MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Linear Operators and Applications

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Die Tagung wurde geleitet von I. Gohberg (Tel Aviv), B. Gramsch (Mainz) und H.H. Schaefer (Tübingen). Im Mittelpunkt der Interessen standen neuere anwendungsbezogene Entwicklungen in der Operatorentheorie auch in Verbindung mit Integralgleichungen, Differential– und Pseudodifferentialoperatoren. Die anregende Atmosphäre hat zu vielen mathematischen Diskussionen geführt, die bereits einige Vorträge beeinflußt haben.

Die allgemeine Operatorentheorie im Banachraum wurde durch Vorträge über Interpolation, invariante Unterräume, Approximationszahlen und der Geometrie der Banachräume vertreten. Weiter gab es Vorträge zur Hilbertraumtheorie, der Theorie der Halbgruppen, der Theorie der Matrixfunktionen, zur Frage nach der Existenz von stetigen linearen Rechtsinversen zu Differentialoperatoren, der Theorie der Funktionenräume, der Pseudodifferentialoperatoren, der ψ^*–Algebren und der Differentialgeometrie in Fréchetalgebren von Operatoren. Schwerpunkte der Vorträge waren unter anderem:

– Reproduzierende Kerne

– Surjektive Isometrien von $L^1 \cap L^\infty [0,\infty)$ und $L^1 + L^\infty [0,\infty)$

– Berezin–Toeplitz–Quantisierung

– ψ^*–Störungstheorie

– Hahn–Zerlegung operatorwertiger Maße

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– Interpolationsprobleme

– Komplettierung und maximale Entropie

– Singuläre Störungstheorie

– Wirtschaftstheorie

– Biomathematik

– Eigenfunktionsentwicklungen

Vortragsauszüge

E. ALBRECHT. Spectra of Linear Operators on Interpolation Spaces.

Let $X=(X_0,X_1)$ and $Y=(Y_0,Y_1)$ be two compatible pairs of Banach spaces and write $X_{\theta,q}$, $Y_{\theta,q}$ for the corresponding real interpolation spaces ($0<\theta<1$, $1\leq q\leq\infty$). If $T_j\in\mathscr{L}(X_j,Y_j)$ ($j=0,1$) satisfy $T_0|X_0\cap X_1=T_1|X_0\cap X_1$ we denote the operator obtained by the real interpolation method in $\mathscr{L}(X_{\theta,q},Y_{\theta,q})$ by $T_{\theta,q}$. Fix $0<\theta<1$.

Theorem 1: If $X=Y$ then the mapping $q\mapsto\sigma(T_{\theta,q})$ is constant on $[1,\infty]$.

This improves an earlier result by M. Zafran (1980). For the essential spectrum we have

Theorem 2: If $X=Y$ then the mapping $q\mapsto\sigma_e(T_{\theta,q})$ is constant on $[1,\infty)$.

This is a direct consequence of the following

Theorem 3: For $T=(T_0,T_1)$ are equivalent:

(a) For some $p\in[1,\infty)$ the operator $T_{\theta,p}$ is Fredholm.

(b) For all $p\in[1,\infty]$ the operator $T_{\theta,p}$ is Fredholm. Moreover, in this situation we have $\ker T_{\theta,p}=\ker T_{\theta,q}$ for all $p,q\in[1,\infty]$ and there is some finite dimensional subspace H_θ of $Y_{\theta,1}$ such that $Y_{\theta,q}=H_\theta\oplus\operatorname{ran} T_{\theta,q}$ for all $q\in[1,\infty]$.

The results have been obtained in joint work with K. Schindler (Saarbrücken).

KHRISTO N. BOYADZHIEV. A Trace Formula for Commutators of Unitary Operators.

Theorem: If U,V are two unitary operators with dim [U,V]=n, then there exists a real measurable bounded function f(t,s) defined for $0 \le t,s \le 2\pi$, $|f(t,s)| \le n$, such that

$$(*)\quad tr[p(U,V),q(U,V)] = \frac{1}{2\pi i}\int_0^{2\pi}\!\!\int \frac{\partial(p,q)}{\partial(t,s)}(e^{it},e^{is})f(t,s)dtds$$

holds for any two polynomials p,q, $p(e^{it},e^{is})=\Sigma\, a_{n,m}e^{int}e^{ims}$, q=similar where $p(U,V)=\Sigma a_{n,m}U^nV^m$, q(U,V)=similar. The function f is given by the formula $f(t,s)=\lim_{r,\rho\to1-}\frac{1}{2\pi i}\log\ \det(U_\lambda V_\mu U_\lambda^{-1}V_\mu^{-1})$ where $\lambda=re^{it}$, $\mu=\rho e^{is}$, $U_\lambda=(U-\bar{\lambda})(1-\lambda U)^{-1}$, $V_\mu=(V-\bar{\mu})(1-\mu V)^{-1}$. This theorem is given a short proof, independent of Larey–Pincus–Helton–Howe's results. The formula (*) implies (via Cayley's transform) a Helton–Howe type formula for pairs of (unbounded) self–adjoint operators.

I. CIORANESCU. Exponential Vectors and Analytic Semigroup Generators.

Let A be an unbounded linear closed operator on the Banach space X; for v>0 we define $X_v=\{x\in X; ||A^kx||\le cv^k, k=0,1,2,...\}$. The space X_v is a Banach space with the norm $||x||_v=\sup_{k\ge0}||A^kx||/v^k$ and its elements are called vectors of exponential type ≤v. We denote $Exp_A=\cup_{v=1}^{\infty}X_v$ and call its elements exponential vectors of A. We note that the sequence $\{X_v\}_{v>0}$ is a Banach–space scale for A. Our main result is:

Theorem: A closed operator A is the generator of an uniformly bounded cosine function iff A generates a bounded analytic semigroup for Re z>0 and $(Exp_A)^{-}=X$.

Application is made to the case of almost–periodic cosine functions, namely:

Corollary: A closed operator A is the generator of an almost–periodic cosine function iff A generates a bounded analytic semigroup for Re z>0 and the set of its eigenvectors is total in X.

L. A. COBURN. The Berezin–Toeplitz Quantization and Geometry on Symmetric Spaces.

For Ω a bounded domain in $\mathbb{C}^n$ with normalized volume measure or $\Omega = \mathbb{C}^n$ with Gaussian measure, consider the L^2 spaces and the associated subspaces, H^2, of holomorphic functions.

For P the orthogonal projection operator from L^2 onto H^2 and M_f the (densely–defined) operator of multiplication by a fixed L^2 function f, the Toeplitz operators on H^2 given by $T_f=PM_f|_P$ and the Hankel operators on L^2, $H_f=(I-P)M_fP$ are of considerable interest. I discuss recently obtained results on the boundedness, compactness and symbol calculus for these operators and their relation to pseudo–differential operators.

R. W. CROSS. Linear Transformations of Tauberian Type in Normed Spaces.

Let $T:D(T)\subset X\to Y$ be a linear transformation where X and Y are normed spaces. The operator T is called Tauberian if $(T'')^{-1}(Q\hat{Y})\subset D(T)^\wedge$ where Q is the quotient map defined on Y" with kernel $D(T')^\perp$. In case of operators in Banach spaces such operators were first studied by N. Kalton and A. Wilansky, Proc. Amer. Math. Soc. 57 (1976), 251–255. This article investigates the basic properties of these operators and characterises Tauberian operators T whose adjoints are continuous (for instance, when T is partially continuous, or when D(T)is complete). For example, let T' be continuous. Then T is Tauberian if and only if for all bounded sets B of D(T), B is relatively $\sigma(D(T),D(T)')$ compact whenever TB is relatively $\sigma(Y,D(T'))$compact. It is shown that if T is Tauberian and dim $T^{-1}(0)<\infty$ then T' is a ϕ_-–operator and that the converse statement is true in the case when T is continuous.

H. DYM. Nevanlinna Pick Interpolation and Reproducing Kernel Hilbert Spaces for Upper Triangular Operators.

Let $\mathcal{X}$ denote the space of bounded linear operators on the sequence space $l^2_{\mathcal{N}}$ of $f=\{\ldots,f_{-1},f_0,f_1,\ldots\}$ with f_i in a complex separable Hilbert space $\mathcal{N}$. For $A\in\mathcal{X}$ let A_{ij} denote the operator from $\mathcal{N}$ into itself which is defined by the rule $(Af)_i=\Sigma_{j=-\infty}^{\infty} A_{ij}f_j$ and let

$$U = \{A\in\mathcal{X}: A_{ij}=0 \text{ for } i>j\} \text{ (= upper trianglar)}$$

$$\mathcal{D}= \{A\in\mathcal{X}: A_{ij}=0 \text{ for } i\neq j\} \text{ (= diagonal).}$$

Also, let Z denote the shift: $(Zf)_i=f_{i+1}$. Z is unitary and for every $F\in U$, there exists a unique set $F_{[0]},F_{[1]},\ldots$ in $\mathcal{D}$ such that $F-\Sigma_{j=0}^{n-1} Z^iF_{[j]} \in Z^nU$.

An important role in the development is played by the "transform" $\hat{F}(V)=\Sigma_{j=1}^{\infty} V^{[j]}F_{[j]}$ of $F\in U$, where $V\in\mathcal{D}$, $V^{[j]}=VV^{(1)}\cdots V^{(j-1)}$ (for $j\geq 2$ with $V^{[1]}=V$, $V^{[0]}=I$) and $V^{(t)}=Z^{*t}VZ^{t}$ (for $t=0,\pm1,\pm2,\ldots$).The transform is well defined when $l_V:=\lim_{n\uparrow\infty} \|V^{[n]}\|^{1/n}$ is less than one. A major step in the development of the theory was the discovery of the analogue $U_V=(Z-V)(I-L_V V^* Z)^{-1}L_V^{-1/2}$ of the Blaschke product in the present setting. Here L_V is a certain positive definite invertible operator in $\mathcal{D}$, which reduces to the identity in the special case that V is Toeplitz and normal. The recipe for L_V and full details of the analogues of the lossless inverse scattering problem, reproducing kernel Hilbert spaces with special forms of kernel of the type introduced by de Branges and of the Nevanlinna Pick interpolation problem in the present setting will appear in a joint paper with D. Alpay and P. Dewilde.

J. ESCHMEIER. Operators with Rich Invariant Subspace Lattices.

In 1987 S. Brown proved that each hyponormal operator with thick spectrum has a non–trivial invariant subspace. In the same year E. Albrecht and B. Chevreau showed that under a slightly stronger condition on the spectrum more generally each operator occuring as a restriction or a quotient of a decomposable operator and acting on a quotient of closed subspaces of l^p ($1<p<\infty$) has a non–trivial invariant subspace. In the intended talk it will be indicated that the same result is true on completely arbitrary Banach spaces and even under the original, weaker, richness condition on the spectrum used by S. Brown. If the richness conditions are posed for the essential spectrum instead of the spectrum, one obtains operators with extremely rich invariant subspace lattices. Some applications to the problem, whether certain classes of operators are reflexive in the sense of Sarason, will be given.

K.–H. FÖRSTER. The Spectral Theory of Operator Polynomials with Non–Negative Operators.

Let $A_0, A_1, \ldots, A_{l-1}$ be linear operators in a vector space ($l\in\mathbb{N}$), then the monic operator polynomial $L(\lambda)=\lambda^l-\lambda^{l-1}A_{l-1}-...-\lambda A_1-A_0$ and the companion matrix operator C have the same point spectrum. We prove the follwing

Theorem: Let E be a Banach lattice, $A_0,...,A_{l-1}$ be linear compact positive operators in E such that $r=r(L)=r(\tau)>0$. Define the positive compact operator $S_r=A_{l-1}+\frac{1}{r}A_{l-2}+...+\frac{1}{r^{l-2}}A+\frac{1}{r^{l-1}}A_0$; then the follwing 4 assertions hold:

1) $r(S_r)=r(L)=r \in \sigma_p(L)\cap\sigma_p(C)\cap\sigma_p(S_r)$.

2) r is a pole of the resolvent of S_r and of L^{-1} of the same order; let p be this mumber.

3) $\dim N((r-S_r)^p)=\dim N((r-C)^p)=\dim\{x\in E$:it exists an L–chain $x_0,x_1,...,x_k=x$ to $r\}$

4) Each L–chain $x_0, x_1,...,x_k$ to r with $x_0\neq 0$ is linear independent.

None of these statements is true if the assumption of the positivity of the coefficients $A_0,...,A_{l-1}$ is dropped. This is a joint work with B. Nagy (TU Budapest).

L. S. FRANK. Linear Dispersive Singular Perturbations.

Dispersive perturbations appear in different fields of Pure and Applied Mathematics and are an essential part in the propagation of waves theory. They are characterized by the absence of viscosity terms and, thus, describe dynamical systems with conservation laws. The reduced operators are hyperbolic (in the sense of I.G. Petrovski). Solutions to the perturbed problems exhibit an asymptotic behaviour which in some regions is characterized by fast oscillations and in other regions solutions are exponentially small as the parameter vanishes. These and other aspects concerning this class of operators will be discussed in the talk.

I. GOHBERG. Interpolation Problems for Rational Matrix Functions.

The talk is planned to be a review of the book of J. Ball, I. Gohberg and L. Radman "Interpolation for rational matrix functions", which is in the last stage of preparation.

This book presents the theory as a recently matured independent mathematical subject with its own problems, methods and applications. The realization approach which comes from systems theory serves as a tool to reduce interpolation problems to problems about operators and matrices and helps to get explicit expressions for solutions. Contained in the book are the matrix valued rational function analogues of the main classical problems as: Lagrange–Sylvester, Nevanlinna–Pick, Carathéodory–Toeplitz, Nehari and others.

The book includes applications in modern systems theory and control. Special attention in the talk will be paid to the analysis of the model reduction problem.

B. GRAMSCH. Local Differential Geometry in Fréchet Algebras of Operators.

Starting from results of Salinas (1988) on the geodesics in Grassmann spaces of a C^*–algebra also the case of special Fréchet algebras of operators can be considered. We derive some results for the locally rational manifolds in the perturbation theory of C^*–algebras and ψ^*–algebras concerning the local existence of geodesics for these infinite dimensional spaces. For Fréchet algebras continuously embedded in a Banach algebra some consequences of the spectral invariance and the submultiplicativity are discussed. Ideas from Hamilton (BAMS 1982) also concerning covariant differentiation go into the method. The main examples to treat are the set of idempotent elements, the set of Fredholm operators $\phi_{m,n}$ and $C^\infty(\Omega,M_{n,k})$, where $M_{n,k}$ is the set of (nxn)–matrices of rank k. This work is done (partly) in collaboration with K. Lorentz and J. Scheiba.

H. ISAEV. Multidimensional Complex–Analytical View on the Joint Singularity of the System of Multiparameter Operators and its Applications.

We consider the multiparameter spectral (MPS) system $P(\lambda)=(P_1(\lambda),\dots,P_n(\lambda))$, $\lambda=(\lambda_1,\dots,\lambda_n)\in\mathbb{C}^n$ where $P_j(\lambda)=A_j-\lambda_1 B_{j1}-\dots-\lambda_n B_{jn}$, $j=1,2,\dots,n$ acts in the Hilbert space $\mathcal{H}_j$, $j=1,2,\dots,n$. Let $A_1,\dots,A_{n-1}$ are self–adjoint operators and their resolvents are compact. We assume that $\exists$ $\alpha>0$: $\det\{(B_{jk}x_j,x_j)\}_{j,k=1}^{n}\geq\alpha>0$ $\forall x_j\in\mathcal{H}_j$, $\|x_j\|=1$.

Theorem: $\sigma[P_j(\lambda)]\cap\mathbb{R}^n$ consists of at most countably number analytic surfaces in $\mathbb{R}^n$. $\sigma[P_1(\lambda)]\cap\dots\cap\sigma[P_{n-1}(\lambda)]\cap\mathbb{R}^n$ consists of at most c. n. a. curves intersecting at most c. n. of the points which are not accumulate at the finite part of $\mathbb{R}^n$.

Because of this theorem we use the spectral measures of $P_j(\lambda)$, $\lambda\in\mathbb{R}^n$ to construct the joint spectral measure.

H. JARCHOW. Composition Operators on Classical Hardy Spaces.

It is well–known that every analytic function φ: $D\to D$ ($D=\{|z|<1\}$) induces, via $f\to f\circ\varphi$, a bounded linear map $C_\varphi:H^p\to H^p$ for all values of p. This talk reports about some results concerning the general problem of how properties of φ are reflected by properties of C_φ, and vice versa. The results are mainly in terms of a Carleson measure canonically associated with φ and in terms of the behaviour of the function $p\to\|\varphi\|^p_{H^p}$ or, equivalently, of the sequence $(\|\varphi^n\|_{H^1})$. For example, if $\beta\geq 1$ and $(\|\varphi^n\|_{H^1})\in l_{1/\beta,\infty}$, then C_φ even maps H^p into $H^{\beta p}$ (for all p); and this result is sharp. Moreover, $(\|\varphi^n\|_{H^1})\in l_{1/\beta,1}$ is equivalent to C_φ being a majorizing operator $H^p\to L^{\beta p}(\frac{dt}{2\pi})$, or else to $(1-|\varphi(e^{it})|)^{-\beta}$ being integrable, or else to C_φ being a $(1+\beta)$–nuclear operator $H^{1+1/\beta}\to H^{1+\beta}$, etc.

M. A. KAASHOEK. A General Framework for Extension Problems and Maximum Entropy.

This talk concerns positive and contractive extension problems, of Carathéodory–Toeplitz and Nehari type, for matrix functions and infinite operator matrices. A general scheme for dealing with such problems is discussed. In the setting of the general scheme a linear fractional description of all solutions is given. Also an abstract maximum entropy principle is derived. The talk reports on joint work with I. Gohberg and H. J. Woerdeman.

W. KABALLO. Small Ideals of Nuclear Operators and Some Applications.

In this talk ideals of φ–nuclear (N_φ) and φ–approximable (S_φ) operators are considered, where φ is a special function $\psi_{p,\gamma}(t)=(\log\frac{1}{t})^{-p}(\log\log\frac{1}{t})^{-\gamma}$, t near 0, where $0<p<\infty$, $\gamma\in\mathbb{R}$, or $p=0$, $\gamma>0$. These ideals occur in a decomposition theorem for meromorphic Fredholm resolvents in N variables due to B. Gramsch–W. Kaballo (1989), which holds for $N_{\psi_{p,0}}$,

$p>N-1$, but fails for $N_{\psi_{N-1,0}}$. S. Castello (1989) remarked that the theorem also holds or $N_{\psi_{N-1,\gamma}}$, $\gamma>1$. Here the theorem is also proven for $S_{\psi_{N-1,\gamma}}$, $\gamma>1$ by just showing $S_{\psi_{p,\gamma}}=N_{\psi_{p,\gamma}}$ for all p, γ. Then this and H. Königs Weyl–inequality (1977) are used to prove that the eigenvalues of an operator $T\in N_{\psi_{p,\gamma}}(X)$ satisfy $\sum_{n=1}^{\infty}\psi_{p,\gamma}(|\lambda_n(T)|)<\infty$. Similar results are obtained for $\Lambda_R(n^{1/t})$–nuclear operators. For integral operators $Tf(x)=\int_K k(x,y)f(y)dy$, $K=cl(\text{int } K)\subset\mathbb{R}^N$ compact, with kernels $k\in\mathcal{E}^s(K;L_q(K))$ satisfying a Gevrey condition in the first variable, the eigenvalues decay as $|\lambda_n(T)|\le C_\alpha e^{-\alpha n^{1/sN}}$, $\alpha>0$. Here a recent result of J. Bonet, R.W. Braun, R. Meise and B.A. Taylor on the surjectivity of restriction $\mathcal{E}^s(\mathbb{R}^N)\to\mathcal{E}^s(K)$ is used.

H. KÖNIG. Fourier Coefficients of Vector–Valued Functions.

We study the decay of the Fourier–coefficients of functions on the circle with values in a Banach space X. Differentiable functions generally have absolutely summable Fourier–coefficients if and only if X is K–convex. In the scalar case X= K, by Bernstein's classical result, much less, namely α–Hölder–continuity with $\alpha>1/2$ suffices. More precise statements on the decay of the (norms of the) Fourier–coefficients can be given if the space X has Fourier–type p, i. e. a Hausdorff–Young–type inequality holds. If a function $f:T^m\to X$ then belongs to the Besov–space $B^{\lambda}_{u,v}(X)$, the sequence of norms of the Fourier coefficients belongs to the Lorentz sequence space $l_{t,v}$ where $1/t=\lambda/m+1/\max(u',p')$. This result is the best possible in the vector–valued case and generalizes the well–known scalar results.

L.E. LABUSCHAGNE. Characterisations of Partially Continuous, Strictly Cosingular and ϕ Type Operators.

Let X and Y be normed linear spaces and let L(X,Y) denote the class of all linear operators T with domain D(T) a subspace of X and range a subspace of Y. In [1] und [2] R.W. Cross defined a partially continuous operator to be an operator which is continuous on some finite codimensional subspace of its domain and showed that an operator T exhibits this property if and only if each infinite dimensional subspace M of D(T) contains an infinite dimensional subspace N on which T is continuous. In this present paper we establish analogous characterisations of partially continuous, strictly cosingular and a class of ϕ_type operators in terms of *closed infinite codimensional* subspaces of Y. As an application we show that under certain conditions all unbounded strictly cosingular operators are at least partially continuous.

1. R.W. Cross, Properties of some norm related functions of unbounded linear operators, Math. Z. 199(1988), 285–302.

2. R.W. Cross, Some continuity properties of linear transformations in normed spaces, Glasgow Math. J. 30(1988), 243–247.

H. LANGER. Definitizing Polynominals of Hermitian Operators in Pontrjagin Spaces (jointly with Z. Sasvári).

Let A be a Hermitian operator in a π_κ–space Π_κ with indefinite inner product $[\cdot,\cdot]$. A monic polynomial $p=q\bar{q}$ with some polynomial q ($\bar{q}(\lambda)=\sum_{j=0}^{n}\bar{\alpha}_j\lambda^j$ if $q(\lambda)=\sum_{j=0}^{n}\alpha_j\lambda^j$) is definitizing for A if $[p(A)x,x]\geq 0$ ($x\in D(p(A))$). A selfadjoint operator A has a unique definitizing polynomial of minimal degree ($\leq 2\kappa$), which divides any other definitizing polynomial (Iohvidor/Krein). If A is essentially selfadjoint and some additional condition is satisfied, it has unique definitizing polynomial of minimal degree, but it can have more irreducible definitizing polynomials of higher degree. Also the case of a not essentially selfadjoint Hermitian operator is considered, and applications to the indefinite Hamburger moment problem and to the continuation problem for indefinite Hermitian functions on $[-2a,2a]$ are considered.

K. LORENTZ. Similarity Orbits in ψ^*–Algebras.

Using the characterization of those Hilbert space operators which admit a norm–continuous local cross section for the similarity operation by the set of nice Jordan operators done by D.A. Herrero, E. Andruchow and D. Stojanoff proved the existence of a holomorphic local structure on the similarity orbit of a nice Jordan operator. We carry these kind of results over to the case of a ψ^*–algebra using algebraic methods of B. Gramsch (1984). One main idea of this approach is to consider the homogeneous topology instead of the algebra–topology. As a consequence we can give to the similarity orbit of a general Jordan operator a very nice local structure, namely a locally rational one (notation of B. Gramsch), by constructing rational local cross sections. Moreover, the homogeneous topology can be characterized as a special kind of "gap"–topology.

G. LUMER. Generalized Solutions, Integrated Semigroups, Applications to Biomathematics and Engineering.

Let X be a Banach space. We consider the general inhomogenous evolution problem

$$(*)\ u'=Au+F(t),\ u(0)=f,\ (F\in L^1_{loc}([0,+\infty[),$$

where A is a linear operation in X, closed, and such that 0 is a unique solution of $u'=Au$, $u(0)=0$. For $n\geq 1$ (here n is integer, but one can also work with n real), we say that $\exists$ an n–strong generalized solution (n–s. g.s.) v_n, and say then that v_n' is a (n–1)–mild generalized solution ((n–1)–m. g.s.), iff $\exists$ v_n classical solution of: $v_n'(t)=Av_n(t)+(t^{n-1}/(n-1)!)f+F_n(t)$, $v_n(0)=0$, where $F_n(t)=\int_0^t((t-s)^{n-1}/(n-1)!)F(s)ds$; 0–s. g.s. means classical solution. $Z_n=\{f\in X:\exists$ a n–s. g.s., $v_n(t,f)=v_n$ of $(*)$ with $F=0$ and $u(0)=f\}$; for $f\in Z_n$ set $\varphi_{n-1}=v_n'(t,f)$. One can show under rather mild assumptions on F(t), that for $f\in Z_{n+1}$ (n=0,1,2,...) the formula

$$(1)\qquad w_n(t) = \varphi_n(t) + \int_0^t \varphi_n(t-s)F(s)ds$$

gives a n–m. g.s. of the inhomogenuous equation (*). Using this one can show that: $\varphi_n(t):Z_{n+1}\to Z_{n+1}$, and on Z_{n+1}

$$\varphi_n(t)\varphi_n(s)=\varphi_{2n}(t+s)-\sum_{k=0}^{n-1}\frac{1}{k!}(s^k\varphi_{2n-k}(t)+t^k\varphi_{2n-k}(s)).$$

Also using (1) certain general perturbation results (A into A+B) can be obtained. We then consider in particular locally lipschitz integrated semigroups (Z_2=X, $t\longrightarrow\|\varphi_1(t)\|$ loc. lipschitz), and their explicit representation (after replacing A by A–ω, some ω>0, if needed):

(2) $S(t)=(e^{tA_0}-1)A^{-1}$, (now we use S(t) instead of $\varphi_1(t)$),

where A_0 is the part of A in $\overline{D(A)}$. We show how this applies immediately to solving, via bounded perturbation of A, problems of age–structured population dynamics of renewal type (where A is now replaced by A^* dual of a semigroup generator, in X^*, or by $A^*+B^*-\omega$, in (2) with X replaced by $X^* - X^*$ being, say, $(C_0([0,1[))^*$ and $D(A^*)^-=L^1([0,1[))$. Another application is made to diffusion problems in, say, $C(\overline{\Omega})$, with discontinuous boundary behavior (like in a heat equation where $U(0,\cdot)=f\in C(\overline{\Omega})$, $f\neq0$ on $\partial\Omega$, but $u(t,\cdot)=0$ on $\partial\Omega$ for all $t>0$); these can be solved via integrated semigroups and explicit solutions obtained from (2) in even rather general situations; strong results on the behavior of $u(t,\cdot)$ as $t\downarrow0$ are obtained. Still other applications can be given for equations of type (*) with operators A not densely defined.

F. MANTLIK. Differentiable Bundles of Subspaces of a Banach Space.

The concepts of R. Janz [1], [2] form an elegant framework for the study of unbounded operators which depend on a parameter. Motivated by this we investigate bundles of closed linear subspaces of a Banach space which are "differentiable" in a suitable sense. As a consequence it is possible to solve the vector function equation T(x)e(x)=f(x) for differentiable data, where the operator function T takes its values in the set of closed linear operators from a Banach space E into a second Banach space F. This extends earlier results for bounded operators T(x) which have been established by the author in [3], [4].

[1] R. Janz: Holomorphic families of subspaces of a Banach space; Operator Theory: Advances and Applications 28 (1988), 155–167.

[2] R. Janz: Perturbation of Banach spaces; (to appear).

[3] F. Mantlik: Linear equations depending differentiably on a parameter; Integral Equations and Operator Theory; (to appear).

[4] F. Mantlik: Isomorphic classification and lifting theorems for spaces of differentiable functions with Lipschitz conditions; (to appear).

R. MEISE. Phragmén–Lindelöf Conditions and Continuous Linear Right Inverses for Partial Differential Operators (joint work with B.A. Taylor (Ann Arbor) and D. Vogt (Wuppertal)).

For a non–constant polynomial $P \in \mathbb{C}[z_1,\dots,z_n]$ let $V:=\{z \in \mathbb{C}^n : P(-z)=0\}$. Then P satisfies the Phragmén–Lindelöf condition PL if the following holds:

$$\forall r>0 \;\exists R>r \;\forall \rho>R \;\exists A>0 \;\forall u \in PSH(\mathbb{C}^n):\; (\alpha) \wedge (\beta) \Rightarrow (\gamma)$$

(α) $u(z) \le r|\mathrm{Im}\, z| + O(\log(1+|z|)) \quad \forall z \in \mathbb{C}^n$

(β) $u(z) \le \rho|\mathrm{Im}\, z| \quad \forall z \in V$

(γ) $u(z) \le R|\mathrm{Im}\, z| + A \log(1+|z|) + A \quad \forall z \in V$

P satisfies APL if the above holds for all $u=\log|f|$, f an entire function. The following theorem and some of its consequences were presented:

Theorem: For $P \in \mathbb{C}[z_1,\dots,z_n]$ t. f. a. e.

1) P(D) admits a continuous linear right inverse on $\mathcal{E}(\mathbb{R}^n)/D'(\mathbb{R}^n)$

(2) P satisfies APL

(3) P satisfies PL.

R. MENNICKEN. Expansion of Analytic Functions in Berson Series and Carlitz Series.

F.W. Schäfke et al. identified several expansions of analytic functions in series of higher transcendental functions (Bessel functions, Whittaker functions, Legendre functions, hypergeometric functions, confluent hypergeometric functions, Mathieu functions and spheroidal wave functions) as eigenfunction expansions of certain boundary eigenvalue problems for differential equations in the complex domain. The common feature of all these expansions is the fact that the corresponding boundary value problems are linear in the eigenvalue parameter λ. W. Krimmer, R. Mennicken and J. Karl recently studied expansions in series of special functions, which are related to λ–nonlinear boundary eigenvalue problems, and developed a thorough spectral theory of such eigenvalue problems.

In the present lecture this theory is applied to certain expansions of analytic functions in series of Bessel functions and products of such functions. Expansions of this type were established by F.Ja. Berson in 1976 and L. Carlitz in 1962. These expansions are related to quadratic operators pencils in certain spaces of holomorphic functions. They are linearized by appropriate transformations. The lecture presents recent results due to H. Langer, R. Mennicken, M. Möller and A. Sattler.

R. NAGEL. Operator Matrices.

Systems of linear evolution equations with values in a product $\mathcal{E}:=E_1 x...xE_n$ of Banach spaces E_i lead to operator matrices $A=(A_{ij})_{nxn}$ with unbounded entries $A_{ij}:E_j \longrightarrow E_i$. The following basic questions are addressed:

1. What is an appropriate domain making A a closed operator on $\mathcal{E}$?
2. How can one compute the spectrum of A from the spectrum of the entries A_{ij}?
3. When does A generate a strongly continuous semigroup?
4. When is this semigroup positive or/and stable?

Some answers can be found in Math. Z. 201, 57–68 (1989).

A. PIETSCH. Approximation Numbers of Nuclear Operators and Geometry of Banach Spaces.

For every Banach space E the n.th Grothendieck number is defined by

$$\Gamma_n(E):=\sup\left\{ |\det(\langle x_i,a_j\rangle)|^{1/n} \;\begin{matrix} \|x_1\|\leq 1,...,\|x_n\|\leq 1 \\ \|a_1\|\leq 1,...,\|a_n\|\leq 1 \end{matrix} \right\},$$

where $x_1,...,x_n \in E$ and $a_1,...,a_n \in E'$. Given $0\leq\rho\leq 1/2$, we let

$$\Pi_\rho:=\{E:\Gamma_n(E)\leq cn^\rho \text{ for some } c\geq 1\}.$$

It turns out that $\Pi_{1/2}$ consists of all Banach spaces, while Π_0 is the class of all weak Hilbert spaces, in the sense of G. Pisier.

Theorem: let $E\in\Pi_\alpha$ and $F\in\Pi_\beta$. Then for every nuclear operator T from E into F, we have

$$a_n(T)\le\frac{c}{n^{1-(\alpha+\beta)}}.$$

D. PRZEWORSKA–ROLEWICZ. Algebraic Analysis.

Let X be a Banach space (over $\mathbb{R}$ or $\mathbb{C}$). Let $D\in L(X)$ be a right invertible operator with $\dim\ker D=1$. Let R be a bounded right inverse of D such that $X=X^+\oplus X^-$, $X^+=\mathrm{cl}(\mathrm{lin}\{R^{2k}e:k\in\mathbb{N}_0\})$, $X^-=\mathrm{cl}(\mathrm{lin}\{R^{2k+1}e:k\in\mathbb{N}_0\})\cup\{0\}$, $e\in\ker D$. Let $A\in L_0(X)$ be bounded. Then $DA=AD$ on dom D if and only if there is a scalar a such that $A=aI$.

If X is a real Banach space then $D^nA=AD^n$ on dom D^n $(n\in\mathbb{N})$ if and only if

$$A = a_0I, \quad \text{when } n = 2m+1$$
$$A = a_0I + b_0S + c_0R(I+S), \quad \text{when } n = 2m, \qquad (m\in\mathbb{N})$$

where S is an involution defined as follows:

$$Sx = \begin{cases} x \text{ for } x\in X^+ \\ -x \text{ for } x\in X^- \end{cases}$$

A.C.M. RAN. Matrix Polynomials with Prescribed Zero Structure in the Finite Complex Plane.

We consider the following inverse problem for a matrix polynomial. Given a full range pair of matrices (A,B), A is nxn, B is nxm; a number $\alpha\notin\sigma(A)$ and an invertible matrix D, construct all matrix polynomials such that

(1) (A,B) is a zero pair for $L(\lambda)$,

(2) $L(\alpha) = D$,

(3) $L(\lambda)^{-1}$ is analytic at infinity.

It turns out that $L(\lambda)$ satisfies (1), (2), (3) if and only if

$$L(\lambda)=D+(\lambda-\alpha)\sum_{j=0}^{\omega-1}\lambda^jFT^j(\alpha-A)^{-1}BD,$$

where (T,F) is a pair of matrices such that AT+BF=I and T is nilpotent of order ω.The equation AT+BF=I is analysed further, in particular all possiblities for the invariant polynomials of T are characterized.

W. RUESS. Ergodic Theorems for Periodic Evolution Systems.

The subject of the talk is the result (joint work with W.H. Summers, Israel J. Math. 64 (1988), J. Functional Analysis, to appear) on the asymptotic behaviour of solutions to periodic evolution systems $u'(t)+A(t)u(t)\ni f(t)$ $(t>0)$, $u(0)=u_0$, ascerting that under certain conditions (both in the linear and the nonlinear case) the solution u decomposes into the sum $u=U(\cdot+\tau,0)y+\varphi$ of an almost periodic motion $U(\cdot+\tau,0)y:\mathbb{R}^+\to X$ and an Eberlein–weakly almost periodic function $\varphi\in W_0(\mathbb{R}^+,X)$. This result improves on the corresponding (nonlinear) ergodic theorems by J.B. Baillon, H. Brézis, F. Browder, R.E. Bruck, A. Pazy, S. Reich and others.

Two problems related to the techniques of proof and the asymptotic behaviour of $\varphi\in W_0(\mathbb{R}^+,X)$ are being discussed:

(I) Measure of non–affineness of non–expansive operators in "nice" Banach spaces: true for uniformly convex X, not true for $L^1[0,1]$ and l_1^4.

(II) Characterizations of unitary operators $T\in B(L^2[0,1])$ $(E\subset\mathbb{N})$ such that $(T^{\cdot}f,f)\in W_0(\mathbb{N})$, $f\in L^2[0,1]$, $(\chi_E\in W_0(\mathbb{N}))$ with a prescribed type of vanishing at infinity.

H.H. SCHAEFER. Surjective Isometries of $L^1\cap L^\infty$ and L^1+L^∞ over $[0,\infty)$.

Let m denote Lebesgue measure on $\mathbb{R}_+=[0,\infty)$. As usual, L^1+L^∞ is supplied with the norm $\|h\|:=\inf(\|f\|_1+\|g\|_\infty)$ where the inf is taken over all $h=f+g$ $(f\in L^1,\ g\in L^\infty)$. Through an explicit determination of the extreme boundary of the unit ball $W=\{h:\|h\|\le 1\}$, it is shown that every surjective isometry of L^1+L^∞ induces a surjective isometry on each of L^1, L^∞, and $L^1\cap L^\infty$ (with respect to the appropriate norms). By a recent result of R. Grzaslewicz that determines the isometries of $L^1\cap L^\infty$ onto $L^1\cap L^\infty$ (norm $\|k\|=\max(\|k\|_1,\|k\|_\infty)$), all these isometries are of the form

$$Tf(s)=r(s)f(\varphi(s)),$$

where r is measurable and $|r|\equiv 1$, φ a bimeasurable, m–preserving transformation of $\mathbb{R}_+$ onto itself.

E. SCHROHE. Boundedness and Invertibility for Pseudodifferential Operators on Anisotropically Weighted L^p–Sobolev Spaces.

Pseudodifferential operators (pdo) with symbols in one of the Hörmander classes $S^0_{\rho\delta}$, $0\le\delta\le\rho\le 1$, $\delta<1$, form a Fréchet algebra of bounded operators on each of the L^p–Sobolev spaces $H^s_p(\mathbb{R}^n)$, $s\in\mathbb{R}$, $1<p<\infty$ (for $p\neq 2$ assume $\rho=1$). For $\rho>0$, these algebras are spectrally invariant: if one of these operators has an inverse in $\mathcal{L}(H^s_p)$, then the inverse is again a pdo with a symbol in the same class. So they form ψ–algebras in the sense of Gramsch (1984) in $\mathcal{L}(H^s_p)$.

For a function $\gamma\in C^\infty(\mathbb{R})$ with $\gamma(x)\ge c>0$ and $\partial^\alpha\gamma$ bounded for all $\alpha\neq 0$ define the weighted space $H^{st}_{p\gamma}=\{\gamma^{-t}u: u\in H^s_p\}$, $t\in\mathbb{R}$. One can then show that the pdo's mentioned above are all bounded on $H^{st}_{p\gamma}$. This is a property that not even so closely related an operator as the Hilbert transform on $\mathbb{R}$ has; cf. Coifman and Fefferman 1974. If in addition $\rho=1$, then, these operators form ψ–algebras in $\mathcal{L}(H^{st}_{p\gamma})$. Moreover, the spectrum of an operator in one of these classes is completely independent of the choice of s, t, p and γ.

H.G. TILLMANN. Infinitedimensional Commodity Spaces in Mathematical Economies.

We consider $\mathcal{X}=l^\infty(X)$ as a global commodity space for an economy with unlimited time horizon, $X(\tau_0)$ is a loc. convex space (time independent commodity space), τ_0 the Mackey topology for the duality $\langle X,Y\rangle$, $Y=X(\tau_0)'$. For $\bar{x}=(x_0,\dots,x_n,\dots)$ we denote the tale $(0,\dots,0,x_n,x_{n+1},\dots)$ by $\hat{x}_n$ and for $x\in X$ is $\iota_n(x)=(0,\dots,0,\underset{(n)}{x},0,\dots)$ the natural imbedding of X in $\mathcal{X}$.

A topology τ on $\mathcal{X}$ is called regular, if $\iota_n: X(\tau_0)\longrightarrow\mathcal{X}(\tau)$ is continuous. We follow Brown–Lewis: Econometica (1981):

Def.1: A preference relation $<$ is myopic, iff $\bar{x}<\bar{y}\Rightarrow\forall\,\bar{z}$: $\bar{x}+\hat{z}_n<y$ if $n\ge n_0(\bar{x},\bar{y},\bar{z})$.

Def.2: A topology τ on $l^\infty(X)$ is myopic iff every τ–continuous preference relation is myopic.

Examples: τ_∞ is not, the product topology τ_π and the strict topology β are myopic.

Theorem.1:a) There is a strongest regular and myopic topology τ_{SM} on $\mathscr{X}=l^{\infty}(X)$.

b) $\mathscr{X}(\tau_{SM})'=\mathscr{Y}:=l_b^1(Y)=\{\bar{u}=(u_n):\sum_0^{\infty}\|u_n\|_B=\|\bar{u}\|_B<\infty\}$, B bounded in X.

c) $\mathscr{X}(\tau_{sm})''=l_e^{\infty}(X'')=\{\bar{x}=(x_n): x_n\in X''$, bounded and equicontinuous$\}$.

Remark: τ_m=Mackey topology on $\mathscr{X}:\tau_m=\tau_m<\mathscr{X},\mathscr{Y}>$ satisfies: $\tau_m\supset\tau_{SM}\supset\beta$ (For $X=\mathbb{R}^1$ Brown–Lewis and for $X=L^{\infty}(S\Sigma\mu)$ Raut (JET 1986) proved $\tau_m=\tau_{SM}=\beta$).

Theorem 2: a) $\tau_m=\tau_{SM}=\beta$ if X is a reflexive Banach lattice (Tillmann 1988).

b) $\tau_m=\tau_{SM}$ if X is an arbitrary barreled space (Flügel 1989).

Essential in the proof of Theorem 2b) is the operator $T_x:l_b^1(Y)\to l^1=l^1(\mathbb{R}^1)$, $\bar{y}\mapsto(<x_n,y_n>)_{n\in\mathbb{N}}$ and

Lemma: T_x is $(\sigma(\mathscr{Y},\mathscr{X}),\sigma(l^1,l^{\infty}))$ continuous.

Therefore, it maps $\sigma(\mathscr{Y},\mathscr{X})$–compact sets $\mathscr{K}$ into $\sigma(l^1,l^{\infty})$–compact sets $K\subset l^1$. It follows that $\mathscr{K}^o$ is a τ_{SM}–neighborhood and then $\tau_m=\tau_{SM}$.

H.TRIEBEL. Atomic Decompositions of Function Spaces, Pseudodifferential Operators.

Atoms are smooth building blocks in function spaces which attracted much attention in the last 10 years or so. We give a brief description of atomic representations of distributions belonging to spaces F_{pq}^s of Hardy–Sobolev type. We sketch applications: Mapping properties of pseudodifferential operators in function spaces.

D. VOGT. Continuous Linear Inverses for Partial Differential Operators with Constant Coefficients.

Let $P(D)=\sum_{|\alpha|\leq m}a_{\alpha}D^{\alpha}$ and $\Omega\subset\mathbb{R}^n$ open. The old problem of L. Schwartz, under which conditions there exists a continuous linear right inverse for the operator $P(D):C^{\infty}(\Omega)\to C^{\infty}(\Omega)$ is solved by the following theorem.

Theorem: The following are equivalent:

(1) P(D) admits a continuous linear right inverse in $C^{\infty}(\Omega)$.

(2) P(D) admits a continuous linear right inverse in $D'(\Omega)$.

(3) Ω is P(D)–convex with bounds, i. e. $\forall \omega \subset\subset \Omega\ \exists \omega' \subset\subset \Omega\ \forall \omega'' \subset\subset \Omega\ \exists k,C$:

$$v \in \mathcal{E}'(\omega''),\ P(-D)v|_{\omega''\setminus\omega'} \in B \Longrightarrow v|_{\omega''\setminus\omega'} \in CB^{-k}.$$

Here B^{-k} denotes the unit ball in the Sobolev space H^{-t}, $B=B^{0}$. If e. g. Ω is an open convex set with C^{1}–boundary (for instance the unit ball) then this is equivalent to: P(D) is hyperbolic and the principal part decomposes (up to a constant) into real linear factors.

Report on joint work with R. Meise (Düsseldorf) and B.A. Taylor (Ann. Arbor).

L. WEIS. Strongly Affine Projections on M(X).

Let X be a Polish space. M(X) (P(X)) denotes the space of finite (probability) measures on X. A bounded operator $T:M(X) \longrightarrow M(X)$ is strongly affine if T is universally measurable with respect to the w^{*}–topology and for every probability measure Q on P(X) with barycenter μ the image measure $Q \circ T^{-1}$ has barycenter $T(\mu)$, equivalently, there is a w^{*}–universally measurable kernel $(\mu_x)_{x\in X}$ of measures on X such that for all $\mu \in M(X)$

$$T_{\mu} = w^{*}\!\int \mu_y \, d\mu(y).$$

Let L be a norm closed, non–separable sublattice of L, $U=L\cap P(X)$ and $E=\mathrm{etr}U=\{\mu\in U:\mu$ an atom of $L\}$. Consider the following conditions:

a) There is a strongly affine order isometry J of M(0,1) onto L.

b) There is a positive, contractive and strongly affine projection P of M(X) onto L.

c) For every $\mu \in U$ there is unique w^{*}–probability measure on E with μ as its barycenter.

d) There is a countably generated σ–algebra Σ of universally measurable subsets of X such that Σ is H–sufficient for U.

e) U has the w^{*}–Radon–Nikodym property.

f)There are w^{*}–universally measurable, w^{*}–measure convex sets M_n so that $P(X)\setminus U = \bigcup_n M_n$.

Theorem: We always have a)⇔c)⇔f) and b)⇔d)⟹f)⟹a). If we assume Martin's axiom we can show a)⟹b) and all the conditions are equivalent.

The proofs use a general Choquet–type representation theorem of Bourgin and Edgar and results on orthogonal kernels by Mauldin, Preiss and v. Weizäcker.

H. WIDOM. The Heat Expansion for Integral Operators.

We consider compressions to $L_2(\Omega)$ (Ω a compact subset of $\mathbb{R}^n$) of positive elliptic ψdo's of negative order $-r$ on $\mathbb{R}^n$. Examples are integral operators with kernels of the form $c|x-y|^{r-n}$(Riesz potentials) or $c|x-y|^{r-n/2}K_{r-n/2}(|x-y|)$ (Bessel potentials). If λ_i are the eigenvalues one seeks an asymptotic expansion as $t\to 0+$ of $\sum_i e^{-t/\lambda_i}$. A formel expansion $\sum_{k=0}^{n} a_k t^{k-n/r}+o(1)$ is obtained using the calculus of ψdo's, where a_k are given by integral formulas in terms of the symbol of the operator. The expansion is sometimes wrong but is correct under an extra assumption which is satisfied in the two examples mentioned. In the general case only the coefficient a_n might be wrong and a conjecture is stated concerning its correct value.

G. WITTSTOCK. Hahn Decomposition of Operator Valued Measures.

Let X be a compact Hausdorff space and $\varphi:\mathrm{Bor}(X)\to B(\mathscr{H})_{sa}$ countable additive with respect to the weak operator topology. If φ has bounded semivariation then $\phi:C(X)\to B(\mathscr{H}),\phi(f):=\int f d\varphi$ is a bounded operator. We define a finer semivariation by

$$\|\varphi\|_{cb}=\sup\{\Sigma\|h^*\varphi(A_i)h\|_{T(\mathscr{H})}:\dot\cup A_i=X,h\in HS(\mathscr{H}),\|h\|_{HS}\le 1\}.$$

The following are equivalent:

(1) $\|\varphi\|_{cb}<\infty$.

(2) ϕ is completely bounded and $\|\phi\|_{cb}=\|\varphi\|_{cb}$.

(3) φ has a Hahn decomposition $\varphi=\varphi^+-\varphi^-$, $\|\varphi^+(x)+\varphi^-(x)\|=\|\varphi\|_{cb}$.

(4) φ has a dilation to a spectral measure i. e. there exist a spectral measure $e:\mathrm{Bor}(X)\to B(\mathscr{K})$, an imbedding $a:\mathscr{H}\to\mathscr{K}$ and $y\in e(\mathrm{Bor}(X))'$, $y=y^*$, $\|y\|=\|\varphi\|_{cb}$ s.th. $\varphi=a^*y\,e(\cdot)\,a$.

V. WROBEL. Sprectra of C_0–Semigroups.

Among other things the failure of the spectral mapping theorem

(*) $$\sigma(U_A(t);X)\setminus\{0\}=e^{t\sigma(A;X)} \quad (t\geq 0)$$

for C_0–semigroups $(U_A(t))_{t\geq 0}$ on complex Banach spaces is analyzed. It turns out that if (*) does not hold, then $\sigma(U_A(t);X)\setminus e^{t\sigma(A;X)}$ contains nontrivial connected subsets and even rings for almost all $t\geq 0$. So, in any case, if (*) is not true, then (*) must fail dramatically.

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