MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1989

Topics in Pseudodifferential Operators

11.6. bis 17.6.1989

Die Tagung wurde geleitet von H.O. Cordes (Berkeley), B. Gramsch (Mainz), und H. Widom (Santa Cruz). Sie setzte die erfolgreiche Tagung gleichen Titels vom 2. bis 8. Februar 1986 fort (vgl. *Pseudo-Differential Operators* Proceedings Oberwolfach 1986, Springer Lecture Notes in Mathematics 1256, Berlin, Heidelberg, New York, Tokyo 1987). Im Mittelpunkt des Interesses standen zum einen natürlich die aktuellen Entwicklungen auf dem Gebiet der Pseudodifferentialoperatoren unter Einschluß der Fourierintegraloperatoren. Insbesondere spielte die Untersuchung dieser Operatoren auf nichtkompakten und singulären Mannigfaltigkeiten sowie auf Mannigfaltigkeiten mit Rändern, Ecken und Kanten eine zentrale Rolle, häufig in Verbindung mit einem Kalkül vom Boutet de Monvelschen Typ.

Darüber hinaus war aber auch die gegenseitige Beeinflussung der Theorie der Pseudodifferentialoperatoren und anderer Disziplinen ein wichtiges Thema. Berührungspunkte ergeben sich in natürlicher Weise im Bereich der Operatoralgebren und in der theoretischen Physik, besonders, wenn es um Schrödingeroperatoren oder semiklassische Grenzwerte geht, aber auch bei der Betrachtung singulärer Störungen.

Es war den Veranstaltern gelungen, wiederum eine große Anzahl exponierter Vertreter aus den verschiedenen Gebieten für die Konferenz zu gewinnen, so daß sich viele Möglichkeiten zur Diskussion und zum Austausch mathematischer Ideen ergaben. Darüber hinaus wurden vor 45 Teilnehmern 38 Vorträge gehalten. Schwerpunkte waren unter anderem:

- Operatoren auf nichtkompakten und singulären Mannigfaltigkeiten, sowie Mannigfaltigkeiten mit Rändern, Ecken und Kanten
- Pseudodifferentialoperatoren in Gevreyklassen
- Banach–, Fréchet– und Ψ^*–Algebren von Operatoren
- Störungstheorie und Stetigkeitseigenschaften
- Mikrolokalisation, auch höherer Ordnung
- Anwendungen in der theoretischen Physik.

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Auf Grund des besonderen Charakters und der ausgezeichneten Resonanz unter den Teilnehmern wird eine Fortsetzung dieser Tagungsserie ins Auge gefaßt.

Vortragsauszüge

F. ALI MEHMETI. Interaction Problems for Wave Equations on Ramified Spaces

We consider wave equations on n domains Ω_i of possibly different dimensions. From the spatial parts we construct a selfadjoint operator by taking any closed subspace of $\prod_{i=1}^{n} H^1(\Omega_i)$ as domain of its square root (interaction space). In this way we can describe various kinds of influence between the waves on the domains as interface problems, but also integral, pseudodifferential and Fourier integral conditions. Special cases are also transmission problems on ramified spaces (c.f. G. Lumer, S. Nicaise). The C^∞–operators with respect to an interaction operator form a Ψ–Algebra (notion of B. Gramsch '84) and are thus candidates for 'generalized pseudodifferential operators'. Local or global existence for nonlinear evolution equations with (nonlinear) interaction is treated using abstract results of Kato, Brezis, Shatah, Minty. Fourier integral operators are used to construct parametrices for interface problems with Fourier integral transmission conditions (c.f. J.Cl. Nosmas, S. Hansen).

J. ALVAREZ. H^p Continuity Properties of Pseudo–differential Operators

The titel refers to the typical continuity properties to be expected from a Calderon-Zygmund type operator.

In order to apply the standard techniques, two types of information are required. Locally, some L^{p_0}, L^{q_0} continuity. At infinity, some decay of the gradient of the distribution kernel.

The purpose here is to follow this pattern with a pseudo–differential operator L having symbol or amplitude in the Hörmander class $S^m_{\rho,\delta}$, $m \in \mathbb{R}$, $0 \le \delta < 1$, $0 < \rho \le 1$, and distribution kernel k(x,y). Typical results are

If $m + n + |\alpha + \beta| > 0$, there exists $C = C_{\alpha\beta} \ge 0$ such that

$$|D_x^\alpha D_y^\beta k(x,y)| \le \frac{C}{|x-y|^{\frac{m+n+|\alpha+\beta|}{\rho}}}$$

This estimate is proved to be optimal at least when $|\alpha+\beta| = 0$. Also, integral estimates of the Hörmander type are obtained.

Concerning continuity, L is continous from L^p into itself if $1 < p < \infty$, $m \le -n\left((1-\rho)\cdot\left|\frac{1}{p} - \frac{1}{2}\right| + \lambda\right)$, where $\lambda = \max\left(0, \frac{\delta-\rho}{2}\right)$.

All this implies pointwise estimates for the sharp maximal function and H^p, L^q contunity properties for some $p,q > 0$, which extend known results.

The above estimates are also used to show that an operator having a distribution kernel worse than a standard one will not satisfy in general the T(1) Theorem.

M. ARSENOVIC. C^*–Algebras of Singular Integral Operators on the Poincaré Plane

The structure of a certain C^* algebra $\mathcal{A}$ of singular integral operators on the Poincaré plane $\Pi = \{(x,y) \mid y > 0\}$ is described. The algebra $\mathcal{A}$ is a perturbation of a translation invariant algebra on Π. The commutator ideal $\mathcal{C}$ of it is strictly larger than the class of compact operators K. Two symbol maps, σ and γ, are defined on $\mathcal{A}$. σ is scalar valued, γ is operator valued. They suffice to characterize Fredholm operators in $\mathcal{A}$, $\ker \sigma \cap \ker \gamma = K$. In essence, this gives a calculus of S.I.O. in $\mathcal{A}$, modulo K. Results on essential spectra of (pseudo)-differential operators and Fredholm properties of (pseudo)–differentail operators on Π can be obtained from this calculus.

E.L. BASOR. Toeplitz Operators on Weighted ℓ_p Spaces and Associated Asymptotics

Consider the operator T_γ defined on $\ell_2(\mathbb{Z}_+)$ by

$$(T_\varphi(a))_k = \sum_{n=0}^{\infty} \varphi_{k-n}\, a_n \qquad k = 0,1,\ldots$$

where $a = (a_0, a_1, \ldots)$ and $\varphi_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \varphi(\theta)\, e^{-ik\theta} d\theta$.

The finite Toeplitz matrices are of the form

$$(\varphi_{j-k}) \qquad j,k = 0,\ldots,n.$$

These operators can also be considered on $\ell_p^{\,\mu}$, weighted ℓ_p spaces, where

$$\ell_p^{\,\mu} = \{x \mid \|x\|^p = \Sigma |x_n|^p (n+1)^{p\mu} < \infty\}.$$

On these weighted spaces the spectrum of T_φ can be precisely described for a function φ having one jump discontinuity and piecewise C^2. The spectrum depends on p and μ.
Using this result, it can be shown that the eigenvalues of the finite matrices for large n lie near the range of the function φ.

B. BOJARSKI. Regular and Irregular Interior Boundary Value Problem (I.B.V.P.)

The study of interior boundary value problems for elliptic equations was initiated by Sobolev (1937) with the variational treatment of the Dirichlet problem for the polyharmonic equation $\Delta^m u = 0$ in a domain $\Omega \subset R^n$, whose boundary $\partial\Omega = \Gamma$ consists of submanifolds of codimension ν, $1 \le \nu \le 2m-1$, $\Gamma = \bigcup_{k=1}^{2m-1} \Gamma_{n-k}$ in correspondence with the Sobolev trace theorems for functions in $W^{m,2}(\Omega)$. In the talk I gave a short review of results on I.B.V.P for a general class of elliptic operators acting on vector bundles on closed manifolds M, based on papers by Sternin, Sternin–Novikov and myself. Special classes of distribution spaces $H^s_\Gamma(M)$, adapted to the study of generalized potentials $P(\tau \otimes \delta_\Gamma^{(\alpha)})$ were defined (P is a pseudodifferential operator on M, δ_Γ the Dirac distribution of Γ, $\tau \in D'(\Gamma)$); also their analogues in the context of vector bundles. The generalized potentials are used to reduce the elliptic I.B.V.P. $\mathcal{A} = (A, B_\Gamma)$, (A is a differential operator on M, B_Γ is a differential operator defining the boundary conditions) to a system $\mathcal{X}(A,B)$ of elliptic pseudodifferential operators on Γ and express the index formula for $\mathcal{A}$, ind $\mathcal{A}$ = $\mathrm{ind}_M A + \mathrm{ind}_\Gamma \mathcal{X}$. These I.B.V.P are called regular. Another class of I.B.V.P. arises if the self–adjoint extensions of a symmetric differential operator A_0 are studied, where A_0 is the restriction of a selfadjoint operator A in $L^2(\mathbb{R}^n)$, or $L^2(M)$, to the domain $D_0(M\setminus\Gamma)$.By recent results of Popov for constant coefficient operators in $\mathbb{R}^n$, (Mat.Sb. 1988) the extension process, of von Neumann-Krein-Naimark type, may lead to asymptotic type "boundary conditions" on Γ, which cannot be formulated in terms of Sobolev trace theorems and which are called irregular I.B.V.P's. This type of problems arises in quantum modelling of various diffraction and scattering processes in crystalic, polymeric and superconductor phenomena. Many of these models are explicitly solvable. An important problem arises of describing the class of irregular I.B.V.P. in terms of microanalysis. Also a broad class of explicit solutions of regular I.B.V.P's for the equation $\Delta^m u = 0$ in the ball or half space in $\mathbb{R}^n$ was described.

L. CATTABRIGA. Global Analytic and Gevrey Surjectivity of the Mizohata operator $D_{x_2} + ix_2^{2k} D_{x_1}$

Let $E^{\{s\}}(\mathbb{R}^n)$, $s \le 1$, be the space of all C^∞ functions f on $\mathbb{R}^n$ such that for every compact subset K of $\mathbb{R}^n$ there exists A > 0 such that

$$\sup_{x \in K} \sup_{\alpha \in \mathbb{Z}_+^n} A^{-|\alpha|} \alpha!^{-s} |\partial_x^\alpha f(x)| < +\infty.$$

Then for every $s \ge 1$

i) $(D_{x_2} + i\, x_2^{2k} D_{x_1})\, E^{\{s\}}(\mathbb{R}^2) = E^{\{s\}}(\mathbb{R}^2)$

ii) $(D_{x_2} + i\, x_2^{2k} D_{x_1})\, E^{\{s\}}(\mathbb{R}^3) \subsetneq E^{\{s\}}(\mathbb{R}^3)$, where $D_{x_j} = -i\partial_{x_j}$, $j = 1,2$.

L. COBURN. Toeplitz Operators, Quantum Mechanics, and Mean Oscillation in the Bergman Metric

For Ω a bounded domain in $\mathbb{C}^n$ with normalized volume measure or $\Omega = \mathbb{C}^n$ with Gaussian measure, consider the L^2 spaces and the associated subspaces, H^2, of holomorphic functions. For P the orthogonal projection operator from L^2 onto H^2 and M_f the (densely-defined) operator of multiplication by a fixed L^2 function f, the Toeplitz operators on H^2 given by T_f =

$PM_{f|P}$ and the Hankel operators on L^2, $H_f = (I - P)M_fP$ are of considerable interest. I discuss recently obtained results on the boundedness, compactness and symbol calculus for these operators and their relation to pseudo–differential operators.

R.G. DOUGLAS. Elliptic Operators on Manifolds with Boundary

A cycle in the K–homology $K_0(M)$ of a compact manifold M without boundary is dertermined by an elliptic differential operator D following Atiyah. If $\partial M \neq \emptyset$ then D dertermines a class in the relative group $K_0(M,\partial M)$. Based on joint work with Baum and Taylor, I described a concrete approach, whose principal advantage is an explicit boundary map in the long exact sequence for K–theory which takes the form $\partial[D] = \text{Ker } D - \text{Ker } D^*$ with the appropriate interpretations.

Applications of this work include a generalization of the index theorem for Toeplitz operators of Boutet de Monvel, the reduction of the Atiyah–Singer Index Theorem to the case of Dirac operators, and a new proof that operators cobordant to zero have index zero.

A. DYNIN. Pseudodifferential Operators on Open Riemannian Manifolds

Geodesic compactifications of open (complete or not) Riemannian manifolds often have a natural Thom stratified structure. They may be of arbitrary finite depth and not nessecarily conormally conical.

Basic examples are regular parts of many singular spaces and SBB–compactifications of locally symmetric spaces.

I introduce a class of pseudodifferential operators with stratified symbols on such manifolds. They generate a C^*–algebra. The main results are its solvability and a complete description of all irreducible representations, both reflecting the geometry of the underlying manifold.

Many Cordes comparison C^*–algebras on open manifolds as well as Plamenevskij-Senichkin C^*–algebras on manifolds with compound conical singularities are special cases of the developed theory.

Y.V. EGOROV. Generalized Functions

The main deficiency of the Schwartz distribution theory consists in the impossibility of the multiplication of distributions, which is very important for physics. In the recent works of J.F. Colombeau a new approach was proposed, helping in the solving of this problem. Unfortunately his theory is too complicated, what does make its applications difficult and restricted.

I would like to state some new theory, more clear and transparent and at the same time more general. It is possible to define not only the products of the generalized functions, but any functions of them, even generalized. Very general theorems on the existence and the uniqueness of the solution for the Cauchy problem can be proved. Moreover this theory can be considered as the natural reflection of the real process of computations.

A. ERKIP. Boundary Value Problems for Non–Compact Boundaries

On non–compact manifolds ellipticity may no longer be sufficient for normal solvability as can be observed in the case of the Laplacean, $\Delta: H^2(\mathbb{R}^n) \to L^2(\mathbb{R}^n)$. Similar behavior occurs in boundary value problems. To obtain Fredholm criteria we construct a class of non-compact manifolds with non–compact boundary of the SG–type of Schrohe. Roughly speaking for such a manifold the coordinate maps extend to conic neighborhoods and changes of coordinates are of linear growth. A Riemannian metric satisfying certain growth conditions is chosen so that compatible normal coordinates can be introduced in a conic neighborhood of the boundary. The Calderon projector method can be applied via Schrohe's results on ψdo's of the SG–class, and we obtain a sufficient condition for the normal solvavility of the boundary value problem with weighted L^2–Sobolev data. The sufficient condition turns out to be an md–version of the usual ellipticity condition. By the md-concept we refer to a global parametrix construction of Cordes; a ψdo of order (m,0) in the class $SG(\mathbb{R}^n)$ is md–elliptic if

$$|a(x,\xi)| \geq c(1+|\xi|)^m \quad \text{for large} \quad |x|+|\xi|.$$

This in turn is necessary and sufficient for a(x,D) to be Fredholm: $H^{s+m}(\mathbb{R}^n) \longrightarrow H^s(\mathbb{R}^n)$. As a step towards showing the necessity, we prove that if the manifold contains a cone and the symbol $p(x,\xi)$ vanishes at some finite ξ_0 as $x \to \infty$ within that cone, no BVP $(p(x,D),b^1,\ldots,b^m)$ can be normally solvable. The cone here seems necessary, as the Dirichlet problem for Δ on the strip $S = [0,1] \times \mathbb{R}$ is invertible as a map $H^2(S) \longrightarrow L^2(S) \times H^{3/2}(\partial S)$. Even though the boundary ∂S is of SG–type, the strip does not satisfy to the cone condition.

L.S. FRANK. Coercive and Dispersive Singular Perturbations

The coerciveness concept for one parameter families of singular perturbations was introduced in 1976 and the equivalence of this algebraic condition with the validity of two–sided a priori estimates (uniformly with respect to the small parameter) was proved. It turned out that the same coerciveness condition guarantees that any coercive singular perturbation can be reduced to a regular perturbation by constructing explicitly a reducing operator S^ε, which for each value of the small parameter $\varepsilon \in (0,\varepsilon_0]$ belongs to the class of operators, introduced by L. Boutet de Monvel (joint work with W.D. Wendt). As some direct applications of the constructive reduction procedure for coercive singular perturbations, one should mention the following ones:

1. Asymptotic formulae for their solutions with smooth or non–smooth data.
2. Asymptotics for their eigenvalues and eigenfuntions under the assumption that the reduced operator has only a discrete spectrum.
3. Asymptotic analysis of the bifurcation phenomena for coercive singular perturbations.

Recently estimates of the error term in the reduction of a coercive singular perturbation $\mathcal{A}^\varepsilon$ to a regular perturbation of the reduced operator $\mathcal{A}^0$ was improved (joint work with J. Heijstek) and it was shown that one has:

$$S^\varepsilon \mathcal{A}^\varepsilon = \mathcal{A}^0 + 0(\varepsilon), \qquad \varepsilon \to 0,$$

where S^{ε} is the reducing operator mentioned above and $0(\varepsilon)$ is interpreted in the corresponding operator norm.

Dispersive singular perturbations (introduced in 1986) are special classes of singular perturbations of strictly hyperbolic first order systems.

For dispersive singular perturbations mixed problems are well–posed (uniformly with respect to the small parameter), provided that it is so also for the reduced hyperbolic system.

Construction of a parametrix, based upon Fourier Integral Operators with singularly perturbed amplitudes and phase functions with real and complex values, is given. Singular support of the corresponding Green and Poisson kernels is described (some of them being spread over on one side of the characteristics of the reduced hyperbolic system and some of them having a localized singularity of boundary layer type).

Some applications are indicated (e.g. asymptotics of the spectral function of a self–adjoint elliptic singular perturbation under various assumption on the relation between the spectral function of a self–adjoint elliptic singular perturbation under various assumptions on the relation between the spectral and small parameters.

D. FUJIWARA. Feynman Path Integral as an Improper Integral on the Path Space

Let $L(\gamma) = \frac{1}{2} m \dot{\gamma}^2 - V(\gamma)$, where $\gamma \in H_0^1(0,T)$. Let $\{e_\nu\}_{\nu=1}^{\infty}$ be a c.o.n.s. in $H_0^1(0,T)$ such that $\{e'_\nu\}_\nu^{\infty} = 1$ is Haar's c.o.n.s. in $L^2(0,T)$. We use coordinates

$$x_\nu = (\gamma, e_\nu), \text{ (the inner product in } H_0^1(0,T), \quad \nu = 1,2,...$$

We denote by $N(dx,0,Q)$ the Gaussian measure on $H_0^1(0,T)$ with mean 0 and variance $Q(x) = \sum_{\nu=1}^{\infty} \lambda^\nu x_\nu^{\,2}$ $(\lambda < 1)$. Following K.Ito, we consider the integral:

$$I_n = \prod_{\nu=1}^{\infty} \left(1 + \frac{n\lambda^\nu}{hi}\right)^{1/2} \int_{H_0^1(0,T)} e^{ih^{-1}\int_0^T L(\gamma)dt} N(dx,0,nQ).$$

Theorem Assume that the potential V is real valued and satisfies the following estimates:

$$\sup_x |V^{(\alpha)}(x)| < \infty, \quad (\forall\, \alpha \geq 2).$$

And assume that T is small. Then $I = \lim_{n\to\infty} I_n$ exists.

This gives a mathematical meaning to the Feynman path integral.

B. GRAMSCH. Analytic Functions with Values in Certain Algebras of Pseudo–Differential Operators

The spectral invariance of many Fréchet algebras of pseudo–differential operators is a useful tool to understand manifolds of Fredholm and semi–Fredholm operators. Results on the mero–

morphic inversion and division of operator distributions are presented (work together with W. Kaballo, 1989) in the context of Ψ^*–algebras. Connected to results of Davie (1971) the Arens–Royden–Oka–principle is applied to Fredholm operators in submultiplicative Fréchet algebras of ψdo's. An approach of Salinas (1988) and others to the existence of geodesics for the manifold of symmetries in C^*–algebras can be used to define some geodesics for the standard Fréchet manifolds in the perturbation theory of Ψ^*–algebras. This is related to some work of K. Lorentz.

A. GREENLEAF. Estimates and Composition Calculi for Singular Fourier Integral Operators
The composition of FIOs, which are singular in the sense that they are associated with Lagrangian manifolds which are not local canonical graphs, is in general not an FIO. Since such operators arise naturally in integral geometry and in the study of pseudodifferential operators with multiple characteristics, it is of interest to obtain a good description of the composition of such opeators. We will describe joint work with G. Uhlmann on some particular classes.

S. HANSEN. Computation of Amplitudes Along Rays
For systems of real principal type in the sense of N. Dencker, e.g. the system of isotropic elastodynamics, we consider solutions which are Lagrangian distributions. Their principal symbols, which we call amplitudes, satisfy transport equations on the Lagrangian manifold, Λ. For a given way γ in Λ we want to compute numerically the amplitude of a solution along γ. A method for this is proposed. It consists in the following:
(1) Compute the tangent spaces $T_{\gamma(t)}\Lambda$. This involves the solution of ordinary differential equations of Ricatti–type for symmetric matrices L(t), the coordinates of $T_{\gamma(t)}\Lambda$. L(t) blows up when $T_{\gamma(t)}\Lambda$ leaves the coordinate patch in the Lagrangian Grassmannian. A numerically stable algorithm, developed by the speaker, is used to perform the necessary change of coordinates.
(2) Express the Lie derivative of the amplitude, a, with respect to the Hamilton–field in terms of L(t). Solve the resulting differential equation for $a \circ \gamma$.– The Maslov index is correctly accounted for, too.
The method is hoped to be useful in the numerical computation of high–frequency waves. Similar methods are used in geophysics for computing synthetic seismograms (Cerveny, Hanyga).

J. HOUNIE. Local Integrability of Systems of Vector Fields
Consider $L_1,\dots,L_m$, m linearly independent vector fields in a nbhd. $\Omega \subset \mathbb{R}^{n+1}$ of the origin, with smooth, complex coefficients. We assume:
i) there exist smooth functions $\alpha_{jk\ell}$ such that

$$[L_j,L_k] = \sum_{\ell=1}^{n} \alpha_{jk\ell} L_\ell \qquad \text{(Frobenius condition)}$$

ii) The origin is not an elliptic point for the system and the Levi form of the system is non–degenerate, having eigenvalues of the same sign (strong pseudo–convexity),
then if $n \geq 2$, the system is locally integrable, i.e., there exists a function Z in a nbhd. of the origin such that

$$L_j Z = 0, \quad j = 1,\dots,m$$

$$dZ(0) \neq 0.$$

(This is part of a joint work with Malagutti)

M. LANGENBRUCH. Continuous Linear Right Inverses for Partial Differential Operators
The problem of right inverses for partial differential operators is solved in the space S' of tempered distributions on $\mathbb{R}^n$: P(D) has a right inverse in S' iff $P = Q_0 .. Q_k$, where Q_j is real, irreducible and admits a right inverse in S', while Q_0 has no real roots.

Let P be irreducible and real. Then P(D) has a right inverse in S' iff $PC^\infty(\mathbb{R}^N) = I(P)(\mathbb{R}^N) := \{f \in C^\infty(\mathbb{R}^N) : f|_{V_P} = 0\}$.

This is the property of zeroes of R. Thom.
The proof is based on the characterization of the Thom property by J. Bochnak and on Bierstone's division theorem, which is proved via Hironaka's theorem on the resolution of singularities and Glaeser's theorem on composition of C^∞–functions. For polynomials in two variables a considerably simpler proof can be given.

O. LIESS. Higher Microlocalization and Conical Refraction
The set–up from which we start is: let $\mathbb{R}^n = M_0 \supsetneqq M_1 \supsetneqq \cdots \supsetneqq M_{k+1} \neq \{0\}$ be a sequence of linear subspaces in $\mathbb{R}^n$, denote by $\Pi_j : \mathbb{R}^n \to M_j$ the orthogonal projection and by $\mu_j(t)\xi = t \cdot \Pi_j \xi + (1 - \Pi_j)\xi$ if $t \in \mathbb{R}$. A function is called polyhomogeneous of polydegree $m = (m_0,\dots,m_{k-1})$ if $f(\mu_j(t)\xi) = t^{m_j} f(\xi)$, $m_j \in \mathbb{R}$, and we write $m < m'$ if $m_0 < m_0'$, or if there is s with $m_i = m'_i$ for $i < s$, $m_s > m'_s$. The results presented refer to a calculus of wave front sets which localize to sets of the form $A = \{\xi : \Pi_j\xi \in G_j,\ |\Pi_{i+j}\ \xi| > c\ |\Pi_i\ \xi|^{1+\beta}/|\Pi_{i-1}\xi|^{\beta},\ |\Pi_\ell\xi| > c'(1 + |\Pi_{i-1}\xi|)^{\delta}\}$, where the G_j are open cones in M_j, $\delta < 1$, $\beta > 0$. Further we consider analytic pseudo–differential operators for which elliptic polyhomogeneous symbols of polydegree m dominate polyhomogeneous symbols of polydegree m' on sets of type A if $m' < m$. As a consequence we obtain a result on conical refraction.

K. LORENTZ. Jordan Operators in Ψ^*–Algebras

Recent results show that many algebras Ψ of pseudo–differential operators of order zero are Ψ^*–algebras, i.e.

1) $\Psi \subset L(H)$, Hilbert space, Ψ is a Fréchet–*–algebra, continuously embedded in L(H)

2) $L(H)^{-1} \cap \Psi = \Psi^{-1}$ (property of spectral invariance).

Following the explicit algebraic methods of B. Gramsch, we have

Thm. 1: The similiarity orbits of Jordan Operators contained in Ψ are locally Ψ–rational homogeneous manifolds.

This is even new in the Hilbert–space case (i.e. $\Psi = L(H)$), for which we further have

Thm. 2: If for $T \in L(H)$ there exists a norm continuous local cross section for the map $\pi^T: L(H)^{-1} \to \{gTg^{-1}: g \in L(H)\}$, $\pi^T(g) = gTg^{-1}$, then this can be chosen to be rational.

Thm. 2 is a generalisation of a theorem due to D.A. Herrero.

R. MAZZEO. Spectral Theory of Hyperbolic Manifolds

Let $M = \mathbb{H}^n/\Gamma$ where Γ is a geometrically finite group of isometries acting (freely and) properly discontinuously on hyperbolic n–space. It is possible to give a fairly complete description of the spectrum of the Laplacian $\Delta_k = d\delta + \delta d$ acting on L^2 differential k–forms: the essential spectrum occupies the same region as in the case of all of $\mathbb{H}^n$, namely it fills out the intervals $[(n-2k+1)^2/4,\infty)$, $[(n-2k-1)^2/4,\infty)$ and also $\{0\}$ when $k = n/2$. If M has infinite volume, the only eigenvalues embedded in the continuum occur in the interval $[\min(n-2k\pm1)^2/4, \max(n-2k\pm1)^2/4)$.

Finally, the dimension of the eigenspace at 0 is given by a purely topological quantity (i.e. there is a Hodge theorem).

The tools used are a calculus of degenerate pseudo–differential operators developed jointly with Richard Melrose, and for the embedded eigenvalue problem a unique continuation theorem for certain degenerate operators was also needed. Part of the Hodge theory (in the case when Γ has parabolic elements) was done jointly with Ralph Phillips.

G. MENDOZA. Laplacians on Strictly Pseudoconvex Manifolds

(jointly with C. Epstein and R. Melrose) Let M be a compact manifold with boundary, let $\Theta \in C^\infty(\partial M, T^*M|_{\partial M})$ have nowhere vanishing restriction θ to ∂M. Let ρ be a defining function for ∂M and extend Θ to a form $\tilde{\Theta}$ on M. The space $V_\Theta = \{V \in C^\infty(M,TM) | \tilde{\Theta}(V) \in \rho^2 C^\infty(M)\}$ is a Lie algebra independent of the extension $\tilde{\Theta}$ of Θ. Differential operators based on this algebra form a natural context in which to study Laplacians with respect to Kähler metrics on smooth strictly pseudoconvex domains Ω in $\mathbb{C}^n$, $n > 1$ with the potential given by a strictly psh function ϕ blowing up at the boundary as log dist$(y,\partial\Omega)$. Given such Ω, let M be the manifold with boundary which is topologically $\bar{\Omega}$ and for which the identity map I: $M \to \bar{\Omega}$ in local coordinates is $(y_1,\dots,y_{2n-1},\rho) \to (y_1,\dots,y_{2n-1},\rho^2)$. Let r be a defining function for $\partial\Omega$. Let $\Theta = I^*(\sqrt{-1}\partial r)$ on ∂M. Then the Laplacian with respect to r is an element in Diff^2_Θ.

There is a natural vector bundle ${}^\Theta TM$ and v.b. map ${}^\Theta TM \to TM$ which converts every C^∞ section of ${}^\Theta TM$ to an element of V_Θ. Each fiber ${}^\Theta T_pM$, $p \in \partial M$, is a Lie algebra naturally, which in the case of a strictly pseudoconvex domain corresponds to a semidirect product of the Heisenberg group $\mathbb{H}^{n-1}$ and $\mathbb{R}^+$ acting nonisotropically on $\mathbb{H}^{n-1}$, ${}^\Theta T_pM$ acts on T_pM as vector fields.

The normal operator $N_p(\Delta), p \in \partial M$, which is obtained by freezing the coefficients in the "right" way, acts on T_pM^+ and is in fact equal to the Bergman Laplacian Δ_B for the ball in $\mathbb{C}^n$ under certain indentifications. The precise knowlegde of the resolvent for Δ_B can now be used to obtain meromorphic extension and Fredholm properties for the resolvent of Δ.

S. MELO. A Comparison Algebra on a Cylinder with Semi-periodic Multiplications

A necessary and sufficient Fredholm-criterion is found for operators in a certain C^*-algebra of bounded operators on $L^2(\Omega)$, where Ω is the product of a compact Riemannian manifold and $\mathbb{R}$. This algebra contains the operators of the form $L\Lambda^N$, where L is an N-th order differential operator on Ω with asymptotically periodic coefficients and $\Lambda = (1-\Delta_\Omega)^{-1/2}$.

T. MURAMATU. Boundedness of Pseudo-differential Operators on Besov Spaces

As a continuation of Muramatu (Springer Lecture Notes 1256 (1987)), we have found boundedness results under minimal assumptions on regularity of symbols:

MAIN THEOREM. Let σ be a real number, $1\le p\le\infty$, $1\le q\le\infty$, $1/q+1/q'=1$ ($q'=\infty$ if $q=1$ and $q'=1$ if $q=\infty$), $0\le\delta\le1$, $0\le\rho\le1$, X and Y Hilbert spaces, and set

(0.1) $$\lambda = \frac{n}{p\vee 2}\cdot\frac{1-\delta\vee\rho}{1-\delta},\ \mu = -\frac{n(1-\rho)}{p\wedge 2} + \frac{n(1-\delta\vee\rho)}{p\vee 2}$$

where $s\vee t = \max(s,t)$, $s\wedge t = \min\{s,t\}$

Then the pseudo-differential operator A with symbol $a(x,\xi)$ is bounded from $B^\sigma_{p,q}(\mathbb{R}^n;X)$ to $B^\sigma_{p,q}(\mathbb{R}^n;Y)$ if one of the following conditions is satisfied;

(I) $\sigma>0$ and $a(x,\xi) \in S^\mu_{\rho,\delta}B^{(\sigma\vee\lambda,n/p\wedge2)}_{(\infty,\infty),(1,1)}(\mathbb{R}^n\times\mathbb{R}^n;\ \mathcal{L}(X,Y))$, with the exception of the case where $0<\sigma=\lambda$, $\sigma=0$, $q>1$;

(II) $\sigma<0$, $\delta<1$ and $a(x,\xi) \in S^\mu_{\rho,\sigma}B^{(\lambda-\sigma/(1-\delta),n/p\wedge2)}_{(\infty,\infty),(q',1)}(\mathbb{R}^n\times\mathbb{R}^n;\ \mathcal{L}(X,Y))$;

(III) $\sigma=0$ and $s(x,\xi) \in S^\mu_{\rho,\delta}B^{(\lambda,n/p\wedge2)}_{(\infty,\infty),(q',1)}(\mathbb{R}^n\times\mathbb{R}^n;\ \mathcal{L}(X,Y))$ with additional conditions that $\delta<1$, $\rho<1$, $1<p<\infty$, and $p\vee2 \le q$.

Here $S^\mu_{\rho,\delta}B^{(\sigma,\tau)}_{(\infty,\infty),(q,1)}(\mathbb{R}^n\times\mathbb{R}^n;\ \mathcal{L}(X,Y))$ denotes the Besov-version of the operator-valued symbol class. In this notation σ denotes the order of regularity with respect to x, while τ denotes that with respect to ξ. $\mathcal{L}(X,Y)$ denotes the space of all bounded linear operators from X into Y.

Sugimoto has proved the Main Theorem when symbols are numerically valued, $1<p<\infty$ and $\sigma<\lambda$. He also proved it for the case where $1<p<\infty$, $\sigma=0$ and $\sigma>\lambda$ or $\sigma=\lambda$, $p<1$ holds. Our method is rather different from Sugimoto's, and it does not depend on the theory of Hardy spaces and Fourier analysis except the Hausdorff–Young inequality.

M. NAGASE. Essential Self-adjointness of Pseudo-Differential Operators and Quantum Hamiltonians

We give a class of essentially self-adjoint pseudo-differential operators, and also we give some applications to the proof of essential self-adjointness of quantum Hamiltonians of relativistic (or non-relativistic) spinless particles in magnetic fields.

N. PEYERIMHOFF. The $\bar{\partial}$-Operator on Algebraic Curves

(with J. Brüning and Herbert Schröder) Let M be a Kähler manifold, $m = \dim_{\mathbb{C}} M$ and

$$\Omega^{p,q}_{(2)}(M) = \{s \in C^\infty(\Lambda^{p,q}M) \mid \int_M |s|^2 < \infty, \int_M |\bar{\partial} s|^2 < \infty\}.$$

One can consider the complex

$$0 \xrightarrow{\bar{\partial}} \Omega^{0,0}_{(2)}(M) \xrightarrow{\bar{\partial}} \Omega^{0,1}_{(2)}(M) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \Omega^{0,m}_{(2)}(M) \longrightarrow 0 .$$

If its cohomology vector spaces $H^{0,k}_{(2)}(M)$ are of finite dimension, one calls

$$\chi_{(2)}(M) = \sum_{k=0}^{m} (-1)^k \dim H^{0,k}_{(2)}(M)$$

the L^2-arithmetic genus. If M is compact, this definition coincides with the "ordinary" arithmetic genus.

We considered irreducible algebraic curves $M \subset \mathbb{CP}^k$ with singular set Σ. $M - \Sigma$ obtains a metric by inducing the Fubini–Study–metric of the $\mathbb{CP}^k$ onto it. Using methods of Brüning and Seeley we obtained the following result: Let M, Σ be as mentioned above and $\Pi: \tilde{M} \to M$ the desingularisation of M. Then $H^{0,0}_{(2)}(M-\Sigma)$ and $H^{0,1}_{(2)}(M-\Sigma)$ are always of finite dimension

and (a) $\operatorname{ind} \bar{\partial}_{max} = \chi(\tilde{M})$, (b) $\operatorname{ind} \bar{\partial}_{min} = \chi_{(2)}(M-\Sigma)$,

$\bar{\partial}_{min}$ and $\bar{\partial}_{max}$ denoting the minimal and maximal L^2-closed extensions of $\bar{\partial}$: $\Omega^{0,0}_0(M-\Sigma) \to \Omega^{0,1}_0(M-\Sigma)$. Moreover there is an explicit formula for the difference $\operatorname{ind} \bar{\partial}_{max} - \operatorname{ind} \bar{\partial}_{min}$ only depending on the branching orders of the branches of the desingularisation near the singularities Σ. This result yields a negative answer to a question of McPherson in the case of algebraic curves, which he asked in the paper "Global questions in the topology of singular spaces".

B.A. PLAMENEVSKY. On Solvable C*–Algebras of Pseudo–differential Operators

This report considers C*–algebras generated by pseudo–differential operators on a smooth m–dimensional manifold. The operators act in spaces L_2 with weighted norms. The symbols of the operators are allowed to have discontinuities along submanifolds. All irreducible representations (to within equivalence) are given for such algebras and the topology on the spectrum is described.

D. ROBERT. High Energy Asymptotics for Scattering Phases for Perturbations of the Laplace Operator

In this lecture we present extensions, to non compact support perturbations of the Laplace operator on $\mathbb{R}^n$, of results proved about ten years ago by several people: Buslaev, Colin de Verdière, Guillopé, Popov, for $-\Delta + V$, $V \in C_0^\infty(\mathbb{R}^n)$ and by Majda–Ralston for the Laplace-Beltrami operator, $-\Delta_g$, associated to a metric $g = \{g_{jk}\}$ such that $g_{jk} = \delta_{jk}$ outside a compact of $\mathbb{R}^n$. Let us denote: $L_0 = -\Delta$ and L a perturbation of L_0 ; $L - L_0$ being small enough for defining the scattering matrix at energy $\lambda > 0$: $S(\lambda)$ and the scattering phase $\theta(\lambda)$ by

$$(1)\ \det S(\lambda) = e^{2i\theta(\lambda)}.$$

Birman–Krein theory defines the spectral shift function $s(\lambda)$ by:

(2) $\mathrm{tr}(f(L) - f(L_0)) = -\int f'(\lambda)\, s(\lambda)d\lambda, \quad f \in C_0^\infty(\mathbb{R})$.

We have the remarkable fact: $\theta(\lambda) = -\pi\, s(\lambda)$, $\forall\lambda > 0$.

Let us consider the following perturbations of L_0:

$$L = L(g,A,V) = -G^{-1/4} \sum_{1\le j,k\le n} (\partial_j + iA_j)G^{1/2} \cdot g^{jk}(\partial_k + iA_k)\, G^{-1/4} + V,$$

where $g = \{g_{jk}\}$, $g^{ik} = \{g\}^{-1}$, $G = \det(g)$, A_j, V are smooth on $\mathbb{R}^n$, with values in Hermitean matrices in a complex Hermitean space E_m of dimension m, $A = (A_1,\dots,A_m)$: $\partial_j = \partial/\partial x_j$.

Assume:

($H\rho$) ($\rho > n$) For every $\alpha \in \mathbb{N}^n$ there exist $C_\alpha > 0$ such that:

$$|\partial^\alpha g(x) - \mathrm{Id}| + |\partial^\alpha A(x)| + |\partial^\alpha V(x)| \le C_\alpha(1+|x|^2)^{-(\rho+|\alpha|)/2}$$

for every x in $\mathbb{R}^n$.

(N–T) The Riemannian metric g is non trapping, i.e. all the geodesic curves leave every compact set of $\mathbb{R}^n$.

Our result is:

Theorem: Under the assumptions $(H_\rho)(\rho>n)$ and (N–T) we have:

$$\frac{ds}{d\lambda}(\lambda) \approx \lambda^{n/2+1}\left(\Sigma_{j\ge 0}\, \alpha_j \lambda^{-j}\right) \quad \text{as } \lambda \nearrow +\infty.$$

The α_j depend on g,A,V. In particular, $\alpha_0 = 2m\pi^{n/2+1}/\Gamma(\frac{n}{2}+1) \cdot \int (\sqrt{G(x)}-1)\, dx$. It is possible to compute α_1 and α_2. The α_j are connected with the usual spectral invariants appearing in spectral geometry. (See Gilkey).

Remark: This theorem answers an implicit question raised by R. Schrader in 1980 (Z. Physik C. Particles and Fields 4 (1980) 27–36).

L. RODINO. Pseudo–Differential Operators in Gevrey classes

The calculus of Gevrey infinite order pseudo–differential operators a(x,D) is presented, with symbol a (x,ξ) in the classes $S^{\infty,s}_{\rho,\delta}$ (L. Zanghirati 1985).

As an application, results of hypoellipticity are obtained for operators of the type

$$P = (D_{x_1} \pm i x_2^h D_{x_2})^\ell + R(x,D),$$

where $h \geq 1$, $\ell \geq 2$ are positive integers and R(x,D) is a classical analytic pseudo–differential operator of order $\ell-1$, with $x = (x_1, x_2, \dots, x_n)$. Precisely (L. Cattabriga, L. Rodino, L. Zanghirati): the (non)hypoellipticity of $D_{x_1} \pm i x_1{}^h D_{x_2}$ implies the (non)hypoellipticity of P in the analytic class and in the Gevrey classes G^1, $1 < s < \ell/(\ell-1)$, independently of the lower order terms R(x,D).

R. SCHNEIDER. The Reduction of Order for Pseudo–differential Operators on Lipschitz–Domains

The reduction of order for a pseudo–differential operator on a Lipschitz–domain

$$P_\Sigma \sigma(x,D): \mathring{H}^s(\Sigma) \to H^{s-m}(\Sigma),$$

(p_Σ denotes the restriction to Σ), requires the construction of ψdo's, which are isomorphic between $\mathring{H}^s(\Sigma)$ and $\mathring{H}^{s-r}(\Sigma)$ (resp. $H^s(\Sigma)$ and $H^{s-r}(\Sigma)$). It turns out that it is sufficient in the local case, to construct convolution operators which are alliptic of order $r \neq 0$ and the convolution kernel is supported in a certain cone. By this means one can construct a ψdo $p_\Sigma \sigma'(x,D)$ equivalent (up to smoothing ops.) to $p_\Sigma \sigma(x,D)$, which is of order zero acting on $L^2(\Sigma)$. As an application, the Fredholm property of a special class of operators on a Lipschitz domain occuring in many applications, is proved by means of reduction of order and Gårdings inequality.

ELMAR SCHROHE. Boundary Value Problems on Noncompact Manifolds

A classical result states that on a compact C^∞ manifold X with boundary, a system $(P, B^1, \dots, B^m)$, consisting of a differential operator P of order 2m and m boundary operators of orders $r_1, \dots, r_m$, defines a Fredholm operator

$$(P, B^1, \dots, B^m): H_{2m}(X) \to H_0(X) \times \prod_{i=1}^{m} H_{2m-r_j-1/2}(\partial X)$$

if and only if P is elliptic and the system satisfies the Lopatinski Shapiro condition. This result

extends to the case of elliptic elements in the Boutet de Monvel algebra over X.
On the other hand, the Fredholm property generally fails to hold if the manifold (and particularly the boundary) is noncompact. In order to recover it, a class of weighted symbols and Sobolev spaces together with a stronger notion of ellipticity was introduced for a suitable class of manifolds [Erkip & Schrohe: Normal Solvability of Elliptic Boundary Value Problems on Asymptotically Flat Manifolds, preprint Mainz 1988]. It was shown that this concept is compatible with the usual framework. In particular, the geometrically defined normal derivative is a pseudodifferential operator within the new class. This gives all the ingredients to construct the analogue of the Boutet de Monvel algebra with pseudodifferential, Green, potential and trace operators of the above type [E. Schrohe: A Boutet de Monvel Type Calculus for Boundary Value Problems on Manifolds with Asymptotically Flat Boundary, in preparation].

B.–W. SCHULZE. Pseudo–Differential Boundary Value Problems on Manifolds with Edges

A manifold with edges is locally described as $X^\wedge \times \mathbb{R}^q$ with X being a closed compact C^∞ manifold (the base of the model cone), $X^\wedge = X \times \mathbb{R}_+$ the (stretched) model cone, and $\mathbb{R}^q$ the edge. If X = {point}, then we have a manifold with boundary, locally described by the half space $\mathbb{R}_+ \times \mathbb{R}^q$ with $\mathbb{R}_+$ as inner normal to the boundary $\mathbb{R}^q$. This makes it natural to establish on $X^\wedge \times \mathbb{R}^q$ (or globally on a (stretched) manifold W with edges) a pseudo–differential calculus analogous to Boutet de Monvel's algebra with extra trace and potential conditions along the edge and a ψdo part along the edge Z, which are matrices

$$\begin{bmatrix} A & K \\ T & Q \end{bmatrix} : \begin{matrix} W^{s,\gamma}(W) \\ + \\ H^s(Z,G_-) \end{matrix} \to \begin{matrix} W^{s-\Gamma,\gamma-\Gamma}(W) \\ + \\ H^{s-\Gamma}(Z,G_+) \end{matrix} \qquad (1)$$

with "wedge Sobolev spaces" $W^{s,\gamma}(W)$ of smoothness s and weight γ, and finite–dimensional vector bundles $G_\pm$ over Z. A is a ψdo with symbol ot the form $t^{-\mu}a(x,t,y,\xi,t\tau,t\eta)$, $(x,t,y) \in X^\wedge \times \mathbb{R}^q$. Algebra and parametrix aspects show that in general we have as left upper corner also operators A+M+G with M being a sort of Mellin ψdo and G a Green operator. A admits two leading symbolic levels $\sigma_\psi^\mu(A)\,(x,t,y,\xi,\tau,\eta)$ (= the usual ψdo symbol of A) and an edge symbol $\sigma_\wedge^\mu(A)\,(y,\eta)$ as an operator family

$$\sigma_\wedge^\mu(A)(y,\eta)\colon K^{s,\gamma}(X^\wedge) + G_{-,y} \to K^{s-\Gamma,\gamma-\Gamma}(X^\wedge) + G_{+,y},$$

$(y,\eta) \in T^*X, 0$, with $K^{s,\gamma}(X^\wedge)$ being weighted Sobolev spaces on $X^\wedge$. Ellipticity means bijectivity of both symbol components. Under this condition (1) is a Fredholm operator for all $s \in \mathbb{R}$, and there is a parametrix B of analogous structure with $\sigma_\psi^{-\mu}(B) = \sigma_\psi^\mu(A)^{-1}$, $\sigma_\wedge^{-\mu}(B) = \sigma_\wedge^\mu(A)^{-1}$. A special case of this calculus is the theory of ψdo boundary problems without transmission property which contains in particular Boutet de Monvel's theory as a special case.

M.A. SHUBIN. Weak Bloch Property and Weight Estimates for Pseudo-differential Operators on Non-compact Manifolds

The weak Bloch property (WBP) for a differential operator on a Riemannian manifold is the following implication:

$$\{\text{there exists a bounded } \Psi \text{ such that } A\Psi = \lambda\Psi\} \Rightarrow \lambda \in \sigma(A),$$

where $\sigma(A)$ is the spectrum of the closure of A in $L^2(M)$.

Theorem: Let M be a Riemannian manifold with bounded geometry and subexponential growth of the volume, A a uniformly elliptic differential operator with C^∞-bounded coefficients. Then WBP holds.

The same statement is true also for some appropriate pseudo-differential operators.

The theorem generalizes a result of T. Kobayashi, K. Ono and T. Sunada (1989) who introduced WBP and considered the case of Schrödinger operators with potentials which are invariant with respect to a free action of a discrete group of isometries with a compact quotient manifold.

The proof uses the ideas developped by G. Meladze, Ju. Kordjukov and the author. We deduce WBP from the exponential decay of the Green function off the diagonal. The coincidence of spectra in all spaces $L^p(M)$ automatically follows.

T. UMEDA. The Weyl Quantized Hamiltonian for a Relativistic Spinless Particle in an Electromagnetic Field

Let H be the Weyl quantized Hamiltonian for a relativistic spinless particle in an electromagnetic field. The Hamiltonian H depends on the velocity of light c, and on the mass of the particle m. First I show that $H - mc^2$ converges to the usual Schrödinger operator in the strong resolvent sense as $c \to \infty$. From this fact I conclude that a solution of the Schrödinger equation for the Hamiltonian H converges to the solution of the usual Schrödinger equation. In addition, I discuss the pure-imaginary time Schrödinger equation. Next, I show that H converges to a Hamiltonian H_0 in uniform operator topology as $m \to 0$. This implies that e^{-itH} converges to e^{-itH_0} in uniform operator topology as $m \to 0$.

A. URIBE. Some Results on the Semi-Classical Limit

The following new result exemplifies what one means by the semi-classical limit. Let $V \in C^\infty(\mathbb{R}^n)$, $V \geq 1$. Pick $E > 0$ such that $E^2 < \liminf_{|x|\to\infty} V(x)$. It is known that for small $c > 0$, the spectrum of the Schrödinger operator $-h^2\Delta + V$ in $(0, E^2 + c)$ consists of finitely many eigenvalues, $\{\lambda_j(h)\}$. Let $\{\phi_t\}$ denote the Hamilton flow of $H(x,\xi) = \sqrt{|\xi|^2 + V(x)}$ on the energy surface $\{H = E\}$.

Theorem: Under a generic condition on $\{\phi_t\}$, and assuming E is a regular value of H, one has,

$\forall\varphi \in \mathcal{S}(\mathbb{R})$ such that $\hat{\varphi} \in C_0^\infty(\mathbb{R})$,

$$\sum_j \varphi\left(\frac{\sqrt{\lambda_j(h)}-E}{h}\right) \underset{h\to 0}{\sim} \sum_{j\geq 0} c_j(\varphi,h)h^{-d+j}$$

Moreover, the right–hand side is governed by the geometry of the flow $\{\phi_t\}$.

<u>Corollary</u> Under generic assumptions $\forall c > 0$

$$\#\{j \mid |\sqrt{\lambda_j(h)}-E| \leq c\} = 2c \text{ Vol } \{H=E\}h^{-n+1} + o(h^{-n+1}).$$

These results are proved using the Hörmander theory of Lagrangian distributions.
In the previous results, the phase space $(\mathbb{R}^{2n}, \Sigma dp\wedge dq)$ is considered as $T^*\mathbb{R}^n$. It could also be considered as $\mathbb{C}^n$, and the symplectic form the imaginary part of the standard Kähler structure on $\mathbb{C}^n$. In general, it is an interesting question to study the semiclassical limit on Kähler manifolds. Let (X,ω) be a Kähler manifold, where $[\omega] \in H^2(X,\mathbb{Z})$. (Assume X compact, for simplicity). Then there exists a holomorphic line bundle $L \to X$ with a hermitian structure with curvature $\frac{1}{i}\omega$. $\forall m \geq 0$, let $H_m = H^0(X,L^m)$. Then one can develop an analogue of the Hörmander theory, in the following sense. To every Lagrangian $\Lambda \subset X$ satisfying a quantization condition, one can associate spaces $J^k(\Lambda)$ of sequences $\{\phi_m \in H^m, m \geq 1\}$ such that its asymptotic behavior as $m \to \infty$ is governed by a symbol of Λ. Here $1/m$ plays the role of Planck's constant. Using this theory, one can effectively study the semi–classical limit on Kähler manifolds. The theory is based on the Boutet–de–Monvel/Guillemin theory of Hermite operators.

XUE PING WANG. <u>Asymptotic Expansions of Widths of Resonances for Schrödinger Operators with Stark Effect</u>

This work is a continuation of our previous one on bounds of widths of resonances in Stark effect. By constructing precise asymptotic solutions in forms of complex oscillatory integral (in sense of J. Sjöstrand), we obtain complete asymptotic expansions for widths of resonances for two particular classes of potentials: $V(x) \equiv 0$, $(x_1,x') \in \mathbb{R}\times\mathbb{R}^{n-1}$, $x_1 < -R$, for some $R > 0$ and in the one dimensional case: $V(x) \sim \frac{1}{|x|^\sigma}(c_0 + \sum_{j\geq 1} c_j x^{-j})$, $x\to-\infty$, $c_0\neq 0$, $\sigma>0$. Our results show that the leading term of the asymptotic of widths depends on quantum wave functions through the vanishing order of the partial Fourier transform (in x' variables) of eigenfunctions of $-\Delta + V(x)$, at $\xi' = 0$. The proof of these results is based on a priori energy estimates for distorted Stark Hamiltonians, which are non–selfadjoint and non–semibounded.

M. YAMAZAKI. <u>Non–Kowalevskian Partial Differential Equations</u>

We are concerned with the singularity of solutions of dispersive partial differential equations; namely, non–kowalevskian differential equations of evolution which are globally well–posed in both directions of time. Typical examples are the Schrödinger equation and the KdV equation. It is known that these equations admit a version of microlocal analysis, but it tells only the location of singularity of solutions restricted to some fixed time. That is, it does not tell the time evolution of the singularity.

On the other hand, several authors found that the decay of the initial data implied the local regularity of the solution after a finite time.

Here we are going to show that the decay of the initial data in some direction guarantees the microlocal regularity of the solution of such equations.

B. ZIEMIAN. Second Microlocalization and Asymptotic Expansions

In my talk I am interested in the behaviour of the multi-dimensional Mellin transforms of solutions to linear PDE's with smooth coefficients $Ru = w$ where $R = R(x_2,\ldots,x_n, x_1 \frac{\partial}{\partial x_1},\ldots,x_n \frac{\partial}{\partial x_n})$ is a singular operator with regular singularities (in the sense of Kashiwara), elliptic in some sense. I study 2-microlocal regularity of such solutions which amounts to the study of the growth order of the Mellin transforms along the pure imaginary planes. Further, I show by means of explicit examples that Mellin transforms of solutions extend to large sets in $\mathbb{C}^n$ and it is possible to compute their jumps on the complements of those sets. It turns out that outside certain singular points the jumps are analytic functions which extend to holomorphic functions with certain singularities. This is the phenomenon called "resurgence" by J. Ecalle. Finally I comment on "continuous asymptotics" of solutions at the point zero.

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