MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 46/1989

Computational Methods in Solid Mechanics

29.10. bis 4.11.1989

The meeting was organized by D.Braess (Bochum), P.Ciarlet (Paris) and E.Stein (Hannover) and brought together mathematicians and engineers dealing with numerical problems in solid mechanics. So the considerations moved between methods from real analysis and the use of physical knowledge and intuition. The encounter between these two approaches led to a fruitful exchange of ideas. In the lectures results of recent research were presented which showed that current interest centers around the following topics:

— finite element analysis of Mindlin-Reissner plates

In the last years some "heuristic tricks" to overcome the locking phenomenon (such as reduced integration and discrete Kirchhoff techniques) have been shown to be equivalent to mixed methods. Some applications of these techniques were discussed: Uniform convergence with respect to the thickness t was proved for suitably designed plate elements. Moreover the consequences for multigrid methods were investigated.

— shell analysis

The discrete Kirchhoff approach mentioned above was generalized to the classical linear shell model of Koiter and rigorously analysed. In the field of nonlinear models lectures dealt with the calculation of shells with finite rotations. Besides the numerical considerations a general asymptotic analysis of the behaviour of thin shells for $t \to 0$ was given.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

— mesh refinement

The necessity to refine the finite element mesh near local singularities was emphasized. Error estimators for the control of the refinement were analysed which were based on the local residuals with respect to the strong form of the differential equation. In more complex nonlinear models an additional control of the linearization error, the plastic behaviour etc. was suggested. Moreover, stability and approximation properties of finite elements were generalized to that kind of degenerate meshes which occur during the refinement process.

— analysis of junctions

A junction between a plate of thickness 2ϵ and a three-dimensional solid was considered. In the limit $\epsilon \to 0$ this leads to a 2d- and a 3d- problem which are coupled by junction conditions for the displacements (a so called "pluri-dimensional" problem).

— treatment of nonlinearities

Several lectures dealt with the modelling of nonlinear behaviour caused by nonlinear cinematics (such as finite deformations) or nonlinear constitutive assumptions (e.g. plasticity, special properties of the lung tissue, etc.). A further point of interest was the stability analysis of nonlinear structures. Here, some bounding properties of the eigenvalues obtained by linearized finite element methods were given.

A. Kirmse and C. Blömer

Vortragsauszüge

Michel Bernadou

Approximations of General Thin Shells by D.K.T. Methods

In our presentation we will detail how to extend and to implement the D.K.T. method for shell problems. We use a representation of the middle surface S of the shell as an image of a plane reference domain Ω through a mapping $\vec{\phi}$:

$$\vec{\phi} : (\zeta^1, \zeta^2) \in \Omega \to S = \vec{\phi}(\zeta^1, \zeta^2)$$

so that we do not introduce any error in the definition of the geometry of the shell. When the analytical definition of the mapping $\vec{\phi}$ is not available, it is always possible to introduce very accurate approximations of $\vec{\phi}$ by using B-spline functions.

We will present some numerical results on benchmarks from a joint paper with P. Mato Eiroa and we will report the results obtained in the study of the convergence and in the obtention of corresponding error estimates.

Heribert Blum

Defect Correction Techniques in Mixed Finite Element Discretizations

High order schemes for mixed finite element discretizations are constructed by means of defect correction. The theoretical basis for this approach is the existence of an asymptotic error expansion for the basic low order scheme with respect to the mesh size parameter. This can be shown to hold on piecewise uniform triangulations. No LBB stability estimate is required for the defect defining operators. In several numerical tests the method yields better results than direct approximation with high order finite elements and Richardson h-extrapolation.

Frederic Bourquin

Modal Synthesis Methods: Convergence Analysis

Modal synthesis methods enable to compute the eigenpairs of a differential operator on a domain that can be subdivided into different subdomains on each of which the eigenpairs of the same operator are assumed to be partially known. A new "fixed interface" method is presented, which extends the Hurty, Graig and Bampton one. The coupling strategy is based on a special choice of the "static modes". Error bounds are derived for the model problem of the heat equation in a non-homogeneous medium occupying a bounded domain in $\mathbf{R}^n, n \geq 2$; numerical tests show the efficiency of the proposed method.

Dietrich Braess

Multigrid Methods for the Timoshenko Beam and the Mindlin-Reissner Plate

Numerical computations for the Timoshenko beam or the Mindlin-Reissner plate give rise to locking when standard finite element methods are applied. Often SRI (selected reduced integration) is a good remedy. A good step forward in the mathematical analysis was done, when SRI was detected to be equivalent with some mixed methods. In a joint paper with C. Blömer it turned out, that this equivalence does not hold in the framework of multigrid methods. Then SRI although simpler does not lead to efficient algorithms, but the mixed formulation may be treated in the (standard but) adapted multigrid framework. The convergence analysis is strongly supported by numerical results. — The same holds for the Mindlin-Reissner plate with the Arnold-Falk elements. The analysis is more involved due to singular perturbation effects.

Philippe G. Ciarlet

Junctions in Elastic Multi-Structures

In this joint work with H. le Dret and R. Nzongwa, we consider a problem in three-dimensional linearized elasticity, posed over a domain consisting of a plate with thickness 2ϵ, inserted into a solid whose Lamé constants are independent of ϵ. If the Lamé constants of the material constituting the plate vary as ϵ^{-3}, we show that, as $\epsilon \to 0$, the solution of the three-dimensional problem converges (up to appropriate "scalings") in the H^1-norm to the solution of a coupled, "pluri-dimensional" problem of a new type, simultaneously posed over a three-dimensional open set with a two-dimensional slit, and a two-dimensional open set.

These results have been recently extended to junctions between plates (folded plates, possibly with corners), or between plates and rods, and also to eigenvalue and time-dependent problems.

Martin Costabel

Coupling Methods of Finite Elements and Boundary Elements for some Nonlinear Interface Problems

Recent progress in the understanding of coercivity properties of boundary integral operators led to new ("symmetric") formulations of coupling methods for finite element methods and boundary element methods. They have the advantage that the convergence proofs work also for nonsmooth interfaces and for many systems where previous coupling methods did not work. As a first example for a nonlinear interface problem, the coupling of an elasto-plastic material with a linear elastic material was treated. It allows a formulation with a functional with a nondegenerate saddle-point. The symmetric coupling method leads to discretized equations which still show this saddle-point structure. Elimination of the unknowns on the boundary gives a convex minimization problem. Quasioptimal convergence of the Galerkin approximations can be shown.

Philippe Destuynder

Use of Piezo-electric Devices for the Control of Flexible Structures

Piezoelectricity has been discovered a century ago by the Curie brothers and then applied by P. Langevin. But this phenomenon has been essentially used for sensors (dynamic analysis of structures). Recently developed by the chemistry industries, large sheets of Polyvinil Difluor (which is a piezo-electric material) permit to use this effect for the control of flexible structures (i.e.: actuators). The goal of this talk is to describe few plate models including piezo-electric devices and to analyze the properties of different control laws based on a numerical computation.

Bernhard Kawohl

Regularity, Uniqueness and Numerical Experiments for a Relaxed Optimal Design Problem

Consider the problem of designing a cylindrical bar of maximal torsional rigidity out of prescribed proportions of two different elastic materials. Moreover, the cross-section Ω of the bar is prescribed. An energy approach leads in a canonical way to a relaxed variational problem, whose solution displays a free boundary. There are subdomains $\Omega_i, i = 1,2$ and H of Ω in which $|\nabla u|$ lies in certain ranges. The Euler equations are elliptic in Ω_1 and Ω_2, but not in H, the homogenized region.

The lecture contains recent results on numerical experiments (joint work with G. Wittum), uniqueness (open problem posed by Murat and Tartar; joint work with J. Stara) and regularity of solutions. The proofs use rearrangement techniques, variational arguments and the coarea formula. In particular, the uniqueness proof is nonstandard. The computations were done with a multigrid method.

Michal Kleiber

Numerical Assessment of Stresses in the Human Lung during Artificial Ventilation

The importance of stress assessment in human lungs during artificial ventilation is pointed out. The complexity of the problem results from difficulties in constitutive modelling of the lung tissue, complex geometry of the lung and complex gas-solid effects. After reviewing the biomechanical background, the fundamental set of equations describing air flow through a finitely deformed pseudo-elastic porous media is formulated. Finite element approximation is introduced resulting in a set of coupled ordinary differential equations with strong nonlinearities. Numerical algorithms based on both explicit and implicit formulations and employing a quasi-Newton iterative scheme are described. Numerical examples illustrate sensitivity of the stress assessment to the constitutive assumptions adopted in the analysis.

B. Kröplin

Stability of Dynamically loaded Structures

A method is suggested, which simplifies the investigation of the safety of structures under short time loading. Representing the deformation behaviour and the loading by some energetic measures a local stability criteria is derived based on the balance of the external energy and the critical strain energy.

Lin Qun

Fourth Order Accuracy for Bilinear Finite Element Eigenvalues on Reentrant Domains

As a model problem the eigenvalue of the Laplace operator on a reentrant domain consisting of rectangles is considered. The problem may be solved by bilinear finite element methods with a rectangular mesh. It can be seen that if the mesh has $O(h^{-2})$ points and is graded appropriately (say, with a graded index $q > 2$ for a slit domain), then $O(h^2)$ convergence is obtained for the eigenvalues. We present a simple extrapolation scheme (with a graded index, say $q > 4$ for the slit domain) which increases these rates of convergence to $O(h^4)$.

Herbert Mang

On Bounding Properties of Eigenvalues from Linear Initial Finite Element Stability Analysis of Thin, Linear-Elastic Shells

There is consensus in the literature that the eigenvalues of smallest absolute value from linear stability analysis of thin, elastic shells by the finite element method (FEM) do not possess bounding properties with respect to corresponding stability limits from geometrically nonlinear stability analyses. A "linear initial stability analysis" by the FEM represents the first step of an "accompanying linear stability analysis" by this method. In the lecture, two modes of such stability analyses of thin, elastic shells are presented. It is proved that for mode 1, by contrast to mode 2, bounding properties of eigenvalues of smallest absolute value with respect to corresponding stability limits from geometrically nonlinear stability, in fact DO EXIST. Moreover, bounding properties of such eigenvalues from mode 1 relative to corresponding eigenvalues from mode 2 are shown to exist. The existence of these properties is important from the standpoint of engineering practice.

Hans D. Mittelmann

Stability Bounds in a Crystal Growth Problem

We consider the float-zone process used to manufacture crystals for semiconductors. Thermocapillary convection arising due to the temperature-dependence of surface tension may become unstable, causing material imperfections, through the onset of dynamic oscillatory convection (Hopf bifurcation). The Marangoni number Ma is used as governing parameter. An energy stability analysis is carried out which yields a bound Ma_E below which the steady convection pattern is stable with respect to disturbances of arbitrary magnitude. Ma_E is computed as the smallest positive eigenvalue of a highly nonlinear eigenvalue problem. A numerical method is presented and results for a wide range of physical regimes including in particular low gravity environments (space shuttle). They compare favourably with model experiments.

Nina Müller-Hoeppe

Finite Elastic and Finite Inelastic Deformations and Multigrid Solvers

Finite element formulations are presented for four different material models:

— compressible Mooney Rivlin material
— elastic plastic material
— elastic viscoplastic material
— Norton law.

They are all used for finite deformation calculations and formulated in the current configuration. For these material laws different types of finite elements were coded, elements with pure displacement approach and mixed models. For the multigrid solver the elements were used, which contained the displacement approach.

The finite element formulation for large elastic strains fulfils all fundamental conditions for applying the multigrid method and it works well. Using the inelastic finite element formulations in multigrid was an ad hoc attempt, which didn't succeed.

Joachim Nitsche

Convergence of Nonconforming Elements for Curved Boundaries

The weak finite element method due to Ciarlet-Raviart for the clamped plate problem is:

Find a $u_h \in S_h^0$ and $v_h = (-\Delta u) \in S_h$ recording to

$$(1)\ \ D(u_h, \chi) = (v_h, \chi) \ \text{ for all } \ \chi \in S_h$$

$$(2)\ \ D(v_h, \psi) = (f, \psi) \ \text{ for all } \ psi \in S_h^0.$$

For curved boundaries it is proposed to add the term

$$- < u_h, \chi_{|n} >= \oint_{\partial\Omega} u_h \frac{\partial\chi}{\partial n}$$

to the left hand side in (1), to insure linear finite elements vanishing at the knots of $\partial\Omega$ and to take $S_h = S_h \oplus \mathsf{H}_n$ where H_n is a proper space of harmonic functions of dimension $n \sim h^{-1}$. Then quadratic convergence of v_h holds true.

Petra Peisker

A Multigrid Method for the Kirchhoff Plate

The numerical solution of the linear equations arising from a discretization of the plate bending problem based on Zienkewicz's or Adini's element is studied. The finite element scheme proceeds from the Mindlin Reissner formulation with modified shear energy and the Kirchhoff condition is imposed on discrete points. We analyse a multigrid algorithm and provide numerical examples which confirm that the convergence rate is bounded away from one independently of the meshparameter h. Furthermore we establish a suitable preconditioning for the use of conjugate gradients.

Ekkehard Ramm

Shape Optimization and Sensitivity Analysis of Structures

Starting with some basic remarks to structural optimization the presentation addresses mainly form finding methods for structures with "optimal" mechanical response. The concept of a design element using different interpolation schemes for the geometrical description of the structures (CAGD) is used allowing to reduce the number of design variables substantially. An essential feature is the coupling of geometry, optimization and structural analysis, here the f.e.m.. For gradient methods as well as engineering judgement the sensitivity analysis is introduced. The presentation concentrates on shape optimization of plate and shell structures where different objectives, like min weight, min strain energy or stress levelling is used. Several examples underline the developed procedures.

Rolf Rannacher

Finite Element Discretization on Degenerate Meshes

Finite element discretizations are usually analysed under the assumption that the underlying family of meshes satisfies the "uniform shape condition" and sometimes additionally the "uniform size condition". This excludes certain classes of (regularly) degenerate meshes which naturally occur for instance in approximating boundary layer solutions. The lecture summarizes some of the results on the approximation and stability properties of finite element schemes which remain valid even on strongly degenerate meshes, e.g., those satisfying only the "maximum angle condition". The key in this analysis is a careful balance of local interpolation and "inverse" properties. It turns out that elements using interior nodal parameters ("bubble functions") are not suitable for being used on general meshes.

Enrique Sanchez-Palencia

Asymptotic Problems in Thin Shells

The Koiter model of shells is considered in the variational formulation of Bernadou and Ciarlet. The elastic bilinear form involves two terms, the membrane one and the flexion one. As the thickness h tends to zero, the membrane term is of order h, whereas the flexion one is of order h^3. Asymptoticaly the solution tends to be contained in the subspace of displacements keeping invariant the intrinsie matrics of the surface (this amounts to the kernel of the membrane form). If this kernel reduces to the null element (i.e. if the pure flexions of the surface are impossible), the asymptotic problem is mostly concerned with the membrane form.

Karl Schweizerhof

Quasi-Newton Algorithms in Nonlinear Mechanics, Current Status — Improvements?

Only the BFGS method is found today in engineering applications despite continuing research in mathematics on Quasi-Newton methods. There is also evidence that other Quasi-Newton schemes can be more or at least evenly effective and robust. This can be also extended to the range of almost indefinite Hessian matrices. In the presentation we show the algorithmic features of the "vectorized" versions of the Quasi-Newton schemes and the combination with general continuation methods for postlimit analyses. Some numerical tests with materially and geometrically nonlinear problems were carried out to compose the well known standard schemes with recently published schemes.

Juan C. Simo

A Class of Mixed Assumed Strain Methods and the Method of Incompatible Modes

A three-field mixed formulation in terms of displacements, stresses and an *enhanced strain* field is presented which encompasses, as a particular case, the classical method of incompatible modes. Within this framework, incompatible elements arise as particular 'compatible' mixed approximations of the enhanced strain field. The conditions that the stress strain interpolation contain piece-wise constant functions and be L_2-orthogonal to the enhanced strain interpolation, ensure satisfaction of the patch test and allow the elimination of the stress field from the formulation. The preceding conditions are formulated in a form particularly convenient for element design. As an illustraion of the methodology three new elements are developed and shown to exhibit good performance: A plane 3D elastic/plastic QUAD, an asisymmetric element, and a thick plate bending QUAD. The formulation described herein is suitable for nonlinear analysis.

Erwin Stein

A Priori and a Posteriori Indicators for Adaptive Finite-Element Mesh Refinements of Thin-Walled Shells

Based on a gometrically nonlinear co-rotational shell theory — of the Reißner-Mindlin type — with finite rotations and elastic-plastic material, a priori and a posteriori error indicators are presented. They are used to obtain efficient adaptive refinements of the real geometry given by Bernstein polynomials, e.g. (ex.: airplane wings), as well as of the Finite-Element discretization using isoparametric quadrilaterals. For both refinements, irregular nodes are not permitted.

The **a priori strategy** is based on "Shape Preserving Recursive Mapped Meshing", controlling the skewness and the taper of the refined elements and other requirements.

The **a posteriori refinements** control the following indicators:

1) **Linear-elastic part**: The quadratic membrane- and moment-jumps across adjacent element boundaries, derived from the error in the energy norm, i.e. in H^1 (Babuška), indicator η_{LM}, η_{LB}.
2) **Geometrical nonlinearity**: The relative rotations of the element nodes due to deformation as a control of the linearization assumption, given by

$$\eta_{NL} = \max_{nodes} \|\underline{\underline{R}}^D_{lin} - \underline{\underline{I}}\|,$$

where $\underline{\underline{R}}^D_{lin} - \underline{\underline{I}}$ is the linearized skew-symmetric relative rotation matrix.
3) **Plastified zones**: The heuristic choice of the von Mises-Yield condition $J_2 = \|s^{Dev} - \alpha^{Dev}\| - \sqrt{\frac{2}{3}}\kappa(e_r)$ with $\eta_{PL} > \kappa\sigma_v$; $\kappa \in (0.9, 1.0)$.

Computed results and graphics are presented for a couple of test problems, combined with accuracy and efficiency comparisons.

Rolf Stenberg

Nonconforming Finite Element Methods for Reissner-Mindlin Plates

We generalize a recent method introduced by D.Arnold and R.Falk. The analysis of the original method is based on a discrete Helmholtz decomposition theorem which does not seem to be easily generalized to other methods. Therefore, we introduce a way to analyze the Arnold-Falk method without use of their Helmholtz theorem. This enables us to formulate several methods which can be shown to be stable. The key is to use (as in the original method) non conforming approximations for the deflection. These elements should be combined with the standard "bubble-function" technique, or even better, the recently introduced (by T.Hughes, L.Franca & al. among others) Galerkin-Least-squares technique.

L. Trabucho

Asymptotic Analysis, Homogenization, Spectral Methods in Beam Theory

A subject of major discussion in classical beam theories is the proper definition and calculation of the so called Timoshenko and warping constants, taking into account the additional bending effects due to the variation of the shear stresses on each cross section.

In this work, using the asymptotic expansion method, spectral method and homogenization techniques, we are able to give a precise definition of these quantities for almost any type of cross section, including open and closed sections, multicellular and both the thick and thin walled cases.

Finally, some numerical examples are also considered.

Rüdiger Verfürth

A Posteriori Error Estimators and Adaptive Mesh Refinement for Flow Problems

In many physical and technical problems, the solution exhibits local singularities, e.g., singularities near re-entrant corners, interior or boundary layers, or shocks. These singularities deteriorate the overall accuracy of a finite element approximation if the finite element mesh is not sufficiently refined near the singularities. One possibility of controlling adaptive mesh refinement is to use a posteriori error estimators. This is of particular interest for problems where no a priori information on the location and strength of the singularities is available. Here, we present an apparently new error estimator which is based on evaluating the local residuals of the finite element approximation with respect to the strong form of the differential equation. Numerical examples show the efficiency of adaptive mesh refinement based on this error estimator. The method can be generalized to non-conforming approximations, to linear elasticity, and to plate problems.

Juan M. Viaño

Asymptotic Analysis of Torsion Theories in Elastic Beams

Using the asymptotic expansion method introduced by Ciarlet and Destuynder for plates and by Bermudez and Viaño for beams, we obtain a very general one-dimensional model for extension-bending-torsion of linear elastic beams. The model is given by the second order terms of asymptotic expansion and it constitutes a generalization of classical theories of Bernouilli-Navier, Timoshenko, Saint-Venant and Vlassov. Moreover, the method allows us to define in a precise way the torsional and warping constants. The torsional constant is the classical one but the warping constant depends on Poisson's ratio (ν) and the classical one corresponds to take this coefficient equal to zero. Finally, the asymptotic technique is once more applied in order to obtain the torsional and warping constants to be used in thin-walled beams. In general, for $\nu = 0$, these constants are close to the classical ones (primary and secondary warping included). This work is a collaboration with L. Trabucho.

W. Wagner

On Geometrically Nonlinear Axisymmetric Shells with Finite Rotations

The behaviour of geometrically nonlinear elastic shells of revolution with finite rotations is discussed. In contrast to the approach of Reissner and to standard classical shell theories a new straightforward strategy is derived for the introduction of fully nonlinear strain measures for the axisymmetric case. The formulation is similar to that given by Simo for the general case. Within this approach the computation of the deformed director vector **d** is a main assumption which is essential to describe the fully nonlinear bending behaviour. An associated efficient but very simple finite element formulation is given. A consistent linearization of the weak form leads to a quadratically convergence behaviour. The discussed examples show the good performance of the derived finite element formulation.

Kaspar Willam

Computational Plasticity & Failure Mechanics

Failure analysis of solids involves "hard" discontinuities in space and time. Elastic-plastic operators exhibit discontinuous unilateral constraints. Spatial localization of shear bands requires special provisions beyond the traditional extension of fixed grid analysis to capture progressive failure zones. The paper focused on a hierarchy of failure diagnostics which can be utilized to separate diffuse from discontinuous bifurcations in case of symmetric and non-symmetric constitutive operators. The presentation was illustrated by several model problems which depict the sensitivity of the response behaviour with regard to mesh-layout in the presence of localized failure.

Peter Wriggers

Theory of Shells and its Numerical Formulation fo Large Rotations

A bending theory for thin shells undergoing finite rotations is presented, and its associated finite element model is described. The kinematic assumption is based on a shear elastic Reissner-Mindlin theory. The starting point for the derivation of the strain measures is the three-dimensional principle of virtual work. Here, the polar decompositon of the shell material deformation gradient leads to symmetric strain measures. The associated work-conjugate stress resultants and stress couples are integrals of the Biot stress tensor. This tensor is invariant with respect to rigid body motions and therefore appropriate for the formulation of constituive equations. The rotations are described through Eulerian angles.

The finite element discretization of arbitrary shells is based on the isoparametric concept. The advantage of the proposed shell formulation and its numerical model is shown by application to different nonlinear plate and shell problems. Finite rotations can be computed within one load increment. Thus the step size of the load increment is only limited by the local convergence behaviour of Newton's method or the appearance of stability phenomena.

Berichterstatterin: C. Blömer

Tagungsteilnehmer

Prof. Dr. H.W. Alt
Institut für Angewandte Mathematik
Abteilung Funktionalanalysis und
Numerische Mathematik
Wegelerstr. 6

5300 Bonn 1

Prof. Dr. M. Bernadou
INRIA
Bat. 8, Domaine de Voluceau-
Rocquencourt
B. P. 105

F-78153 Le Chesnay Cedex

C. Blömer
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Dr. H. Blum
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 293

6900 Heidelberg 1

Prof. Dr. F. Bourquin
Service de Mathematiques
Laboratoire Central des Ponts et
Chaussees
Blvd. Lefevre

F-75015 Paris

Prof. Dr. D. Braess
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof. Dr. P. Ciarlet
Laboratoire d'Analyse Numerique,
Tour 55
Universite P. et M. Curie
4, Place Jussieu

F-75005 Paris

Dr. M. Costabel
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. Ph. Destuynder
Laboratoire de Mecanique des
Structures
Ecole Centrale
Grande Voie des Vignes

F-92290 Chateney-Malabry

Dr. W. Dörfler
Institut für Angewandte Mathematik
Universität Bonn
Wegelerstr. 4

5300 Bonn 1

Dr. B. Kawohl
Sonderforschungsbereich 123
"Stochastische math. Modelle"
Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Prof. Dr. B. H. Kröplin
ISD Institut für Statistik und Dynamik
Universität Stuttgart
Pfaffenwaldring 27

7000 Stuttgart 80

Prof. Dr. K. Kirchgässner
Mathematisches Institut A
der Universität Stuttgart
Pfaffenwaldring 57
Postfach 560

7000 Stuttgart 80

Prof. Dr. Lin Qun
Institute of System Science
Academia Sinica

Beijing 100080
CHINA

A. Kirmse
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof. Dr. H. Mang
Institut für Festigkeitslehre
Technische Universität
Karlsplatz 13/202

A-1040 Wien

Prof. Dr. M. Kleiber
Institute of Fundamental
Technological Research
Polish Academy of Sciences
Swietokrzyska 21

00-049 Warszawa
POLAND

Dr. H. Matthies
Germanischer Lloyd AG
Hauptverwaltung Hamburg
Postfach 11 16 06

2000 Hamburg 11

Prof. Dr. F. G. Kollmann
Fachgebiet Maschinenelemente und Getriebe
Technische Hochschule Darmstadt
Magdalenenstr. 8 - 10

6100 Darmstadt

Prof. Dr. H. D. Mittelmann
Department of Mathematics
Arizona State University

Tempe ,AZ 85287
USA

N. Müller-Hoeppe
Institut für Baumechanik und
Numerische Mechanik
Universität Hannover
Appelstr. 9A

3000 Hannover 1

Prof. Dr. E. Sanchez-Palencia
Laboratoire d'Analyse Numerique,
Tour 55
Universite P. et M. Curie
4, Place Jussieu

F-75005 Paris

Prof. Dr. J. Nitsche
Institut für Angewandte Mathematik
der Universität Freiburg
Hermann-Herder-Str. 10

7800 Freiburg

Prof.Dr.-Ing. K. Schweizerhof
Institut für Baustatik
Universität Karlsruhe
Postfach 6980

7500 Karlsruhe

Dr. P. Peisker
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof. Dr. J. C. Simo
Dept. of Mechanical Engineering
Division of Applied Mechanics
Stanford University

Stanford , CA 94305-2391
USA

Prof. Dr. E. Ramm
Institut für Baustatik
Universität Stuttgart
Pfaffenwaldring 7

7000 Stuttgart 80

Prof. Dr. E. Stein
Institut für Baumechanik und
Numerische Mechanik
Universität Hannover
Appelstr. 9A

3000 Hannover 1

Prof. Dr. R. Rannacher
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 293

6900 Heidelberg 1

Prof. Dr. R. Stenberg
Institute of Mathematics
Helsinki University of Technology
Otakaari 1

SF-02150 Espoo

Prof. Dr. L. Trabucho
Instituto Nacional de Investigacao
Cientifica, Centro de Matematica e
Aplicacoes Fundamentais
2, Av. Prof. Gama Pinto

P-1699 Lisboa Codex

Dr. W. Wagner
Institut für Baumechanik und
Numerische Mechanik
Universität Hannover
Appelstr. 9A

3000 Hannover 1

Prof. Dr. R. Verfürth
Institut für Angewandte Mathematik
der Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof. Dr. K. William
Institut für Mechanik
Universität Karlsruhe
Kaiserstr. 12

7500 Karlsruhe

Prof. Dr. J. M. Viano
Departemento de Matematica Aplicada
Facultad de Matematicas
Universidad de Santiago

E-15706 Santiago de Compostela

Prof. Dr. P. Wriggers
Institut für Baumechanik und
Numerische Mechanik
Universität Hannover
Appelstr. 9A

3000 Hannover 1