MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Special complex varieties

27.8. bis 2.9.1989

Die Tagung fand unter der Leitung von Herrn W. Barth (Erlangen) und Herrn A. Van de Ven (Leiden) statt. Es nahmen 40 Mathematiker aus 8 Ländern teil. Die Tagung repräsentierte nur einen Ausschnitt der gesamten Forschungsativität in der Algebraischen Geometrie, denn um sie möglichst produktiv zu gestalten war der Kreis der Eingeladenen bewußt begrenzt worden. Gleichzeitig wurde aber darauf geachtet jüngere Mathematiker einzubeziehen. So konzentrierten sich die Vorträge vor allem auf Eigenschaften algebraischer Untermannigfaltigkeiten des projektiven Raumes kleiner Dimension.

Vortragsauszüge

P.Wilson:

The Kähler cone of Calabi–Yau manifolds

For V a Calabi–Yau manifold (3-dim.), it is conjectured that $e(V) = 2(h^{1,1}(V) - h^{1,2}(V))$ lies in some bounded range. We observe that known examples with high Picard number $\rho = h^{1,1}$ all arise as resolutions of singular 3-folds with relatively small ρ. A normal projective 3-fold is a Calabi–Yau manifold if it has a Calabi–Yau manifold as desingularisation (the singularities of such a model are necessarily rational Gorenstein).

Theorem 1: Any Calabi–Yau manifold is the resolution of a Calabi–Yau model with $\rho \leq 19$.

In particular it follows that a Calabi–Yau manifold with $\rho > 19$ must contain rational curves.

The proof of Theorem 1 splits into two parts, the first of which is valid without restriction on ρ and gives useful rationalty information concerning the Kähler cone $\mathcal{K}$ (the cone of Kähler classes in $Pic(V) \otimes \mathbb{R}$) for a Calabi–Yau manifold V. As a byproduct, we also get:

Theorem 2: The Kähler cone of a Calabi–Yau manifold does not move in $H^2(V, \mathbb{R})$ if we deform the complex structure.

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The second part of the proof of Theorem 1 consists of analyzing points on the boundary $\partial\overline{\mathcal{K}}$ of $\mathcal{K}$ (the nef cone) which are non-singular points of the cone $W^* = \{D \in Pic(V) \otimes \mathbb{R}\, s.t.\, D^3 = 0\}$. Except for points on the hyperplane defined by the linear form $c_2(V)$, any rational such point will give rise to an elliptic fibre space structure on V over some normal rational surface; such fibre spaces are investigated. Letting $W \subset \mathbf{P}^{\rho-1}(\mathbf{R})$ be the cubic hypersurface corresponding to W^*, the theorem follows by using results of Davenport to show that the rational points are dense in W for $\rho > 19$.

C. Okonek:

Instanton homology of Seifert fibred homology 3-spheres

Let Σ be a closed, oriented (smooth) manifold of (real) dimension 3. Σ is a ($\mathbf{Z}$-)homology sphere if $H_*(\Sigma, \mathbf{Z}) \cong H_*(S^3, \mathbf{Z})$, i.e. $\pi_1(\Sigma)$ is a perfect group. For every such hmology 3-sphere Σ Floer has defined the instanton homology $I_*(\Sigma)$, a $\mathbf{Z}/8$ graded, finitely generated Abelian group. These groups, which are the natural target of relative Donaldson invariants, are defined via gauge theoretic techniques and are therefore hard to compute in general. For Seifert fibered Σ, i.e. for $\mathbf{Z}$-homology spheres which occur as links of Brieskorn complete intersections, it is possible to calculate $I_*(\Sigma)$ by studying the representation space

$$R(\Sigma) := Hom^*(\pi_1(\Sigma), SU(2))/conj.$$

Theorem (Bauer/Okonek): Let Σ be a Seifert fibered homology 3-sphere. Then $R(\Sigma)$ is an algebraic variety over $\mathbf{C}$ s.t. every component R_λ is complete, smooth and rational. There exists an algorithm for computing the Betti numbers $b_i(R_\lambda)$, in particular $b_{2j+1}(R_\lambda) = 0$.

This result "solves" a conjecture of Fitushel/Stern "up to torsion". Together with the work of Fitushel/Stern and the main result of Kirk/Klassen it yields an algorithm which computes the instanton homology for all Seifert fibered Σ. The proof uses a translation into a question about moduli spaces of stable bundles over certain algebraic surfaces and the Weil conjectures (applied to these moduli spaces).

S. Verra:

Trisecants to Enriques surfaces

Let X be an Enriques surface over $\mathbf{C}$; $|C|$ a Fano polarization on X. Then $\phi_{|C|}$ embeds X in $\mathbf{P}^5$ as a (smooth) surface of degrree 10 and sectional genus 6. $|C + K_X|$ has the same property. We say that $|C|$ is a Reye polarization in the special case $\phi_{|C|} \subseteq G$ a smooth quadric of $\mathbf{P}^5$ (the Grassmannian of lines in $\mathbf{P}^3$).
We want to study the variety Tris(X) of trisecant lines to $\phi_{|C|}(X)$. One obtains:

(a) $|C + K_X|$ is not a Reye polarization: Then $L \in$ Tris(X) if and only if L is trisecant to one of the 20 plane cubic curves contained in $\phi_{|C|}(X)$.
(b) $|C + K_X|$ is a Reye polarization: Let $\Sigma \subset |\mathcal{O}_{\mathbf{P}^3}(2)|$ be the 6-fold parametrizing rank 2 quadrics of $\mathbb{P}^3$. Then $X = \Sigma \cap \Lambda$ (Λ a 5-dimensional subspace of $|\mathcal{C}_{\mathbf{P}^3}(2)|$). Λ is a

linear system of quadrics of $\mathbf{P}^3$. For all $p \in \mathbf{P}^3$ let Λ_p be the sublinear system of quadrics singular at p. Then $\dim \Lambda_p = 1$ (except for 20 points p s.t. $\dim \Lambda_p = 2$). Now $L \in \mathrm{Tris}(X)$ if and only if $L \subseteq \Lambda_p$ for some p. Projection of $\phi_{|C|}(X)$ from a general one of its points yields a smooth surface in $\mathbf{P}^4$ whose existence was previously conjectured.

S.Katz:

Varieties which are the base locus of a Cremona transformation

Let $V \subset \mathbb{P}^r$ be a smooth, irreducible variety. Let $\tilde{\mathbf{P}}^r$ be the blow-up of $\mathbf{P}^r$ along V. Let $E, H \in \mathrm{Pic}(\tilde{\mathbf{P}}^r)$ be the classes of the exceptional divisor and the pullback of a hyperplane. When is there a base point free linear system $|nH - mE|$ such that the morphism $\phi_{|nH-mE|} : \tilde{\mathbb{P}}^r \to \mathbb{P}^N$ is birational with $N = r$?
In joint work with Bruce Crauder, a complete classification is given if $\dim V \leq 2$:

-$\dim V = 1, V \subset \mathbf{P}^3, \deg(V) = 6$, genus$(V) = 6$, V artihmetically Cohen–Macaulay, $n = 3, m = 1$ (classical cubo-cubic transformation)
-$V \subset \mathbb{P}^4$ is an elliptic normal curve, $n = 2, m = 1$
-$V \subset \mathbb{P}^4$ is a quintic elliptic scroll, $n = 3, m = 1$
-$V \subset \mathbb{P}^4$ is the zero locus of the maximal minors of a general 4×5 matrix of linear forms, $n = 4, m = 1$
-$V \subset \mathbb{P}^5$ is the Veronese surface, $n = 2, m = 1$
-$V \subset \mathbf{P}^6$ is ruled over an elliptic curve with invariant $e = -1$, embedded via $|C_0 + 3f|$ where C_0 is a section with $C_0^2 = 1$, and f is a fiber, $n = 2, m = 1$
-$V \subset \mathbb{P}^6$ is rational, the blow up of $\mathbf{P}^2$ at 8 general points, embedded via $|4H - \sum_{i=1}^{8} E_i|$ where E_i are the exceptional divisors, $n = 2, m = 1$.

It is also proven that if Hartshorne's conjecture on complete intersections is true, then $m = 1$.

F.O. Schreyer:

Construction of surfaces in $\mathbb{P}^4$

Only finitely many components of the Hilbert scheme of smooth surfaces in $\mathbb{P}^4$ contain surfaces which are not of general type by a theorem of Ellingsrud and Peskine. This poses the problem to find these components.
One construction method of surfaces is given by the Eagon–Northcott complex, i.e. we define $S = \{rk\,\phi < rk\,\mathcal{F}\}$ where $\phi : \mathcal{F} \to \mathcal{G}$ is a morphism between appropriate vector bundles $\mathcal{F}, \mathcal{G}$ on $\mathbb{P}^4$ with $rk\,\mathcal{F} = rk\,\mathcal{G} - 1$. We illustrate the strength of this method by constructing all known families of surfaces of degree $d \leq 10$, including Enriques surfaces of degree 10 and sectional genus $\pi = 8$ and a second family of rational surfaces with $d = 10, \pi = 9$ (the two cases whose existence remained open in a recent paper of Ranestad). Except for the abelian and bielliptic surfaces of degree 10 suitable bundles $\mathcal{F}$ and $\mathcal{G}$ can be found easily from the Beilinson spectral sequence applied to $\mathcal{J}_s(4)$. The method seem's to be still powerful for larger degree, e.g. we get a rational surface with $d = 11, \pi = 11$.

A. Beauville:

Jacobians of spectral curves and completely integrable Hamiltonian systems

In Tata lectures on Theta II, Mumford describes an explicit construction of the Jacobian of an hyperelliptic curve (originally due to Jacobi), and uses it to show that a classical completely integrable Hamiltonian system, the so–called Neumann system, can be linearized on these Jacobians.
The Jacobi–Mumford approach is computational. I propose a simple geometric interpretation of this construction, which moreover extends to a more general class of curves, the so–called spectral curves (by this I mean a covering $C_P \to \mathbb{P}^1$ defined by an equation of the type $P(y,x) = y^r + s_1 i(x) y^{r-1} + \ldots + s_r(x) = 0$, with $\deg s_i \le id$ for some fixed integer d). This gives a concrete description of the Jacobian $J(C_P)$ in terms of polynomial matrices whose characteristic polynomial is equal to P, which coincides with the Jacobi–Mumford construction in the case $r = 2$. Then I compute the holomorphic vector fields on this Jacobian using this description. By varying the polynomial P, I obtain a completely integrable system which generializes the Neumann system. The Hamiltonian flows are given by Lax–type equations

$$\frac{\mathrm{d}}{\mathrm{d}t} A(x) = [A(x), \frac{A^i(a)}{x-a}]$$

for $a \in \mathbb{P}^1$ and $1 \le i < r$.

C. Peters:

Generalizations of Castelnuovo–De Franchis' Theorem

The generalizations alluded to in the title are the following:

Theorem 1: (M. Green) Let X be a compact n-dimensional Kähler manifold, $\beta \in H^0(\Omega_X^k)$, $\omega \in H^0(\Omega_X^1)$ with $\beta \wedge \omega = 0$. Then either $\beta = \gamma \wedge \omega$, $\gamma \in H^0(\Omega_X^{k-1})$ or there exists a morphism $f : X \to Y$, $\dim Y \le k$ s.t. $\omega = f^*\tilde{\omega}, \tilde{\omega} \in H^0(\Omega_Y^1)$ and if $\dim Y = k$ then $\beta = f^*\tilde{\beta}, \tilde{\beta} \in H^0(\Omega_Y)$.

Theorem 2: (C.Peters, M.Green, F.Catanese) Let X be as in Theorem 1, and $\omega_1, \ldots, \omega_{k+1} \in H^0(\Omega_X^1)$ linearly independent s.t. $\omega_1 \wedge \ldots \wedge \omega^{k+1} \equiv 0$. Then there exists a torus T and a holomorphic map $f : X \to T$ s.t. $\dim f(X) \le k$ and $\tilde{\omega}_i \in H^0(\Omega_T^1)$ s.t. $f^*\tilde{\omega}_i = \omega_i$.

Both theorems specialize to Castelnuovo–De Franchis' Theorem if $n = 2, k = 1$.
There is a topological characterization for the existence of non–trivial maps $f : X \to T$, T a torus, $\dim f(X) = k$ (k optimal) which can be proved as a consequence of theorem 2.

J. Spandaw:

Picard numbers of linked surfaces in $\mathbb{P}^4$

The following theorem is an analogue of a classical theorem of Noether, Lefschetz and Deligne.

Theorem: Let $S \subset \mathbb{P}^4$ be a smooth, irreducible, nondegenerate surface of degree d. Suppose S' is linked to S via two sufficiently general hypersurfaces of degrees $m' \geq m \geq d$. If $2m + m' \geq 5d + 1$ then $\mathrm{Pic}(\mathbb{P}^4) \to \mathrm{Pic}(S')$ is an isomorphism.

This is almost a special case of a theorem of L.Ein about the Picard groups of certain degeneracy loci of general linear subspaces of the vectorspace of global sections of sufficiently ample vector bundles on projective manifolds.
I showed that his arguments were still valid in our special case and obtained the explicit bounds on m, m' with the help of a theorem of R.Lazarsfeld on the regularity of the idealsheaf of surfaces in $\mathbb{P}^4$.

J. Winkelmann:

Automorphisms of complements of analytic subsets in $\mathbb{C}^n$

W.Kaup has shown that $X = \mathbb{C}^2 \setminus (\mathbb{Z} \times \{0\})$ is homogeneous in the sense that $\mathrm{Aut}_{\mathcal{O}}(X)$ acts transitively on X, although no real Liegroup acts transitively on X. We show that there are more such examples. In particular the same statement is true for $X = \mathbb{C}^n \setminus A$ with A algebraic and $\mathrm{codim}(A) \geq 2$ and for $X = \mathbb{C}^2 \setminus \{zw = 1\}$.
Furthermore we describe obstructions for homogenity: Assume A is analytic in $\mathbb{C}^n$. Let $X = \mathbb{C}^n \setminus A$ and assume that $\mathrm{Aut}_{\mathcal{O}}(X)$ acts transitively on X, and let $f : X \to Y$ be a holomorphic map to a manifold Y which is hyperbolic modulo an anlytic subset $S \subset Y$. Then f is constant or $f(X) \subset S$.

Consequences: 1. If A is a union of hyperplanes, then A is homogeneous if and only if A is the union of at most $n = \dim_{\mathbb{C}} X$ hyperplanes in general position.

2. If f is a non-constant holomorphic function on $\mathbb{C}^n$ and $X = \mathbb{C}^n \setminus \{f = 0, 1\}$ then X is not homogeneous.

S. Bauer:

Parabolic rank-2 bundles on $\mathbb{P}^1$

Parabolic structures on bundles over a curve C essentially describe bundles with vanishing Chern classes on elliptic surfaces X over C. If the Euler characteristic $\chi(\mathcal{O}_X)$ is nonzero, then the notions of (semi-) stability of parabolic bundles on C and of bundles with vanishing Chern classes actually coincide.
This can be used to compute the moduli spaces of semistable bundles with parabolic structures on the projective line. One starts with a projective space $\mathbb{P}^n$ where C is naturally embedded as a rational normal curve. Any component $M(\alpha)$, with fixed weights α, will be obtained by a sequence of blowing up embedded projective spaces $\mathbb{P}^l$ and subsequently blowing down the exceptional divisor ($\cong \mathbb{P}^l \times \mathbb{P}^{n-l-1}$) onto the second factor. The embedded $\mathbb{P}^l$ are strict transforms of joins $\mathrm{Sec}(C) * p_1 * \ldots * p_s$. Here $\{p_i\} \subset I$ is contained in the set of points I where the parabolic structure is concentrated.

C. Viosin:

On the Chow ring of the surface of conics of a Fano threefold

Let X be a Fano threefold. We assume that the variety of conics of X is a smooth surface Σ. Let $\Phi : \mathrm{Alb}(\Sigma) \to JX$ be the Abel–Jacobi map. We study the following questions:

i) Is Φ an isomorphism?
ii) Is the following diagramm commutative:

$$\begin{array}{cccc} \Lambda^2 D: & \Lambda^2 H^1(\Sigma,\mathbf{Z})^* & \longrightarrow & \Lambda^2 H^1(\Sigma,\mathbf{Z}) \\ & \uparrow & & \uparrow \\ & H^2(\Sigma,\mathbf{Z})^* & \longrightarrow & H^2(\Sigma,\mathbf{Z}) \end{array}$$

where $D : H^1(\Sigma,\mathbf{Z})^* \to H^1(\Sigma,\mathbf{Z})$ is the duality map given by the pull-back of the intersection form on $H^3(X,\mathbf{Z})$.

For this we introduce two incidence relations on Σ:

a) $D : CH_0(\Sigma) \to \mathrm{Pic}(\Sigma)$, s.t. for $C \in \Sigma$, $D(C)$ is the curve of conics touching C.
b) $I : CH_0(\Sigma) \to CH_0(\Sigma)$, s.t. for $C \in \Sigma, I(C)$ is the zero–cycle of conics touching C twice.
We prove then the following relation: There exist constants $R \in \mathrm{Pic}(\Sigma), K \in CH_0(\Sigma)$ s.t. for all $C \in \Sigma$, $2(C + I(C)) = D_C^2 - RD_C + K$ in $CH_0(\Sigma)$. From this we deduce that Φ has finite kernel and the answer to ii) is positive, when I is empty, which is the case for many Fano threefolds.

T. Peternell:

Ample vector bundles on Fano manifolds

The following two theorems are discussed:

Theorem 1: Let X be a projective manifold, $n = \dim X$, E an ample vector bundle of rank $n+1$ on X with $c_1(E) = c_1(X)$. Then $X \simeq \mathbf{P}^n$ and $E = \mathcal{O}(1)^{n+1}$.

Theorem 2: Let X be a projective manifold, $n = \dim X$, E an ample vector bundle of rank n on X with $c_1(E) = c_1(X)$. Then

$$(X,E) \simeq \begin{cases} (\mathbf{P}^n, \mathcal{O}(2) \oplus \mathcal{O}(1)^{n-1}) \\ (\mathbf{P}^n, T_{P^n}) \\ (Q_n, \mathcal{O}_{Q_n}(1)) \end{cases}$$

Here Q_n is the n–dimensional smooth quadric.

F. Bardelli:

Intermediate Jacobians and their maximal Hodge subtorus for some special threefolds

We let $J_M(X)$ be the maximal Hodge subtorus of the intermediate Jacobian of a smooth projective threefold X. Let $\pi : \mathcal{X} \to U$ be a family of threefolds, smooth and with trivial canonical class $K_{X_t} = 0, \pi^{-1}(t) = X_t$. Then, if the family π depends on d moduli, we prove that at the general point $t \in U$ we have $\dim J_M(X_t) \leq \dim J(X) - d - 1$. Furthermore if there is a finite group G acting freely on the fibers of π and $Y_t = X_t/G$, $p : X_t \to Y_t$ the natural quotient map, $\tilde{p} : J(X_t) \to J(Y_t)$ the induced map of Jacobians and if we assume $\pi_+(X_t) = 0$ we have that for a general $t \in U$

$$J_M(X_t) = (\,\mathrm{Ker}\,\tilde{p}\,)^0$$

We apply the results above to compute $J_M(X_t)$ for the family of complete intersections of four quadrics in $\mathbf{P}^7$ admitting a fixed point free involution σ. In this case we explicitly construct a family of algebraic cycles that parametrizes $J_M(X_t)$ for t general, thus checking the generalized Hodge conjecture for the general threefold of this family.

C. Peskine:

Decomposition of the normal bundle of smooth surfaces in $\mathbf{P}^4$

We prove (with B.Basili) the following results:

Theorem 1: Let S be a smooth surface in $\mathbb{P}^4$. If the normal bundle N_S of S splits then S is a complete intersection.

Theorem 2: Let V be a smooth threefold in $\mathbb{P}^5$. If there exists a locally complete intersection double structure on V, then this double structure is a zero section of a rank two bundle on $\mathbf{P}^5$; furthermore if this rank two bundle is decomposable then V is a complete intersection.

Corollary: If a smooth projectively Cohen–Macaulay solid of $\mathbf{P}^5$ has a locally complete intersection double structure it is a complete intersection.

Main lemmas for the proof of theorem 1:
Lemma 1: Assume $N_S = \mathcal{L}_1 \oplus \mathcal{L}_2$. Let S_1, S_2 be the double structures induced by $\mathcal{L}_1$ and $\mathcal{L}_2$. If $\mathcal{L}$ is a line bundle on S, the following conditions are equivalent:
1) The classes of $\mathcal{L}$ and H (the hyperplane section) are dependent.
2) There exists n s.t. the class of $\mathcal{L}^n$ in $Num(S)$ lifts to $Num(S_1)$ and $Num(S_2)$.

Key Lemma: $o_S^2(5)$ lifts to S_1 and S_2 (and is therefore numerically dependent of H).

Main lemma for the proof of theorem 2:

Lemma 2: If V_1 is a locally complete intersection double structure on V then a line bundle lifts to V_1 if and only if it is a multiple of H.

C.T.C. Wall:

Singular Del Pezzo surfaces

These surfaces X have a long history and are well understood. One can construct a moduli space for pairs (X, E) with E a smooth hyperplane section (elliptic): It consists of $\mathrm{Hom}(L, Jac\,E)$, where $L = K_X^{\perp} \subset \mathrm{Pic}X$. If $R \subset L$ is the set of roots and for $\Phi : L \to Jac\,E$, $R_0 = R \cap \mathrm{Ker}\,\Phi$, the root system R_0 determines the singularities on X_Φ. From this one obtains an algorithm: R_0 is obtained from R by ≤ 9 "elementary transformations".

Write L_0 for the sublattice spanned by R_0, $T = \text{Tors}(L/L_0), R_1 = R \cap (\mathbb{Q} \otimes L_0)$, $L_1 = L \cap (\mathbb{Q} \otimes L_0)$. Then R_1 spans L_1. R_0 is obtained by $\leq g$ elementary transformations if and only if T admits g generators.
Individual surfaces X_Φ have interesting geometry. For $n = 5$, X_Φ is the intersection of two quadrics in $\mathbb{P}^4$ and our list of cases corresponds bijectively to Segre symbols of pencils. For $n = 6$ we have cubics in $\mathbf{P}^3$ and interest centres on the geometry of the 27 lines, which are listed by images in $\text{Pic}X_\Phi = \text{Pic}\tilde{X}/L_0$ of classes ξ with $K\xi = \xi^2 = -1$. For $n = 7$, X is a double cover of $\mathbf{P}^2$ branched over a reduced quartic Γ. We find that Γ is a union of 2 conics if and only if $\mathbf{Z}_2 \subset T$. "Lines" give bitangents to Γ. For $n = 8$, X is a double cover of a quadric cone C in $\mathbf{P}^3$, branched over a sextic Γ. The above have anlogues here; also, blowing up the vertex gives an elliptic surface. T is the group of torsion sections. The cases A_2^4, A_4^2 give universal elliptic curves with certain level structures.

C. Birkenhake:

Cubic theta relations

Let X be an abelian variety over $\mathbb{C}$ and L an ample line bundle on X. Then $L^n, n \geq 3$ embeds X into $\mathbf{P}^N$ with image $\bar{X}$. According to a theorem of Kempf the ideal $I_{\bar{X}}$ of $\bar{X}$ is generated by quadrics if $n \geq 4$, and by quadrics and cubics if $n \geq 3$. If n is even and $n \geq 4$, the qudratic equations describing $\bar{X}$ are the well-known Riemann theta relations. In the case that $n = 3m, m \geq 1$, explicite cubic theta relations were presented, which describe $\bar{X}$ completely.

A.T. Huckleberry:

$|\text{Aut}(X)| \leq c_2^{11}$ for X a surface of general type

Theorem: (A.T.Huckleberry, M.Sauer) There is a constant k s.t., for X a surface of general type, $|\text{Aut}(X)| \leq kc_2^{11}$

Remarks: 1) we actually prove $|\text{Aut}(X)| \leq k\,(\log c_2)\, c_2^{\frac{32}{3}}$. The log–part occurs for a real reason, but the $\frac{32}{3}$ can certainly be lowered by looking more closely at the structure of certain finite groups.
2) The constant k arising in the proof is roughly 10^{45}. Since the result is certainly not sharp, i.e. the exponent, we did not worry too much about minimizing the constant.
3) At a recent conference in Göttingen, Catanese explained to us geometric methods (completely different from ours) which also give a polynomial estimate $\approx c_2^{13}$. This will appear in a paper of Corti.

Idea of the proof: The only geometric ingredient is the

Centralizer Theorem: Let A be an abelian group in Aut (X) and suppose that Fix $(A) \neq \emptyset$. Let $Z(A)$ be the centralizer of A in Aut (X). Then $|Z(A)| \leq c_2^4$.

The proof of the estimate for Aut (X) goes by applying the centralizer theorem to various group–theoretic situations. The case which requires the most work is where we must estimate $|E|$ for E a simple group of Lie type contained in Aut (X).

Remark: After the lecture Beauville called our attention to a result on abelian subgroups of Aut (X) ($|A| \leq c_2$). Using this, our results now read: $|Z(A)| \leq c_2^3$ and Aut $(X) \leq (\log c_2)c_2^8$.

M. Mulase:

Infinite dimensional Grassmannians, vector bundles on curves and commuting differential operators

The study of commutative algebras consisting of ordinary differential operators has a long history in mathematics; it started in 1903 by Wallenberg. Since then there have been many contributions by various mathematicians including Schur (1905), Burchnall–Chaundy (1923), Krichever (1977), Mumford (1978) and Verdier (1979). In this talk, I gave a complete geometric classification of all commutative algebras of ordinary differential operators. Namely, let $\mathcal{B}$ be the category of all such algebras amd $\mathcal{G}$ be the category whose objects are quintets $(C, p, \mathcal{I}, \pi, \Phi)$ consisting of an arbitrary algebraic curve C , a point p on it, a torsion free sheaf of any rank satisfying $h^0(\mathcal{I}) = h^1(\mathcal{I}) = 0$, a local covering π defined near p and a local trivialization Φ of $\mathcal{I}$ around p. Then there is a natural functor between $\mathcal{B}$ and $\mathcal{G}$ which makes them categorically equivalent. The proof of this theorem involves the theory of pseudo–differential operators and infinite dimensional Grassmannians.

W. Hoyt:

Neron–Severi groups and intersection products for twisted Legendre surfaces

Let $W = \{(a,b,c) \in \mathbb{C}^3$ with disticnt $a,b,c \neq 0,1\}$, and let X be the minimal resolution for $(*)$ $\quad y^2 = (t-a)(t-b)(t-c)x(x-1)(x-t)$ with $(a,b,c) \in W$.

Problem: Determine the Neron–Severi group $NS(X)$ for various $(a,b,c) \in W$.
Partial results: 1)X is a K3 surface with an elliptic fibering over the t–sphere and with Picard number and Mordell–Weil rank related by $\rho = r + 17 = 17, \ldots, 20$.
2) There are countably many irreducible surfaces $W_i \subset W$ s.t. $\rho > 17$ if and only if $(a,b,c) \in W_i$ for some i. The W_i can be defined as components of algebraic sets consisting of (a,b,c) for which $(*)$ has a $\mathbb{C}(t)$–rational solution (x,y) of height $\leq$ various n.
3) Several W_i and also some components of $W_i \cap W_j$ can be described by configurations of six lines with suitable extra contact with rational curves of degrees ≤ 3; self intersections and intersection products can be computed for algebraic cycles in some cases
4) The K3 lattice Λ is spanned over $\mathbb{Q}$ by representatives relative to suitable isomorphisms $H^2(X,\mathbb{Z}) \simeq \Lambda$, of sections and components of fibers for at most five different choices of X. This may make it possible to determine all W_i by repeatedly applying chord and tangent operations to pairs of sections correspondig to $W_i \cap W_j$.
5) There are rational maps $A \to X$ from abelian surfaces A which induce i) isogenies of transcendental lattices, ii) an algebraic correspondence between W and the Siegel modular threefold, iii) algebraic correspondences between the W_i and generalized Hilbert modular surfaces.

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M.Schneider:

On two conjectures of Hartshorne's

The object of the talk was to report on joint work with D. Bartlet and T. Peternell concerning the following two conjectures of Hartshorne's:

C1: Let Z be a smooth projective variety and let $X \subset Z$ be a submanifold with ample normal bundle. Then kX moves in a large family, $k \gg 0$.

C2: Let Z be a smooth projective variety and let $X, Y \subset Z$ be submanifolds with ample normal bundle. Then X and Y intersect if $\dim X + \dim Y \geq \dim Z$.

C1 is false in general, but we have:

Theorem 1: Let C be a curve and let E be an ample 2–bundle on C. Then kC moves in a family covering $\mathbb{P}(\mathcal{O} \oplus E^\vee)$, $k \gg 0$.

This is used to prove the following

Theorem 2: Let Z be a $\mathbf{P}^2$–bundle over a surface. Let $X, Y \subset Z$ be submanifolds whose normal bundles are positive (in the sense of Griffiths). Then X and Y intersect provided $\dim X + \dim Y \geq 4$.

U. Persson:

Torsion groups of elliptic surfaces

Consider a Jacobian elliptic surface $X \to C$ with non–constant j–invariant. If T denotes the torsion of the Mordell–Weil group, and r its rank we are interested in bounding the possibilities of T in terms of the genus g of C, the Euler characteristic e of X and r.
In a joint work with H.P. Miranda we define a Size of abelian groups with at most two generators and prove the formula

$$\mathrm{Size}\,(T) \geq \frac{1}{6} + \frac{R}{e} - \frac{2(g-1)}{e},$$

where

$$\mathrm{Size}\,(\mathbf{Z}_N \times \mathbf{Z}_M) = \frac{1}{N} \prod_{p|N} (1 + \exp_p(\frac{N}{M}) \frac{p-1}{p+1}).$$

The idea is just to consider the translation of a section, to note its fixed points (nodes of certain singular fibers) and make an elementary Euler computation (If the section has order p, then there are $\frac{e}{p+1}$ fixed points) and to combine this with a lower estimate of the number of singular fibers.
The formula gives exactly the 19 possible torsion groups determined by Cox and Parry, over C rational; all possible information on Mordell–Weil groups for rational and K3 surfaces determined by Cox, and the bound on the order of torsion groups for large e as obtained by Hindrey and Silverman.

C. Schoen:

Detecting non–trivial elements of Chow groups by reduction mod p

Joint work with Jaap Top extending Bloch, Crelle vol. 350

1) Survey CH^r, CH^r_{hom} and Beilinson–Bloch conjecture: $[k:\mathbb{Q}] < \infty$, $\operatorname{rank} CH^r_{hom}(W) = \operatorname{ord}_{s=r} L_k(H^{2r-1}(W), s)$
2) In order to check the conjecture – for very little evidence presently exists – one needs a method to show that certain null–hom. cycles are not rational equivalent to 0. We take for W the motive $\mathrm{Sym}^3 H^1(E)$, E elliptic curve without complex multipication. For the cycle we take $C - i_* C$, $C \subset E^3$ a certain genus 3 curve. We extend the methods of Bloch to this situation. The main point is that $\mathrm{Sym}^3 H^1(E)$ splits off an "algebraic" piece when End $\neq \mathbf{Z}$. This is always the case when the base field is finite. For a projector $Q : H^3(E^3) \to \mathrm{Sym}^3 H^1(E)$ the class of $Q(C - i_* C) \in H^1_{cont}(G_k, \mathrm{Sym}^3 H^1(E))$ is studied by reduction mod p. The class $Q(C - i_* C) \in H^1_{cont}(G_{\mathbf{F}_p}, \mathrm{Sym}^3 H^1(E))$ can be shown to be non-zero in favorable instances by classical geometric techniques. The class will in most instances have infinite order since $H^1_{cont}(G_k, \mathrm{Sym}^3 H^1(E, \mathbf{Z}))$ is usually torsion free.

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