MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 32/1989

Konstruktive Approximation

31.7 bis 4.8.1989

Tagungsleiter: C. de Boor (Madison)
R. DeVore (Columbia)

The conference brought together researchers from approximation theory and related areas such as computer aided geometric design, finite elements, harmonic analysis, mathematical statistics, mathematical physics, and nonlinear differential equations. This resulted in many stimulating lectures and informal discussions. Some of the themes addressed were: generalizations of Bernstein polynomials in one and more dimensions; applications of approximation to the computer generation of curves and surfaces; approximation in the complex; approximation methods in mathematical statistics; multivariate polynomial interpolation; nonlinear approximation methods and their application to nonlinear differential equations and image compression; multivariate splines; n-widths; smoothness spaces; wavelet decompositions and their applications; Padé and rational approximation.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

Bob Barnhill:

COMPUTER AIDED GEOMETRIC DESIGN

Computer Aided Geometric Design is the representation and approximation of curves, surfaces, and solids in an interactive computer graphics environment. This survey of the use of approximation theory in CAGD includes intrerpolation methods for surface design and representation and a mention of applications of such methods. Open CAGD questions with approximation theory aspects conclude the presentation.

Hubert Berens:

ON BERNSTEIN-DURMEYER POLYNOMIALS WITH JACOBI WEIGHTS

This is joint work with Yuan Xu. In 1967, J.L. Durmeyer introduced modified Bernstein polynomials on $L_1[0,1]$ which show some remarkable properties. The operators were brought to the attention of the mathematical community by M.M. Derriennic in 1981 who made a first systematic study. A second paper of interest is a recent one by Z. Ditzian and K. Ivanov who study the approximaton behavior in greater detail.

Let $w^{(\alpha,\beta)}(x) := x^\alpha(1-x)^\beta$, $\alpha,\beta > -1$, and let $L^1_w(0,1)$ be the (equivalence class) of Lebesgue measurable functions f on $(0,1)$ for which $\|f\|_{1,w} := \int |f|w$ is finite. For $f \in L^1_w(0,1)$ and $n \in \mathbb{N}_0$,

$$V_n^{(\alpha,\beta)}(f;x) := \sum_{k=0}^{n} a_{k,n}^{(\alpha,\beta)}(f)p_{k,n}(x) \quad \text{where} \quad a_{k,n}^{(\alpha,\beta)} := \frac{\int f p_{k,n} w}{\int p_{k,n} w}$$

and $p_{k,n}(x)$ are the Bernstein basis functions. For $\alpha = \beta = 0$, we have Durmeyer's original definition. The sequence $\{V_n\}_{n=0}^{\infty}$ shares many of the properties associated with the Bernstein polynomials, e.g., they form an approximate identity of positive contractions on $L^p_w(0,1)$, $1 \le p < \infty$, and $C[0,1]$. One special property is that the Jacobi polynomials $Q_k^{(\alpha,\beta)}$, $k = 0,1,\ldots,n$, are the eigenfunctions of $V_n^{(\alpha,\beta)}$ with eigenvalues

$$\lambda_{k,n} = \frac{n!}{(n-k)!}\,\frac{\Gamma(n+\alpha+\beta+2)}{\Gamma(n+k+\alpha+\beta)}.$$

It follows that

$$V_n^{(\alpha,\beta)}(f;x) = \sum_{k=0}^{n} \lambda_{k,n}^{(\alpha,\beta)} A_k^{(\alpha,\beta)} h_k^{(\alpha,\beta)} Q_k^{(\alpha,\beta)}(x)$$

where $\{A_k^{(\alpha,\beta)}\}_{k=0}^n$ are the Jacobi coefficients of f and $\{h_k^{(\alpha,\beta)}\}$ are the normalization factors; $Q_k^{(\alpha,\beta)}$ is uniquely determined by setting $Q_k^{(\alpha,\beta)}(0) = 1$. Setting

$$D_w^p(0,1) = \begin{array}{l}\{f \in L_w^p(0,1) : f, f' \text{ loc. abs. cont. on } (0,1), [w^{(\alpha+1,\beta+1)}f'](x) \to 0,\ x \to 0,1, \\ \text{and } [w^{(\alpha+1,\beta+1)}f']'/w^{(\alpha,\beta)} \in L_w^p(0,1)\},\end{array}$$

we characterize the approximation behavior of $\{V_n^{(\alpha,\beta)}\}$ via the Peetre K-functional $K(\cdot, 1/n, L_w^p, D_w^p)$ and for $1 < p < \infty$, via a weighted modulus of smoothness of Ditzian and Totik.

M.M. Derrienic, *J. Approx. Theory* **31**(1981), 325–343.
Z. Ditaian - K. Ivanov, *J. Approx. Theory* **56**(1989), 72–90.

Wolfgang Boehm:

SPLINES IN CAGD WITH AN APPLICATION TO CAR DESIGN

A short historical review is given: the change from the classical design methods over the Bernstein-Bezier method to the spline method. The dominating algorithm in the spline method is the "inserting algorithm" of a new knot. It allows to construct points and tangents of a spline curve as well as to pass over to the well-known and popular Bernstein-Bezier method. Tensor product surfaces are easily built. Some pictures from some steps of a car design illustrate an application.

Carl de Boor:

MULTIVARIATE POLYNOMIAL INTERPOLATION

In joint work with Amos Ron, a map $\Theta \mapsto \pi_\Theta$ from finite subsets Θ of $\mathbb{R}^d$ to linear subspaces π_Θ of polynomials is constructed, with the following properties:

(i) The pair (Θ, π_Θ) is **correct** in the sense that it is possible to interpolate, uniquely, from π_Θ to arbitrary data on Θ, i.e., the map $f \mapsto f_{|\Theta}$ on π_Θ is invertible.

(ii) The map $\Theta \mapsto \pi_\Theta$ is continuous (where possible), and (iii) leads to osculatory interpolation (where possible) as points in Θ coalesce.

(iv) $\pi_{\Theta+a} = \pi_\Theta$; (v) $\pi_{\alpha\Theta} = \pi_\Theta$; (vi) for any invertible matrix A, $\pi_{A\Theta} = \pi_\Theta \circ A^T$.

(vii) For all correct (Θ, P) and all j, $\dim(\pi_j \cap \pi_\Theta) \geq \dim(\pi_j \cap P)$, i.e., π_Θ is **of minimal degree**.

(viii) $\pi_\Theta \subset \pi_{\Theta'}$ for $\Theta \subset \Theta'$, leading to a Newton form for the interpolant. (ix) $\pi_{\Theta \times \Theta'} = \pi_\Theta \otimes \pi_{\Theta'}$.

(x) π_Θ is constructible in finitely many arithmetic operations, as the **least** $(\exp_\theta)_\downarrow$ of $\exp_\theta := \operatorname{span}(e_\theta)_{\theta\in\Theta}$, with $e_\theta : x \mapsto e^{\theta x}$, with $H_\downarrow := \operatorname{span}\{f_\downarrow : f \in H\}$, and $f_\downarrow$ the first nontrivial term in the expansion $f = f_0 + f_1 + f_2 + \cdots$ of f into homogeneous polynomials f_j of degree j.

Peter Borwein:

DENSENESS QUESTIONS IN MARKOV SYSTEMS

I prove the following theorem:

THEOREM. *A Markov system is dense in* $C[a, b]$ *if and only if the zeros of the associated Chebyshev polynomials are dense in* $[a, b]$.

Len Bos:

ON MARKOV'S INEQUALITY IN N-SPACE

This is joint work with P.D. Milman. Suppose that E is a compact subset of Euclidean n-space. We will say that E admits a Markov inequality if, for any polynomial p, the uniform norms of the first partials of p are bounded by a constant times a power of deg(p) times the uniform norm of p. We discuss the relationship between a set admitting a Markov inequality, the Gagliardo-Nirenberg inequality (from the theory of Sobolev spaces), the extension of smooth functions, and bounds on the uniform norm of polynomials orthogonal on E.

Zbigniew Ciesielski:

APPROXIMATION BY POSITIVE SPLINE OPERATORS

To define the operators, tensor product B-splines corresponding to the uniform mesh are used. In the B-spline expansions of the values of the given operator, the coefficient functionals are represented by (possibly different) B-splines but such that the linear functions are invariant. Direct, inverse and saturation theorems are obtained for the approximation process defined by such operators (as the mesh size goes to zero.) The results are used by the author for the non-parametric density estimation in the multivariate case.

Antonio Córdova:

THE MAXIMAL RANGE PROBLEM FOR POLYNOMIAL SPACES IN THE UNIT DISK

This is joint work with Stephan Ruscheweyh. Let $\Omega \subset \mathbb{C}$ be a domain in the complex plane, and $w \in \Omega$. For the family of complex polynomials $\mathrm{P}_n^w(\Omega) := \{p(z) = \sum_{k=0}^n a_k z^k : p(0) = 0,\ p(\mathbb{D}) \subset \Omega\}$ with $\mathbb{D}$ the unit disk, we define the maximal range of this family as

$$\Omega_n^w := \bigcup_{p \in \mathrm{P}_m^w(\Omega)} p(\mathbb{D}).$$

We calculate explicitly the maximal ranges Ω_n^w for different types of Ω such as: half-planes, strips, interior and exterior of disks, and the slit domains $\mathbb{C} \setminus ((-\infty, a) \cup [b, \infty))$, $a, b > 0$. We also calculate the corresponding extremal polynomials, i.e., the ones with $p(\zeta) \in \partial\Omega_n^w \setminus \partial\Omega$ for some $\zeta \in \partial\mathbb{D}$. For the general problem, all the extremal polynomials have all the zeros of the derivative on $\partial\mathbb{D}$. As applications, we obtain sharp estimates relating $\|p\|_{\mathbb{D}}$, $\|Re\ p\|_{\mathbb{D}}$, $\|Im\ p\|_{\mathbb{D}}$, $\min_z |p(z)|$, etc. and a subordination theorem for complex polynomials.

W. Dahmen:

MONOTONE EXTENSIONS OF BOUNDARY DATA

This is joint work with R. DeVore and C. Micchelli. Given any set $\Gamma \subset \mathbb{R}^s$ a function $f : \Gamma \to \mathbb{R}$ is called monotone on Γ if for any $x, y \in \Gamma$ such that $x - y \in \mathbb{R}_+^s$ one has $f(x) \geq f(y)$. Given any bounded domain $\Omega \subset \mathbb{R}^s$ with continuous boundary $\partial\Omega$ and any function f which is monotone on $\partial\Omega$, any function F that agrees with f on $\partial\Omega$ and is monotone on $\bar{\Omega}$ is called a monotone extension of f to $\bar{\Omega}$. After pointing out that there is no linear mapping L such that for all monotone boundary data f, Lf is a monotone extension of f to $\bar{\Omega}$, several nonlinear constructions of monotone extensions for essentially arbitrary domains are described. The common drawback of these constructions is that regardless of how smooth the boundary data f are, the extensions F will generally only be Lipschitz continuous. For the case $\Omega = [0,1]^2$, however, we prove that for boundary data $f(t,0), f(t,1), f(0,t), f(1,t)$ with strictly positive slopes and any given degree of smoothness, there exists a monotone extension with the same degree of smoothness.

R. DeVore:

SURFACE COMPRESSION

In this joint work with Björn Jawerth and Brad Lucier, we present an algorithm for surface and image compression based on wavelet decompositions into box splines and Haar functions. By a wavelet decomposition, we mean a representation

$$f = \sum_{I \in D} a_I \varphi_I$$

where φ is a function defined on R^d, $\varphi_I(x) := \varphi(2^j x - k)$ is its dilate associated with the cube $I := 2^{-j}k + 2^{-j}\Omega$, $\Omega := [0,1]^d$, and $\mathcal{D}$ is the set of dyadic cubes. In some cases, such as the representation in Haar function in more than one variable, one uses more than one function φ.

To compress a surface or image, we write it in a wavelet decompostion and choose a finite expansion $\sum_{I \in \Lambda} a_I \varphi_I$ among all such finite sums with at most n terms. Our algorithms for selecting the sets Λ and the coefficients a_I are based on earlier work of DeVore, Jawerth and Popov. Examples are given for compression of images using box splines and Haar functions (in two variables) for φ.

Zeev Ditzian:

THE LAPLACIAN AND THE RELATED K-FUNCTIONAL

The boundedness of the Laplacian of $f(x)$, $x \in \mathbb{R}^d$ (given in the weak sense) is equivalent to the boundedness of the discrete Laplacian. A new K-functional given by

$$\tilde{K}(f,t^2) := \inf(\|f-g\| + t^2\|\Delta g\|)$$

is shown to be equivalent to the modulus

$$\tilde{\omega}(f,t) := \sup_{0<h\leq t} \|2df(x) - \sum_{i=1}^{d}(f(x+he_i) + f(x-he_i))\|$$

where e_i is a fixed orthonormal basis of $\mathbb{R}^d$ and the norm is one for which translation is a continuous isometry.

Nira Dyn:

OPTIMAL DISTRIBUTION OF KNOTS FOR TENSOR-PRODUCT SPLINE APPROXIMATION

This joint work with Itai Yad-Shalom deals with optimal asymptotic distribution of knots for various types of spline approximation. Known results on the optimal distribution of knots, which asymptotically minimizes the $L_p[0,1]$ error in spline approximation of a sufficiently smooth function, are extended to the following cases:
(1) An optimal distribution for a simultaneous $L_p[0,1]$ approximation of a finite set of functions or a set which depends on a continuous parameter, e.g. a bivariate function.
(2) An optimal distribution on each axis for $L_2[0,1]^2$ tensor-product spline approximation. This result extends to $L_2[0,1]^n$, $n > 2$.

A method of knot placement for tensor-product spline approximation is proposed, based on the theoretical optimal distribution.

Willi Freeden:

BEST APPROXIMATION BY SPHERICAL BLENDING FUNCTIONS

Let Ω be the unit sphere in three dimensional Euclidean spaces and let P_r be the set of all linear combinations of spherical harmonics of order $\le r$. Then the following approximation problem is considered: Given a function h defined on $\Omega \times \Omega$ of sufficiently high order of differentiability, calculate (bounds of) a best approximation to h with respect to the space $\mathsf{P}_r \otimes C(\Omega) + C(\Omega) \otimes \mathsf{P}_s$ in the L^p norms $(1 \le p \le \infty.)$ The main tools are the Green's functions of iterated Beltrami-operators and corresponding (blending type) integral formulas on the unit sphere. As an application, blending type representations of geophysically relevant kernel functions on the sphere are considered in more detail.

Manfred von Golitschek:

APPROXIMATION BY SEPARABLE FUNCTIONS

Let S and T be compact real sets and let $C(S \times T)$ be the space of continuous functions on $S \times T$. Two finite dimensional subspaces $U \subset C(S)$ and $V \subset C(T)$ are given, with the basis functions $\{u_1, \ldots, u_m\}$ and $\{v_1, \ldots, v_n\}$ respectively. They generate the linear subspace

$$W := \{w \in C(S \times T) : w(s,t) = \sum_{j=1}^{m} u_j(s) x_j(t) \; + \; \sum_{k=1}^{n} y_k(s) v_k(t)\}$$

where $x_j \in C(T)$ and $y_k \in C(S)$ are arbitrary functions. The function classes W are called "Tensor Product Subspaces" or "Separable Functions" or "Blending Functions". We shall aproximate functions $f \in C(S \times T)$ by the elements of W in the uniform norm on $S \times T$ and denote the infimum (distance) by

$$\operatorname{dist}(f, W) := \inf_{w \in W} \|f - w\|_{S \times T}.$$

We shall talk about the size of $\operatorname{dist}(f, W)$, the existence and computation of a best approximation for f from W.

Klaus Höllig:

BOX-SPLINE TILINGS

Let $f : \mathbb{R}^d \to \mathbb{R}$ be a real analytic function such that, for $j \in \mathbb{Z}^d$, $|f(x+j)| \to \infty$, as $|j| \to \infty$, for almost all $x \in \mathbb{R}^d$. Then the translates of the set $\{x \in \mathbb{R}^d : |f(x)| < |f(x+j)|,\ j \in \mathbb{Z}^d \setminus 0\}$ over the lattice $\mathbb{Z}^d$ generate a tiling (i.e. an essentially disjoint partition) of $\mathbb{R}^d$. Such sets arise in box-spline theory and generate rather interesting patterns.

Arieh Iserles:

ORTHOGONALITY AND APPROXIMATION IN SOBOLEV SPACES

We consder polynomials $p_n^{(\lambda)}$ which are orthogonal with respect to the Sobolev inner product

$$(f,g)_\lambda = \int_{\mathbb{R}} fg\,d\varphi + \lambda \int_{\mathbb{R}} f'g'\,d\psi,$$

where $\lambda \geq 0$. Let $\{p_k\}$ be the OPS w.r.t the distribution $d\varphi$ (in the classical sense.) It can be demonstrated that , subject to an extra condition on $\{\varphi,\psi\}$ (whose specific formulation is that $\int_{\mathbb{R}} p_n' p_m' d\psi$ is a function of $\min\{m,n\}$ for $m,n \geq 1$), it is possible to expand $p_n^{(\lambda)}$ in $\{p_n\}$ such that

$$p_n^{(\lambda)} = \sum_{\ell=1}^{n-1} r_{\ell-1}(\lambda) p_\ell(x) \ + \ \sigma_n(\lambda) p_n(x)$$

where the r_ℓ's are independent of n. Moreover, each r_ℓ is a polynomial of degree ℓ and $\{\sigma_\ell\}$ is an OPS w.r.t. some distribution α. We identify some instances whereby our condition holds and single out the underlying distribution α.

The framework is generalised to cater for the case when $p_n^{(\lambda)}$ is of the same parity as n — this caters for the case of both φ and ψ being Legendre distributions. We identify the underlying α's with atomic measures linked with Bernouli numbers.

Finally, we present an application of Sobolev orthogonality to spectral methods for linear parabolic PDE's with variable coefficients

Kamen Ivanov:

BETWEEN FREE KNOT AND FIXED KNOT SPLINES

A characterization of the order of approximation by free knot spines in terms of Besov spaces has recently been obtained by P. Petrushev, while the corresponding characterization for fixed knot splines has been known for years.

We link the free knot and fixed knot splines by splines which can be represented as linear combinations of dyadic B-splines with controlled behavior of their supports. Direct and converse theorems for the approximating properties of these spline families are proved.

Thus, we get a unified view of free knot and fixed knot splines and give an answer to a question of J. Peetre posed at the US-Swedish Seminar in Lund, 1986.

Björn Jawerth:

THE φ–TRANSFORM AND WAVELETS: A SURVEY

In joint work with M. Frazier we have shown that for a large class of functions φ and ψ it is possible to write a general function f on R^n as

$$f = \sum_{\nu \in Z} \sum_{k \in Z^n} \langle f, \varphi_{\nu,k} \rangle \psi_{\nu,k}.$$

Here $\varphi_{\nu,k}(x) = 2^{\nu n/2}\varphi(2^\nu x - k)$ and similarly for $\psi_{\nu,k}$. The operator $f \to \{\langle f, \varphi_{\nu,k}\rangle\}$ is called the φ–transform. Furthermore, for most of the classical distribution spaces F there is a corresponding sequence space $\mathcal{F}$ such that $f \in F$ if and only if $\{\langle f, \varphi_{\nu,k}\rangle\} \in \mathcal{F}$. The φ–transform includes certain spline expansions and, when $\{\psi_{\nu,k}\}$ forms an orthonormal basis, the so-called wavelets. The φ–transform and the wavelets have a number of interesting consequences in theoretical mathematics and applications.

Rong-Qing Jia:

MULTIVARIATE DISCRETE SPLINES, PARTIAL DIFFERENCE EQUATIONS AND LINEAR DIOPHANTINE EQUATIONS

In this talk, we demonstrate the close relationship between multivariate discrete splines and linear diophantine equations. On the one hand, we use linear diophantine equations to investigate the algebraic properties of discrete box splines. In particular, we obtain necessary and sufficient conditions for the multiinteger translates of a discrete box spline to be linearly independent. On the other hand, we use multivariate discrete truncated powers to study linear diophantine equations. In particular, we solve a conjecture of Stanley concerning symmetric magic squares. The link between linear diophantine equations and multivariate discrete splines is certain systems of linear partial difference equations, for which we establish a general existence and uniqueness theorem on the solutions.

Iain Johnstone:

MINIMAX ESTIMATION IN NON-PARAMETRIC REGRESSION

We survey recent work by subsets of Donoho, Liu, MacGibbon, and the speaker. In the first problem, data $y = \theta + \epsilon$ is observed, where the unknown signal θ is to be estimated, but is contaminated by white Gaussian noise of standard deviation σ. Prior information that θ belongs to Θ is available. We compare minimax risks of linear and non-linear estimators for estimating θ under squared error loss. For linear estimators, these are approximately determined by Kolmogorov linear n-widths, and for non-linear estimates by certain Bernstein n-widths. Precise asymptotic evaluations of minimax risk are available when Θ is an ℓ_p ball in R^n as $\sigma \to 0$ and $n \to \infty$. In the second problem, approximation of bivariate functions by ridge functions ("plane waves") is compared with kernel smoothing. A duality is exhibited between the two methods. Function classes are constructed in which the methods respectively achieve faster convergence rates than polynomial approximation. This illustrates quantitatively the heuristic that multivariate regression procedures can be tailored to prior informaiton about the structure of the regression function.

D. Leviatan:

ON THE MÜNTZ-JACKSON THEOREM

We are interested in obtaining estimates of the Jackson type in the approximation of $f \in L_p[0,1]$, $1 \le p \le \infty$ (where L_∞ denotes the space $C[0,1]$) by means of Müntz polynomials $\sum_{k=0}^n a_k x^{\lambda_k}$ where the λ are taken from a sequence $\Lambda = \{\lambda_k\}_{k=0}^\infty$ of distinct real numbers satisfying

$$0 = \lambda_0 < \lambda_1 < \lambda_2 < \ldots < \lambda_n < \ldots$$

Let the Blaschke product $B_n^p(z)$ be defined as

$$B_n^p(z) := \prod_{k=1}^n \frac{z - \lambda_k - \frac{1}{p}}{z + \lambda_k + \frac{1}{p}}$$

and let

$$\mathcal{E}_p(\Lambda_n) := \max_{\Re z = 1} \left| \frac{B_n^p(z)}{z} \right| .$$

We prove

THEOREM. *For each $1 \le p \le \infty$ and integer $r \ge 1$, there exists a constant $A_p = A_p(r)$ such that if Λ satisfies $\lambda_k = k$, $k = 0, \ldots, r-1$, then for all $f \in L_p[0,1]$ and all $n \ge r-1$*

$$E_p(f, \lambda_n) := \inf_{a_k} \left\| f - \sum_{k=0}^n a_k x^{\lambda_k} \right\|_p \le A_p w_\varphi^r(f, \mathcal{E}_p(\lambda_n))_p$$

where $\varphi(x) = (1-x)^{\frac{1}{2}}$ and $w_\varphi^r(f,\cdot)_p$ is the Ditzian - Totik r-th modulus of smoothness in L_p.

Rudi Lorentz:

BIRKHOFF INTERPOLATION IN AN ARBITRARY NUMBER OF DIMENSIONS

The author and G.G. Lorentz have concentrated their work on Birkhoff interpolation primarily to bivariate polynomials. Some of these results have now been generalized to an arbitrary number of dimensions.

One of the results is the characterization that a Birkhoff interpolation is regular if and only if its incidence matrix is an Abel matrix. This immediately implies the regularity of the Kergin interpolation in Micchelli's formulation and allows a large class of generalizations.

Another result which can be generalized to an arbitrary number of dimensions is a sufficient condition for regularity of Birkhoff interpolation with prescribed knots. This is used to strengthen a theorem of Séveri on the minimal total degree of a space of polynomials in which given Hermite interpolation functionals are linearly independent.

Finally, singularity results for Hermite interpolation with a small number of knots are given. Even in two dimensions, these results are new.

Bradley J. Lucier:

APPROXIMATING DISCONTINUOUS SOLUTIONS OF HYPERBOLIC CONSERVATION LAWS WITH FREE-KNOT SPLINES

The solution $u(x,t)$ to the differential equation

$$u_t + f(u)_x = 0, \qquad x \in \mathbb{R},\ t > 0,$$

can develop discontinuities (called "shocks") even when the flux f and the initial data $u(x,0) = u_0(x)$, $x \in \mathbb{R}$, are smooth. However, it has been known since the fifties that regularity is preserved in the space BV of functions of bounded variation, that is, $u_0 \in \mathrm{BV}$ implies that $u(\cdot,t) \in \mathrm{BV}$ for all positive t and $\|u(\cdot,t)\|_{\mathrm{BV}} \le \|u_0\|_{\mathrm{BV}}$. Therefore, the best rate of approximation to the solution $u(\cdot,t)$ in $L^1(\mathbb{R})$ by splines of any order with uniformly placed knots is $O(1/n)$, where the grid spacing is $1/n$.

The situation is different when one considers approximation with free knots. It was shown by Petrushev and DeVore and Popov that some of the regularity spaces associated with approximation by free knot splines are Besov spaces. In particular, $u \in B^\alpha := B^\alpha_\sigma(L^\sigma)$ with $\sigma = 1/(\alpha+1)$ if and only if $n^\alpha E^r_n(u)_1 \in l^\sigma(1/n)$, where $E^r_n(u)_1$ is the least error in $L^1(\mathbb{R})$ when approximating u by piecewise polynomials of degree $< r$ $(\alpha < r)$ with n free knots. One might hope that by moving the knots with time one can approximate to high order solutions $u(x,t)$ even after discontinuities occur. This is indeed the case:

Theorem (DeVore and Lucier). Assume $f'' \geq C > 0$, $f \in C^{r+1}$, and $\alpha < r$. Then $u_0 \in \mathrm{BV} \cap B^\alpha$ implies that for all later times t, $u(\cdot, t) \in \mathrm{BV} \cap B^\alpha$.

The proof is constructive for $\alpha < 2$; in this case one generates piecewise rational approximations (linear in x) to $u(x,t)$ and piecewise algebraic curve approximations to the shock curves in x–t space. For $\alpha \geq 2$ the proof relies on a new Bernstein inequality for free knot approximation in $L^1(\mathbb{R})$ with piecewise algebraic curves of a particular form.

John Mason:

APPROXIMATION BY FUNCTIONS OF LINEAR FORMS

We consider the approximation of a given function or data set y by a form $f(L)$ where f is a chosen function and L is a linear form (and in particular a polynomial), using two linearisation algorithms based on comparisons between a true error $\epsilon = y - f(L)$ and a linearized error $\epsilon^* = w \cdot [f^{-1}(y) - L]$, where $w = \{f^{-1}(y)\}'$. The first algorithm, based on ideas of K. Appel, minimises $\|\epsilon^*\|$ and the second, an iterative procedure based on ideas of D.G. Carta, minimises $\|\epsilon^*_{i+1} - (\epsilon^*_i - \epsilon_i)\|$. The convergence properties of these algorithm are discussed for ℓ_p norms, and it is shown that, although best approximations may be obtained in ℓ_∞, non-best approximations (of Galerkin-type in the case of polynomials) are obtained in ℓ_2. We also discuss the known existence and characterisation theories for best approximation, especially when f is monotonic. Applications of the algorithms are discussed for $f = L^{-r}$ (for integer r) and $f = e^L$, and numerical results are given.

H. Michael Möller:

MAXIMAL LINEAR FUNCTIONALS FOR SYMMETRIC POLOYNOMIAL APPROXIMATION PROBLEMS

Assume that a function $f \in C(B)$ has certain symmetries. We want to approximate it uniformly by polynomials having the same symmetry. Using group representation theory, we specify symmetries which allow this symmmetry preserving approximation. One important example is given by the functions which are invariant under all transformation of a finite transformation group.

We study the maximal functionals which vanish on the (finite dimensional) polynomial spaces having one of these specified symmetries in the bivariate case. We give bounds for the number of point evaluation functionals involved in these maximal functionals and try to describe the geometric configuatrion of these points.

Günther Nürnberger:

THE STRUCTURE OF NONLINEAR APPROXIMATING SETS AND SP LINES WITH FREE KNOTS

An important property of certain nonlinear families is that local and global best uniform approximations are the same. The set $\mathcal{R}_{p,q}$ of rational functions has this property, while the set $\mathcal{S}_{m,k}$ of splines with free knots fails to have this structure. Therefore, given a nonlinear family, we restrict our considerations to a subclass of best approximations, and show that local and global strongly unique best approximations coincide if and only if strongly unique best approximations can be characterized by properties of the linear tangent space. Since splines from $\mathcal{S}_{m,k}$ with simple knots satisfy these properties, we obtain an alternation characterization of strongly unique best approximations. In addition, it is shown that "many" unique best approximations from $\mathcal{S}_{m,k}$ are strongly unique. Finally, we give a stability result for strong unicity and continuity results for the set-valued metric projection onto $\mathcal{S}_{m,k}$.

J. Nuttall:

HERMITE-PADÉ ASYMPTOTICS - INTRODUCTION

For functions $f_j(z), j = 1, \dots, m$, analytic near infinity, diagonal type 1 (Latin) polynomials $p_j(z), j = 1, \dots, m$, of degree n, are defined by

$$\sum_{j=1}^{m} p_j(z) f_j(z) = O(z^{-(m-1)(n+1)}), z \to \infty.$$

We survey their history and mention various special cases and applications, in particular the case of Padé approximants, $m = 2$. A basic problem is to determined the asymptotics of the polynomials as $n \to \infty$. For $m = 2$, for functions with branch points, it is indicated why most of the zeros of the polynomials approach an appropriate set of minimum capacity. We show the importance of a correctly chosen Riemann surface in describing asymptotic results by discussing the example of Dumas. A general conjecture, proved for some functions, gives strong asymptotics of HP polynomials in terms of the solution of a certain Hilbert problem (in the sense of Muskhelishvili) on the appropriate Riemann surface: A recently discovered integral equation promises to lead to a proof of the conjecture for an extended class of functions.

Peter Oswald:

FUNCTION SPACE TECHNIQUES FOR SPLINE APPROXIMATION PROBLEMS AND PROBLEMS OF GRID GENERATION IN NUMERICAL DISCRETIZATIONS

We report on some recent results concerning spline representations in various functions spaces. Especially, Besov spaces turn out to be a proper basis for dealing with L_p $(0 < p < \infty)$ and Sobolev space estimates for spline and similar approximation schemes. The concept developed applies to both linear and nonlinear approximation by one-dimensional and multivariate splines or by finite element functions. Possible applications to error estimates and grid generation for numerical methods are discussed.

Pensho Petrushev:

UNIFORM RATIONAL APPROXIMATION OF FUNCTIONS WITH FIRST DERIVATIVE IN THE REAL HARDY SPACE H^1

Denote by $R_n(f)$ the best uniform approximation of f by means of rational functions of degree n. The following result is obtained together with E. Moskona:

THEOREM. *If the function f is absolutely continuous on the real line and f' is in the real Hardy space ReH^1, then*

$$R_n(f) \le Cn^{-1}\|f'\|_{ReH^1}, \qquad n = 1, 2, \ldots. \tag{1}$$

Moreover, for each individual function f with $f' \in ReH^1$, we have $R_n(f) = o(n^{-1})$, i.e. the "small o" effect appears.

The Jackson estimate (1) and the corresponding Bernstein type inequality proved by V. Russak provide a characterization of the uniform rational approximation. A similar theorem for approximation on the unit disk was proved by A. Pekarskii.

Ted Rivlin:

THE REPRESENTATION OF FUNCTIONS IN TERMS OF THEIR DIVIDED DIFFERENCES

Let B denote an infinite triangular array of complex numbers whose n-th row, $n = 0, 1, \ldots$ is $b_n := (b_1^{(n)}, \ldots, b_{n+1}^{(n)})$. If f is a function defined on the entries in B, let $I_k f$, $k = 1, 2, \ldots$ be the divided differences of f with respect to b_{k-1} Since $I_k f = 0$ for $f(x) = x^m$, $m < k - 1$, and $I_{m+1}x^m = 1$, it is easy to see that there exist unique monic basic polynomials, $P_n(x)$, of degree n, $n = 0, 1, \ldots$, such that $I_{k+1}P_n = \delta_{k,n}$, $k, n = 0, 1, \ldots$. Thus $\{P_n(x), I_{k+1}\}_{k=0}^{\infty}$ is a normalized biorthogonal system, and, given B, each f defined on B has the biorthogonal expansion, $\sum_{j=1}^{\infty}(I_j f)P_{j-1}(x)$ associated to it. In a long paper, "The Representation of Functions in Terms of their Divided Difference at Chebyshev Nodes and Roots of Unity", by K.G. Ivanov, T.J. Rivlin, and E.B. Saff (preprint: ICM-USF # 89–014), polynomials were exhibited for the following choices of b_n: $(n+1)$-st roots of unity; extrema on $[-1,1]$ of the Chebyshev polynomial, $T_n(x)$; and zeros of $T_{n+1}(x)$. In this talk, we consider the case that the entries of B are $b_j^{(n)} = (j-1)/(n+1)$, $j = 1,\ldots,n+1$, $n = 1,2,\ldots$; $b_1^{(0)} = 0$, and we present an explicit expression for the coefficients of the basic polynomials.

Amos Ron:

ON THE INTEGER TRANSLATES OF A COMPACTLY SUPPORTED FUNCTION: DUAL BASES AND LINEAR PROJECTORS

This is joint work with Asher Ben-Artzi. Given a multivariate compactly supported function ϕ, we discuss here linear projectors to the space $S(\phi)$ spanned by its integer translates. These projectors are constructed with the aid of a dual basis for the integer translates of ϕ, hence under the assumption that these translates are linearly independent. Our main result shows that the linear functionals of the dual basis are local, hence makes it possible to contruct local linear projectors onto $S(\phi)$. We then discuss, for a general compactly supported function, a scheme for the construction of such local projectors.

In the second part of the paper we apply these observations to piecewise-polynomials and piecewise-exponentials to obtain a necessary and sufficient condition for a quasi-interpolant to be a projector. The results of that part extend and refine recent constructions of dual bases and linear projectors for polynomial and exponential box splines.

Paul Sablonniere:

BERNSTEIN QUASI-INTERPOLANTS IN ONE AND SEVERAL VARIABLES

Let $\{b_i^n\}$ be the Bernstein basis of polynomials of total degree at most n over a simplex or a hypercube. We study Bernstein Quasi-interpolants $B_n^{(k)}f = \sum_i \mu_{i,n}^{(k)}(f)b_i^n$, where the coefficients are combinations of Dirac measures on the regular lattice associated with the domain. These operators are intermediate ones between classical Bernstein approximants and Lagrange interpolants. Results are given on their norm and their speed of convergence when n tends to infinity together with some practical applications.

Ed Saff:

DISTRIBUTION OF EXTREME POINTS IN BEST POLYNOMIAL AND RATIONAL APPROXIMATION

Let K be a compact set in the complex plane having positive logarithmic capacity and connected complement. For any f continuous on K and analytic in the interior of K we investigate the distribution of the extreme points for the error in best uniform approximation to f by polynomials. More precisely, if

$$A_n(f) := \{z \in K : |f(z) - p_n^*(f;z)| = \|f - p_n^*(f)\|_K\},$$

where $p_n^*(f)$ is the polynomial of degree $\leq n$ of best uniform approximation to f on K, we show that there is a subsequence $\{n_k\}$ with the property that the sequence of $(n_k + 2)$–point Fekete subsets of A_{n_k} has limiting distribution (as $k \to \infty$) equal to the equilibrium distribution for K. Analogous results for weighted polynomial approximation are also given. In the special case when $K = [-1,1]$ and $f \in C[-1,1]$ is real valued, we give some sufficient conditions for the denseness of extreme points in best rational approximation.

Robert Sharpley:

EXTENSION OPERATORS FOR BESOV SPACES: $0 < p < 1$

In joint work with R. DeVore we construct an operator E which extends functions from domains having possibly rough boundaries to all of R^n while preserving smoothness in appropriate Besov space norms. These norms are defined in terms of the modulus of smoothness by

$$\|f\|_{B_q^\alpha(L^p(\Omega))} := \left(\int_0^1 [\omega_r(f,t)_p\ t^{-\alpha}]^q \frac{dt}{t}\right)^{1/q} + \|f\|_{L^p}.$$

In the case $1 \leq p \leq \infty$ these results are well known and follow by applying interpolation theory to the corresponding extension operator for the Sobolev spaces. When $0 < p < 1$ the Sobolev spaces are not available and the operator must be constructed using the local polynomial approximants. As corollaries of this result and its proof, we extend various characterizations and properties of Besov spaces (where Ω is either a cube or all of R^n) recently given by DeVore and Popov to this setting. In particular, atomic decompositions of the functions and interpolation properties of these spaces are provided.

Herbert Stahl:

DAS ASYMPTOTISCHE VERHALTEN ORTHOGONALER POLYNOME UND RATIONALE APPROXIMATION

Der Zusammenhang zwischen orthogonalen Polynomen und Kettenbrüchen, beziehungsweise Padéapproximierenden ist klassisch, und viele Systeme orthogonaler Polynome wurden erstmalig bei Kettenbruchentwicklungen spezieller Funktionen systematisch untersucht. Wir beschäftigen uns mit Polynomen, die orthogonal sind bezüglich eines positiven Masses mit kompaktem Träger in $\mathbb{C}$. In dieser Klasse entsprechen die Polynome mit regulärem asymptotischen Verhalten am ehesten den klassischen Systemen orthogonaler Polynome (wie z.B. Legendre, Chebyshev oder Jacobi). Im Vortrag werden neuere Ergebnisse zu diesem Themenkreis vorgestellt, und aufbauend auf diesen Ergebnissen wird die exakte Konvergenzgeschwindigkeit für interpolierende rationale Funktionen und rationale Bestapproximierende bei Markov-Funktionen abgeleitet. In analoger Weise werden dann, falls es die Redezeit erlaubt, die genauen Konvergenz- und Divergenzgebiete für nicht-diagonale Padéapproximierende bei Markov-Funktionen bestimmt.

Joachim Stöckler:

CUBATURE FORMULAS FOR PERIODIC FUNCTIONS

We investigate the problem of minimizing the error for multivariate integration rules

$$(2\pi)^{-d} \int_{[0,2\pi)^d} f(x)\,dx \sim \sum_{\nu=1}^{N} c_\nu f(t_\nu),$$

where f lies in a subset of a reproducing kernel space of smooth periodic functions. Given fixed nodes t_ν, $1 \le \nu \le N$, the optimal weights can be computed by solving a linear system of equations involving the reproducing kernel. We develop estimates for special cases, when the kernel is a multivariate Bernoulli-spline.

An introduction to the nonlinear problem of placing N nodes in $[0,2\pi)^d$ in order to minimize the interpolation error is given, and differences between the univariate and the multivariate case are demonstrated.

Joszef Szabados:

FINE AND ROUGH THEORY OF INTERPOLATION

A new norm is defined for the r times differentiated Lagrange interpolation operators. By using this norm, we are able to extend the so-called fine and rough theory of interpolation introduced by Erdős and Turán in 1955, to the case of the differentiated operator. A simultaneous approximation theorem for several consecutive derivatives is also considered.

Vladimir Tikhomirov:

SMOTHNESS AND APPROXIMATION

In this lecture, we present some results of the participant's of the author's seminar on approximation theory at Moscow University (including the author, E. Galeev, A. Buslaev, G. Magaril-Iliyaev and A. Hodulev.)

The first part of the lecture is devoted to asymptotic results on n-widths in the sense of Kolmogorov, Fourier n-widths, linear n-widths and n-widths in the sense of P. Alexandrov, for the classes $H^{\Gamma}(\Pi) := \cup_{(1/p,\alpha)\in\Gamma} H_p^{\alpha}(\Pi)$, $\Gamma \subset I \times \mathbb{R}$.

In the second part of the lecture, we discuss some problems of nonlinear analysis connected with generalized differential equations of Sturm-Liouisville type (the spectral numbers of such equations are the n-widths of corresponding Sobolev classes.)

Finally, the problems of mean N dimensions of spaces in $L_p(\mathbb{R})$ or $C^{\ell}(\mathbb{R})$ were considered.

Vilmos Totik:

POTENTIALS OF SIGNED MEASURES AND RATIONAL FUNCTIONS

For two disjoint compact sets A and B on the complex plane and a weight function Q on $A \cup B$, we minimize the logarithmic energy in the presence of the external field Q for signed measures with mass 1 on A and mass -1 on B. The optimal measure will be supported on a subset $A^* \cup B^*$ of $A \cup B$, and the corresponding weighted potential is constant (quasi-everywhere) on A^* and on B^*. The difference F of these constants is closely linked to the problem of finding rational functions of degree n such that their modulus multiplied by the weight $\exp(-nQ(z))$ be as large as possible on A under the condition that their weighted norms on B (with weight $\exp(-nQ(z))$) is at most one. In the unweighted case this in turn solves the problem on the rate of rational approximants to the function f that is 1 on A and 0 on B (the n-th root of this best approximation tends to $\exp(-F/2)$). Asymptotically optimal rational functions can be obtained by dicretizing the weighted energy (Fekete points, Leja points). When A and B are connected sets, then the potential of the extremal measure is just the real part of a conformal mapping of the complement of $A \cup B$ onto a ring while $\exp(-|F|/2)$ equals the ratio of the radii of this ring. Therefore, we get a (numerical) method for finding conformal mappings of ring domains onto rings.

Lars Wahlbin:

POINTWISE STABILITY OF CUBIC SMOOTHING SPLINES WITH NONUNIFORM DATA POINTS

The cubic smoothing spline $s_n(x)$ is the solution of

$$\min_{s_n \in H_2} \left\{ \frac{1}{n} \sum_{i=0}^{n-1} (s_n(x_i) - d_i)^2 + \lambda_n \int_0^1 (s_n''(x))^2 \, dx \right\}.$$

In this joint work with R.S. Anderssen and F.R. de Hoog, a general technique is developed for analyzing the pointwise stability of the above minimization problem when the meshpoints are unevenly spaced. The technique applies e.g. to the cases of (i) meshes which "look quasi-uniform on the scale of $\lambda_n^{1/4}$", and (ii) meshes which are systematically refined or thinned around a point.

The technique also applies to analyzing pointwise convergence when data $d_i = D(x_i) +$ error for some function $D(x)$.

e-mail addresses:

Hubert Berens: mpmal2@derrze0 (bitnet)
Wolfgang Boehm: I1120101@DBSTU1 (bitnet)
Carl de Boor: deboor@cs.wisc.edu
Peter Borwein: PBORWEIN@cs.dal.ca (Canadian net)
Len Bos: LPBOS@UNCAMULT (bitnet)
Dietrich Braess: p150204@dborub01 (bitnet)
Antonio Córdova: math022@dwuuni21 (bitnet)
Wolfgang Dahmen: dahmen@fubinf.uucp
Ron DeVore: N410099@UNIVSCVM (bitnet)
Nira Dyn: niradyn@taurus (bitnet)
Arieh Iserles: na@mhd.amtp.cam.ac.uk (janet)
Björn Jawerth: N410117@UNIVSCVM (bitnet)
Kurt Jetter: hu277je@uuidui.uucp
Iain Johnstone: iainj@playfair.stanford.edu
Rudolph Lorentz: GMAP27@DBNGMD21 (bitnet)
Dany Leviatan: LEVIATAN@TAURUS (bitnet)
Brad Lucier: LUCIER@math.purdue.edu
H. Michael Möller: MA105@DHAFEU11 (bitnet)
Sherm Riemenschneider: SRIEMENS@UALTAVM (bitnet)
T. Rivlin: RIVLIN@YKTVMV (bitnet)
Amos Ron: amos@cs.wisc.edu
E.B. Saff: DKGVAA@CFRVM (bitnet)
Bob Sharpley: N410059@UNIVSCVM (bitnet)
Grace Wahba: wahba@stat.wisc.edu
Lars B. Wahlbin: WHALBIN@MSSUNZ.MSI.CORNELL.EDU

Berichterstatter: C. de Boor and R.A. DeVore

Tagungsteilnehmer

Prof. Dr. R. E. Barnhill
Computer Science Department
Arizona State University
Tempe, AZ 85287
USA

Prof. Dr. L. Bos
Dept. of Mathematics and Statistics
University of Calgary
2500 University Drive N. W.
Calgary, Alberta T2N 1N4
CANADA

Prof. Dr. H. Berens
Mathematisches Institut
der Universität Erlangen
Bismarckstr. 1 1/2
8520 Erlangen

Prof. Dr. D. Braess
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48
4630 Bochum 1

Prof.Dr.-Ing. W. Boehm
Angewandte Geometrie und
Computergraphik
TU Braunschweig
Pockelsstr. 14
3300 Braunschweig

Prof. Dr. Z. Ciesielski
Instytut Matematyczny
Polskiej Akademii Nauk
ul. Abrahama 18
81-825 Sopot
POLAND

Prof. Dr. C. de Boor
Center for the Mathematical Scienc.
University of Wisconsin-Madison
610 Walnut Street
Madison, WI 53705
USA

Prof. Dr. A. Cordova
Fakultät für Mathematik
Universität Würzburg
Am Hubland
8700 Würzburg

Prof. Dr. P. Borwein
Department of Mathematics
Dalhousie University
Halifax, N.S. B3H 4H8
CANADA

Prof. Dr. W. Dahmen
Institut für Mathematik III
der Freien Universität Berlin
Arnimallee 2-6
1000 Berlin 33

Prof. Dr. R. A. DeVore
Dept. of Mathematics
University of South Carolina

Columbia , SC 29208
USA

Dr. J. A. Gregory
Dept. of Mathematics and Statistics
Brunel University

GB- Uxbridge, Middlesex , UB8 3PH

Prof. Dr. Z. Ditzian
Dept. of Mathematics
University of Alberta
632 Central Academic Building

Edmonton, Alberta T6G 2G1
CANADA

Prof. Dr. K. Höllig
Mathematisches Institut A
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 560

7000 Stuttgart 80

Prof. Dr. N. Dyn
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof. Dr. A. Iserles
Dept. of Applied Mathematics and
Theoretical Physics
University of Cambridge
Silver Street

GB- Cambridge , CB3 9EW

Prof. Dr. W. Freeden
Institut für Reine und Angewandte
Mathematik
der RWTH Aachen
Templergraben 55

5100 Aachen

Prof. Dr. K. Ivanov
Inst. of Mathematics
Bulgarian Academy of Sciences
P. O. Box 373

1090 Sofia
BULGARIA

Prof. Dr. M. von Golitschek
Institut für Angewandte Mathematik
und Statistik
der Universität Würzburg
Am Hubland

8700 Würzburg

Prof. Dr. B. Jawerth
Dept. of Mathematics
University of South Carolina

Columbia , SC 29208
USA

Prof. Dr. K. Jetter
Fachbereich Mathematik
der Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Prof. Dr. B. Lucier
Dept. of Mathematics
Purdue University

West Lafayette , IN 47907
USA

Prof. Dr. Jia Rong-qing
Dept. of Mathematics
University of Oregon

Eugene, OR 97403
USA

Prof. Dr. J. C. Mason
Applied & Computational Maths Gro
RMCS

GB- Shrivenham, Swindon Wilts. SN6 8LA

Prof. Dr. I. Johnstone
School of Mathematics
University of Bath

GB- Bath BA2 7AY

Prof. Dr. H.M. Möller
Fachbereich Mathematik
Fernuniversität Hagen/GH
Postfach 940

5800 Hagen 1

Prof. Dr. D. Leviatan
Approximation Theory, The Sackler
Faculty of Exact Sciences
Tel Aviv University

Tel Aviv 69978
ISRAEL

Prof. Dr. G. Nürnberger
Mathematisches Institut
der Universität Erlangen
Bismarckstr. 1 1/2

8520 Erlangen

Dr. R. A. Lorentz
Gesellschaft für Mathematik und
Datenverarbeitung - GMD
Postfach 1240
Schloß Birlinghoven

5205 St. Augustin 1

Prof. Dr. J. Nuttall
Department of Physics
The University of Western Ontario

London, Ontario N6A 3K7
CANADA

Prof. Dr. P. Oswald
Sektion Mathematik
Friedrich-Schiller-Universität
Jena
Universitätshochhaus, 17. OG.

DDR-6900 Jena

Prof. Dr. P. Sablonniere
Laboratoire LANS
INSA
20, av. des Buttes de Coesmes

F-35043 Rennes Cedex

Prof. Dr. P. P. Petrushev
Inst. of Mathematics
Bulgarian Academy of Sciences
P. O. Box 373

1090 Sofia
BULGARIA

Prof. Dr. E. B. Saff
Dept. of Mathematics
University of South Florida

Tampa , FL 33620-5700
USA

Prof. Dr. S. D. Riemenschneider
Dept. of Mathematics
University of Alberta
632 Central Academic Building

Edmonton, Alberta T6G 2G1
CANADA

Prof. Dr. K. Scherer
Institut für Angewandte Mathematik
der Universität Bonn
Wegelerstr. 6

5300 Bonn 1

Prof. Dr. T. J. Rivlin
IBM Corporation
Thomas J. Watson Research Center
P. O. Box 218

Yorktown Heights , NY 10598
USA

Prof. Dr. R. C. Sharpley
Dept. of Mathematics
University of South Carolina

Columbia , SC 29208
USA

Prof. Dr. A. Ron
Computer Science Department
University of Wisconsin-Madison
1210 W. Dayton St.

Madison , WI 53706
USA

Prof. Dr. H. Stahl
Fachbereich 2
Technische Fachhochschule Berlin
Luxemburger Str. 10

1000 Berlin 65

Dr. J. Stöckler
Fachbereich Mathematik
der Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Prof. Dr. V. Totik
Bolyai Institute
Szeged University
Aradi Vertanuk Tere 1

H-6720 Szeged

Prof. Dr. J. Szabados
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15
P. O. Box 127

H-1053 Budapest

Prof. Dr. G. Wahba
Department of Statistics
University of Wisconsin
1210 W. Dayton Street

Madison, WI 53706
USA

Prof. Dr. V. M. Tikhomirov
Dept. of Mathematics
M. V. Lomonosov State University
Moskovskii University
Mehmat

Moscow 117 234
USSR

Prof. Dr. L. B. Wahlbin
Dept. of Mathematics
Cornell University
White Hall

Ithaca, NY 14853-7961
USA