MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1989

Freie Randwertaufgaben, insbesondere numerische Behandlung und optimale Steuerung

9.7. bis 15.7.1989

Die Tagung wurde organisiert von K.–H. Hoffmann (Augsburg) und J. Sprekels (Essen).

Im Mittelpunkt des wissenschaftlichen Programms standen insgesamt 34 Vortragsbeiträge zur mathematischen Theorie, Numerik und Steuerung verschiedener Klassen von freien Randwertproblemen, die bei der Modellierung physikalisch–technischer Vorgänge auftreten. Insbesondere handelte es sich um folgende Problemkreise: Hysteresis–Phänomene bei thermomechanischen Phasenübergängen in Festkörpern, nicht–isotherme spinodale Dekomposition, Phase–Field–Modelle, Stefan– und Muskat– Probleme, Spritzguß, Kristallzucht, Strangguß von Stahl, Oberflächenwellen, Oberflächenspannung, Kapillarflächen, Überschallströmungen und Strömungen durch poröse Medien.

Die Tagung wurde mitgeprägt durch eine intensive Diskussion der Vorträge und angrenzender Problemstellungen, die insbesondere auch während der Abendszeit im kleineren Kreis stattfand.

Die angenehme Atmosphäre der Tagung, die nicht zuletzt der exzellenten Betreuung durch die Mitarbeiter des Instituts zu verdanken ist, sei noch besonders erwähnt. Im Namen der Tagungsteilnehmer danken wir Herrn Prof. Dr. M. Barner und seinen Mitarbeitern herzlich dafür.

Vortragsauszüge

H. W. Alt:

Non–isothermal phase separation in binary systems II

We prove the existence of a weak solution $(u, v, w) \in L^2([0,T];\ H^{1,2}(\Omega;\ \mathbb{R}^3))$ of the phase–separation model presented in the first talk. Here we assume that κ =const. The differential equations are

$$-v + \varphi_{,u}(u,w) - \nabla\cdot(\kappa w \nabla u) = 0,$$

$$\partial_t u + \nabla\cdot\vec{j} = 0, \text{ where } \vec{j} = -(l_{11}\nabla v - l_{12}\nabla w),$$

$$-\partial_t \varphi_{,w}(u,w) + \nabla \cdot \vec{q} + g = 0, \text{ where } \vec{q} = l_{22} \nabla w - l_{21} \nabla v,$$

with boundary conditions

$$\partial_{\vec{n}} u = 0, \quad \vec{j} \cdot \vec{n} = 0, \quad \vec{q} \cdot \vec{n} = p(u,w).$$

The main difficulty which arises is the absence of a maximum principle for temperature, therefore the inequality $w > 0$ must be a result of the construction of a solution. We achieve this by imposing a side condition $\epsilon \leq w \leq \frac{1}{\epsilon}$ for the time discrete problem with step size τ. By the physical assumptions on the potential φ we obtain in the limit $\epsilon \to 0$ that $w > 0$ a.e., and that the variational inequality in fact is an equality. A compactness argument in time then enables us to go the limit $\tau \to 0$.

M. Brokate:

Optimal control of shape memory alloys

We consider an optimal control problem for the shape memory alloy equations

$$u_{tt} - (\theta F_1'(\epsilon) + F_2'(\epsilon))_x + u_{xxxx} = f$$

$$\theta_t - \theta F_1'(\epsilon)\epsilon_t - \theta_{xx} = g, \quad \epsilon = u_x,$$

where f, g and the boundary temperature θ_Γ are the controls. It is proved that the map $(f, g, \theta_\Gamma) \to (u, \theta)$ has a directional derivative in appropriate function spaces, and that necessary optimality conditions in form of a variational inequality and an adjoint equation hold. (joint work with J. Sprekels, Essen)

G. Caginalp:

The phase field model – theory and numerical computation

The sharp interface that arises from any of the major phase transition problems (classical or modified Stefan, etc.) can be smoothed using the phase field approach as a numerical tool. The computations in one dimensional space and n–dimensions with radial symmetry indicate that this in an efficient method for dealing with stiff equations and results in a very accurate interface determination without explicit tracking. The technique also provides a numerical verification of the concept of an unstable critical radius of solidification.

J. N. Dewynne – J. R. Ockendon:

Numerical solution of the continuous casting problem

The process of continuous casting has considerable technological importance, but in practice the solidification is liable to lead to imperfections and instabilities. A thorough understanding of these problems is not yet available, but they are commonly believed to originate in the region of the caster where the solidification first takes place. For a simple one–phase model, with everywhere constant casting velocity, an explicit classical solution is available in the case that cooling is (effectively) a Dirichlet boundary condition. This solution exhibits upstream influence, in which the freezing begins upstream of the point where cooling starts in the mould. For relatively weak cooling, however, there is no upstream influence and the possibility of sensitive dependence on the data (even though for quite general cooling existence and uniqueness of a weak solution is known). A numerical investigation is needed to clarify the matter, and because a fine, unbiased, resolution of the phase boundary is necessary, a boundary integral method has been proposed and implemented.

R. B. GUENTHER:

Damping of surface water waves

The problem of shallow water, edge waves incident on a sloping beach is discussed. A nonlinear model for the wave heights is proposed and a numerical example is given.

J. HASLINGER:

Numerical solution of optimal control problems, governed by variational inequalities

Let $\Omega(\alpha) = \{(x_1, x_2) \in \mathbb{R}^2 | 0 < x_1 < \alpha(x_2),\ x_2 \in (0,1)\}, \alpha \in U_{ad}$, where

$$U_{ad} = \left\{\alpha \in C^{0,1}([0,1]) | 0 < C_0 \leq \alpha(x_2) \leq C_1,\ |\alpha'(x_2)| \leq C_2,\ \text{meas}\ \Omega(\alpha) = C_3\right\} \neq \emptyset$$

is a space of control variables. In any $\Omega(\alpha)$, $\alpha \in U_{ad}$, we assume the unilateral boundary value problem:

$$\begin{cases} -\Delta u(\alpha) + u(\alpha) = f \\ \frac{\partial}{\partial n} u(\alpha) = 0 \text{ on } \Gamma_1 \\ u(\alpha) \geq 0,\ \frac{\partial u}{\partial n} \geq 0,\ u\frac{\partial u}{\partial n} = 0 \text{ on } \Gamma(\alpha) \end{cases}$$

where $\Gamma(\alpha) = \{(x_1, x_2) \in \mathbb{R}^2 | x_1 = \alpha(x_2),\ x_2 \in (0,1)\}$, $\Gamma_1 = \partial\Omega(\alpha) - \overline{\Gamma(\alpha)}$. Let $I(\alpha) = \frac{1}{2}\|\frac{\partial u(\alpha)}{\partial n}\|^2_{-1/2, \partial\Omega(\alpha)}$ be a cost functional. We consider the problem

$$\text{(P)} \qquad \begin{cases} \text{Find } \alpha^* \in U_{ad} \text{ such that} \\ I(\alpha^*) \leq I(\alpha) \quad \forall \alpha \in U_{ad}. \end{cases}$$

U. HORNUNG:

Numerical calculation of capillary surfaces

If a volume Ω of liquid fills partially a container Φ and is at rest a free surface Γ will separate the liquid from the rest of the container. The equilibrium configurations are minimizers of the energy functional

$$E = \int_\Gamma d\Gamma + \int_\Omega (Bz - \frac{1}{2}Rr^2) d\Omega - \beta \int_\Sigma d\Sigma$$

with the constraint $V = \int_\Omega d\Omega$ =prescribed. Here B is the Bond number, z the vertical coordinate, R the rotational Bond number, r the distance from the axis of rotation, and β a factor describing the adhesion to the wetted part Σ of the container walls $\partial\Phi$. A finite element method is presented that allows to calculate solutions of this variational problem. Several numerical examples are shown.

L.–S. JIANG:

Perturbation of an interface to a diffraction problem and a two–dimensional approximating Muskat problem

In my talk two subjects are talked about. The first one is the perturbation problem of interface to a diffraction problem, and the second one is an approximating Muskat problem.

A new straightening transformation has been found to introduce the perterbution of interface to a perturbation of coefficients of equations and boundary values, then the optimal estimates in space $C^{2+\alpha}$ have been obtained.

We use this perturbation theorem to consider an approximating Muskat problem. The existence and uniqueness of the solution for this second order evolutional elliptic free boundary problem are proved in local by the fixed point theorem.

B. KAWOHL:

Regularity, uniqueness and numerical experiments for a relaxed optimal design problem

Consider the problem of designing a cylindrical bar of maximal torsional rigidity out of prescribed proportions of two different elastic materials. Moreover, the cross–section Ω of the bar is prescribed. An energy approach leads in a canonical way to a relaxed variational problem, whose solution displays a free boundary. There are subdomains Ω_i, $i = 1,2,3$ of Ω in which $|\nabla u|$ lies in certain ranges. The Euler equations are elliptic in Ω_1 and Ω_3, but not in Ω_2. The lecture contains recent results on numerical experiments (joint work with G. Wittum), uniqueness (open problem posed by Murat and Tartar; joint work with J. Stara) and regularity of solutions. The proofs use rearrangement techniques, variational arguments and the coarea formula. In particular the uniqueness proof is nonstandard.

N. KENMOCHI:

A new proof of the uniqueness of solutions to two–phase Stefan problems for nonlinear parabolic equations

We consider the uniqueness of solutions to two–phase stefan problems for a class of nonlinear parabolic equations. Our class includes parabolic equations of the form

$$(*) \qquad \rho(u)_t - (|u_x|^{p-2}u_x)_x + g(u) \ni 0 \text{ in } (0,T) \times (0,1),$$

where $2 \leq p < +\infty$, $\rho(\cdot): \mathbb{R} \to \mathbb{R}$ is a bi–Lipschitz continuous and increasing function and $g(\cdot)$ is a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$ with $0 \in g(0)$ and bounded range $R(g)$ in $\mathbb{R}$. In general, the strong maximum principle, which is the most standard tool in the uniqueness proof for one–dimensional Stefan problems, does not hold for equation $(*)$. Also, we consider the Stefan problem with nonlinear flux conditions such as unilateral boundary conditions (Signorini boundary conditions) on the fixed boundaries $x = 0,\ 1$. In such cases, the uniqueness of solution to the Stefan problem is not obtained in the weak (enthalpy) formulation of S. Kamenotskaja. In this talk we give a new proof of the uniqueness for Stefan problems without using the strong maximum principle.

K. KIRCHGÄSSNER:

Surface waves under small surface tension

It is a long standing open problem whether solitary waves exits on the surface of a liquid if the Bond number is less than one third. A proof is presented, via normal form theory, that, for any order of algebraic approximation, the answer is affirmative. The persistence for the full vectorfield is conjectured and arguments are given. (Joint work with G. Iooss, Nice)

P. KNABNER:

Travelling wave solutions of reactive flow problems in porous media

We study travelling wave solutions (u, v, c), $u \geq 0$, $v \geq 0$, of

$$(TW) \qquad \begin{array}{ll} \partial_t u + \partial_t \psi(u) + \partial_t v - D\partial_{xx} u + q\partial_x u = 0 & \\ \partial_t u = kf(u,v) & \text{in } \mathbb{R} \\ u(-\infty) = u^* > 0, & u(+\infty) = 0 \\ v(-\infty) = v^* > 0, & v(+\infty) = 0 \end{array}$$

This is a model for solute transport in porous media, where the substance reacts with the grain surface (adsorption). ($D > 0$, $q > 0$ are constants)

Ψ and f need not be Lipschitz continuous up to $u = 0$ or $v = 0$; this leads to a degeneration in (TW).

Besides existence and uniqueness we investigate the finiteness of the wave, i.e. the vanishing of u or v for finite downstream values. For $f(u,v) = \varphi_1(u) - v$ and $f(u,v) = u - \varphi_2(v)$ we characterize this situation. Finally we analyse the limit processes
$k \to \infty$, $D > 0$; $k < \infty$, $D \searrow 0$ and $k \to \infty$, $D \searrow 0$.

D. KRÖNER:

About a new upwind scheme for the Euler equation in 2–D

In this lecture we present a new upwind scheme for the compressible inviscid Euler equation in 2–D. The most frequently used schemes to approximate this kind of systems are the dimensional splitting (or fractional step) schemes. They reduce the 2–D problem to two 1–D steps in the direction of the cartesian coordinates. Although they are used very often to simulate flows in 2–D, for some special cases you may run into problems. This may happen for instance if you will compute a shear flow or if you look at the stability conditions for a strongly unisotropic problem. Therefore people tried to develop some new algorithm for the Euler equation in 2–D. Our approach is based on the selection of a locally prefered direction. This one is used to diagonalize the system in order to get a single equation in 2–D. This is then solved with a finite volume method and an upwind dicretization. Finally we show the results of some numerical experiments.

M. Kubo:

Periodicity of saturated–unsaturated flow in porous medium

Unsteady saturated–unsaturated flow in porous medium is described by an equation of parabolic–elliptic type. The boundary of the porous medium is supposed to be in contact with the atmosphere, reservoirs and an impervious layer. We have obtained the existence and uniqueness of the solution of this problem for any prescribed initial data. And the solution is to be proved to have a strong time derivative as an L^2–valued function. The second result is concerned with the large time behaviour of solutions. We have proved that there is one and only one periodic solution which is asymptotically stable, provided that the given data are periodic in time.

F. Kuhnert:

Numerische Lösung von Aufgaben mit freien Rändern aus der Praxis

Das Spritzgießen und Pressen von flachen Kunststofferzeugnissen wird mathematisch durch die Hele–Shaw–Näherung für zähe Strömungen beschrieben. Die Transformation in Variationsungleichungen führt dazu, daß der freie Rand (Fließfront) nicht mehr explizit in der Aufgabenstellung vorkommt. Für die dabei entstehende Klasse von Variationsungleichungen vom Volterraschen Evolutionstyp werden Existenz– und Eindeutigkeitssätze formuliert. Im Falle linearer Aufgaben werden numerische Algorithmen aus der Variationsformulierung abgeleitet, um eine iterative Korrektur des freien Randes zu vermeiden.

X. Liu:

Fréchet differentiability of the free boundary operator for a Muskat–type problem

We consider the one–dimensional Muskat problem. It is well known that there exists a unique global classical solution u, s under the assumptions upon the data

$$f_1(t) \text{ and } f_2(t) \in C^{0,1}[0,T], \quad u_0(x) \in C^{0,1}[0,1],$$
$$f_1(0) = u_0(0), \quad f_2(0) = u_0(1).$$

We prove that under the same assumptions the solution operator S which to each pair of boundary data (f_1, f_2) assigns the correponding free boundary, is Fréchet differentiable and that the F–derivative is Lipschitz continuous. We give the boundary value problem which represents the F–derivative of S.

A. MEIRMANOV:

Principles of simulating the models of phase transitions in multiparametric media

The paper suggests the principles of simulating the mathematical models of phase transition in multiparametric media for the case when the different phases are described by the same number of independent thermodynamical parameters. The main attention is paid to the conservation laws in a divergence form, convenient for calculations, and the axioms of equilibrium thermodynamics.

G.H. MEYER:

Free boundaries with curvature

This talk will be concerned with front tracking for parabolic free boundary problems which contain a curvature term in the free boundary condition. A method of lines discretisation is applied to obtain a coupled system of ordinary differential equations which is solved iteratively. An essential feature in this approach is the algebraic elimination of derivatives orthogonal to the front tracking direction from the normal and tangential derivatives on the free boundary. The method is then applied to the undercooled two phase Stefan problem with a Gibbs Thomson interface condition. We shall conclude with comments on a new convergence proof for this approach for problems with a globally defined free boundary condition.

H.D. MITTELMANN:

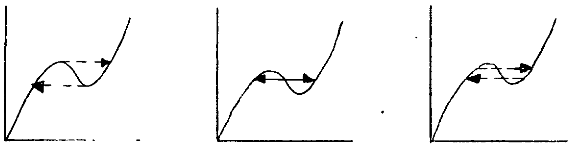
Energy stability of thermocapillary convection in a model of the float–zone crystal growth process

Thermocapillary convection (TC) is a fluid motion driven by surface–tension gradients on a liquid–gas interface. This type of convection plays an important role in materials processing, particularly in crystal growth. The main reason for instability of TC appears to be the onset of time–dependent oscillatory TC. This causes material imperfections in the produced crystal. First, the steady state is computed. Then using the Marangoni number Ma as governing parameter, energy stability is applied to determine sufficient conditions for stability, i.e. the flow is stable for $\mathsf{Ma} < \mathsf{Ma}_E$. The computation of Ma_E is a highly nontrivial numerical problem, for which a complex method is presented involving several nested nonlinear iterations. The core is a nonstandard inverse iteration procedure to compute the smallest positive eigenvalue of $Ax = \lambda Bx$, where A, B are both symmetrics but indefinite. Extensive numerical results are reported which show remarkably good agreement with model experiments for Prandtl numbers of size $O(1)$. The method, however, allows to compute for the materials actually used, as silicon with Prandtl number $O(10^{-2})$, both in terrestrial and microgravity (space shuttle) environments.

I. MÜLLER:

The size of the hysteresis in pseudoelasticity and its dependence on temperature

Many mathematical models for hysteresis phenomena produce non–monotone characteristics. This is the case for instance for pseudoelasticity, where the load–deformation curve is as shown in the figure.



One may then argue that such curves imply a hysteresis that is bounded by the two dashed horizontal lines in the left figure. Actually, however, there are cases when the break–through occurs reversibly along an intermediate horziontal line, see central figure. That raises the question whether there can be a case in which the size of the hysteresis is as shown in the right figure. In this research it is shown that this is indeed the case. The area enclosed by the hysteresis loop is equal to $2A$, where A is a constant that determines the interfacial energy $Az \cdot (1-z)$ of a phase mixture where two phases are mixed in the proportion $z/(1-z)$.

In the Landau theory of phase transitions the load–deformation curves are simple analytic functions, whose downward sloping region becomes less pronounced at higher temperatures. It can then be shown that – as a result of this behaviour – the height of the hysteresis first grows with increasing temperature and then decreases. This behaviour finds some support in tensile experiments with shape memory alloys in the pseudoelastic temperature range.

M. NIEZGÓDKA:

Coupled–field models of the dynamics of phase transitions in shape memory alloys

We consider phenomenological models of coupled thermomechanical processes accompanying structural phase transitions in shape–memory alloys. These are systems alternating between one–loop, non–monotone constitutive relations at low temperatures and the double–loop relations at high temperatures. The proposed models are constructed within an extended Landau–Ginzburg theory of phase transitions, with surface effects taken into account. For the one–dimensional model, discrete approximations are constructed. Results on the numerical stability of the discrete approximations are reported.

We also give some results on the convergence of the discrete solutions, uniform in case of regular initial and boundary data. (Joint work with J. Sprekels, Essen.)

P.D. PANAGIOTOPOULOS:

Free boundary value problems expressed in terms of hemivariational inequalities

In mechanics there is a variety of variational formulations which arise when material laws and/or boundary conditions are derived by nonconvex, generally nondifferentiable energy functions in terms of the generalized gradient of F.H. Clarke – R.T. Rockafellar. They have a precise physical meaning: they express the principle of virtual work (or power) in its inequality form. The material laws and the boundary conditions are generally multivalued and nonmonotone and give rise to free BVPs. The coresponding variational formulations are called hemivariational inequalities, because they differ due to the lack of convexity from the classical variational inequalities. This presentation deals with the existence and approximation theory of the solutions of hemivariational inequalities in Sobolev spaces and in BD-spaces. Some numerical results illustrate the theory.

M. PAOLINI:

An adaptive finite element method for two–phase Stefan problems in two space variables: implementation and numerical results

A description of the implementation of a local refinement strategy for two phase Stefan problems on planar domains based on linear finite element approximation is given. The strategy is based on equidistributing interpolation errors obtaining a typical triangulation which is coarse far from the free boundary, where the discretization parameters satisfy the usual parabolic relation, and refined in its vicinity in order to obtain the hyperbolic relation. Theoretical results compel that a mesh cannot be modified for a number of time steps, so that a prediction of the movement of the free boundary is necessary. Moreover we decide that subsequent meshes are independent from each other, thus forcing the use of an efficient mesh generator for general planar domains, which is based on the advancing front technique. At each time step a number of tests are performed in order to ensure that the computed free boundary does not escape from the refined region and that the local mesh size still satisfies the various constraints arising from the first and second derivatives of the computed solution. Finally three numerical experiments are presented illustrating the various features of the implementation including the presence of mushy regions and the developping of cusps along the free boundary.

I. Pawlow:

Non–isothermal phase separation in binary systems I

A mathematical model of non–isothermal diffusive phase separation in binary systems is proposed. A typical situation there is that the system, initially in one–phase thermodynamical equilibrium state, upon rapid cooling transfers to a non–equilibrium state within the coexistence region of its phase diagram. Activated by a fluctuation, the system evolves towards a new equilibrium state with spatially non–homogeneous structure that locally separates different phases. The model is constructed within the general Landau–Ginzburg theory of phase transitions and is simultaneoulsy based on non–equilibrium thermodynamics approach to desription of coupled mass and heat transport processes. In the case of one space dimension, a method for solving the resulting nonlinear initial–boundary value problem is presented and numerical results are discussed with data that correspond to specific metallic alloys. The simulation experiments deliver information on the kinetics of phase separation at various stages, on the driving factors and the impact of the thermal treatment.

J.–F. Rodrigues:

A remark on the optimal control of a Stefan problem with C^0–observation of the free boundary

Motivated by the continuous casting Stefan probem, in several space dimensions, we describe two recent results which allow to prove the existence of an optimal control for a cost functional involving the C^0–norm of the free boundary: first, we justify the asymptotic model, as the Péclet number $\nu \rightarrow \infty$ (high velocities of extraction), based on the parabolic two–phase Stefan problem (where time is identified with the extraction spacial direction); second, we give, for this problem in a known degenerate case considered by Nochetts (1987), a new sharp estimate for the continuous dependence of the free boundary with respect to the variation of the data. This results is applied to the optimal control with the observation of the nondegenerate free boundary.

J.C.W. Rogers:

Numerical solution of hydrodynamic free boundary problems

A generalized formulation of hydrodynamics as a system of conservation laws subject to a one–sided density constraint has been used to solve underwater explosion problems and free surface problems without applied pressures. Numerical results are shown, and are compared with theoretical predictions in some benchmark cases.

T. ROUBIČEK:

A finite–element approximation of Stefan problems in heterogeneous media

The nonlinear heat transfer problem with phase transitions in heterogeneous, but piecewice homogeneous medium is investigated in the enthalpy formulation. A regularization of the contact conditions between the homogeneous subdomains is employed, and afterwards the problem is approximated numerically by means of the backward Euler formula in time and linear finite elements in space, using also the linear interpolation and a numerical integration to obtain a scheme readily implementable on computers. The convergence of the approximate solutions is proved under the conditions that $\rho \to 0, h \to 0, \tau \to 0$ and, in addition, $\tau^2/\rho \to 0$ and h/τ is bounded, where h is the mesh parameter, τ is the time–step length, and ρ is the thermal resistivity of the surfaces between the adjacent homogeneous subdomains (ρ is the regularization parameter).

J.L. VAZQUEZ:

Free boundaries, uniqueness and asymptotic behaviour of slow diffusion with strong reaction

We consider the reaction–diffusion problem

$$u_t = \Delta u^m + u^p \qquad x \in \mathbb{R}^N, t > 0 \qquad u(x,0) = u_0(x) \geq 0 \qquad \text{for } x \in \mathbb{R}^N$$

with exponents $m > 1 > p$ which imply slow diffusion (since the diffusion coefficient $mu^{m-1} \to 0$ as $u \to 0$) and strong reaction ($R(u) = u^p/u \to \infty$ as $u \to 0$). The effect of the diffusive term on, say, compactly supported initial data leads to compactly supported solution. On the other hand the ODE $\dot{u} = u^p$ has solutions branching away from $u = 0$, i.e. $\bar{u} = c_* t^\alpha, \alpha = 1/(p-1)$. When both terms are combined the result depends critically on the number $p + m - 2 = q$ If $q \geq 0$ there are infinitely many solutions, ranging from a minimal solution with compact support to a positive maximal solution. On the contrary for $q < 0$ either $u_0 \not\equiv 0$ in which case there is only a solution u (and $u > 0$) or $u_0 \equiv 0$ and then you may the homogeneous solutions $\bar{u}$. A travelling wave analysis is performed and shows (in $N = 1$) minimal speed of propagation $c > 0$ for $m + p = 2$ and finite interfaces for $m + p > 2$ $(p < 1, m > 1)$. (Work in collaboration with A. de Pablo, V.A.M.)

C. VERDI:

An adaptive finite element method for two–phase Stefan problems: stability and error estimates

Based on equidistributing interpolation errors, a local mesh refinement strategy is presented. A typical triangulation is coarse away from the discrete interface, where discretization parameters satisfy a parabolic relation, whereas is locally refined in its vicinity for the relation to become hyperbolic. Numerical tests are performed on the computed solution to extract information about first and second derivatives as well as to predict discrete free boundary location. The resulting scheme is stable and necessitates less degree of freedom than previous practical methods on quasi-uniform meshes to achieve the same asymptotic accuray. An $O(\sigma^{1/2})$ rate of convergence holds in the natural energy spaces.

R. VERFÜRTH:

A posteriori error estimators and adaptive mesh refinement for finite element discretizations of the Navier-Stokes equations

We present two a posteriori error estimators for the mini-element discretization of the Stokes equations. One is based on a suitable evaluation of the residual of the finite element solution. The other one is based on the solution of suitable local Stokes problems involving the residual of the finite element solution. Both estimators are globally upper and locally lower bounds for the error of the finite element discretization. Numerical examples show their efficiency both in estimating the error and in controlling an automatic, self-adaptive mesh-refinement process. The methods presented here can casily be generalized to the Navier-Stokes equations and to other discretization schemes.

A. VISINTIN:

Surface tension effects in two-phase systems

Consider a solid-liquid system of a homogeneous substance in a domain $\Omega \in \mathbb{R}^3$. In stationary conditions one can assume that temperature $\theta \in L^1(\Omega)$ is given, and then look for the phase field $\chi(x)(\chi = -1$ in solid, $\chi = 1$ in liquid). Accounting for surface tension effects, we introduce the free enthalpy functional

$$\Psi_\theta(\chi) = -C \int_\Omega [\theta\chi + \tilde{\theta}(\chi^2 - 1)]dx + \frac{\sigma}{2} \int_\Omega |\nabla\chi| \quad \text{for } |\chi| \leq 1 \text{ in } \Omega,$$

where $c, \tilde{\theta}, \sigma$ are constants > 0. Absolute and relative minimia of Ψ_θ in $L^1(\Omega)$ are then interpreted as states of stable and metastable equilibium, respectively. It can also be shown that such states do not have any mushy region (i.e., $|\chi| = 1$ a.e. in Ω). In the evolution problem, the Euler equation

$$\partial\psi_\theta(\chi) \ni 0$$

is coupled with the Stefan equation

$$C_p \frac{\partial\theta}{\partial t} + L \frac{\partial\chi}{\partial t} - \nabla \cdot (k\nabla\theta) = f;$$

existence of a weak solution can be proved.

A.F. VOYEVODIN:

Peculiarities of applying the fractional step method to the numerical solution of the Stokes system

The paper deals with consideration of a system of equations for viscous incompressible fluid. To solve numerically the boundary-value problems by the fractional step method, the use is made of a special method of solving the difference equations that excludes a necessity in iteration methods of determination of the vortex function on the domain boundaries.

W.L. WENDLAND:

Transonic flows around airfoils

The two–dimensional compressible, steady, inviscid flow around an airfoil with given constant subsonic travelling velocity $\vec{v}_\infty$ at ∞ develops pockets of supersonic flow with a free shock boundary from supersonic to subsonic velocity. Here, we consider the model of the second order full potential equation for irrotational, isentropic flows which admits weak solutions with discontinuous gradient. Circulation is included and the Kutta–Youkowski condition at the trailing edge. The fundamental questions of existence and uniqueness for this problem are still open.

Here we present a finite element conjugate gradient method which is based on an optimal control problem asociated with the Bateman principle where a nonconvex functional is minimized on a convex admissible set of functions. The weak solution is not unique without an additional selection principle which here is modelled by a mesh dependent penalty term penalizing large positive accelerations excluding expansion shocks. Berger proved recently that this finite element method converges in $W^{1,p}$ for any $p \in (0, \infty)$, provided the transonic "entropy" solution to the full potential equation would exists and be unique. This is the completest convergence result currently known. As a consequence, one would also have uniform convergence of the approximate shock curves. Several numerical test computations are shown.

Literature:

[H. Berger, G. Warnecke, W. Wendland: Finite elements for transonic potential flows. To appear in "Numerical Methods for Partial Differential Equations"]
[H. Berger: A convergent finite element formulation for transonic flow. To appear in "Numerische Mathematik"]

S. ZHENG:

Global existence and stability of solutions to the phase field equations

The global existence of solutions to the phase field equations which were proposed by Caginalp to describe the phase transitions with finite thickness is proved which improved corresponding results by Caginalp. The asymptotic behavior of solution and the corresponding stationary problem are also extensively studied.

Berichterstatter: J. Sprekels

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