MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 36/1989

Differentialgeometrie im Großen

20.8. bis 26.8.1989

Die Tagung fand unter der Leitung von Herrn Ballmann (Bonn), Herrn Bourguignon (Palaiseau) und Herrn Klingenberg (Bonn) statt. Es wurden aktuelle Entwicklungen in der globalen Differentialgeometrie behandelt, wobei vor allem die Anhänger von Mannigfaltigkeiten nichtpositiver Krümmung und die der Spektralgeometrie auf ihre Kosten kamen. Daneben ermunterte die angenehme Atmosphäre die Teilnehmer, sich im persönlichen Gespräch neue Anregungen zu holen.

Vortragsauszüge

I.G. NIKOLAEV (Inst. Math. Novosibirsk)

ALEXANDROV'S SPACES OF BOUNDED CURVATURE AND LIMIT SPACES IN GROMOV'S COMPACTNESS THEOREM.

The metric spaces of bounded curvature were introduced by A.D. Alexandrov more than 30 years ago. The restrictions of bounded curvature are given by means of purely synthetic methods. Due to Gromov's compactness theorem it is useful to consider metrics being limits if C^∞ — Riemannian ones having uniformly bounded curvature $|K_\sigma| \leq \Lambda$. One easily observes that the limit spaces in Gromov's compactness theorem are Alexandrov spaces of bounded curvature . Our principal result states that conversely, each metric space of bounded curvature can be d_L — approximated by C^∞ — Riemannian manifolds with the same restrictions upon curvature : $|K_\sigma| \leq \Lambda$. Hence the natural closure of the set of all C^∞ — Riemannian manifolds consists excactly of Alexandrov's spaces of bounded curvature. As an application of this approach we get metric versions of the well-known sphere and Schur's theorem.

P. SCHMUTZ (Zürich / Lausanne)

ON THE NUMBER OF SMALL EIGENVALUES ON RIEMANN SURFACES

We treat eigenvalue problems of the Laplace Operator on compact, connected Riemann surfaces R.S. of curvature $K \equiv -1$. In 1977, P. Buser obtained the following results:

Theorem 1: Given any $\varepsilon > 0$ and integer $g \geq 2$, there exists a $R.S.$ of genus g with $2g-2$ small (that is $< \frac{1}{4}$) eigenvalues. (0 is also a small eigenvalue.)

Theorem 2: A closed $R.S.$ has at most $4g-2$ small eigenvalues.

I proved an amelioration of Theorem 2:

Theorem A: A closed $R.S.$ has at most $4g-4$ small eigenvalues.

To prove this decompose the $R.S.$ in $2g-2$ Y-pieces (Riemann surfaces of signature $(0,3)$ such that the boundary components are closed geodesics), then the Theorem A follows from

Theorem B: A Y-piece hat at most 2 small eigenvalues (Neumann boundary condition).

To prove Theorem B, I showed first that a rectangular hexagon has at most 2 small eigenvalues.

Then I proved: If a Y-piece M has 3 small eigenvalues, then the rectangle H has 2 small eigenvalues for Neumann boundary conditions and 1 small eigenvalue for Neumann boundary conditions and 1 small eigenvalue for mixed boundary conditions.

Then I showed that this situation is impossible for all rectangular hexagons.

M. TROYANOV (Ecole Polytechnique - Paris)

A CONCENTRATION - COMPACTNESS PRINCIPLE FOR SEQUENCES OF RIEMANNIAN SURFACES

The problem treated is that of understanding divergent sequences of metrics on a compact surface. If the curvature stays pinched between two negative constants $(-c_1 \leq K \leq -c_2 < 0)$ then a sequence of metrics will diverge if and only if the conformal structure diverges (this is a well understood theory).

In the presence of positive curvature, the same behaviour does not necessarily occur, the reason being that some positive curvature may concentrate:

Theorem: Let $\{g_j\}$ be a sequence of metrics on a compact surface S such that the associated conformal structure stays bounded. Then
either $\{g_j\}$ has a convergent subsequence (in the Lipschitz topology)
or there is a point p such that

$$\limsup_{j\to\infty} \int_u K^t \geq 2\pi$$

for all neighborhoods U of p.

The above holds true under the two following assumptions:

$$(1) \qquad \|K_j\|_{L^p} < c$$

$$(2) \qquad (0 <) a \leq \text{ Area of } (S, g_j) \leq a'.$$

K. FUKAYA (Tokyo, Japan)
(Joint work with J. CHEEGER and M. GROMOV)

A SYMMETRIZATION THEOREM FOR RIEMANNIAN MANIFOLDS

For each Riemannian manifold M with $|K_M| \leq 1$ and $p \in M$ we can find $U_p \ni p$ s.t. $U_p \supset B_p(\varepsilon_M, M) = \{x | d(p,x) < \varepsilon_M\}$ and U_p is diffeomorphic to $N/\Lambda\tilde{x}\mathbf{R}^k$, where N is a nilpotent Lie group, $\Lambda \subset N\tilde{x}$ Aut N a discrete subgroup with $[\Lambda : \Lambda \cap N] < k_M$. Here ε_M depends only on M.

In this talk, it is discussed how these structures are patched on $U_p \cap U_q \neq \emptyset$.

Theorem $\forall \varepsilon > U$, $n \in \mathbb{N}\ \exists\ \rho > 0,\ k > 0$ s.t., if $\dim M = n$, $|K_{(M,g)}| \leq 1$, then we can find a metric g_ε on M s.t.

1) $$\forall p \in M\ \exists U_p \supset B_p(\rho, M),$$

$$\exists \tilde{U}_p \to U_p. \quad U_p = \tilde{U}_p/\Lambda_p, \quad \Lambda_p \subset N_p \quad \text{as above},$$

and the action of Λ_p on $(\tilde{U}_p, \tilde{g}_\varepsilon)$ is extended to an isometric action of N_p.

2) The injectivity radius of $\tilde{U}_p$ at $\tilde{p}$ is greater than ρ.

3) $|g - g_\varepsilon|_{c^{1,\alpha}} < \varepsilon$.

JENS HEBER (Augsburg)

HADAMARD MANIFOLDS OF HIGHER RANK

The talk describes joint work with Patrick Eberlein (Chapell Hill, North Carolina). We consider a complete , simply connected Riemannian manifold $\tilde{M}$ of non-positive curvature, i.e. a Hadamard mfd. For such spaces the rank $rk(\tilde{M})$ is defined as the maximal integer k such that every geodesic is contained in a totally geodesic $\mathbb{R}^k$ in $\tilde{M}$. We present a proof of the *rank rigidity theorem:* Let $\tilde{M}$ have a large isometry group in the sense of duality condition (e.g. if a quotient of finite volume exists). Then, if $rk(\tilde{M}) \geq 2$,

$\tilde{M}$ is a Riemannian product space
or symmetric of non-compact type and higher rank.

The first proofs of this theorem were given independently by Ballmann and Burns / Spatzier under the additional assumption that $k \geq -b^2$. Our short proof in the general case is essentially based on Gromov's Tits-distance at infinity.

VIKTOR SCHRÖDER (Münster)

EXISTENCE OF FLAT TORI IN ANALYTIC MANIFOLDS OF NONPOSITIVE CURVATURE

A k -flat in a complete Riemannian manifold M is a totally geodesic and isometric immersion $F : \mathbb{R}^k \to M$ of the flat $\mathbb{R}^k$. F is closed if it is periodic with respect to a cocompact lattice. The following theorem answers a question raised by Yau:

Theorem (Bangert, Schröder): Let M be a compact real analytic Riemannian manifold with nonpositive sectional curvature. If M contains a k-flat, then M contains also a closed k-flat.

As a corollary one obtains immediately:
Let M be as in the theorem, then M is a visibility manifold if and only if every abelian subgroup of $\pi_1(M)$ is isomorphic to $\mathbb{Z}$.

The proof of the theorem is very involved and uses methods from

(i) synthetic geometry of nonpositively curved manifolds,

(ii) the theory of subanalytic sets,

(iii) the theory of dynamical systems.

We give a short outline of the proof:
In the first step we construct flats with an additional structure of singular subspaces similar to the case of symmetric spaces. This allows us to define an $\mathbf{R}^k$-operation on the set of all flats. Using the theory of subanalytic sets we can show that $\mathbf{R}^k$ operates on an analytic submanifold and this operation is normally hyperbolic in certain directions. Using a closing lemma from the theory of normally hyperbolic systems we finally obtain a closed flat.

URSULA HAMENSTÄDT (Calif. Inst. of Technology)

RIGIDITY OF GEOMETRIC MEASURES

On the unit tangent bundle T^1M of a compact manifold M of negative sectional curvature there are two natural Borel-probability measures which are invariant under the geodesic flow Φ^t : The Bowen-Margulis measure μ and the harmonic measure ν. Both are ergodic with respect to Φ^t, hence either they coincide or else they are singular. We show that if these measures coincide then the mean curvature of the horospheres in the universal covering $\tilde{M}$ of M is constant.

PATRICK EBERLEIN (University of North Carolina, Chapel Hill)

MANIFOLDS OF NONPOSITIVE CURVATURE

Let $\tilde{M}$ denote a complete manifold, simply connected, $K \leq 0$ and let M denote a compact quotient of $\tilde{M}$. For M one defines integers $r(M)$ = the geometric rank and $r(\pi_1 M)$ = algebraic rank of $\pi_1 M$ and one can prove that these integers are the same. It is known that if $\tilde{M}$ is irreducible and $r(M) = k \geq 2$, then $\tilde{M}$ is symmetric of rank k. (Ballmann and Burns-Spatzier).

As corollaries we obtain

<u>Cor. 1</u> Let M be compact such that

1) $r(\pi_1 M) = k \geq 2$

2) $\pi_1(M)$ contains no finite index subgroup of the form $A \times B$.

Then $\tilde{M}$ is symmetric of rank k

Cor. 2 Let M be compact, $K \leq 0$, without Euclidean factor. Then M has a finite cover M^* with

$$M^* = M_s \times M_1 \times .... \times M_k$$

M_s locally symmetric, M_i rank 1 for $1 \leq i \leq k$.

A discussion of rank 1 spaces followed, presenting results of Ballmann, including the solution of the Dirichlet problem on $\tilde{M}(\infty)$, and the theorem of Bangert-Schroeder presented also in this conference.

BRUNO COLBOIS (Lausanne)
(Joint work with GILLES COURTOIS, Grenoble)

CONTROL OF THE SPECTRUM OF A CONVERGING FAMILY OF HYPERBOLIC 3-MANIFOLDS

Let X be a complete, connected, hyperbolic 3-manifold of finite volume. It follows, from a result of Thurston, that X is the limit for the pointed Lipschitz distance, of a sequence $(X_i)_{i=1}^{\infty}$ of compact, hyperbolic 3-manifolds with

$$\lim_{i \to \infty} \text{Vol } X_i = \text{Vol } X \ ; \ \text{Vol } X_i < \text{Vol } X.$$

Consider the spectrum of the Laplace operator on X and X_i:

$$\text{Spec } X = \{0 = \mu_0 < \mu_1 \leq .... \leq \mu_N\} \cup [1, \infty[.$$
$$\text{Spec } X_i = \{0 = \lambda_0(X_i) < \lambda_1(X_i) \leq .... \leq \lambda_{m(i)}(X_i) < 1 \leq \lambda_{m(i)+1}(X_i) \leq ... \uparrow \infty\}$$

Theorem

(1) If i is sufficiently great, $m(i) \geq N$

(2) $\lambda_k(X_i) \to \mu_k$ as $i \to \infty$ for $0 \leq k \leq N$

(3) If there exist arbitrarily great i with $m(i) > N$, $\lambda_k(X_i) \to 1$ for $N < k \leq m(i)$.

Ivan Sterling (Technische Univ. Berlin)

Constant mean curvature tori and soliton theory

Constant mean curvature (CMC) tori were classified in 1988 by Pinkall + Sterling. Based on this work Bohenko gave explicit formulas in terms of theta functions for CMC-tori in $\mathbf{R}^3$ and CMC + minimal tori in S^3. Ferus + Pedit classified equivariant Willmore tori and Ferus + Pedit + Pinkall + Sterling classified (1990) all Willmore tori. Pinkall + Sterling classified curves in $\mathbf{R}^3$ by analyzing the smoke ring flow. Videos of deforming soap bubbles and smoke rings were produced in 1988/89. CMC cylinders with embedded cylinder ends and CMC proper planes in $\mathbf{R}^3$ were found.

Valery Marenich (Novosibirsk)

The geometric structure of open manifolds of nonnegative sectional curvature

It is a well-known theorem by Cheeger and Gromoll which states that every open manifold V^n of nonnegative sectional curvature admits a convex exhaustion $V^n = \cup_{t>0} C_t$ which converges to the totally geodesic submanifold $S : C_t \searrow S,\ t \to 0, \partial S = \emptyset (S = \text{ soul } V^n)$; and the whole V^n is diffeomorphic to the normal bundle νS. In the report we sketch the proof of Cheeger-Gromoll and Toponogov conjectures for analytical manifolds. Namely: if $\dim S > 0$, so V^n is not diffeomorphic to euclidean space $\mathbf{R}^n$, then in every point in V^n there exists a two dimensional direction of zero curvature. These directions may be constructed in the following way : take a point p on S, any vectors e tangent to S and v normal to S at p. Let $l_v(\rho) = \exp_p(\rho v)$ and $e(\rho), v(\rho)$ be the parallel vector fields along l_v such that $e(0) = e, v(0) = v$. Then $K_{\sigma(p,v,e,\rho)} \equiv 0$ where $\sigma(p, v, l, \rho)$ is the two dimensional direction generated by $e(\rho), v(\rho)$.

In the second part of the report we consider the holonomy operator of νS and derive some statements of the following type : if the holonomy of νS is trivial, then the whole V^n is isometric to the direct product: $V^n \overset{\text{iso}}{\equiv} S \times W,\ w \overset{\text{diff}}{\equiv} \mathbf{R}^k$.

J. Dodziuk (Flushing)

Small eigenvalues of hyperbolic manifolds

I reviewed results about eigenvalues of Laplace-Beltrami operator in the interval $(0, \frac{(n-1)^2}{4})$ for a complete Riemannian manifold of n dimensions, finite volume and with sectional curvatures in $[-a^2, -1]$. The talk consisted of some answers to and conjectures concerning the following questions.

- What is the significance of eigenvalues in this range?
- How close to zero can the smallest positive eigenvalue be?
- What is the number of eigenvalues in $(0, (n-1)^2/4)$?
- What is a good upper bound for the function $\#\{\lambda \mid \lambda \text{ an eigenvalue } \leq x\}$?

The new results discussed are joint work with P. Buser and B. Colbois.

ROBERT BARTNIK (Bonn)

GEOMETRIC QUESTIONS ARISING FROM THE POSITIVE MASS THEOREM

Let (M, g) be a complete 3-manifold, asympt. flat in the sense there are coordinates near ∞ s.t. $|g_{ij} - \delta_{ij}| \leq c/r$, $|\partial g| + r|\partial^2 g| \leq c/r^2$.... and having scalar curvature $R(g) \geq 0$. The ADM mass of (M, g) is defined by

$$m_{\text{ADM}} = \frac{1}{16\pi} \oint_{S(\infty)} (\partial_j g_{ij} - \partial_i g_{jj})\, ds^i$$

This quantity is well-defined, indep. of the AF coordinates, or of the limiting process used to define $\oint_{s(\infty)}$, the integral over the sphere at ∞.

The positive mass theorem proves that $m_{\text{ADM}} \geq 0$ and that $m_{\text{ADM}} = 0$ iff (M, g) is flat. Recently I have used this do define the (quasi-local) mass of a bounded region $\Omega \subset M$: First define

> $PM = \{(M, g) : (M, g)$ satisfies the conditions of the positive mass theorem, and, (M, g) contains no horizon (stable minimal S^2) $\}$
>
> Then for $\Omega \subset M \in PM$ with $\partial\Omega$ connected, define
>
> $$m(\Omega) = \inf\{m_{\text{ADM}}(\tilde{M}) \; : \; \Omega \subset \tilde{M} \in PM\}$$

Then $m(\Omega)$ is well defined, ≥ 0, and monotonic : if $\Omega_1 \subset \Omega_2 \subset M$, then $m(\Omega_1) \leq m(\Omega_2) \leq m_{\text{ADM}}(M)$. Apart from spherical symmetry, I can say almost nothing about this definition:

Q1: Is $m(\Omega) > 0$ for some Ω?

Q2: Is there a metric (not C^2) which reaches the inf?

I conjecture yes, and that this extension is a static (spacetime) metric. I do not understand PM very well, so I ask

Q3: Construct (or prove existence of) $(M,g) \in PM$ s.t. (M,g) has a metrically flat open subset $((M,g) \neq (\mathbf{R}^3,$ flat) !).

PATRICK FOULON (Palaiseau)

ANOSOV FLOWS WITH DIFFERENTIABLE LIAPUNOV DISTRIBUTIONS

We prove a result of differentiable rigidity for contact Anosov flows on a compact manifold. Ergodic theory applied to this contact shows that there exists a full measure invariant set Ω s.t.

for every $x \in \Omega$ there is an invariant decomposition

$$T_x M = E_{-\gamma_s} \oplus \dots \oplus \mathbf{R}X \oplus E_{\gamma_i} \oplus \dots \oplus E_{\gamma_s}. \qquad (\bigstar)$$

$$\text{For } Z_i \in E_{\gamma_i} \;:\; \lim_{t\to\pm\infty} \frac{1}{t} \log \|T\Phi_t(Z_i)\| = \gamma_i \quad (i^{th} \text{ Liapunov exponent}).$$

If we assume that the decomposition $(\bigstar)$ extends to a C^3 decomposition then we show that such flows are C^3 orbit equivalent to algebraic flows on homogeneous spaces.

Theorem 1. (—, Labourie). Let Φ_t be a C^3 contact Anosov flow on M^{2n+1}, a compact manifold without boundary. If the decomposition $(\bigstar)$ is everywhere defined and C^3 then Φ_t is C^3 conjugate to our algebraic flow in the following sense.

1) There exists a C^3 diffeomorphism $\psi : M \to \Gamma \setminus G/H$
2) The group G is semi-simple, H is a closed subgroup, $\Gamma \subset G \times \mathbf{R}$ is discrete .
3) The Lie algebra $\mathcal{G}$ contains a semi-simple element X with real eigenvalues. The corresponding graduation is:

$$\mathcal{G} = \mathcal{G}_{-s} \oplus \dots \oplus \mathcal{G}_{-1} \oplus \mathcal{G}_0 \oplus \mathcal{G}_1 \dots \oplus \mathcal{G}_s$$
$$ad(X)_{/\mathcal{G}_i} = \gamma_i Id_{/\mathcal{G}_i}$$

4) The subalgebra $\mathcal{G}_0$ decomposes in $\mathcal{G}_0 = \mathbf{R}X \oplus \mathcal{H}$.
5) The subalgebra

$$\mathcal{G}^+ = \mathcal{G}_0 \oplus \mathcal{G}_1 \oplus \dots \oplus \mathcal{G}_s$$

is maximal parabolic.

Theorem 2: Under the same assumptions there exists a C^3 diffeomorphism such that Φ_t is orbit equivalent to an algebraic flow. But now Γ is a discrete subgroup of G.

DENNIS DETURCK (Philadelphia)

DIFFERENTIAL SYSTEMS WITH EXTRA SYMBOL INVARIANCE

When trying to prove existence for geometric systems of Partial Differential Equations, one introduces a gauge-group-valued unknown to overcome degeneracy induced by the invariance of the system. Often this does not work because the symbol of the system is "more invariant" than the system itself. Even in this case, the degeneracy can often be overcome. This is illustrated for the system $Rc(g) - \frac{1}{2(n-1)}Sc(g)g = T$, for the prescribed curvature of a connection problem, and for the inhomogeneous Yang Miles equations.

JÜRGEN JOST (Ruhruniv. Bochum)

HARMONIC MAPS AND GEOMETRY

Let M, N be Riemannian manifolds with isometry groups $I(M), I(N)$ resp. $f : M \to N$ is called harmonic, if trace $\nabla df \equiv 0$.

We always assume that N has nonpositive sectional curvature and is complete, and 1-connected. Let Γ be a discrete, cocompact subgroup of $I(M), \rho : \Gamma \to I(N)$ a homomorphism. This question of the existence of a ρ-equivariant harmonic map $f : M \to N$, i.e. $f(\gamma x) = \rho(\gamma)f(x) \quad \forall x \in M, \gamma \in \Gamma$ was studied in special cases by Diederich - Ohsawa and Donaldson and in general by Corlette, Jost-Yau, and Labourie. Such a map exists if $\rho(\Gamma)$ is not contained in a parabolic subgroup of $I(N)$ or if $\rho(\Gamma)$ centralizes a totally geodesic flat. If N is symmetric this holds if the Zariski closure of $\rho(\Gamma)$ is reductive.

In the second part, various applications of the existence of harmonic maps to Kähler geometry are given, based on work of Siu, Jost-Yau, and Mok. These results partially overlap with ones obtained by Margulis by a different approach.

MARC BURGER (Basel)

BEHAVIOUR OF SMALL EIGENVALUES ON GEOMETRICALLY FINITE RIEMANNIAN SURFACES

Let $M(g,p,f)$ be the moduli space of Riemannian surfaces of signature (g,p,f). The set

$$\{S \in M(g,p,f): \quad \text{Inj.rad.}(S) < \varepsilon\}$$

can be covered by a finite union of neighborhoods $V_\varepsilon(\mathfrak{g},m)$ each one associated to a finite graph $\mathfrak{g}$ with weight function m on the vertex set of $\mathfrak{g}$. A surface $S \in V_\varepsilon(\mathfrak{g},m)$ puts on $\mathfrak{g}$ the structure of a weighted graph $(\mathfrak{g},m,L)$.

Theorem: For $\varepsilon < \alpha_1$ and $S \in V_\varepsilon(\mathfrak{g},m)$ we have:

$$\frac{1}{\pi}(1-o(V_\varepsilon)) \leq \frac{\lambda_i(S)}{\lambda_i(\mathfrak{g},m,L)} \leq \frac{1}{\pi}(1+o(\varepsilon ln\varepsilon))$$

for all $1 \leq i \leq N$. Here all implied constants only depend on (g,p,f).

Corollary: There is a constant $c = c(g,p,f)$ such that for all $\lambda_2(S) < c$:

$$\text{mult } \lambda_2(S) \leq \frac{2}{3}[2g-2+p+f]+2.$$

JÜRGEN EICHHORN (Greifswald)

THE SPACE OF RIEMANNIAN METRICS ON AN OPEN MANIFOLD OF BOUNDED GEOMETRY.

For M^n open all the methods of Ebin break down. One has to start with a completly new approach. We consider $\mathcal{M}(B_k,I) = \{g|g \text{ complete, injrad} > 0, |\nabla^i R| \leq C_i\ ,\ 0 \leq i \leq k\}$. We endow $\mathcal{M}(B_k,I)$ with a very natural locally metrizable topology such that each component has a natural Banach manifold structure. Consider ${}^{bb,k+3}D(M) = \{f \in \text{Diff}M|\ |\nabla^m\ df\ | \leq C_m, 0 \leq m \leq k-2\ ,\ \inf_x |\ df|_x > 0\}$. Then ${}^{bb,k+3}D(M)$ has the structure of a completely metrizable topological group, acting on each component of ${}^{b,k+2}\overline{\mathcal{M}(B_k,I)} \cap C^{k+2}\mathcal{M}$.

VLADIMIR GOL'DSHTEIN (Novosibirsk)

L_p-COHOMOLOGIES OF NONCOMPACT RIEMANNIAN MANIFOLDS AND ITS APPLICATIONS

My lecture is a survey about the cycle of works related with L_p-theory of integration (in the Whitney sense) and L_p-cohomologies. L_p-cohomologies are the cohomologies of the differential complex (W_p^*, d). Here W_p^* is the space of forms $\omega \in L_p^k$ with $d\omega \in L_p^{k+1}$ $(\|\omega\|^2_{W_p^k} = \|\omega\|^2_{L_p^k} + \|d\omega\|^2_{L_p^{k+1}})$. Except the traditional cohomologies we investigate its reduced variant $\overline{H}_p^k(M) =$ Ker $d^k/[\text{Im } d^{k-1}]$ where $[\text{Im } d^{k-1}]$ is the closure of Im d^{k-1}.

My talk concernes the following questions:

1) Theorem about change of variable for W_p^k and classes of differential forms on Lipschitz manifolds;

2) L_p-integration theory, "imbedding theorem";

3) L_p-cohomologies of compact manifolds;

4) Poincaré duality in L_p-sense;

5) L_p-cohomologies of warped products.

THOMAS WOLTER (Univ. Zürich)

HOMOGENEOUS MANIFOLDS OF NONPOSITIVE CURVATURE

Homogeneous Hadamard manifolds correspond to a class of solvable Lie groups, which have been classified by Azencott-Wilson in 1976. Their geometry resembles in many aspects the well-known picture for symmetric spaces of noncompact type; however, in the nonsymmetric case, the isometry group is small, i.e. it fixes points at infinity. One can find many nonsymmetric Einstein metrics on these groups, some examples even with nonpositive sectional curvature. The algebra of invariant differential operators is commutative if and only if the space is symmetric; but there may be nonsymmetric examples with volume preserving geodesic symmetries.

G. THORBERGSSON (University of Notre Dame)

ISOPARAMETRIC SUBMANIFOLDS AND THEIR BUILDINGS

In the talk we associate a topological Tits building to any isoparametric submanifolds in a Euclidean space. A consequence of this construction is the following theorem that answers a question of Hsiang, Palais and Terng:

Theorem: Let M^n be an isoparametric submanifold in $\mathbf{R}^{n+r}$ that is compact irreducible and full with codimension $r \geq 3$. Then there is a symmetric space $X = G/K$ and an isometry $A : \mathbf{R}^{n+r} \to T_{(K)}X$ that carries M^n into a principal orbit of the isotropy representation of X.

GILLER COURTOIS (Univ. of Grenoble)
(Joint work with B. COLBOIS)

THE SMALL EIGENVALUES OF THE p-FORMS

The geometry of compact manifolds which have a very small first non zero eigenvalue for functions is well described by the following Buser-Cheeger inequalities: $h^2(M)/4 \leq \lambda_1(M) \leq C_1(\delta h^2(M) + h(M))$ where $h(M)$ is the Cheeger isoperimetric constant, C_1 a constant depending on the dimension of M and δ a lower bound of the Ricci curvature. In particular, a compact manifold whose curvature and diameter are bounded cannot have very small first eigenvalue for functions.

The case of p-forms, $p \in]1, n[$ is very much different:

Theorem 1: In any dimension n, there exists an n- dimensional compact manifold M^n and a family of metrics g_ε on it with bounded curvature, diameter and Cheeger's constant, such that the first non zero eigenvalue for the p-forms is going to zero when ε goes to zero, for $1 \leq p \leq (n-1)$.

The examples of theorem 1 are constructed using collapsing manifolds, and they are the only ones in the following sense. Let $\mathcal{M}(n, A, B)$ be the set of all compact Riemannian manifolds such that the sectional curvature is bounded by A and the diameter by B.

Theorem 2: $\exists\ \varepsilon(p, A, B, n) > 0$ such that if $M \in \mathcal{M}(n, A, B)$ has its first non zero eigenvalue for p-forms less than $\varepsilon(p, A, B, n)$, then M must admit collapsing.

K. SUGIYAMA (Kanazawa Univ.)

AN INEQUALITY BETWEEN CHERN NUMBERS OF MINIMAL VARIETIES

In this talk, we generalize the following Miyaoka-Yau's inequality in the higher dimensional case.

Fact (Miyaoka)

Let S be a surface of general type. Then we have the following inequality:

$$c_1(S)^2 \leq 3c_2(S)$$

and " $=$ " holds if and only if K_s is ample.

Fact (Yau)

Let M be a $n-$ dim. projective variety with ample canonical divisor. Then we have

$$(-1)^n\{nc_1(M)^2 - 2(n+1)c_2(M)\} \cdot c_1(M)^{n-2} \leq 0.$$

And moreover " $=$ " holds if and only if the universal covering $\tilde{M}$ of M is a unit ball in $\mathbf{C}^n$.

Now our result is as follows.

Theorem: Let M be an $n-$ dim. projective minimal variety of general type. Then we have

(1) The regular part M^{reg} of M admits an Einstein-Kähler metric.

(2) Let $N \xrightarrow{\mu} M$ be a desingularization of M. Then

$$\{(n-1)c_1(N)^2 - 2n \cdot c_2(N)\} \cdot (\mu^* K_M)^{n-2} \leq 0.$$

The idea of the proof is as follows. We first show that (1) implies the tangent circle J_N of N is $\mu^* K_M$-semistable, and then, using algebraic geometric arguments, we shall show (2).

V. TOPONOGOV (Novosibirsk)
(joint work with S. AKLABAROV)

COMPARISON THEOREM FOR TRIANGLES

We define a new class of Riemannian manifolds for which the triangle comparison theorem holds in the following sense. If the triangle ABC is sufficiently large then it has angle not less than the angles of the comparison triangle $A'B'C'$. We compare the manifold with the surface of constant curvature, but the condition on the curvature of the manifold is expressed in terms of integrals.

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