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Dirichlet Spaces

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This conference, worldwide the first one completely devoted to the theory of Dirichlet spaces, was organized by Prof. H. Bauer (Erlangen) and Prof. M. Fukushima (Osaka). There had been 44 participants from twelve countries (including France, Japan, Norway, the People's Republic of China, Portugal, Rumania, Spain, Switzerland, the United Kingdom, the United States of America and Vietnam), 35 of them gave lectures. In seven survey lectures an introduction to those fields was given in which Dirichlet spaces now play a significant role.

The lectures covered the whole scope of the theory of Dirichlet spaces with particular emphasis to the connections with partial differential equations, symmetric Markov processes, complex analysis, analysis in infinite dimensional spaces, analysis on fractals and mathematical physics.

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Vortragsauszüge

MASATOSHI FUKUSHIMA

Dirichlet forms in the spectral analysis on fractals

Brownian motion on the Sierpinski gasket has been constructed by Kusuoka and later by Barlow and Perkins, who also derive a detailed bound of the transition density. On the other hand, Rammal and Toulouse discovered "the decimation method", relating the eigenvalues of the discrete Laplacian H_m on the m-th step pregasket V_m to those of H_{m+1} by a quadratic map Φ.

In this talk, I first show, that the Dirichlet norm on the gasket can be very simply defined as an increasing limit of a renormalized sequence of Dirichlet norms, corresponding to H_m. This gives us a shortest access to the Brownian motion on the gasket. This approach, combined with the decimation method, makes it possible to determine the eigenvalues of the Laplacian on the gasket completely. The distribution of eigenvalues is shown not to be varying regularly. The integrated density of states is also described explicitly, which turns out to be purely discrete. Every jump point of it is then proven to be the eigenvalue of the Laplacian on the infinite gasket with infinite multiplicity.

ZHIMING MA

On the Perturbation of Dirichlet forms

Consider a regular Dirichlet form $(\mathcal{E},\mathcal{F})$, defined on $L^2(X;m)$, where X is a secondly countable Hausdorff space and m is an everywhere dense Radon measure on X. We denote by S the totality of smooth measures (in the sense of M. Fukushima's definition). For $\mu \in S - S$, define $Q_\mu(f,f) := \langle f,f\rangle_\mu := \int f^2\,\mu(dx)$ and consider the following perturbation:
$\mathcal{E}^\mu(f,f) := \mathcal{E}(f,f) + Q_\mu(f,f) \quad \forall f \in \mathcal{F}^\mu := \mathcal{F}\cap L^2(|\mu|)$. $(\mathcal{E}^\mu,\mathcal{F}^\mu)$ is in general a quadratic form. If $\mu \in S$ (i.e. μ is nonnegative), $(\mathcal{E}^\mu,\mathcal{F}^\mu)$ is again a Dirichlet form. We remark that therefore there are smooth measures which are nowhere Radon. (A measure μ is said to be nowhere Radon, if $\mu(G) = \infty$ for all non-empty open sets G.) By this perturbation we get non-regular Dirichlet forms and this fact enables us to study non-regular Dirichlet forms with concrete examples in mind (e.g. consider the construction of Markovian processes associated with non-regular Dirichlet problems).

In the case that μ is not nonnegative (i.e. perturbed with a negative potential), the most important thing is the criteria for lower semiboundedness and for closability. We obtained several criteria for the lower semiboundedness. In particular, $(\mathcal{E}^\mu, \mathcal{F}^\mu)$ is lower semibounded, if and only if the corresponding Feynman-Kac functional forms a strongly continuous semigroup on $L^2(X;m)$. The criterion for lower semiboundedness and closability is also obtained under the condition that the corresponding operator domain is contained in $\mathcal{F}^\mu$. Various results, concerning the core of $(\mathcal{E}^\mu, \mathcal{F}^\mu)$ (when it is bounded from below), are also studied. (Joint work with S. ALBEVERIO.)

STEFAN RICHTER

A formula for the local Dirichlet integral

The local Dirichlet integral of an analytic function on the unit disc $\mathbf{D}$ is defined by

$$D_\xi(f) = \frac{1}{2\pi}\int\left|\frac{f(e^{it})-f(\xi)}{e^{it}-\xi}\right|^2 dt = \iint_{\mathbf{D}} |f'(z)|^2 P_z(\xi)\, dA(z)\,.$$

Here $|\xi| = 1$ and $P_z(\xi)$ denotes the Poisson kernel for z at ξ. An analytic function f must be in H^2, if $D_\xi(f)$ is finite for at least one ξ.

I discussed a generalization of a formula of Carleson for the Dirichlet integral of f:

> The local Dirichlet integral of an H^2-function f equals the sum of three nonnegative terms.

corresponding to the Blaschke, the singular inner, and the outer factor of f. (Joint work with CARL SUNDBERG.)

NIELS JACOB

Dirichlet spaces and pseudo differential operators

Let $a : \mathbf{R}^n \to \mathbf{R}$ be a continuous negative definite function and $L(x,D) = \sum_{i,j=1}^m P_i(D)(a_{ij}(x)Q_j(D))$ be a pseudo differential operator with L^∞-coefficients. We assume that P_i and Q_j are also continuous negative definite functions and bounded from above by $a^{1/2}$. We consider the bilinearform $B(u,v) = \sum_{i,j=1}^m(a_{ij}(\cdot)Q_j(D)u\,,\,P_i(D)v\,)_\circ$ and give criteria in order that it is bounded from below in the Dirichlet space $H_\circ^{1,a^{1/2}}(\Omega) := \overline{C_\circ^\infty(\Omega)}^{\|\cdot\|_{1,a^{1/2}}}$, where $\|u\|_{1,a^{1/2}} = \|(1+a^{1/2}(D))u\|_\circ$. If further B turns out to be a Dirichlet form on $H_\circ^{1,a^{1/2}}(\Omega)$, then we find L^∞-bounds for solutions of a generalized Dirichlet problem associated with $L(x,D)$ in the space $H_\circ^{1,a^{1/2}}(\Omega)$. These L^∞-bounds do imply, that the transition probabilities of the Hunt process associated with the Dirichlet form B have densities.

HIDEO NAGAI

Ergodic control and symmetric diffusion processes

Let us consider the following quasi-linear equation: $-a\Delta v_\alpha + b|\nabla v_\alpha|^2 + \alpha v_\alpha = V(x)$ on $\mathbf{R}^N$, where a and b are fixed positive constants, α is a positive constant and $V(x)$ is a given function, such that $V(x) \geq 0$ and $V(x) \uparrow \infty$ as $|x| \to \infty$. We consider the asymptotic behaviour of the positive solution v_α of the equation, as $\alpha \to 0$. We have the results, that $\alpha v_\alpha \longrightarrow \lambda_1$ as $\alpha \to 0$ and $v_\alpha - \int v_\alpha \Phi^2\, dx \longrightarrow -\frac{1}{k}\log\Phi + \frac{1}{k}\int \Phi^2 \log\Phi\, dx$ as $\alpha \to 0$, where λ_1 is the principal eigenvalue of the Schrödinger operator $-\frac{a^2}{b}\Delta + V$ in $L^2(\mathbf{R}^N)$ and Φ is the principal eigenfunction normalized as $\int \Phi^2\, dx = 1$. The convergence is in the space $\{ f : \int f^2\Phi^2\, dx + \int |\nabla f|^2 \Phi^2\, dx < \infty \}$ strongly.

DOMINIQUE BAKRY

Dirichlet forms and "opérateurs carré du champ"

Let $\mathcal{E}$ be a Dirichlet form on a measure space $(E, \mathcal{E}, \mu)$. In many cases, one can express $\mathcal{E}(f,f)$ as the integral over E of $\Gamma(f,f) = \frac{1}{2}[Lf^2 - 2fLf]$, where L is the generator of the semigroup attached to $\mathcal{E}$. This operator Γ, called 'opérateur carré du champ', allows us to give a metric structure on E, namely $d(x,y) = \sup_{\Gamma(f,f)\leq 1}[f(x) - f(y)]$.

In the first part of our talk, we give examples of this metric for various Markov chains on a finite space. In most of the cases, we cannot embed isometrically these Markov chains in a Euclidean space.

Then, we present some of Davies' results, relating a Sobolev inequality with upper bounds on the heat kernels in terms of this metric. We show that, if we have a weak Sobolev inequality of the form $\forall f \in D(\mathcal{E}) : \int f^2 \log f^2\, d\mu \leq \|f\|_2^2 \log\|f\|_2^2 + \frac{n}{2}\|f\|_2^2 \log[1 + c\frac{\mathcal{E}(f,f)}{\|f\|_2^2}]$ with a measure μ, being a probability measure, then $c \geq \frac{4\,\mathrm{diam}(E)^2}{n^2\pi^2}$, where $\mathrm{diam}(E)$ is defined in terms of the distance associated with $\mathcal{E}$: $\mathrm{diam}(E) = \sup_{\Gamma(f,f)\leq 1} \operatorname{ess\,sup}_{E\times E} |f(x) - f(y)|$.

FRANCIS HIRSCH

On exponential martingales of measures

Let $(\Omega, \mathbf{P}, (B_t)_{0\leq t\leq 1})$ be the standard one dimensional Brownian motion and, for $t \in [0,1]$, $\mathcal{F}_t = \sigma(B_s;\, 0 \leq s \leq t)$. Let τ be a Watanabe distribution on Ω, which is positive and, therefore, can be considered as a positive measure. Suppose $\int d\tau = 1$. The distribution τ admits an *adapted gradient* in the sense of J.A. Yan (Sem. Proba. XXI, 86) denoted by $D_{ad}\tau$. Let $(\gamma_t)_{0\leq t\leq 1}$ be a measurable $(\mathcal{F}_t)$-adapted process, satisfying some weak integrability condition w.r.t. τ, and $\mathcal{L}_t$ be a

space of $\mathcal{F}_t$-measurable continuous "test-functions". The main result, which can be considered as an extension of the Girsanov theorem, is the following:
Theorem: The following are equivalent:
(I) For a.e. t in $[0,1]$ $\forall \varphi \in \mathcal{L}_t \ \langle \varphi, (D_{ad}\tau)_t \rangle = \int \varphi \gamma_t \, d\tau$
(II) For τ-a.e. ω, $\int_0^t \gamma_s(\omega)\, ds$ can be defined, for every t in $[0,1]$, as a semi-convergent integral so that $(B_t - \int_0^t \gamma_s \, ds)_{0 \leq t \leq 1}$ is an $(\mathcal{F}_t)$-Brownian motion w.r.t. τ.

In some examples, τ is singular w.r.t. $\mathbf{P}$, and (B_t) is *not* a semi-martingale w.r.t. τ. This is the case if $\tau = \sqrt{2\pi}\, \delta_0 (\int_0^1 h(s)\, dB_s)$ ($\delta_0 =$ Diracmeasure at 0, $\int_0^1 h^2(s)\, ds = 1$, the composition being in the sense of Watanabe) for some choices of h. (These results were obtained in a joint work with N. Bouleau.)

Eugen Popa

Morphisms of Dirichlet Spaces

Let S and T be two cones from a good theory of potential (e.g. H-cones, but also standard spaces of balayage, superharmonic positive functions on a harmonic space, or potentials from a Dirichlet space). $\varphi : S \to T$ is called a *H-morphism*, if it is additive, increasing and continuous (i.e. $s_i \uparrow s \Rightarrow \varphi(s_i) \uparrow \varphi(s)$). When the elements of S and T are functions (real, positive) on some sets X and Y, then $\varphi : Y \to X$ is called *H-map*, if $s \in S \Rightarrow s \circ \varphi \in T$. Clearly, each H-map induces an H-morphism, which has the properties:
$(*)$ $\varphi(s \wedge t) = \varphi s \wedge \varphi t$, $\forall s, t \in T$ and $\varphi(1) = 1$.
Conversely, if X has "sufficiently" many points, then any morphism satisfying $(*)$ is induced by a H-map.

When S and T are H-cones, or cones of potentials from a Dirichlet space, then there exists a good duality theory, i.e. S and T have duals of the same kind, denoted here by S^* and T^*. Moreover, some morphisms admit adjoints: $\varphi^* : T^* \to S^*$ such that: $\varphi^*(\mu)(s) = \mu(\varphi s)$, $\forall \mu \in T^*$, $s \in S$.

The aim of the talk is to give a characterization for the property of being induced by a H-map, in terms of the adjoint. In the case of Dirichlet spaces, this reads as follows:
Theorem: $\varphi^*(\mu \wedge \nu) = \varphi^*\mu \wedge \varphi^*\nu$, $\forall \mu, \nu \in T^*$ iff $\forall s \in S\ \forall t \in T$ such that $t \preceq \varphi s$, $\exists s_1 \in S$, $s_1 \preceq s$ and $\varphi s_1 = t$.

(Here $s \preceq t$ means: $\exists u \in S$ such that $s + u = t$). The main point in the proof is a simultaneous extension result, based on a lemma of Choquet.

Shintaro Nakao

Stochastic calculus associated with Dirichlet spaces

Let $(\mathcal{E},\mathcal{F})$ be a regular Dirichlet space on $L^2(X;m)$ and let $\mathbf{M}=(X_t,P_x)$ be the Hunt process associated with $(\mathcal{E},\mathcal{F})$. Here X is a locally compact separable metric space. We consider a continuous martingale additive functional M_t of $\mathbf{M}$, satisfying that $\mu_{\langle M\rangle}\ll m$ and $d\mu_{\langle M\rangle}/dm$ is bounded. Set $d\hat{P}_x = \exp\{-M_0-\frac{1}{2}\langle M\rangle\}\,dP_x$.
Theorem 1: The Dirichlet space (non-symmetric) $(\hat{\mathcal{E}},\hat{\mathcal{F}})$ associated with $(X_t,\hat{P}_x)$ is the following:
$\hat{\mathcal{E}}(u,v) = \mathcal{E}(u,v)+\int_X v\,d\mu_{\langle M^{[u]},M\rangle}$ for $u,v\in\hat{\mathcal{F}}$, $\hat{\mathcal{F}}=\mathcal{F}$.

From now on we assume that $(\mathcal{E},\mathcal{F})$ is irreducible. Let ν be a smooth Radon measure on X and let A_t be the positive continuous additive functional associated with ν. The support of ν is denoted by Y and the fine support of A_t is denoted by $\tilde{Y}$. Let's consider the extended Dirichlet space $(\mathcal{E}^\nu_e,\mathcal{F}^\nu_e)$ of $(\mathcal{E}^\nu,\mathcal{F}^\nu)$, where $\mathcal{E}^\nu = \mathcal{E}(u,v)+\langle u,v\rangle_\nu$ and $\mathcal{F}^\nu = \mathcal{F}\cap L^2(X;\nu)$. The projection operator from $\mathcal{F}^\nu_e$ onto $\{u\in\mathcal{F}^\nu_e;\ u=0 \text{ q.e. on } \tilde{Y}\}^\perp$ is denoted by P.
Theorem 2: The Dirichlet space $(\tilde{\mathcal{E}},\tilde{\mathcal{F}})$ on $L^2(Y;\nu)$ associated with $(X_{A_t^{-1}},P_x)$ is the following:
$\tilde{\mathcal{E}}(Pu|_Y,Pv|_Y) = \mathcal{E}^\nu_e(Pu,Pv)-\langle Pu,Pv\rangle_\nu$ for $u,v\in\mathcal{F}^\nu_e$,
$\tilde{\mathcal{F}} = \{Pu|_Y;\ u\in\mathcal{F}^\nu_e\}$.

Masoyoshi Takeda

On a martingale method for symmetric diffusion processes and its applications

In the study of symmetric Markov processes, T. Lyons and W. Zheng proposed a useful formula, which reflects the symmetry of Markov processes faithfully. By using this formula, we obtain the conservativeness criterion for diffusion processes associated with general Dirichlet forms with the strongly local property.

To give examples, we deal with more concrete Dirichlet forms. Let $\mathcal{E}(u,v) = \frac{1}{2}\sum_{i,j=1}^d\int_{\mathbf{R}^d}a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j}\,dm$, $\mathcal{F}=\overline{C_0^\infty}^{\mathcal{E}_1}\subset L^2(\mathbf{R}^d,m)$, where the coefficients a_{ij} are locally integrable Borel measurable functions, satisfying *i)* $a_{ij}=a_{ji}$ *ii)* $\sum_{i,j=1}^d a_{ij}(x)\xi_i\xi_j\ge 0$. Then, by the application of the above criterion, we have the following examples, that corresponding diffusion processes are conservative:
Example 1: $m=dx$ (Lebesgue measure),

$$\sum_{i,j=1}^d a_{ij}(x)\xi_i\xi_j \le k(2+|x|)^2\log(2+|x|)|\xi|^2 .$$

Example 2: $m(|x|\le r)\le e^{kr^2}$, $\sum_{i,j=1}^d a_{ij}(x)\xi_i\xi_j\le\lambda|\xi|^2$.
(Example 2'): Let (M,g) be a complete Riemannian manifold. Then, if $v_g(B_p(r))\le e^{kr^2}$, the Brownian motion on M is conservative. Here v_g is a volume element and $B_p(r)$ is a geodesic ball with radius r.

Finally, we have also the tightness criterion for some class of symmetric diffusion processes.

E. Brian Davies

Pointwise bounds on Heat Kernels

A survey of recent results concerning the heat kernels K of second order uniformly elliptic operators on $\mathbf{R}^N$, and of Laplacians on complete Riemannian manifolds. Typical bounds are:

$$0 \leq K(t,x,y) \leq ct^{-N/2}\Big(1+\frac{d^2}{t}\Big)^{N/2} e^{-d^2/4t}$$

$$\left|\frac{\partial K}{\partial t}\right| \leq ct^{-1-N/2}\Big(1+\frac{d^2}{t}\Big)^{1+N/2} e^{-d^2/4t}$$

$$|\nabla_x K| \leq ct^{-\frac{1}{2}-\frac{N}{2}}\Big(1+\frac{d^2}{t}\Big)^{\frac{1}{2}+\frac{N}{2}} e^{-d^2/4t}$$

in the first case, and

$$K(t,x,y) \sim |B(x,t^{1/2})|^{-1/2}\,|B(y,t^{1/2})|^{-1/2}\,e^{-d^2/(4+\varepsilon)t}$$

in the second case, when the Ricci curvature is non-negative. In the first case $d = d(x,y)$ is a certain metric defined from the second order coefficients, and in the second case d is the Riemannian metric. Such estimates can often be transferred from one metric to any other metric which is Lipschitz equivalent to it, since they are proved by establishing equivalences between the heat kernel bounds and certain Sobolev or log-Sobolev inequalities.

Pawel Kröger

Comparison of heat kernels on piecewise smooth manifolds

Take a simply connected smooth Riemannian manifold N of non-positive curvature with a metric tensor (g_{ij}). We establish heat kernel bounds for Riemannian manifolds M, which are obtained by replacing the metric tensor (g_{ij}) by a metric tensor $(\tilde{g}_{ij})$ with $(1-\varepsilon)(g_{ij}) \leq (\tilde{g}_{ij}) \leq (1+\varepsilon)(g_{ij})$ for some $0<\varepsilon<1$. Our estimates can be considered as perturbations of the estimates given by Debiard, Gaveau and Mazet in 1976. In order to take care on the large time behaviour of the heat kernel (in particular, the drift term corresponding to the curvature), we give estimates with constants tending to the constants of Debiard, Gaveau and Mazet for $\varepsilon \downarrow 0$.

Santiago Carrillo Menendez

Properties of the resolvent associated with elliptic operators degenerated on the boundary

Let $Lu = -\sum \frac{\partial}{\partial x_i}(\sum a_{ij}\frac{\partial u}{\partial x_j} + d_i u) + \sum b_i \frac{\partial u}{\partial x_i} + cu$ be a differential operator, defined on an open set $E \subseteq \mathbf{R}^m$ $(m \geq 3)$, relatively compact, in the sense of distributions. We suppose that $E = \bigcup_1^\infty E_n$, where $\overline{E}_n \subseteq E_{n+1} \ldots$ are open sets with regular boundary, and that L, on each E_n, is a uniformly elliptic operator, whose resolvent, i.e. the resolvent associated to the Dirichlet form on $H_0^1(E_n)$ defined by the restriction of L to E_n, is noted by $(V_\lambda^n;\ \lambda \geq 0)$. Let $V_\lambda f = \sup_n V_\lambda^n f$ for $f \in C_k^+(E)$. We show that, if $c - \sum \frac{\partial b_i}{\partial u} \geq 0$ and $c - \sum \frac{\partial d_i}{\partial x_i} \geq 0$, $(V_\lambda;\ \lambda > 0)$ is a strongly continuous resolvent on L^2, submarkovian, borelian, such that $\|\lambda V_\lambda\| \leq 1$ on each L^p space ($p \in [0,\infty]$); moreover, V_λ maps L^2 on H^1_{loc} and $V_\lambda f$ is a solution of $(\lambda I + L)u = f$ in the sense of distributions and, also, V_λ maps $L^{q/2}$, $q > m$, in the space of functions which are Hölder continuous on each compact set.

A consequence is, that $(V_\lambda;\ \lambda > 0)$ satisfies the Kunita-Watanabe conditions for constructing the Martin boundary, if $c - \sum \frac{\partial b_i}{\partial x_i} \geq c_0 \geq 0$ and $c - \sum \frac{\partial d_i}{\partial x_i} \geq 0$. Moreover, $(V_\lambda;\ \lambda > 0)$ is the resolvent of a diffusion process, whose killed processes on ∂E_n have $(V_\lambda^n;\ \lambda > 0)$ as resolvent.

Yves Le Jan

Longitudinal energy for isotropic stochastic flows

We consider the class of isotropic stationary measure preserving flows on $\mathbf{R}^d$. They define a field of unstable directions, defined by a random field of projectors $A_\infty^{ij}(x,\omega)$, which appears to be the limit of a field of matrix valued martingales $A_t^{ij}(x,\omega)$.

The longitudinal Dirichlet form is defined as $\int A_\infty^{ij}(x,\omega)\, D_i f(x)\, D_j g(x)\, dx$, and appears as the limit of Dirichlet forms, defined by the A_t's in the same way. The closability of the form can be proved and the generator computed. It should be viewed as the generator of a diffusion on the unstable foliation. The law of the normal drift (curvature) can be computed.

Remark: For a conservative Dirichlet form, it is easy to prove, that the two following properties are equivalent:

(L) $\forall f \in \mathrm{E},\ e(f^+, f^-) = 0$. (D) $\forall f \in \mathrm{E} \cap L^\infty,\ 4e(f^3, f) - 3e(f^2, f) = 0$.

Theo Sturm

Gauge Theorems for Compact Resolvents with Application to Perturbation of Markov Processes

Let $\mathbf{V} = (V^\alpha)_{\alpha\geq 0}$ be a family of kernels on a Radon measurable space $(E,\mathcal{E})$, satisfying the strong resolvent equation: $V^\beta = \sum_{n=1}^\infty (\beta-\alpha)^{n-1}(V^\alpha)^n$ for all $0 \leq \beta < \alpha < \infty$. For example, let $V^\alpha = \int_0^\infty e^{-\alpha t}\cdot Q_t\,dt$ with some measurable semigroup $\mathbf{Q} = (Q_t)_{t\geq 0}$ of kernels on $(E,\mathcal{E})$ (not necessarily bounded or sub-Markovian).

We ask for criteria, which imply $\|V\| := \sup_{x\in E} V1(x) < \infty$. Our essential assumption is, that the kernel V^α for *some* $\alpha \geq 0$ defines a *compact* operator on the set $\mathcal{E}_b$ of bounded, $\mathcal{E}$-measurable functions on E. For simplicity we also assume that V is irreducible.

> If in this case there exists at least one $\mathbf{V}$-excessive function, which is not $\mathbf{V}$-invariant, then $\|V\| < \infty$.

We apply this result to resolvents $\mathbf{U}^M = (U^M_\alpha)_{\alpha\geq 0}$, arising from perturbation of right processes (X_t, P^x) by multiplicative functionals M (not necessarily bounded by 1 or supermartingales), i.e. $U^M_\alpha : f \mapsto E^\bullet[\int_0^\xi M_t\cdot e^{-\alpha t}\cdot f(X_t)\,dt]$. If $\mathbf{U}^M$ satisfies the above compactness and irreducibility assumptions, then $\|U^M\| < \infty$ under each of the following conditions:

- *(non-recurrence)* $\exists f \geq 0 :\ 0 \not\equiv U^M f \not\equiv \infty$
- *(gauge condition)* $0 < \Gamma(x) := E^x[M_\xi \cdot \mathbf{1}_{\{\xi<\infty\}}] < \infty$ for some x
- *(conditional gauge condition)* $0 < \gamma(x,s) := E^{x/s}[M_\xi\cdot\mathbf{1}_{\{\xi<\infty\}}] < \infty$ for some x and some $\mathbf{U}$-excessive function s.

This generalizes the gauge theorem of K.L. Chung and M. Rao and, moreover, contains one direction of the conditional gauge theorem of R. Williams, N. Falkner, Zh. Zao.

Nicolas Bouleau

The algebra of Dirichlet structures

By a Dirichlet structure, we mean a term $(\Omega,\mathcal{F},m,\mathcal{E},\mathbf{D})$, where $(\Omega,\mathcal{F},m)$ is a measured space with m σ-finite and positive, and where $\mathcal{E}$ is a Dirichlet form with domain $\mathbf{D}$, i.e. a positive quadratic form with dense domain in $L^2(m)$, which is closed and on which the unit contraction operates. We recall basic properties and give equivalent definitions for the locality of the form and the existence of a 'carré du champ' operator. We state the results already obtained about the functional calculus associated with local Dirichlet forms and the EID (Energy Image Density) property.

Then we show, that there is a natural notion of *image structure* of a Dirichlet structure, and we give properties, which are kept by this operation, especially, under which assumptions is the EID property preserved by image. We state a theorem about the generator of the image structure and show numerical computations of image structures of Wiener space by functionals of the 2^d chaos.

The notion of *tensor product* of Dirichlet structures is introduced, as well as *infinite tensor product,* which always exists. If each finite product satisfies the EID property, that is also true for the infinite product, and that gives rise to examplas of non gaussian infinite dimensional Dirichlet structures in which the EID property holds. An example of a *projective system* of D-structures is given, which has no limit, whereas systems obtained by images of the same D-structure always possess limits.

D-independence of functions defined on the same D-structure is defined by analogy with probability structures, examples are given. The notion of *convergence in D-law* is then exposed and a Dirichlet version of the central limit theorem is proved, and also of the Gateaux-Lévy theorem. Dirichlet sub-structures and conditionning is then studied. Finally these notions are illustrated by introducing D-stationary processes and the analog of Wiener filtering theory. (This work was made in collaboration with F. HIRSCH.)

ANA BELA CRUZEIRO

Diffusions in Infinite Dimensions

We consider second order differential operators on the classical Wiener space and give a construction of the associated 'diffusion' processes. Let $X = \{x \in C([0;1];\mathbf{R}) : x(0) = 0\}$ be the Wiener space and μ the Wiener measure. We consider Sobolev spaces over X in the sense of Malliavin calculus; namely, for $F : X \to G$ (G Hilbert space), and $h \in H$ (the Cameron Martin space), $D_h F(x) = \lim_{\substack{\epsilon \to 0 \\ (\text{a.e.})}} \frac{1}{\epsilon}[F(x+\epsilon h) - F(x)]$ is the directional derivative of F and $\nabla F(x) \in \mathcal{L}(H;G)$ the linear operator associated. The Sobolev spaces are defined by $W^p_r(X) = \{\varphi \in L^p_\mu : \nabla^i F \in L^p_\mu(X; \mathcal{L}^i{}_{\text{H.S.}}(H;\mathbf{R}))\ \forall 1 \leq i \leq r\}$ (H.S. = Hilbert-Schmidt class). Since there are no immersion Sobolev lemmas in infinite dimensions, a functional in W^p_r is not necessarily continuous. Operators of the form:
$Lf = \frac{1}{2}\sum_{i,j}(\delta_{ij} + A^{ij})D_{e_i}D_{e_j}f + B_i\nabla f$, where e_i is an orthonormal basis of H, are studied. The hypothesis on B (functional with values in $\mathbf{R}$) and (A^{ij}) (matrix operating on H) are, that they belong to the Sobolev space. The operator is also supposed to be elliptic and verifying the condition $\int f L f \, d\mu \leq 0 \quad \forall f$. Remark that the coefficients of L are not only supposed not to be continuous, but they don't even have to be bounded.

In the situation above, and given $k_0 \in L^2_\mu : \int k_o\, d\mu = 1$, we prove the existence of a process X_t taking values in $C(\mathbf{R}^+; X)$ with initial distribution given by $k_0\, d\mu$, whose law at time t is absolute continuous w.r.t. the Wiener measure μ and that solves the martingale problem of Strook and Varadhan, namely: $f(X_t) - f(X_s) - \int_s^t Lf(X_\tau)\, d\tau$ is a $\mathcal{P}_s$-martingale.

Armand de la Pradelle

Sobolev Spaces in Infinite Dimensions

Let E be a locally convex space with a regular centered gaussian measure μ. For any cylindrical function of class C^1 of finite rank we define $Var_p(f) = \iint |\langle f'(u), v\rangle|^p . d\mu(u)\, d\mu(v)$ and the p-energy of f: $e_p(f) = Var_p(f) + \int |f|^p\, d\mu$. The completion with respect to $\|f\|_{1,p} = (e_p(f))^{1/p}$ is $W^{1,p}(E,\mu)$. By recurrence we define $(p > 1)$: $\|f\|^p_{n+1,p} = \|f'\|^p_{n,p} + \int |f|^p\, d\mu$, where the norm of f' is relative to $E \times E$, $\mu \otimes \mu$. The completion is $W^{n,p}(E,\mu)$. For φ cylindrical continuous, $c_{n,p}(\varphi) = \inf\{ \|f\|_{n,p} \mid f \geq |\varphi| \}$ is the capacity. We then study these capacities and the quasi-continuous functions. A key result is, that any seminorm $\leq +\infty$ and borelian, finite μ-a.s., belongs to $W^{1,p}(E,\mu)$ and to all Riesz-Banach spaces associated to $c_{n,p}$ for $n > 1$. (This work has been done with D. Feyel.)

Hélène Airault

Factorization of the Wiener space and Dirichlet forms

Let μ be the Wiener measure on the space X of continuous functions from $[0,1]$ to $\mathbf{R}^q$, vanishing at zero; let $\mathcal{L}$ be the Ornstein-Uhlenbeck operator on X and define the Dirichlet form on X: $\mathcal{D}(f,g) = \int \mathcal{L}f \cdot g\, d\mu$. For a regular map $p : X \to \mathbf{R}^n$ and a measure ν on X, denote by $p_*\nu$ the image measure on $\mathbf{R}^n$ (i.e. $p_*\nu(A) = \nu(p^{-1}(A))$) and let $d(p_*\nu)/d\xi$ be its density with respect to the Lebesgue measure $d\xi$ on $\mathbf{R}^n$. The map p induces a Dirichlet form on $\mathbf{R}^n$:
$\mathcal{D}_p(\tilde{f},\tilde{g}) = \mathcal{D}(\tilde{f} \circ p, \tilde{g} \circ p) = \int A_p \tilde{f}(\xi)\, \tilde{g}(\xi)\, k(\xi)\, d\xi$.
One has $k(\xi) = \frac{d(p_*\mu)}{d\xi}$ and $A_p = \sigma^{ij} \frac{\partial^2}{\partial \xi_i \partial \xi_j} + \beta^i \frac{\partial}{\partial \xi_i}$ is a symmetric operator with respect to the measure $k(\xi)\, d\xi$. The coefficients σ^{ij} and β^i are given by $k(\xi)\, \sigma^{ij}(\xi) = d(p_*(\nabla p_i | \nabla p_j)\mu)/d\xi$ and $k(\xi)\, \beta^i(\xi) = d(p_*(\mathcal{L}p_i)\mu)/d\xi$. $(\nabla p_i | \nabla p_j)$ is the Malliavin covariance matrix. The Dirichlet forms $\mathcal{D}_p$ are closable.

Now, let V be a C^∞-differentiable manifold and $p : X \to V$ be a regular map. For $\tilde{f}$ and $\tilde{g}$ in $C^\infty(V; \mathbf{R})$, define the Dirichlet form on V: $\tilde{\mathcal{D}}_p(\tilde{f},\tilde{g}) = \mathcal{D}(\tilde{f} o p, \tilde{g} o p)$.
Theorem: The map p induces on V a riemannian metric such that

$$\tilde{\mathcal{D}}_p(\tilde{f},\tilde{g}) = \int_V \tilde{f}(v)\, (A_p \tilde{g})(v)\, k(v)\, dv ,$$

where A_p is a differentiable operator on the manifold V, dv is the riemannian volume induced by p on V and k is the density of $p_*\mu$ with respect to dv; moreover, $A_p = \frac{1}{2}\Delta_v + \nabla\vec{u}$, where $\vec{u} = \frac{1}{2}\mathrm{grad}(\log k)$.

On the other hand, let $\Phi : X \to \mathbf{R}^d$ and $V_\xi = \Phi^{*-1}(\xi)$ be a submanifold of finite codimension in X. [Malliavin-Airault]. On V_ξ, one defines the Dirichlet forms

$$\mathcal{D}^\lambda_{V_\xi}(f,g) = \int_{V_\xi}(\nabla f(x)|P_{H_x}\nabla g(x))\,\lambda(x)\frac{da^\Phi(x)}{[Det(\nabla\Phi_i|\nabla\Phi_j)]^{1/2}} \quad ,$$

where $\lambda : X \to \mathbf{R}$; da^Φ is the area measure on V_ξ and P_{H_x} is the orthogonal projection in the Cameron-Martin space H of functions of X onto the tangent subspace H_x at x. These Dirichlet forms on V_ξ are closable.

PATRICK J. FITZSIMMONS

General Theory of Symmetic Markov Processes

The rudiments of a potential theory for symmetric right Markov processes is presented. Since no hypothesis of C_0-regularity is made, it is possible that there are no continuous functions in the Dirichlet space of such a process. Consequently the fine topology plays a crucial role. The methods are more probabilistic than those in the C_0-regular case. In particular, the stationary (Kuznetsov) process associated with a symmetric right process is used to prove an "essential downcrossing" estimate which yields a new proof of the existence of quasi (finely) continuous modifications of elements of the Dirichlet space. This result in hand, one constructs the natural fine outer capacity, and from this base one recovers all of the elements of the theory of Fukushima and Silverstein (martingale and zero-energy additive functional decomposition, smooth measures,etc.). Of note is an absolute continuity result ($\forall\mu$, $\int_0^\infty \mu P_t\, dt << m \Rightarrow \mu P_t << m\ \forall t > 0$) which extends a theorem of Fukushima. (Portions of this work jointly with R. GETOOR.)

MARTIN L. SILVERSTEIN

Normal derivative and the extended approximate Markov process associated to a symmetic Markov process

Let $X \subset \mathbf{R}^n$ be bounded and open, no smoothness of the boundary is assumed. $(E_\infty, \Omega_\infty)$ is the symmetic approximate Markov process associated to the absorbed Brownian motion on X. The class B is defined as the set of all functions φ on an abstract boundary which are "restrictions" of functions with finite Dirichlet norm; $H\varphi$ is harmonic and minimizes the Dirichlet form. Let f have a finite Dirichlet norm $\int_X dx\, |\nabla f|^2$ and $\Delta f \in L^2$. Then

$$\frac{\partial f}{\partial n}(\varphi) = \lim_{k\uparrow\infty} E_\infty(\varphi(X_{\sigma^\bullet}),\ \sigma(D_K) < \infty,\ f(X_{\sigma^\bullet}) - f(X_{\sigma(D_k)})),$$

where σ^* is the birth time, $X = \cup D_K$, each D_K is open and $\overline{D_K} \subset X$ is compact, $D_1 \subset D_2 \subset \ldots$, exists and corresponds to the integral of φ against the classical normal derivative on the boundary, when f and ∂X are sufficiently smooth. As an application one finds: An element f in the reflected Dirichlet space F^{ref} belongs to the domain of the reflected generator A^{ref} iff $\Delta f \in L^2$, $\frac{\partial f}{\partial n}(\varphi) = 0$ for all $\varphi \in B$.

YOICHI OSHIMA

On a criteria of conservativeness of Markov processes

Let $\mathbf{M} = (X_t, P_x)$ and $\hat{\mathbf{M}} = (\hat{X}_t, \hat{P}_x)$ be Markov processes which are in duality with respect to a positive Radon measure m. Let $(V^p)_{p>0}$ and $(W^p)_{p>0}$ be the resolvents of $\mathbf{M}$ and the killed process of $\mathbf{M}$ by the PCAF $\int_0^t I_C(X_s)\,ds$. Define $(\hat{V}^p)$ and $(\hat{W}^p)$ similarly with respect to $\hat{\mathbf{M}}$. Then we have the following criteria: $\mathbf{M}$ is conservative [resp. recurrent] if there exists a sequence $\{u_n\}$ such that $0 \le u_n \le 1$, $\lim u_n = 1$ a.e. and $\lim_{n\to\infty} \mathcal{E}(u_n, v) = 0$ for some $v = \hat{V}^p f$, $p > 0$, $f \in C_0^\infty$ [resp. $v = \hat{W}^0 f, f \in C_0^\infty$]. By using this criteria, we can give conditions which show the conservativeness or recurrence of the process $\mathbf{M}$ corresponding to the generator $L = \frac{1}{m}\left\{\sum \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial}{\partial x_j}) + \sum b_i \frac{\partial}{\partial x_i}\right\}$. Let $V(r) = \int_{S_r}(\langle An, \nabla v\rangle - \langle b, n\rangle v)\,dS$ for a surface S_r. If the domain with boundary S_r increases to the whole space $\mathbf{R}^d$, then a sufficient condition for $\mathbf{M}$ to be conservative [resp. recurrent] is $\lim_{r\to\infty} V(r) = 0$ for $v = \hat{V}^p f$ [resp. $v = \hat{W}^0 f$]. By using Schwarz inequality, we can give a condition which covers the Ichihara's test. The Khasiminskii test can be covered by this method.

TERRY LYONS

Some infinite dimensional and multi-parameter Martingales

Uniform inequalities of Gaussian type relate the tail behaviour of a real continuous martingale to the value of its bracket. Less satisfactory results exist for vector valued and discontinuous martingales. One imagines results can be improved for multi-parameter martingales. An application arises in statistics. Suppose μ maps M (parameter manifold) to a family (μ_m) of dominated probability measures on $(E, \mathcal{E})$: $\mu : m \mapsto p(m,e)\,v(de)$. Impose regularity so that the likelyhood funktion $p(\cdot, e)$ exists as a function (e.g. in $W^{1,n}$), is v-integrable and $\mathcal{E}$-measurable. The likelyhood function descibes the frequency with which e occurs for different sampling rules. Let $t : E \longrightarrow E'$ descibe an aspect of e then we can introduce p^t in the same way. Then $p^t = \mathsf{E}_v(p|\mathcal{E}^t)$ where $\mathcal{E}^t \subset \mathcal{E}$ is the algebra generated by t. So (p^t) is a v-martingale with values in $W^{1,n}$ indexed by the aspects t of interest. If one had full confidence in the model μ one could allow t to range over all possible aspects of E. In practical situations one never has full confidence in μ; but the discribtion one has can usually be specified by describing p^t for a specific class of statistics t. The attraction is, that using any (nice) specified family (p^t) one can obtain uniform estimates without further reference to the original model. These estimates can be sharp. The basic observation is if $l^t = \log p^t$ and dl^t is the derivative of l as a form on M (restrict to $t \in \mathbf{R}$, dl^t weakly continuous) then $dl^t + \langle dl^t, l^t - l^t(m)\rangle$ is a μ_m local martingale for each m and acts as a near pivotal random variable. For example if $X \in (\wedge M)^*$ then one may obtain uniform estimates $\mathsf{P}_{\mu_m}(|X\,dl^t + \langle X\,dl^t, l^t - l^t(m)\rangle|^2) > \lambda^2\Psi(\langle X\,dl^t, X\,dl^t\rangle)$ with sharp gaussian behaviour in λ. One can thus generate confidence intervals, metrics, flows etc.

MICHAEL RÖCKNER

Dirichlet forms on topological vector spaces and applications to infinite dimensional stochastic differential equations

(joint work with S. ALBEVERIO)

A classical Dirichlet form is a quadratic form on $L^2(E;\mu)$ of the type

(1) $\quad \mathcal{E} = \int_E \langle \nabla u, \nabla v\rangle_H \, d\mu$;

u, v smooth bounded cylinder functions, where E is a Souslinean locally convex vector space, μ a probability measure on its Borel sets and H is a Hilbert space, densely and continuously imbedded in E. The first part of the talk consisted of a survey on recent results on classical Dirichlet forms, i.e. a (necessary and) sufficient closability criterion, construction of the associated diffusion and a partial integration formula to determine the corresponding generator. In the second part applications to ∞-dimensinal SDE's were discussed. It was shown that the diffusion $(X_t)_{t>0}$ corresponding to the closure of (1) satisfies an SDE of type

(2) $\quad dX_t = dW_t + \underline{\beta}(X_t)\,dt \quad$ on E

under q.e. starting measure P_z, $z \in E$. Here $(W_t)_{t>0}$ is a Brownian motion on E and $\underline{\beta} : E \longrightarrow E$ is obtained using the above mentioned partial integration result. (2) means that $(X_t)_{t>0}$ can be interpreted as a "distorted Brownian motion" as in the finite dimensional case studied by M. Fukushima. Finally, applications to quantum field theory were discussed. As a special case of (2) we obtain a solution to $dX_t = dW_t + :X_t^3: dt$ on $\mathcal{S}'(\mathbf{R}^2)$ (where ": ^{3} :" means "renormalized" third power) which is the stochastic quantisation of the $:\phi^4:_2$-quantum field.

JÜRGEN POTTHOFF

White Noise Analysis: A Framework for Infinite Dimensional Dirichlet Forms

The White Noise Analysis of T. Hida and his followers was presented in a form which is particularly usefull for the construction and investigation of Dirichlet forms over $\mathcal{S}'(\mathbf{R}^d)$. The two main ingredients are

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- the introduction of spaces of test and generalized functionals on $\mathcal{S}'(\mathbf{R}^d)$
- the development of an efficient calculus on the space of test functionals

Using Yokoi's theorem, which asserts that positive generalized functionals are measures, one arrives at a rich class of forms which - if closable - can be identified as "classical Dirichlet forms" in the sense of Albeverio-Röckner. As a consequence, all their results apply. The presented framework is specially suited to study Dirichlet forms arising in quantum field theory. The talk was a report on various collaborations with S. ALBEVERIO, T. HIDA, M. RÖCKNER, L. STREIT AND J.A. YAN.

LUDWIG STREIT

Energy Forms - White Noise Analysis - Quantum Field Theory

The Dirichlet strategy for quantum dynamics is highlighted. For the purpose of quantum field theory Dirichlet forms are defined in terms of positive Hida distributions. It is shown that this framework is adequate: Gibbs and vacuum states in the standard models of canonical QFT are all given as positive generalized functions of White Noise. In particular the ground state functional of any canonical QFT with a "φ-bound" $\pm\varphi(f) \leq H + \alpha\,|f|^2 + \beta$ is always a Hida distribution. (Joint work with S. ALBEVERIO, T. HIDA, J. POTTHOFF, M. RÖCKNER)

SERGIO ALBEVERIO

A survey of some interactions of the theory of Dirichlet forms and physics

This is a survey lecture, complementing those by J. Potthoff, M. Röckner, L. Streit in the same session. We mention in particular interactions of the theory of Dirichlet forms with quantum mechanics and quantum field theory. After mentioning the basic relations between dynamics in Schrödinger quantum mechanics and in "Dirichlet quantum mechanics", we show the advantage of the latter approach both in discussing singular perturbations as well as the infinite dimensional case. We discuss recent developments as well as prospects and open problems concerning basic aspects of the theory and its quantum mechanical correlates like closability (in particular recent results of J. Brasche, M. Röckner, U. Spõnemann, Zhiming Ma and myself), uniqueness of Dirichlet forms (N. Wielens, M. Takeda), approximations, perturbations, ergodic and spectral properties. We also report on recent developments of the theory of quantum fields, stressing classical Dirichlet forms as a tool to construct the relevant Hamiltonians and processes (recent results by M. Röckner and myself, as well as with T. Hida, J. Potthoff, M. Röckner, L. Streit, the latter concerning the "white noise approach"). We mention the solution by

Høegh-Krohn, Zegarlinski and myself of the problem of the global Markov property of polynomial fields. We discuss the role of probability theory and more specifically the theory of Dirichlet forms in discussing basic constructive problems of quantum fields. We mention the recent obtained construction of simplicial approximations of quantum fields on manifolds (B. Zegarlinski and A.) as well as the construction of certain quantum fields in four space-time dimensions, exploiting the isomorphism of $\mathbf{R}^4$ and the quaternions (based on joint work with R. Høegh-Krohn, K. Iwata and T. Kolsrud). We close by mentioning some other aspects of the theory of Dirchlet forms in its numerous interactions with other domains.

MASAO NAGASAWA

Can Schrödinger's equation be Boltzmann's one?

Yes for $d = 1$ and open for $d \geq 2$. The idea is to combine Schrödinger processes which are equivalent to Schrödinger equation and the propagation of chaos for segregated systems of interacting particles. Let $P_t(x) = \Psi_t(x)\overline{\Psi}_t(x)$ be the distribution density where $\Psi_t = e^{\alpha_t + i\beta_t}$ is a solution of Schrödinger's equation. The drift of McKean-Vlasov's limit process (Boltzmann's equation) should agree with the drift of Schrödinger process. Therefore $\frac{1}{2}\nabla \log P_t + \nabla\beta_t = b[\alpha, P_t]$. One can find $b[u, u]$ solving the above equation in $d = 1$ and show the propagation of chaos.

JUTTA STEFFENS

Integral representation for excessive measures

Let $(V_\alpha)_{\alpha>0}$ be a substochastic resolvent on a measurable space $(E, \mathcal{E})$ where $\mathcal{E}$ is separable and such that the potential kernel V is strictly positive and proper. Let $q > 0$ s.th. $V_q < \infty$. We give a simple elementary new proof of the following well known result: The set K of excessive measures m s.th. $m(q) \leq 1$ is a simplex. Proofs of this fact under stronger assumptions are given by Dynkin ('71) and Rogers ('83). Our proof relies on the fact that the set K can be embedded into a cap C of a metrizable lattice cone, and the unique integral representation in C induces one on K. (The lattice cone consists in the positive cone $p\mathcal{D}'$ of the order dual of an order unit space $\mathcal{D} := \mathcal{P} - \mathcal{P}$ where $\mathcal{P}$ is a certain suitably chosen separable potential cone of excessive functions.)

Klaus Janssen

Integral representation of polysupermedian measures

Let J be an index set, let $(P_t^j)_{t>0}$ for each $j \in J$ be a submarkovian semigroup on a measurable space $(E, \mathcal{E})$ such that $P_t^j P_s^k = P_s^k P_t^j \; \forall s,t,j,k$. A σ-finite measure m on E is called supermedian (i.e. $m \in \mathcal{S}$) if $m(I - P_t^j) \geq 0 \; \forall t > 0, j \in J$. m is called polysupermedian (i.e. $m \in \mathcal{SS}$) if for any finite $H \subset J$ we have $m(\underset{j \in H}{\times}(I - P_{t_j}^j)) \geq 0$ for all $(t_j) \subset]0, \infty[^H$.

Theorem: a) The set $\mathcal{SS}$ of polysupermedian measures is a lattice.

b) Assume that $(E, \mathcal{E})$ is a U-space, and that J is at most countable. Then every $m \in \mathcal{SS}$ admits a "unique" integral representation $m = \int_\Delta \rho \, d\eta(\rho)$, where Δ is the set of extreme points of $\{m \in \mathcal{SS} : m(g) < \infty, m(gh) \leq 1\}$ for suitable functions g, h.

Moreover, some applications and relations with known special cases were discussed, e.g. separately excessive functions in a product setting (c.f. Gowrisankaran, Cairoli), and completely excessive measures (c.f. M. Itô, Beznea).

Jürgen Bliedtner

The Rôle of Simpliciality in Potential Theory

The notion of simplicial cones is used to characterize the main property of potential cones:

Let X be a locally compact space with a countable base and $\mathcal{P} \subset C^+(X)$ be a function cone such that

$$R_f := \inf\{p \in \mathcal{P} : f \leq p\} \in \mathcal{P} \text{ for all } f \in C_{\mathcal{P}}(X) := \{f \in C(X) : \exists p \in \mathcal{P} : |f| \leq p\}.$$

Then the following holds:

Theorem: The following conditions are equivalent:

(1) $\forall p, q \in \mathcal{P} \Rightarrow p - R_{p-q} \in \mathcal{P}$ (i.e. $\mathcal{P}$ is a potential cone)

(2) $\forall p \in \mathcal{P} \Rightarrow S(p) := \mathcal{P} + \mathbf{R}p$ is a simplicial cone.

The proof mainly relies on results obtained jointly with W. Hansen in the context of balayage space.

Setsuo Taniguchi

Applications of Dirichlet spaces to Complex Analysis

Let D be a bounded domain in $\mathbf{C}^n$. We call a diffusion process $\mathsf{M} = (Z_t, \zeta, P_z)$ a holomorphic diffusion process, simply, if it is a $C_0^\infty(D)$-regular symmetic diffusion with the life time ζ such that $h(Z_{t\wedge\tau_K})$ is a P_z-martingale, M-q.e. z for every holomorphic function h on D and compact $K \subset D$, where τ_K is the first exit time from K. Such diffusion processes connect holomorphic functions and martingales and hence give us a way to consider topics in complex analysis in the probabilistic terms. In this talk, several attemps in this direction are reviewed. First we revisit thin sets in complex analysis - analytic sets and pluripolar sets from the probabilistic point of view. We give the stochastic characterization of pluripolar sets. It is seen that a pluripolar set is a common exceptional set for suitable holomorphic diffusion processes. Next we consider the solution to the complex Monge-Ampère equation: $(dd^cu)^n = f\,dV$ in D. This is solved by taking advantage of the minimum principle: if u and v are both bounded, plurisubharmonic and $(dd^cu)^n \leq (dd^cv)^n$ and $\liminf_{\zeta\to z}\{u(\zeta) - v(\zeta)\} \geq 0$, $\forall z \in \partial D$ then $u \geq v$ on D. The third topic is the stochastic approach to the Šilov boundary, the smallest closed subset of ∂D where the maximum principle holds for any holomorphic function continuous up to ∂D. Using the exit distribution of holomophic diffusion processes, the Šilov boundary is studied. The 4^{th} object concerns with the relationship between domains of holomorphy and the conservativeness of holomorphic diffusion processes. First seen is that there is a conservative Kähler diffusion process if D is a domain of holomorphy. Secondly seen is that D is a domain of holomorphy if the Kähler diffusion process associated with the Bergman metric is conservative and ∂D is nice. Finally, the boundary behaviour of plurisubharmonic functions along the sample path of holomorphic diffusion processes is studied.

Hiroshi Kaneko

A stochastic approach to a Liouville type theorem

It is well known that any plurisubharmonic function on $\mathbf{C}^n$ increasing slower than of logarithmic order at infinity must be constant. This kind of the Liouville type property has been extended to certain complex manifolds. The aim is to prove assertions of this type by using weakly recurrent holomorphic diffusion. It will be shown that a Liouville type theorem for plurisubharmonic functions holds on a complex manifold of dimension n possessing a plurisubharmonic exhaustion function Ψ such that $(dd^c\Psi)^n$ tends to zero in a certatin sense as $\Psi \longrightarrow \infty$. By these observations, one can justify the following assertion: Let M be a Kähler manifold with a pole of dimension $n \geq 2$ and r be the distance from the pole. If the radial curvature k satisfies $|k| \leq \delta/(r+a)^2 \log(r+a)$ on $\{r > 0\}$ for some $\delta < 1/(9n-2)$

and $a > \exp\{2(1+2\delta)\}$, then there exists no non-constant plurisubharmonic function u on M such that

$$\lim_{s\to\infty} \frac{\sup_{r(z)\le s} u(z)}{\log\log s} = 0.$$

We'd like to mention that Professor K. Takegoshi gives the proof independently, by establishing a certain energy estimate.

KARL DOPPEL

A regularity result for a boundary value problem defined by non-hypoelliptic partial differential operators

In $\Omega_1 \times \Omega_2 \subset \mathbf{R}^{n_1} \times \mathbf{R}^{n_2}$, $\Omega_l \subset \mathbf{R}^{n_l}$, $l = 1,2$, open, let the elliptic partial differential operators

$$P_l(\cdot, D_{x_l}) := \sum_{i,k=1}^{n} a_l^{ik}(x_l) \frac{\partial^2}{\partial x_i \partial x_k} + \sum_{i=1}^{n} b_l^i(x_l) \frac{\partial}{\partial x_i} - c_l(x_l), \; l = 1,2$$

be given, where

(i) $a_l^{ik}(x_l), b_l^i(x_l)$; $i,k = 1,2,\ldots,n$, $l = 1,2$ are sufficiently smooth functions,

(ii) $\sum_{i,k=1}^{n} a_l^{ik}(x_l)\lambda_i\lambda_k > 0$, $\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbf{R}$, $x_l \in \overline{\Omega_l}$, $l = 1,2$,

(iii) $c_l(x_l) \ge 0 \qquad x_l \in \Omega_l$, $l = 1,2$.

Define on $\Omega := \Omega_1 \times \Omega_2 = \{(x_1,x_2) | x_l \in \Omega_l, l = 1,2\}$ the product operator $P(\cdot, D_x) = P_1(\cdot, D_{x_1})\, P_2(\cdot, D_{x_2})$ and pose the following boundary value problem: To $f \in C(\Omega)$ find $u \in C^{2,2}(\Omega) \cap C(\overline{\Omega})$ such that

$$(1) \begin{cases} P(x, D_x)u(x) &= f(x) & x \in \Omega \\ D^{\alpha_1}u(x) &= 0 & x \in \Omega_1 \times \partial\Omega_2, \; |\alpha_1| \le 1 \\ D^{\alpha_2}u(x) &= 0 & x \in \partial\Omega_1 \times \Omega_2, \; |\alpha_2| \le 1 \end{cases}$$

where $\alpha = (\alpha_1, \alpha_2) \in \mathbf{N}_0^n$, $\alpha_1 = (\alpha_1^{(1)}, \ldots, \alpha_{n_1}^{(1)})$, $\alpha_2 = (\alpha_{n_1+1}^{(2)}, \ldots, \alpha_{n_1+n_2}^{(2)})$. The following theorem is stated:
Theorem: Take $f \in C^\infty(\overline{\Omega}) \Rightarrow$ Problem (1) has a unique solution $u \in C^\infty(\overline{\Omega})$.
(This is a joint work with R. Hochmuth and partly contained in his thesis.)

BERNT ØKSENDAL

Weighted Sobolev inequalities and harmonic measures for quasiregular functions

Let U be an open set in $\mathbf{R}^n$ and let $\phi : U \longrightarrow \mathbf{R}^n$ be a quasiregular (q.r.) function on U. Rešetnjak (1966) studied properties of such functions ϕ by proving that each component of ϕ is a solution of a certain *quasilinear elliptic* differential equation. In this talk a different approach is given, based on the fact that each component of ϕ is a solution of a *linear* second order *degenerate elliptic* partial differential equation. More pricisely, it has been proved by the lecturer that if we define the form

$$\mathcal{E}(u,v) = \frac{1}{2}\int_U \nabla u^T J_\phi \cdot (\phi')^{-1}((\phi')^{-1})^T \nabla v \, dx$$

for $u, v \in C_0^\infty(U) \subset H = L^2(J_\phi \, dx)$ ($J_\phi =$ the Jacobian of ϕ, $\phi' = \left[\frac{\partial \phi_i}{\partial x_j}\right]$ the derivative matrix of ϕ)
then $\mathcal{E}$ is closable and the corresponding diffusion X_t is mapped by ϕ into a Brownian motion in $\mathbf{R}^n$. This opens the way for the use of stochastic methods in the study of quasiregular functions.
A question which is important for the applications of this result is the existence of $X_\zeta := \lim_{t\to\zeta} X_t$ on $\{\zeta < \infty\}$, where $\zeta \leq \infty$ is the life time of X_t. The existence of the limit is proved under the assumptions that 1) $\int_U J_\phi^{1-\frac{2}{n}} \, dx < \infty$ and 2) $\mathrm{Vol}(\phi(U)) < \infty$. This means that the *harmonic measure* associated to ϕ, $\mu_x(F) = P^x[X_\zeta \in F]$, exists. The proof is based on the following weighted Sobolev inequality.

$$\exists C < \infty \text{ s.t. } \int_U |u|^2 J_\phi \, dx \leq C \int_U |\nabla u|^2 J_\phi^{1-\frac{2}{n}} \, dx \quad \forall u \in C_0^\infty(U)$$

Question: Are μ_x and μ_y mutually absolutely continuous? This would follow if one could establish a Harnack inequality for positive solutions u of $Au = 0$, where A is the generator of X_t.

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