MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 47/1989

Fastringe und Fastkörper

05.11. bis 11.11.1989

Die Vorträge und Diskussionen der Tagung behandelten verschiedene aktuelle Richtungen in der Theorie der Fastringe. Planare Fastringe und Verallgemeinerungen, mit ihren Anwendungen in Geometrie und Kombinatorik, bildeten einen thematischen Schwerpunkt. Als weitere Themenkreise seien genannt:

- Struktursätze, neue Typen von Fastringen (z. B. laminated near-rings);
- Fastringe von Gruppenabbildungen, matrix near-rings, distributiv erzeugte Fastringe;
- Radikale von Fastringen;

ferner: Homomorphismen von Halb-Fastkörpern, sowie eine Beziehung von Fastbereichen zur speziellen Relativitätstheorie.

Die Leitung hatte G. Betsch (Tübingen).

Vortragsauszüge

J. ANDRÉ

On Anshel-Clay-nearrings

An Anshel-Clay-nearring (ACN) introduced by H. Ney is a zero-symmetric integral nearring being a generalization of a planar nearring: The finite ACNs are exactly the integral planar nearrings. Any ACN is

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geometric in Theobald's sense. If $(N,+)$ is abelian, $\{0\} \neq (U,+) \leq (N,+)$ and $NU \subseteq U$ imply $N = U$, then the non-commutative space $\mathbb{R}(N)$ is primitive. H. Ney stated a purely geometric characterization of the spaces $\mathbb{R}(N^{(I)})$ for $|I| \geq 3$ which are generalizations of the regular nearaffine spaces (where N is a nearfield).

G. BETSCH

On near-rings linked with bicentralizer near-rings

1. Let $(G,+)$ be a (not necessarily commutative) group, $\Phi \leq \text{Aut } G$, and $M_{\Phi}o(G)$ the 0-symmetric bicentralizer near-ring. For Φ fixed point free there exists a method due to G. FERRERO of constructing by means of Φ a (finite) planar near-ring N with $(N,+) = (G,+)$. We extend FERRERO's method to the far more general case, where G contains elements, the stabilizer of which in Φ reduces to $\{1\}$. We describe the corresponding tactical configuration and the relationship between N and $M_{\Phi}o(G)$.
2. LITOFF's well known theorem on simple rings with minimal one-sided ideals has no easy generalization to near-rings. We establish a series of results in this direction for non-rings with identity, which have a minimal one-sided ideal K such that K is of type 2, and satisfy certain conditions on $(N,.)$.

J.R. CLAY

Double planar nearrings

A double planar nearring is a quadruple $(N,+,*_1,*_2)$ where each $(N,+,*_i)$ is a planar nearring, and where $a*_i(b*_jc) = (a*_ib) *_j (a*_ic)$ for $i \neq j$. In some cases the resulting geometric structures $(N,B_1,B_2^{\#})$ have nice geometric interpretations, where

$B_1 = \{N*_1a + b \mid a,b \in N,\ a \neq 0\}$,

$B_2^{\#} = \{N^{\#}*_2a + b \mid a,b \in N,\ a \neq 0\}$, and

$N^{\#} = N\backslash[0]_m$.

C. COTTI FERRERO

On N-near-rings

A near-ring N is called an N-near-ring if every right ideal of its multiplicative semigroup $N^{\cdot}$ is an N-subgroup of N.

I get a complete characterization of such near-rings, and I prove that if N is an N-near-ring then N_o is an N-near-ring, too.

I define as quasi-local a near-ring N if $\Theta_1 = \{x \in N \mid xN = N\} \neq \phi$ and $L = N \setminus \Theta_1$ is an N-subgroup of N, and I prove several properties of such near-rings; moreover I prove that the N-near-rings are quasi-local.

Other results are obtained under some finiteness conditions; namely if a 0-symmetric N-near-ring N satisfies the ACCR or the DCCR then N has a left identity and I get a characterization of N-near-rings with ACCN. A near-ring N is a 0-symmetric N-near-ring with DCCN iff it is a 0-symmetric N-near-ring with ACCN and L is nilpotent.

Gallina got examples linked to these structures, and using the above results we can show that a 0-symmetric N-near-ring N with ACCN and $A_s(N) = 0$ is integral, it is a Goldie near-ring and moreover we can obtain divisibility results.

G. FERRERO

Near-rings and designs from semigroups

Let N be a (finite) near-ring. We set ϕ_a: $x \to ax$, and get a homomorphism ϕ from the multiplicative semigroup $N^{\cdot}$ of N to a semigroup Φ of endomorphisms of N^+. The mapping ϕ satisfies an obvious condition which reflects the associativity of the product in N. Conversely, if we are given a mapping ϕ: $N^+ \to \Phi$ (N^+ a group, Φ a semigroup of endomorphisms of N^+) satisfying the condition, then it is possible to define a product on N^+ in order to obtain a near-ring linked to ϕ.

We are dealing with cases in which the condition can be treated e.g. with combinatorial tools and group theory to grasp the situation and to obtain interesting near-rings. In some of the sets of type ϕN it is possible to obtain non-trivial geometric structures. We restrict ourselves to some examples (mainly due to Gallina) to give an idea of the applications.

YUEN FONG

On the minimal generating sets of the endomorphism near-rings of the dihedral groups D_{2n} with odd n

In the sequel, all groups are written additively. Let

$$D_{2n} = Gp \langle a,b \mid na = 0 = 2b,\ a+b = b-a\rangle$$

be the dihedral group of order 2n and $E(D_{2n})$ $(A(D_{2n}), I(D_{2n}))$ be the endomorphism (automorphism, inner automorphism) near-ring generated additively by End D_{2n} (Aut D_{2n}, Inn D_{2n}), the set of all endomorphisms (automorphisms, inner automorphisms) of D_{2n}. The purpose of this paper is to show that $E(D_{2n}) = Gp \langle \{\rho_o, \phi_{ia+b}, \phi_{ja+b}\}, +\rangle$ $(0 \le i,j \le n-1,\ i \ne j)$ if $n \in 2\mathbb{N} + 1 = \{3,5,7,9,\ldots\}$ and $(i-j,n) = 1$. Here ρ_o is the identity automorphism of D_{2n} and ϕ_x is the endomorphism of D_{2n} that sends the normal subgroup $\langle a\rangle$ to the additive identity 0 and the coset $\langle a\rangle+b$ to the assigned element x where $x \in \{ia+b \in D_{2n} \mid i \in \{0,1,2,\ldots, n-1\}\}$.

P. FUCHS (und G. F. PILZ)

A new density Theorem for primitive near-rings

In this paper we extend the Wielandt-Betsch density theorem (version for 2-primitive near-rings with identity) to the much bigger class of 1-primitive near-rings. Let $\phi \ne X$ be a set, $(\Gamma,+)$ a group and $\phi: \Gamma \to X$ a map. Define $*_\phi$ on Γ^X by $f *_\phi g = f \circ \phi \circ g$, $f,g \in \Gamma^X$. Then $M(X,\Gamma,\phi) := (\Gamma^X, +, *_\phi)$ is a near-ring. If A is a group of permutations on X, B a subgroup of

$Aut(\Gamma)$ and $\psi \in Hom(A,B)$ such that $\phi \cdot \psi(a) = a \cdot \phi$, $a \in A$, then $M(\phi,\psi) = \{f: X \to \Gamma \mid f(ax) = \psi(a)f(x),\ x \in X,\ a \in A\}$ is a subnear-ring of $M(X,\Gamma,\phi)$. Centralizer near-rings arise as special cases of this concept. Let $M_o(\phi,\psi)$ denote the 0-symmetric part of $M(\phi,\psi)$. We then have the following theorem:

Theorem: Let N be a 0-symmetric non-ring. Then N is 2-primitive if and only if N is dense in some $M_o(\phi,\psi)$ such that

1) $|X| > 1$ and ϕ is onto.
2) A is fixed-point-free on X.
3) $\Gamma_{x_o} = \{\gamma \in \Gamma \mid \phi(\gamma) = \phi(0) = x_o\}$ does not contain a nontrivial subgroup Σ of Γ.

Replacing 3) by a slightly more general condition we obtain a similar result for arbitrary 1-primitive near-rings.

K. KAARLI

On Affine Comleteness of Groups

Affine comleteness is a notion of universal algebra naturally generalizing functional completeness. It is well known that functionally complete groups are precisely the finite non-abelian simple groups and it would be interesting to have a description of affine complete groups. This problem is linked with structure theory of near-rings I(G) generated by inner automorphisms of G. A list of groups known to be affine complete or affine non-complete is presented. A special attention is paid to a method enabling to prove affine non-completeness in several important cases. Some properties related to affine completeness (1-affine and local affine completeness) and some open problems are discussed, too.

H. KARZEL

Near-rings, (MDS)- and Laguerre codes

Let K be a finite set, called an alphabet, with $|K| = q$, let $k \in \mathbb{N}$ and $\mathbb{N}_k := \{1,2,\ldots,k\}$. Then a q-nary code $\mathcal{L}$ of length k is nothing else but a set of functions $C: \mathbb{N}_k \to K$. $\mathcal{L}$ is called an (MDS)-code (Laguerre code) if for all $h,i \in \mathbb{N}_k$, $h \neq i$ ($h,i,j \in \mathbb{N}_k$, $|\{h,i,j\}| = 3$) and for all $\lambda,\mu \in K$ ($\lambda,\mu,\nu \in K$) there is exactly one $C \in \mathcal{L}$ such that $C(h) = \lambda$, $C(i) = \mu$ ($C(h) = \lambda$, $C(i) = \mu$, $C(j) = \nu$).

For (MDS)-codes we have $k \leq q+1$ and for Laguerre codes $k \leq q+2$. If q is the power of a prime then there are (MDS)-codes with $k = q+1$ (defined using punctured projective planes of order q) and with $k = q$ (defined using affine planes of order q), and also Laguerre codes with $k = q+1$, and if q is a power of 2 with $k = q+2$. In case q is not the power of a prime the problems will be discussed whether it is possible to construct (MDS)-codes or Laguerre codes with the help of finite near-rings $(N,+,\cdot)$. In the first case this leads us to the notion of a coding set $T \subset N$. If T is a coding set of a near-ring N, $q := |N|$, $k := |T|+2$ and $T = \{t_3,t_4,\ldots,t_k\}$ then $\mathcal{L} := \{(x,y,xt_3+y,xt_4+y,\ldots,xt_k) \mid x,y \in N\}$ is a q-nary (MDS)-code of length k.

H. KAUTSCHITSCH

Constructions of nearrings, nearfields and composition-rings by quotients of power-series-rings

In this contribution J. Clay and I describe a new way to construct nearrings $(M\times R,+,o)$ from a commutative ring R and an R-module M. For an R-Algebra A one gets a compositon ring $(A\times R,+,.,o)$, sometimes a "double ring", in the sense, that also $(A\times R,+,o)$ is a ring. The idea for these new operations is coming from the quotient structures of the composition-ring of formal power series over R by certain composition-ideals.

If R has an identity, then the group of units with respect to o and the ideals have a particularly nice description. In particular one can determine all maximal ideals in the nearring MxR and compute some of the most popular radicals.

K.D. MAGILL, Jr.

Recent and new results on the Automorphism Groups of Laminated Near-rings

Let N be a right near-ring and let $x \in N$. Define a new near-ring N_x by defining addition to coincide with addition in N but define the product $a \otimes b$ of two elements $a, b \in N_x$ by $a \otimes b = axb$. The near-ring N_x is a *laminated near-ring of* N, the element x is the *laminator* or *laminating element* and N is referred to as the *base near-ring*. Some general abstract results will be presented and these will be followed by some results on laminated near-rings of continuous selfmaps of topological groups. Various laminated near-rings of the base near-rings N(R) and $N(R^2)$ will then be investigated in some detail where N(R) is the near-ring of all continuous selfmaps of the additive topological group of real numbers and $N(R^2)$ is the near-ring of all continuous selfmaps of the additive topological group of the Euclidean plane. The automorphism groups of a number of these laminated near-rings will be completely determined.

G. MASON

Kernels of Covered Groups

If G is a group, a cover of G is a set $\Gamma = \{C_i\}$ of proper subgroups of G such that $\cup C_i = G$ and $C_i \nsubseteq C_j$ for all i,j. If in addition $C_i \cap C_j = (0)$ then Γ is called a fibration. We set $E(G,\Gamma) = \{f \in \text{End } G \mid f(C_i) \subseteq C_i \; \forall C_i\}$ and $\ker(G,\Gamma) = $ d.g. $E(G,\Gamma)$. It is known that if G is fibered then $\ker(G,\Gamma)$ is always a ring. Moreover $\ker(G,\Gamma)$ is a field if G is abelian or G is non-abelian and $|E| > 2$.

For several classes of groups it happens that G has a cover with $C_i \cap C_j$

= C ◁ G. It follows that G/C is fibered and using the canonical map $E(G,\Gamma) \to E(G/C,\Gamma/C)$ we have information about the ideal $M_C = \{f \mid f(G) \subset C\}$.

If G is covered by normal subgroups then $I(G) \subseteq \ker(G,\Gamma)$. Consider the following groups: The quaternion group; $C_9 \rtimes C_3$; $D_4 \times C_3$; any dihedral group of order 2n, n even; any dicyclic group of order 4n, n even. In all cases G can be covered by a set of maximal normal subgroups with $C_i \cap C_j = C$ and in all cases the resulting kernel is precisely I(G). Counter example: There is a group of order 16 (type 16/8 in Thomas and Wood) which has the same kind of cover but for which $I(G) \subsetneq \ker(G,\Gamma)$. Other kernels of dicyclic and dihedral groups are discussed.

C.J. MAXSON

Remarks on the centalizer near-ring $\mathbb{M}_R(G)$, G an R-module

This is a report about some recent investigations of the centralizer near-ring $M_R(G) := \{f \in M(G) \mid f(ar) = (fa)r,\ a \in G,\ r \in R\}$ where R is a ring with identity and G is a unitary R-module. One finds that when D is an integral domain and G is the free D-module D^n on n generators then $M_D(D^n)$ is a ring $\Leftrightarrow$ n=1. Further, the structure of $M_D(D^n)$ is discussed. For example, if $n \geq 2$, $M_D(D^n)$ is a simple near-ring.

On the other hand, there are rings R such that $M_R(G)$ is a ring for every R-module G. We denote this collection of rings by $\mathcal{R}$ and present information about $\mathcal{R}$. Several sufficient conditions for a ring R to be in $\mathcal{R}$ are obtained. In particular, an arbitrary product $R = \prod_\alpha \mathbb{M}_{n_\alpha}(R_\alpha)$ of matrix rings $\mathbb{M}_{n_\alpha}(R_\alpha)$ is in $\mathcal{R}$ whenever $n_\alpha \geq 2$ for each α. The Artinian rings in $\mathcal{R}$ are completely characterized.

J.D.P. MELDRUM

Weakly distributive matrix near-rings

All near-rings considered are right zero-symmetric. We are concerned with the near-ring $\mathbb{M}_n(R)$ of matrices of order n over a near-ring R, defined by Meldrum and Van der Walt using a functional approach. It is possible to show that even when R does not have an identity, (R,+) belongs to a variety V of groups if and only if $(\mathbb{M}_n(R),+)$ belongs to V. In this result, we can replace V by the class of soluble groups or the class of nilpotent groups. A near-ring R is weakly distributive if it has a series of ideals $R = R_o \triangleright R_1 \triangleright R_2 \triangleright \dots \triangleright R_n = \{0\}$ such that R/R_{i+1} distributes over R_i/R_{i+1}. Using results of Fröhlich which link the distributive properties of a d. g. (distributively generated) near-ring with its additive structure, we show that if R is a d. g. near-ring with identity then R is weakly distributive if and only if $\mathbb{M}_n(R)$ is of the same type. A number of properties of ideals in R and corresponding ideals in $\mathbb{M}_n(R)$ are given; in particular the J_o and J_2 radicals of $\mathbb{M}_n(R)$ are characterized. This work was done jointly with S. J. Abbasi.

R. MLITZ

(Near-ring)-radicals defined by prime ideals

Starting from the ideas of a paper by K.I. Beidar (Uspechi Matemat. Nauk 44, 187 - 188 (1989)) heredity properties of Ω-group- resp. nearring-radicals are investigated.

Firstly, a new characterisation of r-heredity (ie. $\rho N \cap I \subseteq \rho I \; \forall I \triangleleft N$) of a radical ρ is given. Necessary and sufficient conditions for a class $\mathbb{M}$ are indicated to ensure that its subdirect closure is the semisimple class of a given r-hereditary radical.

Secondly, it is shown that for every class $\mathbb{P}$ of prime near-rings, the elements of the corresponding upper radical class do not contain essential ideals belonging to $\mathbb{P}$. It follows that in the zero-symmetric case the upper radical class determined by an arbitrary class of prime near-rings is hereditary. For the general case, following the same lines, one obtains that the upper radical class determined by an arbitrary class of equiprime near-rings (in the sense of Booth/Groenewald/Veldsman - Commun. in Algebra - to appear) is hereditary with respect to invariant ideals.

D. NIEWIECZERZAŁ

Some finiteness conditions in near-rings

Let N be a right, zero-symmetric near-ring with identity. Let $(V_i)_{i\in I}$ be a family of unitary N-groups and let $\aleph$ be an infinite cardinal number. We consider some subdirect products, so called $\aleph$-products, which are between the direct sum and the direct product of the family of N-groups $(V_i)_{i\in I}$. We determine a necessary condition for canonical embedding of $\aleph$-products to split in the corresponding direct product. This condition is a chain condition for anihilators in a certain homomorphic image of N.

All the considerations above are based on ring theory papers of Dauns and Laistaunam, who tried to generalize in some way the well known fact that a left noetherian ring R is characterized by the condition that, for any family of left injective R-modules $(E_i)_{i\in I}$, the direct sum $\bigoplus_{i\in I} E_i$ is a direct summand of the direct product $\prod_{i\in I} E_i$.

A. OSWALD

Nearly Nilpotent Nearrings

Nilpotent right ideals in near-rings have the property that they contain a subset X with $X^2 = 0$ and $X \cap B \neq 0$ for every non zero right ideal of the near-ring which is in the right ideal. We abstract from this the

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idea of a nearly nilpotent right ideal and of a nearly nilpotent near-ring. A characterisation of nearly nilpotent near-rings via the singular set is given.

An ideal P of N is prime if $aNb \subseteq P$ implies $a \in P$ or $b \in P$. The intersection of all the prime ideals will be denoted by $\mathcal{S}$. Then $\mathcal{S}$ contains the set W of strongly nilpotent elements of N, i.e. elements a such that every sequence $a_o, a_1, \ldots$ defined by $a_o = a$, $a_{n+1} \in a_n N a_n$. In the case where W is an essential subset of $\mathcal{S}$ we prove that $\mathcal{S}$ is nearly nilpotent if N has finite dimension (i.e. no infinite direct sums of right ideals) or if N has maximum condition on right annihilators.

S. PELLEGRINI

<u>Φ-sums of near-rings: medial, permutable and LRD-near-rings</u>

The Φ-sums of near-rings allow us to classify some classes of near-rings: near-rings with a left permutable idempotent element are special Φ-sums, mixed medial near-rings and left permutable near-rings with an idempotent are characterized as Φ-sums. Moreover we have studied the left permutable Θ-near-rings (satisfying the condition that r(n) is a prime ideal of type 1 and each element is a zero-divisor) and we have shown that a left permutable near-ring N is a Θ-near-ring iff Q (the set of nilpotent elements) is a prime ideal of type 1 and Q = A (annihilator of N). Using the Φ-sums we are able to obtain the left permutable near-rings with an idempotent. In terms of Φ-sums we have studied the LRD-near-rings (satisfying the identities xyz = xyxz = xzyz). If N is an LRD-near-ring, then Q is an ideal, $Q^3 = \{0\}$ and N/Q is Boolean. Moreover if N is a zero-symmetric Boolean LRD-near-ring, then $N = A \oplus B$ where A and B are Boolean LRD-near-rings. If the a.c.c. on the right annihilators holds, then N_o is the direct sum of a finite number of LRD-near-rings whose non trivial elements are left identities. The LRD-near-rings having Id (set of idempotents) as a sub-n.r. are classified as Φ-sums.

G.L. PETERSON

Blocks in Near-rings

If R is an artinian ring, R can be expressed uniquely as a direct sum of indecomposable ideals called the blocks of R. These blocks alternatively can be defined in terms of primitive idempotents of R and play a central role in representation theory. A theory of blocks can be developed for near-rings as well. This was done initially by H. Lausch (Idempotents and blocks in artinian d. g. near-rings with identity element, Math. Ann., 188 (1970), 43 - 52). In this talk, a simpler way of developing the theory of blocks in near-rings using results of the author on lifting idempotents (Lifting idempotents in near-rings, Arch. Math., 51 (1988), 208 - 212) will be outlined.

G. PILZ

Codes obtained from planar near-rings

Given a BIB-design with parameters (v,b,r,k,λ), one can form its incidence matrix A of size $v\times b$. The rows of A (as a subset of $(\mathbb{Z}_2)^b$) form a code, the *row-code* of A. This code consists of b codewords of length v, equal weight k and minimal distance $2(k-\mu)$, where μ is the maximal size of block intersections. In a similar way, the columns of A give rise to the *column-code* of A, a code with v words of length b and weight r. Each two words of this code have the same distance $2(r-\lambda)$.

Results by G. Ferrero and J. Clay show that under certain circumstances one gets BID-designs from a finite planar near-ring N. There are 3 ways to get such designs, by taking the elements of N as the points of the design and as blocks all subsets of the form $aN+b$ or aN^*+b or $(aN \cup (-a)N) + b$, with $a \in N^* = N - \{0\}$, $b \in N$. There are easy ways to construct these designs and the parameter of the resulting row and column codes can be determined. Many of the resulting codes turn out to be very good w.r.t. the usual criteria for codes.

D. RAMAKOTAIAH

Interpolation of self maps of a group

A classical result in interpolation theory is that if f(x) is a real valued function of a real variable x and $x_1, x_2, \ldots, x_n$ are real numbers, then there exists a polynomial function P(x) over the reals such that $f(x_i) = P(x_i)$, $i = 1,2,\ldots,n$. This result can be generalized over any arbitrary field in the usual way. In fact every self map of a finite field is a polynomial function over that field. The concept of polynomial functions over a group has been introduced and some of their properties have been studied, ([1]). In this paper we introduce the concept of a group with interpolation property with respect to a semi-group of its endomorphisms and determine all such groups. As a special case of this, we determine all the groups G, whose selfmaps can be interpolated by polynomial functions over G.

REFERENCES

[1] LAUSCH, H. and NÖBAUER, W.: Algebra of polynomials, North Holland, 1973.

S. RAO

Some remarks on distributors

The distributor D(R) of any (left) near-ring R is an invariant ideal of R, containing the constant part of R, and it is representable as the sum of the normal closures in (R,+) of two subgroups: the commutator subgroup $[R^2,R^2]$, and the subgroup generated by the set $\{xa+ya-(x+y)a \mid x,y \in R,\ a \in A\}$, where A is any set of generators of (R,+).

This reduces to a classical result of A. FRÖHLICH, when R is a distributively generated near-ring.

S. VELDSMAN

Radicals of near-rings

All the (Kurosh-Amitsur) radicals with hereditary semisimple classes in the variety of all near-rings which are known are such that the semisimple class contains only 0-symmetric near-rings. We do not know if this is true in general - what we have proved is that such radicals contain all the constant near-rings. In the variety of 0-symmetric near-rings it is known that J_2, J_3 and the Brown-McCoy radicals are ideal-hereditary. We present many more, based on the equiprime near-rings; some of which are independent of the mentioned radicals. A near-ring N is equiprime if $0 \neq a \in N$ and $anx = any$ for all $n \in N$ $(x,y, \in N)$ implies $x = y$. (This is joint work with G.L. Booth and N.J. Groenewald.) The near-rings N with the property: whenever $J \vartriangleleft I \vartriangleleft A$ and $I/J \cong N$, then $J \vartriangleleft A$ are characterized as the quasi, semi-equiprime near-rings. The latter are near-rings N with $(0:N) = 0$, and if (I, ϕ) is a pre-image of N and $I \vartriangleleft A$, then $x - y \in \ker\phi$ $(x,y \in I)$ implies $ax - ay \in \ker\phi$ for all $a \in A$.

A.P.J. van der WALT

Near-linear transformations of near-vector spaces

The 2-primitive near-rings are usually considered to be very well-behaved near-rings, more or less taking the place in near-ring theory that rings of matrices over division rings take in ring theory. The thesis of this talk is that 2-primitive near-rings can be very "wild", and that it should be worth while to study some especially nice examples of 2-primitive near-rings in order to arrive at useful minimum conditions in a structure theory. It is suggested that near-rings of near-linear transformations of near-vector spaces are excellent examples to study in this context, and some results about such near-rings are presented. One result is that there are 2-primitive, very well-behaved near-rings which have minimal subgroups but no minimal left ideals.

H. WEFELSCHEID

The Ungar loop in the special theory of relativity, and neardomains

The Lorentz transformations $L = L(v,\rho)$ depend on two parameters: the velocity $v = (v_1,v_2,v_3) \in \mathbb{R}^3_c := \{v \in \mathbb{R}^3 \mid |v| < c\}$ and $\rho \in SO(3)$, where c denotes the speed of light and SO(3) the rotation group of $\mathbb{R}^3$.

If $L(u,\rho_u)$ and $L(v,\rho_v)$ are two Lorentz transformations then one can ask, how in the product $L(w,\rho_w) = L(u,\rho_u)L(v,\rho_v)$ the parameters w,ρ_w depend on the parameters u,v,ρ_u,ρ_v.

Now A. Ungar (Iargo) discovered that if we set $u * v = w$ then $(\mathbb{R}^3_c,*)$, the set of admissible velocities is a loop, which satisfies some weak associativity and weak commutativity conditions.

In this talk it is explained, that the Ungar loop $(\mathbb{R}^3_c,*)$ has exactly the additive structure $(F,+)$ of a neardomain $(F,+,\cdot)$. There are also connections to (infinite) sharply 2-transitive groups and (infinite) Frobenius groups. Question: Let G be an infinite Frobenius group operating on a set M and $G_a := \{\alpha \in G \mid \alpha(a) = a\}$ a stabilizer. Is it always possible to find a subset $N \subset G$ such that the representation $G \ni \gamma = \beta\cdot\alpha$, with $\beta \in N$ and $\alpha \in G_a$ is unique?

H.J. WEINERT

Homomorphisms of seminearfields

An algebra $(S,+,\cdot)$ is called a (right distributive) seminearring iff $(S,+)$ and $(S,\cdot)$ are semigroups and $(a+b)c = ac+bc$ holds for all $a,b,c \in S$. If there exists a neutral o of $(S,+)$, we define $S^* = S\setminus\{o\}$ and otherwise $S^* = S$. A seminearring $(S,+,\cdot)$ is called a seminearfield iff $(S^*,\cdot)$ is a group. Several basic statements on seminearfields have been given in the talk, in particular a survey of all finite ones. Moreover, if a seminearfield $(S,+,\cdot)$ has a zero o and at least two more elements, then $oa = ao = o$ holds for all $a \in S$, and $(S,+,\cdot)$ is either a nearfield or

$(S^*,+,\cdot)$ is a subseminearfield of $(S,+,\cdot)$. Therefore, dealing with seminearfields which are not nearfields, we may restrict ourselves to those satisfying $S = S^*$, called proper seminearfields in the following.

Let Φ: $S \to A$ be a homomorphism of a proper seminearfield $(S,+,\cdot)$ into any (2,2)-algebra $(A,+,\cdot)$. Then $(\Phi(S),+,\cdot)$ is again a proper seminearfield, $\kappa = \Phi^{-1} \circ \Phi$ a congruence on $(S,+,\cdot)$, and $(\Phi(S),+,\cdot)$ isomorphic to the congruence class seminearfield $(S/\kappa,+,\cdot)$, consisting of the κ-classes $[a]_\kappa$ with operations given by representatives. Each congruence κ of this kind corresponds to a normal subgroup K of $(S,\cdot)$ such that $a\kappa b \Leftrightarrow ab^{-1} \in K$ is also a congruence on $(S,+)$, called a kernel of $(S,+,\cdot)$. These normal subgroups can be characterized by several conditions, e. g. by $s+t \in K \Rightarrow s+tk \in K$ for all $s,t \in S$ and $k \in K$. Based on that we obtain statements on seminearfield homomorphisms and kernels corresponding to those on group homomorphisms and normal subgroups, e. g. both Isomorphism Theorems, characterizations of direct products of seminearfield, etc. In particular, a kernel of a seminearfield need not be a subseminearfield. But the latter is the case iff the corresponding epimorphic image $(S/K,+,\cdot)$ has idempotent addition.

The results are also applicable to semifields $(S,+,\cdot)$, i. e. to seminearfields satisfying also the other distributive law. Moreover, they provide a new kind of examples in the context of a general radical theory.

For parts of this talk, in particular in the case of semifields, we refer to: Harry H. Hutchins and Hanns J. Weinert, Homomorphisms and kernels of semifields, to appear in Period. Math. Hungar.

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