MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topologische Methoden in der Gruppentheorie

1. 10. bis 7. 10. 1989

Die Tagung fand unter der Leitung von R. Bieri (Frankfurt), K. S. Brown (Ithaca) und K. W. Gruenberg (London) statt. Inhaltlich knüpfte sie an die gleichnamige Tagung im Juni 1986 an. In der Zwischenzeit haben die topologischen - und allgemeiner, geometrischen - Methoden in der Gruppentheorie weiterhin kräftig zugelegt. Richtungsweisender Antrieb war u. a. das Werk von Gromov über Hyperbolizität. Etliche der Tagungsteilnehmer hatten sich im Rahmen des Programms "Combinatorial group theory and geometry" am Mathematical Science Research Institute in Berkeley mit der Gromov'schen Theorie und deren Verallgemeinerung auseinandergesetzt.

Dies kam auch in den 28 Vorträgen zum Ausdruck. Auf der methodischen Seite stand die Operation von Gruppen auf CW-Komplexen, Gebäuden, $\mathbb{R}$-Bäumen etc. im Vordergrund. Die behandelten Gegenstände lassen sich den folgenden Stichworten zuordnen: <u>Hyperbolizität</u> (hyperbolische und automatische Gruppen, $\mathbb{R}$-Bäume und non-standard Analysis, etc.) <u>Endlichkeitsbedingungen</u> (Arithmetische Gruppen, endliche Präsentierbarkeit, PL-Homöomorphismengruppen, Automorphismengruppen, geometrische Invariante) <u>Endentheorie</u>, <u>Gruppencohomologie</u>, <u>homologische Aspekte der Darstellungstheorie</u> und <u>Gruppentheoretische Methoden in der Topologie</u>.

Der letzte Halbtag wurde dazu genutzt, Vermutungen und offene Fragen vorzustellen. Diese sind - soweit sie von den Problemstellern ausformuliert wurden - im Anhang zu den Vortragsauszügen zusammengestellt.

Vortragsauszüge

H. ABELS

Finiteness conditions for arithmetic groups

For locally compact topological groups a sequence of compactness conditions C_n, $n = 1, 2 \dots$ is defined with the following properties:

C_1 holds iff the group has a compact set of generators,

C_2 holds iff the group has a compact presentation.

The following theorem generalizes a theorem of Kneser (1964). Let G be a linear algebraic group defined over a number field k and let S be a finite set of prime divisors of k. Let Γ be the corresponding S-arithmetic group.

Theorem Given n, Γ has the finiteness property F_n (i.e. there is a $K(\Gamma, 1)$ with finite n-skeleton) iff $G_{k_\mathfrak{p}}$ is C_n for every $\mathfrak{p} \in S$.

This theorem may be useful in solving the problem which S-arithmetic groups (for non-reductive G) are F_n.

P. ABRAMENKO

Finiteness properties of $SL_n(\mathbb{F}_q[t])$, $Sp_{2n}(\mathbb{F}_q[t])$ and related arithmetic groups

If Γ is a group, denote by $\Phi(\Gamma)$ the "finiteness length" $\Phi(\Gamma) := \max\{m \in \mathbb{N}_0 \mid \Gamma$ is of type $F_m\}$. For the groups $\Gamma = SL_n(\mathbb{F}_q[t])$ $(n \geq 2)$ the following result was proved by Abels and Abramenko:

$$\Phi\left(SL_n(\mathbb{F}_q[t])\right) = n - 2 \quad \text{if } q \geq \begin{cases} 2^{n-2} & \text{(Abels)} \\ \max\limits_{k=0}^{n-2} \binom{n-2}{k} & \text{(Abramenko)} \end{cases}$$

Conjecture: For every Chevalley group G of rank r there exists a function $f_G : \mathbb{N} \to \mathbb{N}$ such that $\Phi\left(G(\mathbb{F}_q[t])\right) = r - 1$ if $q \geq f_G(r)$.

Having the methods of the proof of the above statement in mind it seems to be likely that the conjecture can be deduced once the following problem is solved:

Question: Let (G,B,N,S) be a Tits system with finite Weyl group $W = \langle S \rangle$, Δ the associated building with standard chamber C_0, standard apartment A_0 and $\bar{C}_0 \subseteq A_0$ the unique chamber opposite to C_0. Denote by P a standard parabolic subgroup $B \subseteq P \subseteq G$. When is the subcomplex $P \cdot \bar{C}_0 \subseteq \Delta$ homotopy equivalent to a bouquet of spheres of dimension $|S| - 1$?

Example: For $G = Sp_{2n}(\mathbb{F}_q)$, $n \geq 3$ and $q \geq 13$ all subcomplexes of type $P \cdot \bar{C}_0 \subseteq \Delta$ are 1-connected (so they are 2-spherical if $n = 3$). This probably implies that in these cases the groups $Sp_{2n}(\mathbb{F}_q[t])$ are finitely presented.

A. ADEM

Exponents in the high dimensional cohomology of discrete groups.

I will discuss how the finite subgroups of a discrete group Γ of finite v.c.d. are related to the torsion in $\hat{H}^*(\Gamma)$. We have a cohomological bound:

Theorem If $\Gamma' \subset \Gamma$ is of finite c.d. and normal in Γ, then there exist classes $y_1, \ldots, y_{r(\Gamma)} \in H^*(\Gamma/\Gamma', \mathbb{Z})$ such that

$$\exp \hat{H}^*(\Gamma) \,\Big|\, \prod_{i=1}^{r(\Gamma)} \exp y_i \quad \text{where } r(\Gamma) = \max_{\substack{p \mid |G| \\ H \subset \Gamma \text{ finite}}} \{p\text{-rank of } H\} .$$

The proof shows that torsion in $\hat{H}^*(\Gamma)$ is related to the finite subgroups of Γ in a much more subtle way than previously thought. In fact we have:

Theorem There exist groups Γ of finite v.c.d. with torsion higher than

$$m = \text{l.c.m.} \{ |H| \mid H \subseteq \Gamma \text{ is finite} \}.$$

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R. BIERI

Action on ended Archimedean trees.

This grew out of joint work with Ralph Strebel. By an ended tree we mean a semi-lattice T whose segments $[a, b] = \{x \in T \mid a \le x \le b\}$ are totally ordered. T is Archimedean if each segment $[a, b]$ is given a length $d(a,b) \ge 0$ which is additive and $= 0$ only when $a = b$. (T is essentially an $\mathbb{R}$-tree with a specified end. For every group action $\tau: G \to \mathrm{Aut}(T)$ there is an associated homomorphism $\chi_\tau: G \to \mathbb{R}_{add}$ by choosing $a \in T$ arbitrary and putting
$\chi_\tau(g) = d(a \wedge ga, ga) - d(a \wedge ga, a)$.
Let G be a group and $\chi: G \to \mathbb{R}_{add}$ a homomorphism. Given a simplicial G-graph Γ with a function $v: \mathrm{edg}\,\Gamma \to \mathbb{R}$ satisfying $v(ge) = \chi(g) + v(e)$, we give a simple construction of an ended Archimedean G-tree $T = T(G,v)$ with $\chi_T = \chi$. In the case when Γ is the Cayley graph we recover Ken Brown's description of the geometric invariant $\Sigma(G)$ in terms of $\mathbb{R}$-tree actions (Invent. math. 90 (1987), p. 478). In the case when Γ is a tree we find a description of $\Sigma(G)$ in terms of the Σ-invariants of the stabilizers.

R. BOLTJE

Induction formulae for Mackey functors, in particular for the character ring of a finite group, and their applications in number theory.

In 1946 R. Brauer proved the following existence theorem:
Theorem Each virtual character χ of a finite group G can be expressed as

$$(*)\quad \chi = \sum_i z_i \,\mathrm{ind}_{H_i}^G \varphi_i$$

where $z_i \in \mathbb{Z}$, $H_i \le G$, φ_i a one-dimensional character of H_i.
In 1986 V. Snaith gave an explicit formula (*) for any actual character (coming from a representation) of G. He used topological methods and determined the z_i's in terms of Euler characteristics of certain spaces coming from the group

of unitary matrices by taking suitable quotients.

Independently the author came up with a formula (*) for any virtual character by an algebraic description.

The question whether these two formulas coincide (at least) on irreducible characters of G led to a more general approach by the common language of Mackey functors which covers both formulas and shows exactly the difference and the relation between both.

C. J. B. BROOKES

Finite presentability and coadjoint orbits of Lie groups.

The aim is to indicate how to generalize the Bieri-Neumann-Strebel invariant Σ using maps of a finitely generated group to Lie groups other than $\mathbb{R}$. The unitary representation theory provides the source of inspiration. Left invariant differential 1-forms on a Lie group correspond to elements of the cotangent space at the identity and in representation theory it is common practice to consider coadjoint orbits in the cotangent space. The easiest non-abelian example of a Lie group is the Heisenberg group H of upper unitriangular real 3×3 matrices; the orbits are simple to describe geometrically and they parametrize its unitary dual. A condition analogous to the one for Σ concerning antipodal points proves to be necessary for the finite presentability of a finitely generated group containing no non-abelian free subgroups. There remains the problem of calculation.

K.S. BROWN

Rewriting systems and homology

Let G be a group or monoid which is presented by means of a complete rewriting system. Then one can use the resulting normal forms to "collapse" the classifying space of G to a quotient complex of the same homotopy type. This leads to an explicit free resolution of $\mathbb{Z}$ over $\mathbb{Z}G$, similar to one obtained by Anick, Groves, and Squier. If the rewriting system is finite, then the resolution has only finitely many generators in each dimension.

J. F. CARLSON

Projective resolutions and Poincaré duality complexes

Joint work with David Benson.

Let k be a field of characteristic $p > 0$ and let G be a finite group of p-rank n. We investigate the relationship between the structure of the cohomology ring $H^*(G,k)$ and the spectral sequences associated to certain Poincaré duality complexes. Each complex is determined by a set $\zeta_1, \ldots, \zeta_n$ of homogeneous elements such that $H^*(G,k)$ is a finitely generated module over the subring generated by the elements. Particularly nice results are obtained when the cohomology ring is Cohen-Macaulay. Evidence suggests several interesting constructions in the general case.

R. CHARNEY

An application of Gromov's work to Coxeter groups

We consider piecewise flat metrics on a manifold M. If these have non-positive curvature (in the sense of Gromov) then M is aspherical. If, in addition, M is compact, then $\pi_1(M)$ is hyperbolic. To determine whether curvature (M) ≤ 0, it suffices to check whether links of points in M (which topologically are spheres, but metrically may be quite strange) are "large". The link is said to be "large" if it has curvature ≤ 1 and all closed geodesics have length $\geq 2\pi$. Interesting examples occur when M is a branched cover of a Riemannian manifold, in which case the links in M are branched covers of the standard sphere S^n.

Motivated by this, we consider metric spheres $\tilde{S}^n$, acted on by a group of isometries G, such that $\tilde{S}^n/G$ is the standard sphere S^n. (Example: G = index 2 subgroup in a finite reflection group). For n = 2 and 3, we give precise criteria for when $\tilde{S}^n$ is large.

I. CHISWELL

$\mathbb{R}$-trees and degeneration of hyperbolic space.

Let G be a finitely generated group, $\mathfrak{H}(G)$ the space of discrete and faithful representations $G \to \mathrm{Isom}_+(\mathbb{H}^n)$, where $\mathbb{H}^n$ is n-dimensional real hyperbolic space (and the "+" denotes orientation-preserving isometries), up to conjugation. Morgan and Shalen have shown that $\mathfrak{H}(G)$ has a compactification in which the ideal points are given by actions of G on $\mathbb{R}$-trees. A version of this result has been given by Bestvina, using Gromov's construction of a limit of a sequence of metric spaces. By generalizing the van den Dries /Wilkie version of Gromov's construction, using non-standard models of the reals, one can simplify part of Bestvina's argument.

F. T. FARRELL

Topological versus smooth rigidity

Mostow showed that any two compact hyperbolic manifolds with isomorphic fundamental groups are diffeomorphic. For dimension > 2 he constructed an isometry. Together with the work on harmonic maps by Eells and Sampson and by Schoen and Yau and the stable results of Cheeger and Gromov, this led to the following conjecture:

Lawson-Yau Conjecture: Any two compact negatively curved manifolds with isomorphic fundamental groups are diffeomorphic.

Lowell Jones and I prove that they are homeomorphic provided their dimension $\neq 3$ and 4. We also construct counterexamples showing they are not always diffeomorphic.

R. GEOGHEGAN

Higher order end structure of groups

The "higher order end structure" of a f. p. infinite group G [e.g. the structure of $H^n(G,\mathbb{Z}G)$, the question of whether G is semistable etc.] is known to be related to shape theory. One believes (and sometimes one knows) that a compactifying set can be added to the universal cover of a nice K(G,1) whose algebraic topology is a model for the end structure of G.

Three new theorems in this area will be described. I will talk about how to translate these theorems from the language of topology to the language of group theory (or vice versa).

P. GREENBERG

Piecewise $SL_2\mathbb{Z}$ geometry in the plane

In the past several years, a group G of homeomorphisms of the circle has been studied: in group theory (e.g. Brown- Geoghegan, Brin-Squier) dynamical systems (Ghys-Sergiescu) and in connection with the braid group (Greenberg-Sergiescu). There is a natural "globalization" of G, to a pseudogroup Γ of piecewise linear homeomorphisms of the plane $\mathbb{R}^2$.
To each polygon P with integer corners, the group $\Gamma(P)$ of restrictions of Γ to P, appears to behave like mapping class groups; there is a natural "complex of curves" for example. A rigidity conjecture about the groups $\Gamma(P)$ is supported (very indirectly) by the fact that, roughly, at most rational points the group of germs of Γ (i) has no torsion (ii) has vanishing Euler class (and hence, is "dynamically simple").

C. HOG-ANGELONI

On the homotopy type of 2-complexes with a free product of cyclic groups as fundamental group

Homotopy classification of 2-complexes has been achieved (as far as I know: only) for π_1 finite abelian and for π_1 free.
The algebraic problem amounts to constructing a chain isomorphism as a completion of the following diagram

$$\begin{array}{ccccccc} C_2\tilde{K} & \to & C_1\tilde{K} & \to & C_0\tilde{K} & \to & \mathbb{Z} \\ & & \| & & \| & & \| \\ C_2\tilde{L} & \to & C_1\tilde{L} & \to & C_0\tilde{L} & \to & \mathbb{Z} \end{array}$$

which arises from the chain complexes of universal covering spaces.

In the case π_1 free (of finite rank) and $\chi(K^2) = \chi(L^2)$ such an isomorphism is first established after passing from the integer coefficients of $\mathbb{Z}\pi$ to a field. This result then can be lifted to $\mathbb{Z}\pi$.

In the present talk the first step is generalized to the case of a free product of arbitrary cyclic groups.

J. HOWIE

Decision problems in one-relator products

Joint work with Andrew Duncan.

Let $G = \frac{A * B}{\langle\langle r \rangle\rangle}$ be a one-relator product of two locally indicable groups A, B, where $r \in A * B$ is a proper power (and does not lie in a conjugate of A or B).

If the genus problem (given $\alpha_1, \ldots, \alpha_n$, find the least $g \leq \infty$ for which $X_1\alpha_1X_1^{-1} \ldots X_n\alpha_nX_n^{-1}[Y_1, Z_1] \ldots [Y_g, Z_g] = 1$ has a solution in the group) is solvable for A and for B, then it is solvable for G.

Corollary: The genus problem is solvable for one-relator groups with torsion.

Y. V. KUZ'MIN

Homology of free abelianized extensions

I should like to sum up results on the topic, to outline probable directions of further development and to set up some problems. Much of the material is contained in my recent paper "Homology theory of free abelianized extensions" (Communications in Algebra 16 No 12, 1988, p. 2447-2533) but there are also some new unpublished results.

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M. LUSTIG

Reducible automorphisms of free groups

Recent work of Bestvina and Handel implies that every outer automorphism ψ of a free group F_n of rank n belongs to one of the following three classes:

(1) ψ has finite order,

(2) there is a decomposition $F_n = H_1 * \ldots * H_r * H$ with $\psi(H_k) = H_{k+1}$ (mod r).

(3) there is an action of F_n on some $\mathbb{R}$-tree with small arc stabilizers, such that for the induced translation length function one has

$$\| \psi(w) \| = \lambda \| w \|$$

for all $w \in F_n$ and a fixed "stretching factor" $\lambda > 1$.

This result comes close to a desired and conjectured classification of free group automorphisms in analogy to Thurston's classification of surface homeomorphisms. In this second view a "reducible" $\psi \in \mathrm{Out}\, F_n$ is an automorphism which fixes some graph of groups decomposition of F_n (up to permutation of the building blocks).

<u>Theorem</u>: ψ: F(a, b, c, d) → F(a, b, c, d);

$a \mapsto a\,c\,a^{-1}c\,a\,b\,a,\ b \mapsto a\,c\,a^{-1}c\,a\,b,\ c \mapsto c\,d\,c,\ d \mapsto c\,d$ belongs to the above class (2) but is not reducible in the second sense.

G. MEIGNIEZ

Ends for groups acting on the real line

We associate to every group π endowed with a homomorphism H into $\mathrm{Homeo}_+(\mathbb{R})$, a set of "ends". Formally, this is very like the classical theory of ends of groups. We find back Bieri-Neumann-Strebel property as a particular case. There is also a strong link with the topology of codimension 1 foliations.

W. METZLER

Simple-Homotopy of (π,d)-complexes and the Andrews-Curtis-Conjecture for nontrivial groups

1) Joint work with Cynthia Hog-Angeloni and Paul Latiolais.
The recent distinction of homotopy type and simple-homotopy type of 2-complexes (Metzler, Lustig) carries over to (π, d)-resolutions and thus to the idea of a simple-homotopy theory of groups. Certain Bias-ideals in $\mathbb{Z}(\pi)$ are crucial in this context which, in the case of π finite, are shown to be independent of the particular resolution.
2) For 2-complexes, the next level of distinction is the one between simple-homotopy and Andrews-Curtis-classes. 3 phenomena are presented:
α) (Joint work with C. Hog-Angeloni) A stabilization theorem: After adding suitably many copies of standard presentations of $\mathbb{Z}_2 \times \mathbb{Z}_4$, simply-homotopy equivalent presentations become AC-equivalent β) the difference between sh-type and AC-classes vanishes after reducing the free group of the generators (or: of the relator subgroup) mod arbitrarily high commutators (extension of a result of W. Browning for $\pi = 1$); γ) a series of presentations of $\mathbb{Z}_n \times \mathbb{Z}_n$ is given, which seem to be good candidates to distinguish sh-type and AC-classes.

M. MIHALIK

Ends of amalgamated products and HNN-extensions

Let $\tilde{K}$ be a 2-complex. Proper rays $r, s\colon [0, \infty) \to \tilde{K}$ converge to the same end of $\tilde{K}$ if for every compact $C \subset \tilde{K}$, there is an integer N such that $r([N, \infty))$ and $s([N, \infty))$ are contained in the same path component of $\tilde{K} - C$. $\tilde{K}$ is semistable at ∞ if every pair of proper rays converging to the same end of $\tilde{K}$ are properly homotopic.

If G is a finitely presented group, and K a finite 2-complex with fundamental group G, then the number of ends of G is the number of ends of $\tilde{K}$, the universal cover of K, and G is semistable at ∞ if $\tilde{K}$ is semistable at ∞. (Both notions are independent of the choice of finite complex with fundamental group G.)

Main Theorem: If $G = A *_H B$ is an amalgamated product of finitely presented groups A and B over a finitely generated group H, and A and B are semistable at ∞ then G is semistable at ∞. [If $G = A *_H$ is an HNN-group with A finitely presented and semistable at ∞ and H finitely generated then G is semistable at ∞.]

Theorem: All one relator groups are semistable at ∞.

F. PAULIN

Outer automorphisms of hyperbolic groups

We prove that word hyperbolic groups Γ, defined by M. Gromov, have a finite outer automorphisms group, unless they isometrically act on an $\mathbb{R}$-tree without a global fixed point and with virtually cyclic edge stabilizers. In particular, hyperbolic Kazhdan groups have a finite outer automorphisms group.

The idea is to consider a sequence of automorphisms of Γ, and to look at the sequence of the actions of Γ by left translation twisted by the automorphisms on a Cayley graph X of Γ. If two of the automorphisms are not conjugate, then, eventually after an extraction, the sequence of actions on X suitably normalized converges in a geometric way to an action on an $\mathbb{R}$-tree, with the desired properties.

S. PRIDE

Groups given by presentations in which each defining relator involves at most two types of generators

Our set up will consist of the following: (i) a graph with vertex set V and edge set E; (ii) for each vertex $v \in V$ a non-trivial group G_v given by a presentation $\langle X_v \mid R_v \rangle$; (iii) for each edge $e = \{u,v\} \in E$ a group G_e given by a presentation $\langle X_u, X_v \mid R_e \rangle$, where R_e consists of the elements of $R_u \cup R_v$ together with some further words on $X_u \cup X_v$. We let $G = \langle X \mid R \rangle$, where $X = \bigcup_{v \in V} X_v$, $R = \bigcup_{e \in E} R_e$. Our aim is to try to describe the structure of G in terms of the groups G_v $(v \in V)$, G_e $(e \in E)$. We show that under suitable conditions the natural homomorphisms $G_v \to G$ $(v \in V)$, $G_e \to G$ $(e \in E)$ are injective; and that there is a short exact sequence

$$0 \to \bigoplus_{v \in V} (\mathbb{Z}G \otimes_{G_v} IG_v)^{|\mathrm{Star}(v)|-1} \to \bigoplus_{e \in E} (\mathbb{Z}G \otimes_{G_e} IG_e) \to IG \to 0$$

(where, for any group H, IH is the augmentation ideal). Some (co-)homological consequences of these results are derived.

S. ROSSET

The higher lower central series

There is an infinite sequence of functors $\Gamma_n(G)$, $\Gamma_n(G)$ is a central extension of $\gamma_n(G)$, Γ_{n+1} map into Γ_n and $\sum \Gamma_n/\Gamma_{n+1}$ is a Lie algebra.
An analogous construction in the group ring kG leads to the universal enveloping algebra of this Lie algebra.

J.P. SERRE

$\tilde{A}_n$ - liftings

For $n \geq 4$, let $1 \to \{\pm 1\} \to \hat{A}_n \to A_n \to 1$ be the unique non split central extension of the alternating group A_n by $\{\pm 1\}$.

If $s \in A_n$ is a product of disjoint cycles of odd order $e_\alpha = e_\alpha(s)$, define $\nu(s) \in \mathbb{Z}$ and $\omega(s) \in \mathbb{Z}/2\mathbb{Z}$ by $\nu(s) = \sum_\alpha (e_\alpha - 1)$, $\omega(s) \equiv \sum_\alpha \frac{e_\alpha^2 - 1}{8}$ (mod 2).

Let $s_1, \ldots, s_k \in A_n$ be such elements with $\Pi s_i = 1$; assume the group $G = \langle s_i \rangle$ generated by the s_i is transitive. Then $\Sigma \nu(s_i) \geq 2n - 2$; more precisely, if g is the genus of the ramified covering of $\mathbb{S}_2$ defined by the s_i, one has

$$\sum \nu(s_i) = 2g + 2n - 2.$$

Let $\tilde{s}_i \in \tilde{A}_n$ be the unique lifting of s_i in $\tilde{A}_n$ which has odd order. It is clear that $\prod_i \tilde{s}_i = \{\pm 1\}$ in $\hat{A}_n$.

<u>Theorem 1.</u> If $g = 0$, one has $\prod_i \tilde{s}_i = (-1)^\omega$, where $\omega \equiv \sum_i \omega(s_i)$ (mod 2).

For an arbitrary g, there is a similar statement, namely:

<u>Theorem 2.</u> $\Pi \tilde{s}_i = (-1)^{\omega + e}$, where e is the invariant in $\mathbb{Z}/2\mathbb{Z}$ of the "theta-characteristic" of the ramified covering $X \to \mathbb{S}_2 = \mathbb{P}_1(\mathbb{C})$ associated to that covering.

(Th. 2 is itself a special case of a result describing the behaviour of the e-invariant of theta-characteristics under ramified coverings $Y \to X$ with odd ramification.)

The motivation for Th. 1 is the construction of $\tilde{A}_n$-extensions of $\mathbb{Q}(T)$, n odd, which are unramified quadratic extensions of the A_n-extensions constructed by J.-F. Mestre.

H. SHORT

Automatic groups and small cancellation theory

The recent definition of automatic groups by Cannon, Epstein, Holt, Paterson and Thurston is an interesting generalization of Gromov's class of hyperbolic groups. Recently, working with S.M. Gersten, we have shown that the C(3) - T(6)', C(4) - T(4) and C(6) small cancellation groups are automatic. The constructions used also give a simple solution to the conjugacy problem for these groups (and for hyperbolic groups).

R. STÖHR

Homology of groups with coefficients in exterior powers of relation modules

Let G be a group given as a quotient G = F/R of a free group F and let R_{ab} denote the relation module associated with this free presentation. The homology of G with coefficients in exterior powers of R_{ab} has been studied by Yu. V. Kuz'min in connection with his work on the homology of free abelianized extensions (Comm. Algebra 16 (1988), 2447-2533). In the talk I discuss a new approach to describing the groups $H_k(G, \Lambda^n R_{ab})$. In particular, I present a new proof of Kuz'min's results on p-torsion in $H_k(G, \Lambda^{mp} R_{ab})$ (p a prime, m < p) and I give a description of the p-torsion in $H_k(G, \Lambda^{p^2} R_{ab})$.

J. THÉVENAZ

Brown's complex and Alperin's conjecture

Let G be a finite group and k an algebraically closed field of characteristic p. A conjecture of Jonathan Alperin expresses the number of non-projective simple kG-modules in terms of p-local information (i.e. in terms of normalizers of non-trivial p-subgroups of G).

Several statements are presented, which are all proved to be equivalent to Alperin's conjecture. Most of them involve Brown's complex (the simplicial complex of chains of non-trivial p-subgroups of G), and they are of a purely combinatorial nature. One of the formulations (joint work with Peter Webb) involves Mackey functors and their structure theory.

P. WEBB

The structure of Mackey functors

Joint work with T. Thévenaz.

We give a complete description of the irreducible Mackey functors and also certain information about their projective covers. This has impact in the different areas in which Mackey functors arise — topology, representation theory, K-theory etc. — and has very much to do with the representations of different subgroups of a group and the way they are inter-related. Arising out of this comes a new approach to Alperin's conjecture.

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Problemliste

1. **(R. Alperin** and **K. Brown)** Let G be a group which is the free product of three finite subgroups A, B, C amalagamated along their intersections. Let X be the associated 1-connected 2-complex (Behr); thus G acts on X with a 2-simplex σ as fundamental domain and with A, B, and C as the stabilizers of the vertices of σ. Assume that G acts faithfully on X. Let 2p, 2q, and 2r be the girths of the links of the vertices of σ. [Thus π/p, π/q, and π/r are the Gersten-Stallings angles of the "triangle of groups" formed by A, B, C and their intersections.] Assume we are in the non-spherical situation, i.e., $1/p + 1/q + 1/r \leq 1$.

Question: Is G residually finite? Is it at least virtually torsion-free? What if we add the hypothesis that X is a building?

Remarks: (a) As the wording suggests, residual finiteness would imply that G is virtually torsion-free. This is a consequence of results of Gersten and Stallings.

(b) For ordinary amalgamated free products (i.e., two subgroups instead of three), there is an easy algebraic argument which shows that the group is residually finite. The same is true if X is the building associated to an algebraic group, since G is then a linear group. But there is a specific example, constructed by Tits, where X is an "exceptional" building and we do not know if G is virtually torsion-free.

(c) The non-sphericity assumption is necessary. K. Brown has constructed examples of spherical triangles of finite groups, such that the amalagam G is an infinite simple group and hence has no proper subgroups of finite index.

2. (Y. Kuz'min) Find a "good" description of the (co)homology of a free-nilpotent group $F/\gamma_n F$, where F is a free group and $\{\gamma_n F\}$ is its lower central series. [The Koszul complex can be viewed as a solution in the free abelian case.] Are the integral homology groups torsion-free? One can ask similar questions about free-soluble groups. In particular, does there exist 2-torsion in the odd-dimensional integral homology of a free-soluble group?

3. (K. W. Gruenberg) If K is a group acting on a group M, let $d_K(M)$ be the minimum number of generators of M as K-group.
Let G, H be finitely generated groups, $E = G * H$ and E^* the kernel of $E \to G \times H$. What is $d_E(E^*)$ in terms of d(G), d(H)? $\left(d(G)d(H) \geq d_E E^*).\right)$

4. (J.- P. Serre, quoted from LNM 573, 1975**)** Recall that a group G is called <u>coherent</u> if every finitely generated subgroup of G is finitely presented.
Is it true that $SL_3(\mathbb{Z})$ is coherent? (Note that $SL_2(\mathbb{Z})$ is obviously coherent, and that $SL_n(\mathbb{Z})$, $n \geq 4$, is not coherent.)
Same question for $SL_2(\mathbb{Z}[\frac{1}{p}])$, where p is a prime.

5. (J. Hillman) If G is coherent, is $\mathbb{Z}G$ coherent? [The converse is true if FP_2 implies finite presentability.]

6. (J. Hillman) Suppose G is finitely presented and cdG = 2. Is def(G) = $\beta_1(G;\mathbb{Q})-\beta_2(G;\mathbb{Q})$? [If so, the 2-complex associated with an optimal presentation is aspherical.]

7. (J. Hillman) Suppose G is PD_3 (3-dimensional Poincaré duality group). Is G virtually indicable?

8. (J. Hillman) Suppose G is finitely generated, has no non-abelian free subgroup, and either is PD_3 or satisfies $\mathrm{cd}G \leq 2$. Is G solvable? [This is true, for example, if the abelianization of G has rank at least 2 and $\mathbb{Z}G$ is coherent.]

9. (J.-P. Serre, from a C. R. note of R. Spector, C. R. 276 (1973), 523-525) Let G be a profinite group (projective limit of finite groups), which is a torsion group: every element x of G has finite order o(x).
Is it true that G has bounded exponent, i.e. that the o(x), $x \in G$, are bounded?

10. (J.-P. Serre, from a letter of J.-P. Serre to K. Vogtman, 6.10.1986) Let F_n be a free group of rank n, $n \geq 3$, and let $\Gamma_n = \mathrm{Out}(F_n) = \mathrm{Aut}(F_n)/F_n$.
Is it true that Γ_n has Kazhdan's T property?

11. (J. - P. Serre, from the same letter as above) If Γ is any group, define a "germ of automorphism" of Γ as a class of isomorphisms φ: $\Gamma' \xrightarrow{\sim} \Gamma''$, where Γ' and Γ'' are subgroups of finite index of Γ (two such φ being identified if and only if they coincide on a subgroup of finite index of Γ). Such germs make a group Γ_Q, the "commensurability " group of Γ; there is a natural map $\Gamma \to \Gamma_Q$.
If one applies this construction to the group $\Gamma = \Gamma_n$ of Problem 3, one gets a group $(\Gamma_n)_Q$. Is the map $\Gamma_n \to (\Gamma_n)_Q$ an isomorphism? If not, is the cokernel finite? (A "yes" answer would make obvious that Γ_n is not arithmetic - not even "S-arithmetic".)

12. (M. Davis) Let (W, S) be a Coxeter system and G a virtually abelian subgroup of W. Is G conjugate to a subgroup of a standard subgroup W_T, $T \subset S$, of the form

$$W_T = W_E \times W_F \times \prod_{i=1}^{m} W_i$$

where W_E is a Euclidean Coxeter group of rank ≥ 2, W_F is finite, and each W_i is an infinite standard subgroup such that the image of G in W_i is of rank 1?

Remarks: Moussang has proved:

(1) If W does not contain a standard subgroup of the above form with $W_E \neq 1$ or $m > 1$, then it is word hyperbolic and hence, any virtually abelian G is of rank 1.

(2) Any solvable subgroup of W is virtually abelian.

13. (P. Webb) Let $\Delta = \Delta\big(S_p(G)\big)$ denote Brown's simplicial complex of finite p-subgroups whose simplices are the chains $P_0 < P_1 < \ldots < P_n$ of non-identity p-subgroups of G. Suppose G is finite. Is Δ/G contractible?

Discussion. The evidence for an affirmative answer comes from

(a) Δ/G is mod-p acyclic (P. J. Webb, A split exact sequence of Mackey functors, preprint).

(b) In the case of a Chevalley group in defining characteristic p, Δ is G-homotopy equivalent to the building, for which the orbit space is a single simplex, hence contractible. (The G-homotopy equivalence comes from Quillen, and J. Thévenaz & P. J. Webb, Homotopy equivalence of posets with a group action, to appear in J. Combin. Theory Ser A)

An affirmative answer would be a help in computing Δ/G.

14. (P. Webb) With G and Δ as above, what is the minimum dimension of a G-simplicial complex which can be G-homotopy equivalent to Δ? More precisely, let

$$B_p(G) = \{H \leq G \mid H = O_p N_G(H)\}$$

and let $\Delta\big(B_p(G)\big)$ be the corresponding simplicial complex. Then $\Delta\big(B_p(G)\big)$ is G-homotopy equivalent to $\Delta\big(\zeta_p(G)\big)$. (Bouc, Thévenaz-Webb) When $O_p(G) = 1$ is $\dim\Delta\big(B_p(G)\big)$ the minimum dimension which can occur?

Discussion. For Chevalley groups this is true, since $\Delta\big(B_p(G)\big)$ is the barycentric subdivision of the building, and has non-zero homology in its top dimension.

15. (J. Thévenaz) With G and Δ as above let $X = |\Delta|$. Assume X is connected. Does X have the homotopy type of a bouquet of spheres (of various dimensions?) Discussion. 1) If G is a Chevalley group in characteristic p, then Δ is homotopy equivalent to the building of G. Thus one gets a bouquet of spheres in the top dimension. 2) There are examples giving bouquets of S^n and S^m with $n \neq m$. 3) By induction, the link of every simplex has the homotopy type of a bouquet of spheres.

16. (M. Lustig) Every surface group $\pi_1 M_g$ (M_g closed of genus g) admits a unique free group surjection $F_{2g} \twoheadrightarrow \pi_1 M_g$. A well known result assures that every outer automorphism φ: $\pi_1 M_g \to \pi_1 M_g$ is induced by some outer automorphism Φ: $F_{2g} \to F_{2g}$. Some such lifts are "good" in the sense that growthrate (Φ) = growthrate (φ), or, more generally, that Φ reflects the geometric properties of the Thurston-representative h: $M_g \overset{\approx}{\to} M_g$ with $h_* = \varphi$. (For example if Φ is obtained by puncturing M_g in a prong of the stable foliation for h).

Question: Given φ, how can one determine algebraically a good lift Φ.

An algorithmic solution to this would allow one to apply techniques from free group automorphisms to surface theory. For example, a fast solution of the conjugacy problem in the mapping class group $\mathrm{Out}(\pi_1 M_g)$ could be derived.

17. (M. Lustig) Assume that $\langle x \mid R(x)\rangle$ and $\langle x' \mid S(x')\rangle$ are two finite presentations of the same group G. Denote by $\langle x \mid R(x)\rangle \amalg \langle y \mid [w(x), y]\rangle$ the presentation obtained from $\langle x \mid R(x)\rangle$ by adding new generators y_i and new relators $[w_i, y_i]$ with $w_i \in F(x)$. Let $\overset{3}{\curlywedge}$ denote a sequence of Andrews-Curtis transformations on presentations (= extended Nielsen transformations on generators and relators).

Question. Does $\langle x \mid R(x)\rangle \amalg \langle y \mid [w(x),y]\rangle \overset{3}{\nearrow\searrow} \langle x' \mid S(x)\rangle \amalg \langle y \mid T(x,y)\rangle$ [$T(x,y)$ stands for a collection of arbitrary elements $T_i \in F(x,y)$] imply $\langle x \mid R(X)\rangle \overset{3}{\nearrow\searrow} \langle x' \mid S(x')\rangle \amalg \langle z \mid W(x',z)\rangle$?(for some $W_j(x',z) \in F(x',z)$, such that the subgroup generated by the z_k is trivial).

If so, the recent simple-homotopy distinction of homotopy equivalent 2-complexes (Lustig, Metzler) together with a strong result of Gerstenhaber-Rothaus could be employed to work out conterexamples to the generalized Andrews-Curtis conjecture (i.e. G is non-trivial).

18. (W. Metzler) If $L^2 \subset K^2$ is a pair of CW-complexes and $\pi_2(K^2) = 0$, does it follows that $\pi_1(L^2)$ is torsion free? Counterexamples would also fulfill $\pi_2(L^2) \neq 0$; hence it seems worth while to know whether this type of counterexamples to the Whitehead- Asphericity-Conjecture can occur.

19. (W. Metzler) Does "π torsion free" imply $Wh(\pi) = 0$?

20. (R. Bieri) Is it true that if a group G contains no free subgroup of rank > 1 and is of type FP_m then so is every metabelian factor group of G?

Remark. This is known to be true for $m \leq 2$ or for subgroups of $GL(\mathbb{Q})$. It is open even when G itself is metabelian. In that case it would follow if one could establish the FP_mconjecture (a metabelian group G is of type FP_m if and only if the geometric invariant $\Sigma(G)$ has the property that every m-point subset of the complement $\Sigma(G)^c = S^{n-1} \setminus \Sigma(G)$ is contained in an open hemisphere of $S^{n-1} = Hom(G,\mathbb{R})/\mathbb{R}_{>0}$).

21. (R. Bieri) Is it true that every finitely generated group of homological dimension $hdG \leq 1$ is free?

Remark. This is true when G is of type FP_2, for in that case the augmentation ideal $I_G \triangleleft \mathbb{Z}G$ is finitely presented flat and hence projective.

22. (R. Geoghegan) 1.)Is there a finitely presented (infinite) torsion free simple group?

Remark. Thurston suggested putting a geometry on a surface S of genus ≥ 2 based on a "small" pseudogroup, and proving that Aut S is the required group.

2.) Let F be the group with presentation

$$\langle x_0, x_1, x_2, \ldots \mid x_i^{-1} x_n x_i = x_{n+1} \quad \forall\, i < n \rangle$$

Is F non-amenable ?

Remarks. (1) I conjectured in 1979 that F is non-amenable and contains no free non-abelian subgroup, thus being a finitely presentable torsion free counter-example to von Neumann's conjecture that non amenable $\Longleftrightarrow$ free non-abelian subgroups. (2) Brin and Squier have proved that F has no free non-abelian subgroup.

3.) Let X be a finite connected CW-complex with universal cover $\tilde{X}$. Let $K_1 \subset K_2 \subset \ldots$ be a sequence of finite subcomplexes of $\tilde{X}$ such that $K_n \subset \mathring{K}_{n+1}$ for all n, and $\tilde{X} = \bigcup_n K_n$. Write $U_n = \tilde{X} \backslash K_n$. Let $w: [0, 1) \to \tilde{X}$ be a proper base ray. There is then an inverse sequence of groups

(∗) $\pi_1(U_1, w) \leftarrow \pi_1(U_2, w) \leftarrow \ldots$ whose bonds are induced by inclusions.

(a) Is (∗) pro-isomorphic to a sequence of finitely generated groups?

(b) Is (∗) pro-isomorphic to a sequence of groups where the bonds are onto?

Remarks. (1) $\{G_n\}$ and $\{H_m\}$ are <u>pro-isomorphic</u> iff $\exists$ subsequences $\{G_{n_i}\}$ and $\{H_{m_i}\}$ and homomorphisms making the following diagram commute:

$$\begin{tikzcd}
{} & G_{n_k} \arrow[l] \arrow[d] & G_{n_{k+1}} \arrow[l] \arrow[d] & {} \arrow[l] \\
{} & H_{m_k} \arrow[l] \arrow[ul] & H_{m_{k+1}} \arrow[l] \arrow[ul] & {} \arrow[l] \arrow[ul]
\end{tikzcd}$$

where the horizontals are bonds.

(2) (b) is equivalent to asking if (∗) is semistable ($\equiv$ Mittag-Leffler).

(3) These are questions about $G = \pi_1(X)$. They are independent of choice of X, and if G has one end they are independent of choice of w.

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