Math. Forschungsinstitut
Oberwolfach
E 20 10L196

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 44/1989

Geometrie

15.10. bis 21.10.1989

Die Tagung fand erstmalig unter der Leitung der Herren V. Bangert (Bern) und U.Pinkall (Berlin) statt. Die 44 Teilnehmer kamen erfreulicherweise auch aus bisher selten vertretenen Ländern. Die Vorträge deckten einen weiten Bereich der Geometrie ab. Neben der Theorie der Untermannigfaltigkeiten, die den breitesten Raum einnahm, sind die Polyedergeometrie und geometrische Anwendungen von dynamischen Systemen weitere Themenbeispiele. Der Erfolg der Tagung beruhte nicht zuletzt auf den anregenden Diskussionen und den abendlichen Arbeitssitzungen.

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Vortragsauszüge

U. Brehm

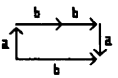
Polyhedral surfaces

Def.: Let K be a simplicial complex. $f : |K| \to \mathbb{R}^3$ is called a polyhedral immersion (polyhedral embedding, respectively) if f is locally injective (injective, respectively), continuous and simplexwise linear.

Theorem: There exists a triangulation of the Möbius strip (with 9 vetices) which does not admit a polyhedral immersion.

We give an overview on the known results concerning the minimal number of vertices of polyhedral embeddings and immerions of given topological type.

We show the following polyhedral models with minimal vertex numbers:

1) A polyhedron of genus 3 with 10 vertices and 3 axes of symmetry (maximal symmetry),
2) Three combinatorial different polyhedral models of the Boy surface (i.e. polyhedral immersions of $\mathbb{R}P^2$ in $\mathbb{R}^3$) with 9 vertices and 10 facets and one axis of symmetry of order three,
3) The polyhedral "Knotbox" K with 8 vertices. K ≡ 

K is a strong deformation retract of the complement of the trefoil knot.

Finally a smooth embedding of K as a minimal surfaces in S^3 is shown (half of Lawson's minimal kleinbottle $\tau_{2,3}$).

Y. Burago

Metric Spaces with Curvatures bounded from below

Let X be a finite dimensional metric space with an inner metric (i.e. "length space") and with curvatures bounded from below. These spaces were introduced by Alexandrov and studied mainly for X with twosided restrictions for curvatures. In fact in this case X is a Riemannian manifold with sufficiently smooth metric g.

We study the spaces X without any a priori topological restrictions. The standard cone under projective space RP^n, $n > 1$ is an example of such "non regular" space X.

For the spaces X the notion of the boundary may be defined. For simplicity we shall suppose that ∂X is empty. We proved that there exists an open dense set $Y \subset X$ with following property: every point $y \in Y$ has a neighbourhood such that it is homeomorphic and "almoust isometric" to Euclidian disk $D^n(0,\delta(y))$.

The proofs are based on the studying of the tangent spaces of X and on the stability theorem for the spherical variant of the Toponogov splitting theorem.

It seems to me this stability theorem has independent interest.

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V. Goldshtein

L_p-cohomology of warped product

For a smooth Riemannian manifold M and the outer differential $d^k : L_p^k(M) \to L_p^{k+1}(M)$ the space $H_p^k(M) = \mathrm{Ker}\, d^k / \mathrm{Im}\, d^{k-1}$ is the space of L_p-cohomology and $\overline{H}_p^k(M) = \mathrm{Ker}\, d^k / [\mathrm{Im}\, d^{k-1}]$ is the space of reduced L_p-cohomology. Here $1 < p < \infty$ and $[\mathrm{Im}\, d^{k-1}]$ is the closure of $\mathrm{Im}\, d^{k-1}$. For example $\overline{H}_p^k(R^n) = H_p^k(R^n)$ for all p.

The main aim is the calculation of L_p-cohomology of both types for warped products $M = T \times_f X$ of a compact manifold X and a manifold T.

For a bounded function f some analogy with the classical Kunneth's formula is right. We used this result and some specific variant of the exact sequence of the pair for the calculation. The main case is $T = [a,b)$, $-\infty < b < \infty$. If the function f is not bounded and "the Kunneth's formula" is not right we also have a possibility to calculate L_p-cohomology for a monotone function f.

Let R_f^{n+1} be the Euclidean space with Riemannian metric coinciding outside of some ball with a rotationally symmetric metric of a warped product. In this case we calculated both type of cohomolgies. Main examples: $f(t) \cong e^t$, $b = \infty$ (hyperbolic space); $f(t) = e^{-t}$, $b = \infty$ (cusp); $f(t) = t^s$.

E. Heintze

Holonomy group of the normal bundle and isoparametric submanifolds

We improve slightly a recent result of Carlos Olmos on the normal holonomy group of a sumbanifold as follows.

Theorem: Let N be conformally flat, $M \subset N$ a submanifold and Φ_* the restricted holonomy group of the normal bundle. Then Φ_* is compact and the action on the normal space is equivalent to the isotropy representation of a symmetric space.

This result is not true for general N. Already $\mathbb{C}P^n \subset \mathbb{C}P^{n+k}$ gives a counterexample.

A submanifold of euclidean space is said to have constant principal curvatures if for any curve and any parallel normal field $\xi(t)$ along that curve the shape operator $A_{\xi(t)}$ has constant eigenvalues. It is called isoparametric if in addition the normal bundle is flat. By use of the above result one gets:

Theorem $M^n \subset \mathbb{R}^{n+k}$ has constant principal curvature if and only if it is the focal manifold of an isoparametric one.

This Theorem is due to Olmos and independently (in a slightly different form) to Thorbergsson and the author.

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N. Hingston

Equivariant topology of free loop spaces and closed geodesics on the two-sphere

We describe an SO(2) and an O(2)-model for the free loop space ΛS^n. From the SO(2)-model one can compute the equivariant cohomology of ΛS^n : It is that which one would expect from ΛX where X is a Finsler manifold with the homotopy type of a point and with exactly one hyperbolic closed geodesic!

We describe a simplification which takes place in the equivariant Morse complex of an orientable surface which means that one can get an O(2)-action on an (ordinary) Morse CW-complex consisting only of summits and tunnels.

We decribe a "morse theoretic" O(2)-model for ΛS^2 and prove the
Theorem: Take a metric on S^2 with all closed geodesics nondegenerate. If there are only three "short" closed geodesics, then there are infinitely many closed geodesics.

G. Knieper

Regularity of entropy

Given a compact Riemannian manifold (M,g), we can associate to it the topological entropy of the geodesic flow $h(\phi_g)$, ($\phi_g^t : SM \to SM$ denotes the geodesic flow).
For a certain class of Riemannian metrics it's possible to describe entropy in terms of geometric quantities.
If we restrict topological entropy to the space of metrics with negative curvature, entropy is C^∞ (resp. C^ω) under C^∞ (resp. C^ω) perturbations of the metric. We derive a formula for the derivative of entropy, which is given by

$$Dh(g)(\xi) = -\frac{h(g)}{2} \cdot \int_{SM} \xi(v,v) d\mu_M(g)$$

where $\xi \in T_g R(M)$, $R(M) = \{g$ Riemannian metric | g has strictly neg. sectional curvature$\}$. If we restrict h to the space of metrics with normalized volume, the locally symmetric spaces are critical points of h.

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P. Kohlmann

Ovaloids in 3-space with mean curvature near 1

We consider the class $M_\varepsilon := \{C \subseteq \mathbb{R}^3 \text{ ovaloid} \mid 1-\varepsilon \le H \le 1+\varepsilon\}$ for $0 \le \varepsilon < 1$ and prove

Theorem 1: For fixed ε, M_ε is uniformly cylindrically bounded.

We define $R(\varepsilon) := \sup\{\text{circumsphere radius of } C \mid C \in M_\varepsilon\}$ and prove with Schwartzsymmetrization and Minkowskiformulas.

Theorem 2: $R(\varepsilon) < \infty$ for $\varepsilon < \frac{1}{7}$.

Theorem 3: For all $t > 0$ there exists $\varepsilon > 0$ that each $C \in M_\varepsilon$ contains a ball with radius 1-t and is contained in a ball with radius 1+t.

O. Kowalski

Curvature-homogeneous and semi-symmetric Riemannian manifolds.

Semi-symmetric spaces are the Riemannian manifolds (M,g) satisfying the identity $R(X,Y)\,R = 0$. They were all classified by Z. Szabó in 1982, but little explicit non-symmetric examples were known. We show that the metric g defined on $U \subset \mathbb{R}^3(w,x,y)$ by $g = \sum_{i=0}^{2} \omega^i \otimes \omega^i$ where $w^0 = f(w,x)dw$, $w^1 = dx - y\,dw$, $w^2 = dy + x\,dw$ is always semi-symmetric. For $f''_{xx} \neq 0$ the space (U,g) is irreducible and non-homogeneous, i.e., $\nabla R \neq 0$. For $U = \mathbb{R}^3$ and $f(w,x) > c > 0$, the space $(\mathbb{R}^3,g)$ is complete. Moreover, each class of locally isometric spaces of the above type depends only on a finite number of parameters. Thus we can say that the locally non-isometric spaces of the above type still depend "on an arbitrary function". We also study the subfamilies of spaces (U,g) given by

a) $f(w,x) = a(w)e^{kx} + b(w)e^{-kx}$,

b) $f(w,x) = a(w)\cos kx + b(w)\sin kx$.

In the case a), the curvature tensor at each point is the same as that of the symmetric space $H^2(-k^2) \times \mathbb{R}$ and in the case b), the curvature tensor at each point is the same as that of $S^2(k^2) \times \mathbb{R}$. These are examples of (so-called) curvature-homogeneouses Riemannian spaces. In each class we can find an infinite dimensional family of mutually non-isometric spaces with the same curvature tensor. Such big families were not known before. The general examples in higher dimensions are also available.

Reference: O. Kowalski, F. Tricerri, L. Vanhecke - a series of papers to appear gradually.

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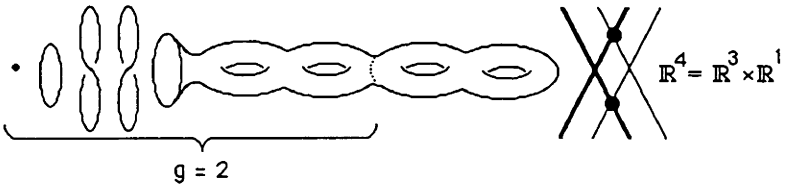
W. Kühnel

On conformal vector fields

A theorem of Obata says that a compact Riemannian manifold of constant scalar curvature admitting an essential non-homothetic conformal vector field is isometric to the standard sphere. This is not true without the assumption "essential". A counterexample of type $S^1 \times_f M_*$ (warped product metric $ds^2 = du^2 + (f(u))^2 ds^2_*$) is due to N. Ejiri 1981. We give an explicit analytic description of this example, and we give a classification of all such vector fields which are locally gradient fields in the case of complete manifolds. Particular examples are the metrics $S^1 \times_f S^3$ with $f(u) = \sqrt{2 + \sin(u)}$ and $\mathbb{R} \times_f S^3$ with $f(u) = \sqrt{1 + u^2}$, the latter being isometric with the 4-dimensional catenoid. The conformal vector field is given by $f \cdot \frac{\partial}{\partial u}$.

N.H. Kuiper

Hyperbolic 4-manifolds



Here is an orientable closed surface of genus 2 resp.4, $\chi = 2\text{-}2g = -2$ resp. -6, immersed in $\mathbb{R}^4$ with one double point. The algebraic intersection with a translated copy is zero as seen by translating far. For a small translation we have a contribution +2 near the intersection, so that the selfintersection of the immersed surface inside an immersed tubular disc bundle neighborhood Σ^4 is $\chi^\perp = \int R^\perp d\sigma/2\pi = -2$, where $R^\perp$ is the normal curvature density.

For g = 2 we have therefore an immersion of the tangent bundle.

For g = 4 we have $|\chi^\perp/\chi| \leq \frac{1}{3}$, sufficient for the total space $\Sigma^4 = \mathbb{H}^4/\pi_1(\text{surface})$ to admit a complete hyperbolic metric with C_{4g} (cyclic group)-symmetry, the lecture at Oberwolfach in October 1988. If we admit "almost-plane" bundles, we can get hyperbolic manifolds with $|\chi^\perp/\chi|$ bigger: C_{4g} symmetric up to $|\chi^\perp/\chi|$ close to $2/\sqrt{3}$, very big if we drop the symmetry condition.

Ref.: Publ. Math. IHES 68(1989) 47-76,

Actes du Congrès de Géométrie d´Avignon 1988 = preprint IHES 89

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F. Labourie

Convex surfaces and hyperbolic geometry

Several constructions, using constant Gaussian curvature surfaces in hyperbolic geometry, are used to parametrize the space of projective (complex) structures on surfaces, and to construct hyperbolic metrics on convex compact hyperbolic 3-manifold with a boundary data.

K. Leichtweiß

Singularities of normal frames

Report on the dissertation of Mrs. U. Hummel (Stuttgart). There the Stiefel-Whitney's obstruction theory for differentiable sections in the tangent bundle of an immersion $f : M_2 \to \mathbb{R}^{2+n}$ ($n \geq 3$) (M_2 closed and orientable) is modified for the normal bundle $\nu(f)$ of f in a differential geometric manner, as this was done by P. Wintgen for $n = 2$. Main result: The normal Euler number $\overline{W}_2 \cdot M_2$ (which is 0) may be expressed in the form $\overline{W}_2 \cdot M_2 \equiv \frac{1}{2\pi} \int_{M_2} \eta (\mathrm{mod}\ 2)$ where η - the so called "obstruction form" - is a 2-form depending on the first n-2 vectors of a test normal frame $\{N_1,\ldots,N_n\}$ with isolated singularities for the last 2 vectors. Such a test frame is indicated for the double Veronese surface $f : S_2 \to \mathbb{R}^5$. There are also inductive methods for the computation of the indices of normal frames at their singularities by mean of suitable orthogonal projections.

M. J. Micallef

The index of minimal surfaces of finite total curvature in Euclidean space

Joint work with N. Ejiri.

Theorem 1. Let $M^2 \to \mathbb{R}^n$ be a complete, oriented minimal surface of finite total curvature. Then $\mathrm{index}(M) \leq \frac{1}{2\pi} \int_{M^2} (-K)dA + 2g - 2$. This estimate is sharp in a number of interesting cases (e.g. Jorge-Meeks surfaces).

Theorem 2. Let $F : M^2 \to N^n$ be a (possibly branched) minimal immersion of a closed Riemann surface into a Riemannian manifold N. Let i_E denote the index of F as a critical point of the energy functional and let i_A denote the index of F as a critical point of the area functional.

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Then $i_E \le i_A \le i_E + r$

where $$r = \begin{cases} 6g - 6 - 2B & \text{if } B \le 2g - 3 \\ 2[-\frac{B}{2}] + 4g - 2 & \text{if } 4g - 4 \ge B \ge 2g - 2 \neq 0 \\ 0 & \text{if } B \ge 4g - 3 \end{cases}$$

where B = total number of branch points of F counted with multiplicity.

Note: $r = 0$ if $g = 0$ and if $g = 1$, then $r = 0$ if $B \ge 1$ and $r = 1$ if $B = 0$.

Corollary: if N has non-positive sectional curvature, then $i_A \le r$.

St. Nölker

Warped products of isometric immersions

We consider a Riemannian manifold M which is a so called warped product $M_0 \times_{\rho_1} M_1 \times \ldots \times_{\rho_k} M_k$ of Riemannian manifolds $M_0,\ldots,M_k$, i.e. M is the C^∞-manifold $M_0 \times \ldots \times M_k$ endowed with the Riemannian metric defined by

$$\langle v, w\rangle^M = \langle v_0, w_0\rangle^{M_0} + \sum_{i=1}^{k} \rho_i^2(p_0)\, \langle v_i, w_i\rangle^{M_i}$$

for all $p = (p_0,\ldots,p_k) \in M$, $v = (v_0,\ldots,v_k)$, $w = (w_0,\ldots,w_k) \in T_pM \cong \bigoplus_{i=0}^{k} T_{p_i}M$

where $\rho_1,\ldots,\rho_k: M_0 \to \mathbb{R}_+$ are positive C^∞-functions on M_0.

Then we have the following decomposition theorem.

Theorem

Let $f : M = M_0 \times_{\rho_1} M_1 \times \ldots \times_{\rho_k} M_k \to N$ be an isometric immersion into a standard space N of constant curvature whose second fundamental form satisfies

$\alpha(v,w) = 0$ for all $p \in M$, $v \in T_pL_i$, $w \in T_pL_j$, $i \neq j$

(where L_i is the foliation of M canonically induced by M_i, $i = 0,\ldots,k$).

Then there exists an isometriy $\psi : N_0 \times_{\bar\rho_1} N_1 \times \ldots \times_{\bar\rho_k} N_k \to N$ of a warped product $N_0 \times_{\bar\rho_1} N_1 \times \ldots \times_{\bar\rho_k} N_k$ onto an open part of N and isometric immersions $f_i : M_i \to N_i$ such that

$\rho_i = \bar\rho_i \circ f_0$ for $i = 1,\ldots,k$ and $f = \psi \circ (f_0 \times \ldots \times f_k)$.

The geometry of such a "warped product of isometric immersions" is described. Applications of the theorem to the classification of certain types of isometric immersions are given.

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K. Polthier

Construction of minimal surfaces in H^3

We construct new minimal surfaces in hyperbolic 3-spaces H^3 using two different approaches. For the first method we assume a rotational symmetric conformal metric, which reduces the minimal surfaces equation to an ODE for the conformal factor. For special second fundamental forms this generalizes the Euclidean k-Enneper surfaces to hyperbolic k-Enneper surfaces, and in addition we get a 2-parameter family of nonisometric translational surfaces. The surfaces we obtain are complete, and all members of their associate family are congruent except for the catenoid. Some of them are embedded.

The second approach extends a conjugate minimal surface construction in S^3 of Karcher, Pinkall and Sterling to H^3. We prove existence of periodic minimal surfaces with fundamental cells in each of the 15 regular polyhedrons (indcluding those with vertices in infinity) and in some Coxeter orthoschemes with vertices beyond infinity. Successive reflections of the fundamental cells in the faces of the polyhedrons lead to complete minimal surfaces.

H.-B. Rademacher

On a generic property of geodesic flows

A Riemannian metric g on a compact manifold is bumpy if all closed geodesics have linear Poincaré maps for which 1 is not an eigenvalue. Let $(c_k)_{k \in A^*}$, $A^* = \{1,\ldots,A\}$, $A \leq \infty$ be the set of prime closed geodesics with linear Poincaré maps $P_k \in Sp(n)$ whose eigenvalues of norm 1 are $e^{\pm \pi i \lambda(k,1)},\ldots,e^{\pm \pi i \lambda(k,l(k))}$, $\lambda(k,j) \in (0,1)$ and set $(\lambda_i)_{i \in B^*} = (\lambda(1,1),\ldots,\lambda(1,l(1),\lambda(2,1),\ldots)$. We say g is strongly bumpy if for every $d \leq B$ and every nontrivial polynomial p of degree d with integer coefficients $p(\lambda_1,\ldots,\lambda_d) \neq 0$ holds. Then one can show that a C^2-generic metric on a compact manifold is strongly bumpy and that a strongly bumpy metric on a simply-connected compact manifold has infinitely many closed geodesics.

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H. Reckziegel

Reelle Hyperflächen der Quaternionen-Raumformen mit konstanten Hauptkrümmungen

Aufgrund der Anisotropie des Quaternionen-projektiven Raumes bzw. Quaternionen-hyperbolischen Raumes $\mathbb{H}P^m$ bzw. $\mathbb{H}H^m$ ist es sinnvoll, reelle Hyperflächen M zu untersuchen, deren Geometrie "geeignet" der Krümmung des umgebenden Raumes angepaßt ist. Hier wird vorausgesetzt, daß in jedem Punkt $p \in M$ der Formoperator A_ξ mit dem "Jacobi-Operator" $\overline{R}(\cdot,\xi)\xi \quad (\xi \in \perp_p^1 M)$ des umgebenden Raumes kommutiert. Diese Beschränkung hat äußerst starke Implikationen für die Hauptkrümmungen von M : Ist M reelle Hyperfläche von $\mathbb{H}P^m$, so sind sie automatisch konstant; sind sie konstant (im Fall $\mathbb{H}P^m$ und $\mathbb{H}H^m$), so ist M offener Teil einer Röhre um eine totalgeodätische Quaternionen-Untermannigfaltigkeit, einer Röhre um eine totalgeodätische komplex-m-dimensionale totalkomplexe Untermannigfaltigkeit oder (im Fall $\mathbb{H}H^m$) einer Horosphäre. Die angeführten totalgeodätischen Untermannigfaltigkeiten sind bestens bekannt. Die hier dargestelllten Ergebnisse wurden von **J. Berndt** aus Köln im Juli 1989 erzielt.

V. Schroeder

Flat Subspaces in Manifolds of Nonpositive Curvature

A k-flat in a Riemannian manifold M is a totally geodesic and isometric immersion $F : \mathbb{R}^k \to M$. F is called closed, if F is periodic with respect to some cocompact lattice. The following theorem is joint work with V. Bangert:

Let M be compact, real analytic and with sectional curvature $K \leq 0$. If M contains a k-flat, then M also contains a closed k-flat.

K.D. Semmler

Two generator torsion-free Fuchsian groups

We give some geometrical insights and proofs of the following results of Purzitsky, Rosenberger and others (collected in Rosenberger 86: All pairs of generators of all 2-generator Fuchsian groups) for the torsion free case:

Notation $1 < b = \mathrm{tr}\,\beta,\ 1 < c = \mathrm{tr}\,\gamma,\ 1 + 2\alpha = \mathrm{tr}\,[\beta,\gamma],\ r = \mathrm{tr}\,\beta\gamma,$
"tr" is half the trace of the corresponding matrices $\beta,\gamma \in SL(2,\mathbb{R})$

Lemma: $a < -1 \Rightarrow \langle\beta,\gamma\rangle = \pi_1(Q)$ Q Riemannsurface type (1,1)
$r < -1 \Rightarrow \langle\beta,\gamma\rangle = \pi_1(Y)$ Y Riemannsurface type (0,3)

Thm: $\langle\beta',\gamma'\rangle$ purly hyperbolic $\Leftrightarrow$ (β',γ') Nielson equivalent to a pair β,γ as in Lemma.

We study the set $\{(a,b,c) \mid \langle\beta,\gamma\rangle = \pi_1(\text{surface})\}$.

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U. Simon

Polar hypersurfaces

The relation of a polar pair of hypersurfaces is a centroaffine duality. To study their properties within Euclidean differential geometry we use the so-called affine conformal structure developed in our paper: Connections and conformal structure in affine differential geometry. Proceedings I. Conf. Diff. Geom. Appl. Brno (CSSR) 1986. There we developed the geometric structure independent of a relative normalization. the application here allows to state properties of a polar pair, e.q.

I. Study of relations between the Euclidean fundamental forms; study of curvature of a polar pair x, x*.

II. Study of the a-c-i and the centroaffine / equiaffine structure, consequences of the centroaffine duality.

III. Local existence and uniqueness theorems as

A. (i) x is l. strongly convex iff x* is ;
(ii) x is spherical iff x* is ;
(iii) x is a quadric iff x* is ;
(iv) x is an affine sphere iff x* is ;
(v) The relative conformal structure of x, x* coinside;
(vi) Existence and uniqueness of a polar pair, determined by a projectively flat affine connection without torsion.

B. Characterization of spherical surfaces by local properties of a polar pair.

IV. Global uniqueness results, characterization of polar pair by properties of their volume elements or curvature functions.

V. Centroaffine-Euclidean isoperimetric inequalities.

G. Stanilov

Curvature Operators in the Riemannian Geometry

Let (M,g) be an n-dimensional Riemannian manifold, R the curvature tensor. For a fixed vector $X \in M_p$ we consider the linear symmetric operator $\lambda_X: M_p \to M_p$, defined by $\lambda_X(\mathfrak{X}) = R(\mathfrak{X},X,X)$. One can consider the characteristic equation, the Hamilton-Jacobi equation of λ_X, and introduce some invariant quadratic forms of X. At least one of the proper values $k_i(p;X)$, i=1,..,n of λ_X must be zero. The Ricci-curvature $\rho(p;X) = \sum_{i=1}^{n-1} k_i(p;X)$, $k_n(p;X) \equiv 0$. We investigate the problem about the constancy of k_i, i.e. $k_i(p;X) = c_i(p)$. In such a case (M,g) is an Einstein manifold. For n = 4 we have the following assertions.

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Th.1. Let the pointwise proper values $c_1(p)$, $c_2(p)$, $c_3(p)$ be different and nonzero numbers. Then for the corresponding proper vectors X_1, X_2, X_3, ($X_4 = X_0$) hold the relations:

(1) $R(X_i, X_j, X_j, X_k) = 0$; i,j,k - different

(2) $R(X_3, X, X_1, X_2) + R(X_2, X, X_1, X_3) = \varepsilon(c_3 - c_2)$
and all permutations of 1,2,3, ($\varepsilon = \pm 1$) hold.

Th.2. The converse is also true in the case $\varepsilon = +1$.

We state the following problem: Do there exist such manifolds (n=4) with the properties (1) and (2)? It is open.

R. Sulanke

On Surfaces in the Möbius Space S^3

It is well-known fact that the homogeneous tori (Clifford tori) have conformal curvature K = 0. We consider the problem if there exist non-homogeneous immersion of the torus into S^3 without umbilical points but with K = 0. Since the corresponding differential equation is hardly to attack directly we tried to find counter-examples in special classes of surfaces. Among other results we proved that every canal surface of toral type with K = 0 must be homogeneous. As a tool we developed a systematic version of the geometry of sphere families as a theory of curves on the unit hypersphere of the pseudo-euclidean vector space V^5.

E. Teufel

Eine Verallgemeinerung der isoperimetrischen Ungleichung in Sphären

Die klassische isoperimetrische Ungleichung wurde für geschlossene Kurven in euklidischen Räumen von Th. Banchoff und W.F. Pohl 1971 verallgemeinert. Entsprechende Resultate für die 2-Sphäre erzielte J.L. Weiner 1974.
Wir zeigen für Immersionen $f : S^1 \to S^3$:

$$0 \le l^2 - \frac{1}{\pi^2} \int_{G \times G} (\Sigma - \text{sign cos } \tau)\, dG_{g_1} \wedge dG_{g_2}$$

mit Gleichheit genau für, evtl. mehrfach durchlaufenen geodätischen Kreis (l = Kurvenlänge; G = Menge der Geodätischen in S^3 mit bewegunsin. Volumenform dG; in der Summe werden die Treffsekanten mit trennender Schnittpunktlage (Sekantenschnittpunkte mit Kurve/Sekantenschnittpunkte mit $g_1 \cup g_2$) gezählt, wobei τ = orient. Winkel zwischen den durch Sekante und Kurventangenten in den Sekantenschnittpunkten def. 2-Sphären). Aufgezeigt wird der Zusammenhang mit, von l abhängigen unteren Schranken für die totale Absolutkrümmung geschlossener Kurven in S^3 (im Sinne der Fenchelschen Ungleichung).

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E. Tjaden

Elastische Raumkurven

Elastische Kurven sind Extremale eines Funktionals, das der Biege- und Torsionssteifigkeit eines Materials Rechnung trägt. Sie sind gleichmäßig tordiert und die Krümmung erfüllt eine Bewegungsgleichung. Geschlossene elastische Raumkurven konstanter Länge treten in Einparameterscharen auf, die einen (eventuell mehrfach überlagerten) ebenen Kreis enthalten.

T. Willmore

The tension field of generalized Gauss maps.

A summary is given of a paper by Cattaneov Romani to be published in the Royal Society of Edinburgh Proceedings 1990. The idea is to generalize the theorem of Ruh-Vilms which says roughly that for an isometric immersion $M^n \to \mathbb{R}^N$, the Gauss map is harmonic if and only if the mean curvature vector is parallel with respect to the normal connexion. Their theorem is the following:

Let $\varkappa : M^n \to \mathbb{R}^N$ be a nicely curved immersion of M^n in $\mathbb{R}^N$. Then for any $k= 0,1,..,N-n-1$ we have

(i) $\nu^k_*(x) = s^k(x) - A^k(x)$

(ii) $\nabla\nu^k_* = \nabla s^k - \nabla A^k + \sigma^{k+1} + \alpha^{k-1}$

where

$\tau(\nu^k) = \text{trace } \nabla\nu^k_*$

$\sigma^{k+1}(X_p,Y_p,\xi^k_p) = s^{k+1}(X_p,s^k(Y_p,\xi^k_p))$

$\alpha^{k-1}(X_p,Y_p,\xi^k_p) = A^{k-1}(X_p,A^k(Y_p,\xi^k_p))$

$X_p,Y_p \in M_p \ , \ \xi^k_p \in \text{Nor}^k M_p$

Here s^k is the second order form of order k, A^k is Weingarten map of order k, and ν^k is the Gauss map of order k, i.e. the map that sends each point p into the k^{th} normal space at p.

This theorem reduces to the Ruh-Vilms theorem for $k = 0$. In a similar way it is possible to refine the theorem of Obata - see Journ. of Diff. Geom. 1967 - which expresses the third fundamental form III in terms of $\langle II(x,y), H\rangle$, the Ricci form of the induced metric and the first fundamental form. We get an analogous result for each value of $k = 0,1,..,N-n-1$.

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L.M. Woodward

A Rigidity Theorem For Minimal Immersions Of Surfaces Into $\mathbb{C}P^n$

Given a minimal immersion $\psi : S \to \mathbb{C}P^n$ of an oriented surface S there is a canonically associated sequence $\{\psi_p\}$ of branched minimal immersions of S into $\mathbb{C}P^n$ called the harmonic sequence of $\psi = \psi_0$. Using this sequence one may introduce globally defined forms on S which vanish identically when ψ is pseudo-holomorphic (i.e. when ψ is an element of the Frenet frame of a holomorphic curve). Using these invariants together with the induced metric and Kähler angle one may prove a rigidity theorem for minimal immersions of a simply connected surface into $\mathbb{C}P^n$ which has the Calabi Rigidity theorem (for the case of holomorphic curves) as a special case. The rigidity theorem may be used to give a simple proof of the classification theorems of Bryant and Kenmotsu for isometric minimal immersion of $\mathbb{R}^2$ into S^n (or more generally $\mathbb{C}P^n$ with constant Kähler angle).

Berichterstatter: Ekkehard-H. Tjaden

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Tagungsteilnehmer

Dr. U. Abresch
Mathematisches Institut
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof. Dr. V. Bangert
Mathematisches Institut
Universität Bern
Sidlerstr. 5

CH-3012 Bern

Prof. Dr. W. Barthel
Mathematisches Institut
der Universität Würzburg
Am Hubland

8700 Würzburg

Dr. U. Brehm
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

Prof. Dr. Yu. D. Burago
Leningrad Branch of Steklov
Mathematical Institute - LOMI
USSR Academy of Science
Fontanka 27

Leningrad 191011
USSR

Prof. Dr. W. Degen
Mathematisches Institut B
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 80 11 407

7000 Stuttgart 80

Prof. Dr. P. Dombrowski
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof. Dr. D. Ferus
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

Prof. Dr. V. M. Gol'dstein
Institute of Mathematics
Siberian Academy of Sciences of the
USSR
Universitetskii pr. 4

Novosibirsk 630090
USSR

Prof. Dr. E. Heil
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Prof. Dr. E. Heintze
Mathematisches Institut
der Universität Augsburg
Memminger Str. 6

8900 Augsburg

Dr. G. Knieper
c/o Prof. Dr. V. Enss
Institut für Mathematik I der
Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Prof. Dr. W. Henke
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

P. Kohlmann
Fachbereich Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. N. Hingston
4, Jeffrey La.

Princeton Jct., N.J.08550
USA

Prof. Dr. O. Kowalski
Department of Mathematics
Charles University
Sokolovska 83

186 00 Praha 8
CZECHOSLOVAKIA

Prof. Dr. H.-C. Im Hof
Mathematisches Institut
Universität Basel
Rheinsprung 21

CH-4051 Basel

Prof. Dr. W. Kühnel
Fachbereich Mathematik
der Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Dr. R. Kellerhals
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

5300 Bonn 3

Prof. Dr. N. H. Kuiper
IHES
Institut des Hautes Etudes
Scientifiques
35, Route de Chartres

F-91440 Bures-sur-Yvette

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Prof. Dr. F. Labourie
Centre de Mathematiques
Ecole Polytechnique
Plateau de Palaiseau

F-91128 Palaiseau Cedex

St. Nölker
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Prof. Dr. K. Leichtweiß
Mathematisches Institut B
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 80 11 407

7000 Stuttgart 80

Prof. Dr. F. Pedit
Dept. of Mathematics
Emory University

Atlanta , GA 30322
USA

E. Leuzinger
Mathematik-Departement
ETH-Zürich
ETH-Zentrum
Rämistrasse 101

CH-8092 Zürich

Prof. Dr. U. Pinkall
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

Prof. Dr. O. Loos
Institut für Mathematik
Universität Innsbruck
Technikerstr. 25

A-6020 Innsbruck

K. Polthier
Mathematisches Institut
der Universität Bonn
Beringstr. 4

5300 Bonn 1

Prof. Dr. M. J. Micallef
Mathematics Institute
University of Warwick

GB- Coventry , CV4 7AL

Dr. H.-B. Rademacher
Mathematisches Institut
der Universität Bonn
Wegelerstr. 10

5300 Bonn 1

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Prof. Dr. H. Reckziegel
Mathematisches Institut
der Universität Köln
Weyertal 86-90

5000 Köln 41

Dr. I. Sterling
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

Dr. V. Schroeder
Mathematisches Institut
der Universität Münster
Einsteinstr. 62

4400 Münster

Prof. Dr. R. Sulanke
Sektion Mathematik der
Humboldt-Universität Berlin
Postfach 1297
Unter den Linden 6

DDR-1086 Berlin

K.-D. Semmler
Dept. de Mathematiques
Ecole Polytechnique Federale
de Lausanne

CH-1015 Lausanne

Prof. Dr. A. Svec
Prehradni 10

635 00 Brno
CZECHOSLOVAKIA

Prof. Dr. U. Simon
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

Prof. Dr. E. Teufel
Mathematisches Institut B
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 80 11 407

7000 Stuttgart 80

Prof. Dr. Gr. Stanilov
Inst. of Mathematics
Bulgarian Academy of Sciences
P. O. Box 373

1090 Sofia
BULGARIA

E.-H. Tjaden
Fachbereich Mathematik / FB 3
der Technischen Universität Berlin
Straße des 17. Juni 135

1000 Berlin 12

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Prof. Dr. L. Vanhecke
Departement Wiskunde
Faculteit der Wetenschappen
Katholieke Universiteit Leuven
Celestijnenlaan 200 B

B-3030 Heverlee

Prof. Dr. K.H. Weise
Reventlouallee 15 a

2300 Kiel 1

Prof. Dr. K. Voss
Mathematik
ETH Zürich
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Prof. Dr. T. J. Willmore
Dale Crest
Crossgate Peth

GB- Durham City,County Durham DH1 4DT

Prof. Dr. R. Walter
Fachbereich Mathematik
der Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Prof. Dr. Woodward
Dept. of Mathematics
The University of Durham
Science Laboratories
South Road

GB- Durham , DH1 3LE

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