightarrow \omega$  such that each homogeneous set is discrete as a subspace of  $X$ .

**P.D. WELCH**

## DETERMINACY AND THE RAMSEY PROPERTY

Equivalences are known for sharps for inner models for every level of the difference hierarchy on  $\prod_1^1$  other than for multiples of  $\omega^2$ . It was shown that  $\text{Det}(\omega^2 - \prod_1^1)$  implies the existence of inner models with many Ramseys and fill in this last equivalence, using games involving mastercodes of mice.

**W.H. WOODIN**

**JUST WHAT CAN LARGE CARDINAL HYPOTHESES PROVE ANYWAY?**

**Theorem.**

Assume there are  $\omega^2$  many Woodin cardinals (below  $\lambda$ ). Assume CBH, UBH (for countable iteration trees on  $V$  below  $\lambda$ ). Then there are partial orders  $P, Q \in V_\lambda$  such that

$$V^P \models \delta_2^1 = \omega_2$$

$V^Q \models$  there is no precipitous ideal on  $\omega_1$ .

In fact  $Q$  can be chosen so that

$V^Q \models$  If  $j : V \rightarrow M \subseteq M(G)$  is a generic embedding with critical point  $\omega_1$  then  $|B| \geq$  least Woodin cardinal,

where  $G \subseteq B$  is  $V$ -generic.

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