MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Combinatorial Convexity and Algebraic Geometry

13.8. bis 19.8.1989

This year's Oberwolfach conference on combinatorial convexity and algebraic geometry was the first of this kind and led by G. Ewald (Bochum), P. McMullen (London), and R. Stanley (Cambridge, MA). It took place in the same week as the conference on "Kombinatorische und reell algebraische Geometrie". Each day of the meeting there were joint lectures in the forenoon giving surveys on topics of both meetings, and on the other hand there were more special lectures in the afternoon separately for each conference. This report contains the abstracts of lectures of participants, who formally belonged to the meeting on combinatorial convexity and algebraic geometry — the other abstracts can be found in the report of the parallel meeting.
One main aspect of the meeting — also discussed in talks giving some survey — was the question how the theory of toric varieties and the theory of convex polytopes are related and what results can be stated by using this connection. Considering this problem there were lectures on homology, intersection homology and f–,h–, or g–vectors; homology, combinatorial invariance and Minkowski–addition, intersection numbers and mixed volumes. Furthermore there were lectures about using linear transforms in the birational geometry of toric varieties and about classification of polytopes with few vertices and the question of projectivity of the associated toric varieties.
A second field of interest consisted in problems of convexity and combinatorics which are interesting both from the point of view of combinatorics and of convexity and algebraic geometry. There were further talks on f–vectors of special classes of cell complexes, on piecewise polynomial functions, which are related to the homology of toric varieties, on arrangements of hyperplanes, which in two ways are connected to varieties, on the Erhard polynomial counting lattice points in polytopes, which are connected to global sections on the associated variety, on the Generalized Lower Bound Theorem and on difference bodies of polytopes.
The third aspect of the conference was that of applications of the theory of toric varieties. Besides a lecture about a concept generalizing the theory of toric varieties there were talks about investigation of singularities, classification of special singular spaces, on divisors, and on the question of compactification.

Vortragsauszüge:

Margaret M. Bayer

Flag Vectors of Polytopes and Intersection Homology

The f–vector of a simplicial polytope can be obtained from the homology ranks of the associated toric variety. Stanley used this to prove that the f–vectors of simplicial polytopes satisfy the McMullen conditions. For nonsimplicial polytopes the relevant ranks are the middle perversity intersection homology Betti numbers. These depend on, but do not determine, the flag vectors of the polytopes. This talk surveys what is known about the connection between the Betti numbers and flag vectors. We discuss the convolutions of gs (introduced by Kalai) and the cd index. Both are attempts to extend the Betti numbers to a set of parameters that incorporates all the flag vector information.

Louis J. Billera

Algebraic, Geometric and Combinatorial Properties of Piecewise Polynomial Functions

We consider the ring $S^r(\Delta)$ of all C^r piecewise polynomial functions over a d–dimensional polyhedral complex $\Delta \subseteq \mathbf{R}^d$. If S^r_m consists of all elements of degree at most m, we consider the generating function $\sum_{m\geq 0} dim_{\mathbf{R}} S^r_m(\Delta)t^m$, describe some of its properties and show how it can be computed by the methods of Gröbner bases.
We then specialize to the case $\Delta = \Sigma_P$, the complete fan associated to a convex d–polytope P with $0 \in \operatorname{int} P$, and show that $S^0(\Sigma_P) \approx A_P$, the face (Stanley–Reisner) ring of P, in the case P simplicial. Thus in the simplicial case we get another geometric description of the Betti numbers of X_P, the projective toric variety associated to rational P. We comment that results of Eikelberg show that the Hilbert series of $S^0(\Sigma_P)$ yields $\beta_2(X_P)$ in the nonsimplicial case as well.

Anders Björner

On f–vectors, Betti numbers and combinatorics

Joint work with G. Kalai was described, giving complete characterisations of the pairs of f–vectors and Betti–number sequences arising from simplicial complexes and various classes of cell complexes. The combinatorial methods used, involving various compression and shifting techniques, were reviewed.

Wolfgang Ebeling

Coxeter–Dynkin diagrams and the Zeta–function of the monodromy

Let $f : (\mathbf{C}^{n+1}, 0) \Rightarrow (\mathbf{C}, 0)$ be the germ of an analytic function with an isolated singularity at 0. A Coxeter–Dynkin diagram of f is a graph with two types of edges, positive and negative ones, which describes the intersection matrix of f with respect to a (strongly) distinguished basis of vanishing cycles of f. It is conjectured that the minimun over all Coxeter–Dynkin diagrams of f of the number of negative edges is equal to the modality of f. A similar conjecture was stated by Il'yuta (Russ. Math. Surveys 42 (1987)). I have almost completed the proof that this conjecture and a related one hold for the 0–, 1–, and 2–modal singularities. For the proof certain numbers related to the zeta–function of the monodromy and the Newton boundary of f are used to derive necessary combinatorial conditions for a graph to be a Coxeter–Dynkin diagram of some singularity.

Markus Eikelberg

On the Picard group of a compact projective toric variety

Let X_Σ be a compact projective toric variety, given by a fan Σ, which is spanned by a convex polytope Δ^* and let X_Σ^h be the associated complex analytic space. We prove that the Picard group $Pic\, X_\Sigma$ is isomorphic to $H^2(X_\Sigma^h, \mathbf{Z})$. Following T. Oda this implies $H^2(X_\Sigma^h, \mathbf{Z}) \cong {}^{SF(N;\Sigma)}/_{M}$. ${}^{SF(N;\Sigma)}/_{M}$ denotes the group of on the elements of Σ linear

functions as defined by T. Oda.
If $HS(\Delta)$ ist the semi–group of translation classes of summands of the convex polytope Δ with Minkowski–addition, Δ denoting the polar polytope of Δ^*, we define $S(\Delta)$ as the group generated by the elements of $HS(\Delta)$ and $\lambda(\Delta) := rk\, S(\Delta)$. We prove ${}^{SF(N;\Sigma)}/_M \otimes \mathbf{R} \cong S(\Delta)$ from which we get $H^2(X_\Sigma^h, \mathbf{Z}) \cong Pic\, X_\Sigma \cong {}^{SF(N;\Sigma)}/_M \cong \mathbf{Z}^{\lambda(\Delta)}$. $\lambda(\Delta)$ can be calculated by a formula of P. McMullen (proved anew by Z. Smilansky). As special case we obtain $Pic\, X_\Sigma \cong \mathbf{Z}^{n-d}$ for simplicial fans with n one–dimensional cones and $Pic\, X_\Sigma \cong \mathbf{Z}$ for fans spanned by simple d–dimensional polytopes with $d \geq 3$. We present classes of non–simplicial 3–polytoptes, for which $Pic\, X_\Sigma$ is determined by the f–vector of Δ, and classes, for which $Pic\, X_\Sigma$ is not even determined by the combinatorial type of Δ (generalizing results of M. McConnell).

Günter Ewald

<u>Mixed volumes and intersection numbers</u>

First we present an introduction into the relationship between mixed volumes of lattice polytopes in $\mathbf{R}^n$ and intersection numbers of varieties associated with the polytopes, called toric varieties.
In a second part we give a new characterization of Alexandrov–Fenchel's inequality and base on it the following contribution to the problem of discussing the equality case (which is open since more than 50 years): If all convex bodies are n–dimensional polytopes, a conjecture made by R. Schneider on the equality case is true. In the lower–dimensional case the conjecture is false.

Jonathan Fine

<u>On Intersection Homology from a combinatorial point of view</u>

Mostly, this talk was concerned with formulae for betti numbers. (Intersection homology, in the singular case).

CASE	METHOD	FORMULA
smooth toric variety	Weil conjectures, counting points	$\sum f_i(-1+t)^i$
general toric variety	— // —	$\sum p(L_\delta)(-1+t)^{\dim \delta}$

general Δ	$f\Delta$ linear combination of polytopes in I and C	$I(a;b;c;b;a) =$ $(a;a+b;b+c;c+b;b+a;a)$ $C(a;b;c;b;a) =$ $(a;b;c;c;b;a)$
general Δ	truncation and Mayer–Vietoris / Inclusion–Exclusion	associated formula not written down
X isolated singularities	analogy to Δ isolated singular	Theorem (Fine + Rao) $hX = h\tilde{X}$ $-(I-C)hE_{(1)}$ $-(I-C)^2hE_{(2)}\dots$
X general	analogy to previous X and Mayer–Vietoris / Inclusion Exclusion	Conjecture (Fine): X= $\sum(-1)^{dim\,E_i}(I-J)^{dim\,E_i}E_i$ sum over all faces of exceptional locus

Also, Mayer–Vietoris / Inclusion–Exclusion was stated and proved, and the construction of $H(\Delta)$ for Δ isolated singular sketched.
Also the join construction J_oE was introduced and defined.

Pierre Goosens

On generalizing the Stanley–Reisner ring construction

The numbers of faces of a simplicial complex Δ can be studied by considering its so-called Stanley–Reisner graded ring, whose Hilbert series is in the form $(h_0 + h_1 + \ldots + h_d t^d)(1-t)^{-d}$. The h–vector $(h_0, h_1, \ldots, h_d)$ of Δ is linearly determined by its f–vector, and conversely.
I consider the question of generalizing these methods to polyhedral complexes. A partial answer is provided by Stanley's generalization of the h–vector to lower Eulerian posets. Is it possible to generalize the Stanley–Reisner ring itself, so that its Hilbert series would be given by this generalized h–vector through the same formula? I specify the properties that such a ring should idealistically satisfy. I stress an inverse limit property stating that the ring associated with a complex should be obtained by "glueing up" those associated with its faces. This property, together with a general construction giving the ring associated with a face given the one associated with its boundary, would be enough to determine the wanted ring.

Takayuki Hibi

The Erhart polynomial of a convex polytope

Let $P \subset \mathbf{R}^N$ be an integral convex polytope, i.e., a convex polytope any of whose vertices has integer coordinates, of dimension d. Given a non–negative integer n, we write $i(P,n)$ for the cardinality of the finite set $nP \cap \mathbf{Z}^N$, where $nP := \{n\underline{\alpha}; \underline{\alpha} \in P\}$. It is known that $i(P,n)$ is a polynomial in n of degree d, called the Erhard polynomial of P. In the talk, after reviewing the algebraic background of $i(P,n)$, a combinatorial self–reciprocity theorem for $i(P,n)$ will be stated. Moreover, from a view point the calculation of $i(P,n)$, we will present the concept of toroidal posets via the theory of algebras with straightening laws.

Masanori Ishida

The stellation of polyhedral cones and invariants of toric divisors

A normal crossing divisor of a complex manifold is said to be a *toric divisor* if its irreducible components are toric varieties. Toric divisors appear as the degenerate fiber of a one–parameter family of abelian varieties, the exceptional divisor of the toroidal desingularization of a Tsuchihashi cusp singularity and the anti–canonical divisors of some non–Kähler complex manifolds.
By using Vinberg's theory of discrete groups generated by reflections of cones, I will give examples of Tsuchihashi cusp singularities obtained by polyhedral cones with stellations.
As important invariants of a toric divisor, there are "the arithmetic genus defect" χ_∞ by Ehlers and Satake and the value at zero of the associated zeta function by Ogata.
I will discuss on the calculation of these invariants of toric divisors especially of those obtained by polyhedral cones with the stellations.

Jerzy Jurkiewicz

r–jets of 1–parameter subgroups at toric singularities

Given a (singular) **T**-fixed point p of a toric variety consider for $r = 1, 2, \ldots,$ the set $J(s)$ (which is always finite) of r–jets of 1–parameter subgroups at p. The question is: how far can the singularity be described by the sequence $\{J(r)\}$? The resulting problem concerning polytopes: given a rational polytope $P \subset \mathbf{R}^l \times \mathbf{R}^m$ search the set

$$K(r) = \{x \in \mathbf{Z}^l : \text{ exist } y \in \mathbf{Z}^m \text{ such that } \frac{1}{r}(x, y) \in \mathbf{Z}^l\}.$$

Result: The sequence $J(1), J(2), \ldots$ is a finite disjoint union of "borded polytopes" of Erhard.
It follows in particular that $\sum_{r=1}^{\infty} \#\{J(r)\}t^r$ is a rational function. The surface case is studied in more details.

Gil Kalai

Algebraic Shifting and f–vectors

Shifted simplicial complexes are defined by the property that their vertices are positive integers, and their k–faces form an initial set with respect to the natural partial ordering on $(k+1)$–subsets of positive integers. A shifting operation $C \rightarrow \Gamma$ in which map an arbitrary simplicial complex C into a shifted complex Γ is described. Γ and C have the same number of k–faces, $k \geq 0$, and homological properties of C are translated to combinatorial properties of Γ. We describe a recent result which asserts that Cohen–Macaulayness is preserved under shifting. An important step is the computation of certain generalized homology groups which give global description to local homological properties (of links of faces). The shifting method imply good description of f–vectors of certain families of simplicial complexes. Results, hopes, speculations and frustrations concerning more applications are given in the lecture.

Peter Kleinschmidt

Projectivity of smooth toric varieties

Oda constructed a complete smooth nonprojective toric 3–variety with Picard number 4. We show that his example is minimal in the sense that every complete smooth toric variety with Picard number at most 3 is projective. In addition we prove a local projectivity criterion for complete smooth toric varieties with arbitrary Picard number.
Using the theory of linear transforms and Helly's theorem we show that our proof can be reduced to the 6–dimensional case which is then solved by a complete classification.
(Joint work with B. Sturmfels (Cornell))

Carl Lee

Winding Numbers and the Generalized Lower–Bound Theorem

Let W be a collection of at least $e+1$ points in $\mathbf{R}^e$ such that no hyperplane contains more than e points of $W \cup \{O\}$, where O is the origin. Choose an integer $0 \leq k \leq \frac{(n-e)}{2}$. For X a subset of W of cardinality e, let us say that X (or conv (X)) is of type k if the hyperplane $H = \mathrm{aff}(X)$ partitions the remaining $n-e$ points into two sets, one of which, say F, has cardinality k. For such a subset X, define the sign of X, sg(X), to be $+1$ if F and O lie on opposite sides of H, and -1 if F and O lie on the same side of H. Finally, define $\alpha(X)$ to be the measure of the solid angel with vertex O determined by X. The value $\alpha(X)$ is normalized to equal the fraction of the surface area of a unit sphere centered at the origin that is intersected by the cone determined by X. Set $\omega_k = \sum_X sg(X)\alpha(X)$ where the sum is taken over all X of type k. Then one can prove that this k^{th} winding number ω_k is in fact a nonnegative integer. In the case that W is a scaled Gale transform of some simplicial $(n-e-1)$–polytope P the result follows by proving that $\omega_k = g_k(P) = h_k(P) - h_{k-1}(P)$. The result for general W then follows readily. A new proof in the case $e \leq 2$ has been found, and as a consequence one obtains another simple proof of the Generalized Lower–Bound Theorem for simplicial polytopes with few vertices.

Ralf Lehmann

Singular Complex–symmetric Torus Embeddings

A compact complex manifold is called complex–symmetric if each point is isolated fixed point of a holomorphic involution. These manifolds are a generalization of the Hermitian symmetric spaces. Examples are the Hirzebruch surfaces and certain $\mathbf{P}^1$–bundles over compact complex tori.
It is shown that a normal complex–symmetric space with a reductive automorphism group is the product of a Hermitian symmetric space and a normal torus embedding satisfying some additional conditions. Among these torus embeddings the smooth ones are classified. Here we will answer the question what might happen in the case that singularities are allowed. Using root systems of Coxeter groups we show that all singular examples are obtained as quotient of a smooth example by a finite subgroup of the torus.

Horst Martini

On difference bodies of convex polytopes

E. Makai Jr. (Budapest) and H. Martini (Dresden):
Let P be a convex polytope ($d \geq 2$) with n vertices. The vertices x_i, x_j of P are called antipodal (strictly antipodal), if there are parellel different supporting hyperplanes H', H'' of P with $x_i \in H'$, $x_j \in H''$ (with $\{x_i\} = P \cap H'$, $\{x_j\} \in H''$).
Now let $a(P)$ ($sa(P)$) denote the number of antipodal (strictly antipodal) pairs of vertices of P. (Thus $sa(P)$ gives the half vertex number of the difference body $DP = P + (-P)$ of P.)
Our interests are devoted to upper and lower bounds for $a(P)$ and $sa(P)$. For example, in 3–space ($d = 3$) one obtains

$$\frac{n^2}{3} - O(1) \leq \max \{sa(P)\} \leq \frac{n^2}{3} + O(n^{\frac{3}{2}})$$

and

$$\min \{sa(P)\} = \begin{cases} 6 & \text{for } n \in \{4,5,7,9\} \\ \lceil \frac{n}{2} \rceil & \text{otherwise} \end{cases}$$

where $\lceil x \rceil$ denotes the smallest integer not smaller than x.
We shall give related bounds for general finite point sets in a corresponding sense, too.

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Mark McConnell

The rational homology of toric varieties is not a combinatorial invariant

The rational homology Betti numbers of a toric variety with singularities are not necessarily determined by the combinatorial type of the fan Σ which defines it; that is, the homology is not determined by the poset formed by the cones in Σ. This holds in all dimensions $n \geq 3$. I will give an algorithm for computing the Betti numbers of a complete toric variety of dimension 3, to use it to prove the stated result.

Peter McMullen

On the f-vectors of simplicial polytopes

The necessity of the known conditions which determine the f-vector of a simplicial polytope at present admits only the deep proof by Stanley. However, a careful check of the details of the proof enables the final polynomial ring, whose numerical invariants yield the conditions, to be expressend in terms of the Gale diagram. The challenge is thus to derive the numerical information about the ring directly from the Gale diagram.

Tadao Oda

Birational geometry of toric varieties

Birational geometry of projective toric varieties is well unterstood thanks to the works of Miles Reid on the toric analog of Mori's theory.
I would like to speak on what possible results we can get for non-projective but complete toric varieties.

Franz Pauer

Spherical varieties and "painted fans"

Let G be a connected reductive algebraic group over $\mathbf{C}$ and let H be a closed subgroup. The homogeneous space G/H is "spherical" iff a Borel–subgroup of G has a dense orbit in G/H. An "embedding of G/H" is an algebraic variety with G–action, containing a dense orbit isomorphic to G/H.
Generalizing the theory of torus embeddings, normal embeddings of spherical homogeneous spaces are classified by finite combinatorical data ("painted fans").

Lauren Rose

The Jacobian Ideal of a Hyperplane Arrangement

(The following is joint work with H. Terao.) Let A be finite central hyperplane in K^d, K a field, and let J in $S = K[x_1, \ldots, x_d]$ be the Jacobian ideal of the union of elements of A. S. Yuzvinski conjectured that for a generic arrangement the depth of the S–module S/J is equal to 0. Equivalently, the homological dimension of S/J is equal to d. We prove this conjecture by considering $D(A)$, the module of logarithmic derivations (vector fields) along A. The study of $D(A)$ was initiated by K. Saito, and many results about the structure of $D(A)$ as an S–module have been proved by Terao. Our main result is that when A is generic, the homological dimension, $\operatorname{hd} D(A) = d - 2$, the maximum possible. In addition, we exhibit a minimal free resolution of $D(A)$ as an S–module and thus obtain its Hilbert series. It turns out that the modules $D(A)$ and S/J are related in such a way that $\operatorname{hd} D(A) = \operatorname{hd}(S/J) - 2$. This proves the above conjecture, and the relation also gives an explicit minimal free resolution of S/J extending that of $D(A)$.

Richard P. Stanley

On f–vectors and toric varieties

Let f_i be the number of i–dimensional faces of a $(d-1)$–dimensional polyhedral complex. We survey what can be said about the f–vector $(f_0, f_1, \ldots, f_{d-1})$, especially in relation to the theory of toric varieties. In particular we discuss (1) the proof of the Upper Bound Conjecture for Spheres using Cohen–Macaulay rings, (2) the proof of McMullen's g–conjecture using toric varieties, (3) generalized h–vectors of nonsimplicial polytopes and the intersection homology of toric varieties, and (4) combinatorial aspects of shellings of polytopes related to non–compact toric varieties.

Bernard Teissier

Introduction to toric varieties

1) An intuitive description of the construction associating a projective toric variety to an integral convex polytope in $\mathbf{R}^d$, starting from the obvious correspondence between monomial ideals on polynomial rings and subsets E of $\mathbf{Z}^d_{\geq 0}$ such that $E + \mathbf{Z}^d_{\geq 0} \subset E$.
2) Some examples of the dictionary between problems in commutative algebra / algebraic geometry and problems in combinatorial convexity: The integral closure of a monomial ideal I corresponds to the convex hull of E. Then the Briançon–Skoda theorem for monomial ideals corresponds to a consequence of Caratheodory's theorem.

Klaus Wirthmüller

Root Systems and Symmetric Jacobi Forms

Let $R \subset V$ be a classical root system, Q its root lattice. For each elliptic curve $E = \mathbf{C}/(\mathbf{Z} + \tau\mathbf{Z})$ the Weyl group W acts on the abelian variety $A = Q \otimes_{\mathbf{Z}} E$. The quotient morphism of this action has been described by E. Looijenga 1974 with the help of the essentially unique minimal ample line bundle $\mathcal{L}$ on A on which W also acts.
In this lecture $\tau \in H$ is allowed to vary and the natural action of $\Gamma = SL_2(\mathbf{Z})$ on the

resulting family $A_H \to H$ taken into account. A toroidal embedding technique is used to compactify the quotient by Γ. The further quotient by W is (in all cases but R of type E_8) described in terms of invariant sections of certain line bundles on A. These sections generalize classical Jacobi forms.

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