MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Random Partial Differential Equations

19.11. bis 25.11.1989

Die Tagung fand unter der Leitung von Ulrich Hornung (München), Peter Kotelenez (Utrecht und Cleveland/Ohio) und George Papanicolaou (New York) statt. Es nahmen insgesamt 37 Personen aus 12 Ländern an der Tagung teil.

Die Schwerpunkte waren zufällige Schrödinger-Operatoren, stochastische partielle Differentialgleichungen, Malliavin-Kalkül, Dirichlet-Formen und stochastische Systeme in zufälligen Medien. Die meisten der gehaltenen Vorträge fallen in diese Gebiete. Die übrigen handelten von Grenzwertesätzen für Teilchensysteme und anwendungsorientierten mathematischen Modellen, wie z.B. durch zufällige Störungen erregte Schwingungen.

Vortragsauszüge

SERGIO A. ALBEVERIO *Some recent developments in the theory of infinite dimensional Dirichlet forms and Markov fields*

We report on some recent developments in the theory of Dirichlet forms and associated diffusions on general infinite dimensional spaces. This is based on joint work with Michael Röckner (Edingurgh). Let E be a Souslin space (e.g. $E = \mathcal{S}'(\mathbf{R}^d)$ or E a Banach space) and let μ be a probability measure on E. A natural "pre Dirichlet form" $\mathcal{E}^0$ associated with μ is given by $\mathcal{E}^0(u,v) \equiv \frac{1}{2}\int \nabla u \nabla v d\mu$ with u, v smooth cylinder functions, looked upon as a form on $L^2(\mu)$. A necessary and sufficient condition on μ for closability of the form is exhibited. This condition solves in particular a conjecture by Fukushima in finite dimensions. We associate to the closure $\mathcal{E}$ of $\mathcal{E}^0$, the so called "classical Dirichlet form given by μ", a symmetric Markov semigroup P_t and a Markov process. Under some additional very general assumptions on E we show that this process is a diffuion on E, satisfying in the weak sense a stochastic differential equation $dX_t = \beta(X_t)dtX + dw_t$, with w_t a Brownian motion on E and β a drift vector uniquely determined by μ and with components in $L^2(\mu)$. The initial condition can be taken arbitrary outside a set of capacity 0. A rich class of interesting examples is provided by the measures μ which give the distribution of homogeneous (Euclidean) Markov scalar fields (over $\mathbf{R}^d$ in the Gaussian case, $d = 2$ in the non Gaussian case). Such measures also fit in the class of positive generalized functionals in the sense of Hida's calculus. The associated quantum fields are local relativistic fields over two-dimensional space-time. Some open problems are also mentioned. We close by indicating the construetion of a class of (non Gaussian) vector Markov homogeneous (Euclidean) random fields over $\mathbf{R}^4$ obtained by solving a stochastic first order partial differential equation. This construction, which leads to models of relativistic local quantum fields over 4-dimensional space-time, is based on recent joint work initiated with R. Høegh-Krohn and pursued with K. Iwata and T. Kolsrud.

PAO-LIU CHOW *Exponential estimates for some parabolic Itô equations*

This talk is concerned with a large deviation type of estimates for some diffusion processes in Hilbert spaces, including the Brownian motion, stochastic integrals and solutions to parabolic Itô equations. First, exponential upper bounds in exit probability are proved for an Itô process to leave a ball of radius r in a Hilbert space before time t. Then estimates are obtained for the solution process to a parabolic Itô equation. We will also indicate some applications of such estimates in the proof of a large deviation theorem for semi-linear parabolic equations perturbed by a multiplicative white noise.

WALTER CRAIG *Trace formulae for Schrödinger operators with ergodic potentials*
We consider the spectral problem for Schrödinger operators

$$L(q)\psi = (-\frac{d^2}{dx^2} + q(x;\omega))\psi = \lambda\psi$$

where $q(x;\omega)$ is ergodic. That is, there is a probability measure P defined on $C(\mathbf{R})$ which is ergodic with respect to translation, and we consider all $q \in supp(P)$. Results are presented which support the conjecture that if the spectrum is entirely absolutely continuous then $q(x)$ is almost periodic and $supp(P)$ is a torus. We prove, under geometrical conditions on the spectrum $\sigma(L(q))$ and under the assumption that the Lyapunov exponent $\gamma(\lambda)$ vanishes almost everywhere on the spectrum, that $supp(P)$ is compact. The methods are inverse spectral theoretic, including a generalization of the trace formulae well known in periodic theory.

GIUSEPPE DA PRATO *Kolmogorov equations in Hilbert space and applications to Hamilton-Jacobi-Bellman equations*
We consider the Kolmogorov equation related to a linear or semi-linear stochastic equation in Hilbert space. We prove a smoothing property of the corresponding semi-group and we apply this result to solve a Hamilton-Jacobi-Bellman equation arising in stochastic control.

G. F. DELL'ANTONIO *Nelson processes cannot attain the zeroes of their density*
A conservative (Nelson) diffusion is (when it exists) a diffusion in $\mathbf{R}^d$ with density ρ and drift $b = v + \frac{1}{2}\nabla \log \rho$, which satisfy a weak form of the continuity equation $\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$. Processes of this type occur when one wants to construct diffusions adapted to solutions of Schrödinger's equation. When v and ρ are sufficiently smooth and ρ is strictly positive, it is easy to prove that such diffusions exist. If $Z_\rho = \{(t,x) : \rho = 0\}$ is not empty, difficulties occur because b becomes singular at Z_ρ. We use Lyapunov stability techniques to prove, under suitable assumptions, that the diffusion a.s. does not reach Z_ρ. We use $-\log\delta$ as a Lyapunov function, where δ is a smooth version of the distance function from Z_ρ. This allows to treat cases when the density is not smooth, and sheds light on Carlen's energy condition.

M. DOZZI, J. B. WALSH *On the Markov property of solutions of parabolic stochastic differential equations*
Let $D \subset \mathbf{R}^d$ $(d \geq 1)$ be a bounded domain with smooth boundary ∂D. We consider the following initial-boundary value problem:

$$\frac{\partial V}{\partial t} = \Delta V + \dot{M} \text{ on } D\times]0,\infty[\,; V = 0 \text{ on } \partial(D \times [0,\infty[), \qquad (1)$$

where Δ is the Laplace-operator on $\mathbf{R}^d$ and $(M_t; t \geq 0)$ is a worthy martingale measure [1]. The solution of 1, in the sense of distributions, is given by

$$V_t(\phi) = \int_{D\times]0,t]} G(\phi,t;y,s)dM_{ys},\ \phi \in C_0^\infty(\bar{D}),$$

where $G(\phi;y,s) = \int \phi(x)G(x,t;y,s)dx$ and $G(x,t;y,s)$ is the Green's function of the homogeneous part of 1, i.e. with $\dot{M} = 0$ (see [1]). Let $A \subset D \times \mathbf{R}_+$ be a bounded domain with smooth boundary ∂A and suppose that M has the germfield Markov property with respect to A. Then also V has the germfield Markov property with respect to A and we show the minimal splitting field can be represented by means of the trace of V on ∂A and the generalized normal derivative of V on ∂A, where the trace is given by

$$\mathrm{tr}_{\partial A} V(\psi) = \int_{D\times\mathbf{R}_+}\int_{\partial A} \psi(x,t)G(x,t;y,s)S(dx,dt)dM_{ys},\ \psi \in C_0^\infty(\bar{D}\times\mathbf{R}),$$

where S is the surface measure on ∂A, and the normal derivative is given by

$$\frac{\partial V}{\partial N_x}(\psi) = \int_{D\times\mathbf{R}_+} \frac{\partial G}{\partial N_x}(\psi; y, s) dM_{ys},$$

where $\frac{\partial G}{\partial N_x}(\psi; y, s) = \int_{\partial A} \psi(x,t) N_x(x,t) \cdot \nabla_x G(x,t;y,s) S(dx,dt)$ and $N_x(x,t)$ is the vector of the space components of the unit outer normal in $(x,t) \in \partial A$. This result is contained in [2].

References:

[1] J. B. Walsh: An introduction to stochastic partial differential equations. Lecture Notes in Math. **1180** (1986) 266-438.

[2] M. DOZZI, J. B. WALSH: Markov property of solutions of stochastic partial differential equations. Manuscript (1989).

RODOLFO FIGARI, S. TETA *Limit theorems for boundary value problems of mixed type*

We analyze the asymptotic behavior of the solution of an elliptic problem with homogeneous mixed boundary conditions on a large number of inclusions randomly distributed. An effective equation is found for the limit problem, as the number of inclusions increases without bound and their linear size converges to zero. The result extends to potentials with negative parts, the class of effective potentials which can be obtained by a limiting procedure. We also establish the relation betweeen this problem and the asymptotic behavior of the Laplacian with zero-range potentials on a set of points invading densly a bounded or unbounded region in $\mathbf{R}^3$.

JEAN-PIERRE FOUQUE *Fluctuations field for the asymmetric simple ex-clusion*

The hydrodynamical limit for the asymmetric simple exclusion process, and more generally for asymmetric attractive processes, has been investigated by several authors during the last years. In Benassi-Fouque [1987] we have shown that the corresponding density profile is the unique entropy solution to the associated nonlinear Euler equation. In this work we study the fluctuations field and we show, at least in a particular case, that the limiting field consists in an initial Gaussian field transported by the linearized equation.

KENNETH GOLDEN *Critical phenomena in random resistor networks*

We consider the bulk conductivity $\sigma^*(p)$ of the bond lattice in $\mathbf{Z}^d$, where the bonds have conductivity 1 with probability p or $\varepsilon \geq 0$ with probability $1-p$. The analytic properties of $\sigma^*(p)$ are investigated, particularly convexity near the percolation threshold p_c, which can be observed in numerical simulations. In $d=2$ it is proved that for every $\varepsilon > 0$, $\sigma^*(p)$ is convex in a neighborhood containing $p_c = \frac{1}{2}$. This result is based on Keller's interchange equality, and analyticity of $\sigma^*(p)$ in a neighborhood containing $[0,1]$ in the complex p-plane. For $\varepsilon = 0$, where it is believed that $\sigma^*(p) \sim (p-p_c)^t$ as $p \to p_c^+$ for a dimension dependent critical exponent t, a simplified model of the conductivity backbone is analyzed. This analysis not only explains convexity near p_c, but indicates in which dimensions $\frac{d^2\sigma^*}{dp^2}$ and $\frac{d^3\sigma^*}{dp^3}$ either diverge or vanish as $p \to p_c^+$. Based on this analysis, we propose the inequalities $1 \leq t \leq 2$ in $d = 2,3$ and $2 \leq t \leq 3$ in $d \geq 4$. In $d=3$, the inequality $t \leq 2$ exclude many numerical estimates of t in $d=3$, which have ranged from 1.5 to 2.36.

I. Ya. GOLDSHEID *Lyapunov exponents and one special representation of the integrated density of states*

The main aim of the first part of this report is to explain that the algebraic language is adequate for describing the properties of Lyapunov exponents.

Let $A_i(\omega)$ be i.i.d. $m \times m$-matrices, $A_i \in GL(m, \mathbf{R})$ with $E|\ \ln ||A_i||\ | < \infty$ and ν being the distribution function of $A_i(.)$. G_ν is the closed group generated by the matrices belonging to the support of ν and $\bar{G}_\nu$ is the Zariski closure of G_ν. Lyapunov exponents of $S_n = A_n(\omega) \cdot \ldots \cdot A_1(\omega)$ are denoted by $\alpha_1 \leq \ldots \leq \alpha_m$.

Theorem 1 If $\bar{G}_\nu = GL(m, \mathbf{R})$ then $\alpha_1 < \ldots < \alpha_m$. So, if for some i, $\alpha_i = \alpha_{i+1}$, then $\bar{G}_\nu$ is a nontrivial algebraic subgroup of $GL(m, \mathbf{R})$.

The results of this type can be found in I. Goldsheid, G. Margulis: "Lyapunov exponents of the product of random matrices" Uspehi Matem. Nauk. **44.5** (1989).

The second part of the report is devoted to some representation of $N(\lambda)$. Let us consider a difference Schrödinger operator H

$$(Hy)_n = -y_{n+1} + Q_n y_n - y_{n-1}, \ -\infty < n < \infty.$$

Here

$$y = (y_n)_{n=-\infty}^{\infty}, \ \sum_{n=-\infty}^{\infty} \|y_n\|^2 < \infty, \ y_n \in \mathbf{R}^m$$

with $m \times m$-matrices $Q_n = Q_n^*$. Let $H^{(k)}$ be a restriction of H to the interval $[0, k]$ with zero boundary conditions. We define

$$N_k(\lambda) = \#\{\lambda_i^{(k)} : \lambda_i^{(k)} < \lambda, \ \lambda_i^{(k)} = \text{eigenvalues of } H^{(k)}\}.$$

We need also the following sequence of selfadjoint $m \times m$-matrices

$$\phi_0 = 0, \ \phi_n(\lambda) = (Q_n - \phi_{n-1} - \lambda I)^{-1}, \ n \geq 1.$$

For any ϕ we define $N(\phi) = \#\{\mu_j : \mu_j < 0, \ \mu_j \text{ is the eigenvalue of } \phi\}$.

Theorem 2 $N_k = \sum_{n=1}^{k} N(\phi_n(\lambda))$.

It follows from theorem 2 and from the results of the mentioned work that if Q_n are i.i.d. selfadjoint matrices (under some natural conditions) then with probability 1 there exist

$$\kappa(\lambda) = \lim_{k \to \infty} \frac{1}{k} N_k(\lambda)$$

and $\kappa(\lambda)$ is Hölder continuous.

LUIS G. GOROSTIZA *Extended solutions of stochastic evolution equations with respect to semimartingales in Hilbert space*

Let $X = \{X_t : 0 \leq t \leq T\}$ be a stochastic process with values in a Hilbert space H, which satisfies the stochastic evolution equation

$$dX_t = A_t X_t dt + dZ(X)_t, \ 0 \leq t \leq T, \tag{1}$$

where $\{A_t : 0 \leq t \leq T\}$ is a family of closed linear operators which generate an evolution system $\{U_{t,s} : 0 \leq t \leq T\}$ on H, and $Z(X)$ is a semimartingale which may depend on X. The variation of constants formula corresponding to 1 is

$$X_t = U_{t,0} X_0 + \int_{]0,t]} U_{t,s} dZ(X)_s, \ 0 \leq t \leq T, \tag{2}$$

which is also an equation for X.

Several definitions of solutions of equations 1 and 2 are given and relationships between them are presented, when $D = \bigcap_{0 \leq t \leq T} \mathcal{D}(A_t^*)$ is dense in H, and they are extended for the case when D is not dense in H.

BRONIUS GRIGELIONIS *Hellinger measures and Hellinger processes for weak solutions of stochastic evolution equations*

Let $(\Omega, \mathcal{F}, \mathbf{F})$ be a filtered measurable space with a family $\{P^\theta : \theta \in \Theta\}$ of probability measures on $\mathcal{F}$. For each $\alpha = (\alpha_1, \ldots, \alpha_k)$, $0 < \alpha_j < 1$, $\sum_{j=1}^k \alpha_j = 1$ and $I = (\theta_1, \ldots, \theta_k)$, $\theta_j \in \Theta$, $j = 1, \ldots, k$, $k \geq 2$, the notions of the Hellinger processes $h_t(\alpha; I)$, $t \geq 0$, and the Hellinger measures $H_t(\alpha; I)(A)$, $A \in \mathcal{F}_t$, $t \geq 0$, are considered. In the case, when the measures P^θ, $\theta \in \Theta$ are weak solutions to the defined stochastic evolution equations, characterized as the unique solutions to the corresponding martingale problems, an explicit relationship of $H_t(\alpha; I)$ and $h_t(\alpha; I)$ is defined using coefficients of the stochastic equations.

ULRICH HORNUNG *A problem from soil physics*

Different approaches are discussed that deal with flow and transport through porous media having random properties. Assuming that the permeability of the diffusivity is a random field, one is led to elliptic and parabolic equations having stochastic coefficients. The mathematical methods used are (1) Monte-Carlo simulation, (2) Fourier-transform methods, (3) stochastic homogenization, and (4) perturbation techniques. Open problems are pointed out.

G. JETSCHKE *Nonlinear reaction-diffusion equations with white noise: Lar-ge deviations, lattice approximation, simulated annealing*

Nonlinear SPDEs of reaction-diffusion type with white noise disturbance arise in several physical and biological applications. The solution of the special equation on a one-dimensional interval $[0, L]$ (see the paper by Manthey) should satisfy

$$U_t = T_t h + \int_0^t T_{t-s} f(U_s) ds + \int_0^t \sigma - T_{t-s} dW_s$$

and can be understood as a Markov process on the state space $C[0, L]$. Here $(T_t)_{t>0}$ denotes the semigroup generated by $\Delta = \frac{d^2}{dx^2}$ and $(W_t)_{t\geq 0}$ is a cylindrical Wiener process.

1) Large deviation properties are derived to study most probable states and trajectories: The process (U_t) has a unique invariant distribution $P_\sigma^0 \ll Q_\sigma^0$, $Q_\sigma^0 = N(0, C)$, $C = \frac{\sigma^2}{2}(-\Delta)^{-1}$, with

$$\frac{dP_\sigma^0}{dQ_\sigma^0}(q) = \mathcal{N} \cdot \exp\Big(-\frac{2}{\sigma^2} \int_0^L F[q(x)] dx\Big), \ F' = f.$$

The family $(P_\varepsilon^0)_{\varepsilon > 0}$, $\varepsilon^2 = \frac{\sigma^2}{2}$, satisfies a large deviation principle (LDP) on $C[0, L]$ with rate function

$$S(q) = \int_0^L \Big(\frac{1}{2}|q'(x)|^2 - F[q(x)]\Big) dx - \text{const}.$$

Hence the most probable states are the local minima of S corresponding to the stable fixed points of the deterministic evolution ($\sigma = 0$). The family of induced measures $(\mathbf{P}_\varepsilon)_\varepsilon > 0$ on the path space $C([0, T]; C[0, L])$ also satisfy a LPD with a certain rate function from which characteristic properties of tunneling between local minima of S can be derived.

2) A lattice approximation $U(t, il) \approx U_i^N(t)$, $i = 0, \ldots, N$, $l = \frac{L}{N}$, $N = 1, 2, \ldots$ is given where Δ is replaced by second order differences and the noise field $\xi(t, x)$ by independent white noise $\frac{1}{\sqrt{l}} \xi_i(t)$. This can be done such that the evolution U^N on $C[0, L]$ associated by piecewise linear interpolation converges in probability, i.e.

$$\lim_{N \to \infty} P\{\sup_{t \leq T,\ x \in [0, L]} |U_{tx} - U_{tx}^N| > \delta\} = 0$$

for all $\delta > 0$.

3) Let the noise strength $\sigma(t)$ be time-dependent. Let μ_0 be the suitably weighted distribution on the global minima of S. The following result on simulated annealing is conjectured: If $\sigma(t) \geq \frac{c}{\log(1+t)}$, c

a certain constant, then $\lim_{t\to\infty} P_{U_t} = \mu_0$. This should be proved by combining lattice approximation and finite-dimensional results.

WERNER KIRSCH *Surface effects for random operators*

Let $V_\omega^+(x)$, $V_\omega^-(x)$, $x \in \mathbb{R}^d$, be two ergodic random fields and set $V_\omega(x) = V_\omega^+(x)$ for $x_1 \geq 0$ and $= V_\omega^-$ for $x_1 < 0$, $x = (x_1, \ldots, x_d)$. We are interested in the random Schrödinger operator $H_\omega = -\Delta + V_\omega$. It is not difficult to see that the spectrum Σ of H_ω (which is a nonrandom set) contains the spectra $\Sigma^\pm$ of $H_\omega^\pm = -\Delta + V_\omega^\pm$. In addition there may be also energies in Σ which are not in $\Sigma^+ \cup \Sigma^-$. We call these energies "surface energies". One may define a "density of surface states" in analogy with the density of states but normalized by a surface term rather than a volume. It is proven that the density of surface states exists and that its support together with the support of the density of (bulk) states agrees with the spectrum Σ.

Reference: H. English, W. Kirsch, M. Schröder, B. Simon: "Random Schrö-dinger operators ergodic in all but one direction", Comm. Math. Phys. (to appear)

V. I. KLYATSKIN *Diffusion approximation used in stochastic equations*

A statistical description of dynamical systems with fluctuating parameters is presented on the basis of a functional approach, of δ-correlated and of diffusive approximations. These approximations are discussed in detail for examples of certain boundary value problems for plane wave propagation in randomly layered media. The application of the invariant imbedding method permits to replace the boundary value problems in question by problems with initial conditions that are characterized by the "dynamic causality" property. On the basis of imbedding equations we analyse δ-correlation and diffusive approximations, the role of boundary conditions on the problem solution and the effects of localization of energy in randomly layered media.

PETER KOTELENEZ *A comparison theorem and positivity for a class of stochastic differential equations*

For a class of stochastic differential equations driven by space-time white niose a comparison theorem is proven under the assumption that the semigroup generated by the linear part of the drift is positivity preserving. As a consequence a sufficient condition for positivity of solutions is derived, which under mild additional assumptions is shown to be necessary.

RALF MANTHEY *Reaction-diffusion equations with white noise*

The Dirichlet and the initial value problem for the reaction diffusion equation

$$\frac{\partial}{\partial t} u(t,x) = (\Delta u)(t,x) + f(u(t,x)) + \sigma \cdot \xi(t,x)$$

are considered. Here ξ is a space-time Gaussian white noise. Physically interesting conditions on the reaction function f are given, which ensure the existence and uniqueness of a solution u to both problems. Two types of approximation of the driving term ξ are discussed. In one case ξ is replaced by a space-time correlated noise ξ_n, in the other a Poissonian point process ϕ_n is considered. It can be shown that the corresponding solutions u_n converge weakly to u.

DAVID NUALART *Stochastic parabolic differential equations with reflection*

We consider the following stochastic partial differential equation of parabolic type

$$\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2}{\partial x^2}(x,t) + f(u(x,t)) = \xi(x,t) + \frac{\partial^2 \eta}{\partial x \partial t},$$

with $0 \leq t \leq T$, $0 \leq x \leq 1$, where ξ is a space-time white noise and f an increasing function. Also we impose Dirichlet boundary conditions on the function u. Then we define a weak solution of the preceding equation as a pair (u, η) such that u is positive and continuous, and η is a random measure (possibly infinite) on $[0,1] \times [0,T]$ whose support is contained in the set where u vanishes, and u obeys the above equation in a weak sense. Intuitively, η represents the minimum amount of pushing upward required to keep the solution u nonnegative. The main result is the existence and uniqueness of such a solution. Moreover the measure η integrates the function $x(1-x)$ and is continuous in the time variable. The proof uses the classical penalization method. The case of a white noise with a nuclear covariance has been studied in detail by Haussmann and Pardoux [Appl. Math. Optim. 1989]. The results described above have been obtained in a joint work with E. Pardoux.

GEORGE PAPANICOLAOU *Pulse reflection of waves by randomly layered media*

When the width of the pulse incident on a randomly layered half space is broad compared to the layer size but narrow compared to the layer scale variations of the layer properties, it is possible to give a very precise and relatively simple characterization of the reflected signals. They are nonstationary Gaussian processes that have a simple structure. The local power spectral density of these processes can be obtained by solving a system of transport equations that arise from a moment hierarchy in the limit. Using this asymptotic theory we address several inverse problems: to detect the large scale properties of the medium from the statistics of the reflected signals.

L. PASTUR, W. KIRSCH *Large time behavior of some moments of fundamental solutions of the random parabolic equation*

Let $V(x)$ be a random ergodic field in $\mathbf{R}^d$ and $P_\pm(t,x,y)$, $t \geq 0$, $x,y \in \mathbf{R}^d$ be the fundamental solution of the equation

$$\frac{\partial P_\pm}{\partial t} = (\Delta \mp V)P_\pm, \ \ P_\pm|_{t=0} = \delta(x-y).$$

We study the behavior for $t \to \infty$ of the quantity

$$\tilde{A}(t) = \int_{\mathbf{R}^d} E\{P_+(t,0,x)P_-(t,x,0)\}dx,$$

where $E\{\dots\}$ denotes the mathematical expectation corresponding to V. Our results are as follows:

(i) if $V(x)$ is the Gaussian field and $E\{V(x)\} = 0$, $E\{V(x)V(y)\} = B(x-y)$, then

$$\ln \tilde{A}(t) = (B(0) - B(x_0))t^2(1 + o(1)), \ t \to \infty$$

where $B(x_0) = \inf_{x \in \mathbf{R}^d} B(x) \leq 0$;

(ii) if $V(x)$ has the form $\sum_j f(x - x_j)$, where $\{x_j\}$ is the Poisson point field with the density η, $f \in L_1(\mathbf{R}^d) \cap C^2(\mathbf{R}^d)$ and $f(0) = \sup_{x \in \mathbf{R}^d} f(x) > 0$, $f(x_0) = \inf_{x \in \mathbf{R}^d} \leq 0$, then

$$\ln \tilde{A}(t) = Ct^{-\frac{d}{2}} \cdot \eta \cdot \exp\{t[f(0) - f(x_0)]\}(1 + o(t)), \ t \to \infty$$

where C can be expressed via the Hessian of f.

The similar asymptotic formula holds for the field $V(x) = \sum_{n \in \mathbf{Z}^d} \xi_n f(x-n)$, where ξ_n, $n \in \mathbf{Z}^d$, are i.i.d. random variables and $f(x)$ is as above. We compare our results with known asymptotical results for $\tilde{N}(t) = E\{P_\pm(0,0,t)\}$ and discuss applications to the theory of disordered systems.

ROSS PINSKY, MARK PINSKY *Transience and recurrence for a class of random diffusions and an application to random parabolic operators*

Let $k(t)$ be an ergodic finite state Markov chain on the state space $E = \{1, 2, \dots n\}$ with invariant probability measure μ. For each $k \in E$ let

$$L_k = \frac{1}{2}\sum_{i,j=1}^{d} a_{ij}(x;k)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{d} b_i(x;k)\frac{\partial}{\partial x_i}$$

be a nondegenerate diffusion generator. Then for each fixed realization $k(t)$, $L_{k(t)}$ may be thought of as a time inhomogeneous diffusion generator. Denote the corresponding time inhomogeneous diffusion process by $X(t;k(.))$. We call $X(t;k(.))$ a random diffusion. We investigate the transience and recurrence properties of the following class of random diffusions:

$$L_k = \frac{1}{2}\Delta + V_k,$$

where

$$V_k \sim |x|^\delta \hat{b}(\frac{x}{|x|};k)\cdot\nabla \text{ as } |x|\to\infty,$$

$\delta \in [-1,1)$ and $\hat{b}(\frac{x}{|x|};k)$ has mean zero with respect to μ. That is, the drift is asymptotically mean zero and homogeneous of degree $\delta \in [-1,1)$. We give necessary and sufficient conditions for the transience or recurrence of $X(t;k(.))$ a.s. with respect to $k(.)$. We also give an application to random parabolic operators.

UWE RÖSLER *The variation diminishing property for diffusions*

The transition probability kernel for one-dimensional birth and death processes or diffusions is totally positive and has the variation diminishing property. A kernel K has the variation diminishing property, if the number of sign changes of a function f is not smaller than those of Kf. In the talk we show how to use these properties to derive results on the geometric shape of the transistion probability densities. It also provides a probabilistic proof that these densities exist in the most general case, even for arbitrary speed measure. This method allows local results from global convergence. As an example we state the following result:

Theorem Let X^n be a sequence of birth and death processes converging weakly to a diffusion. X^n, X are on natural scale. Then

(i) $P_x(X,t,\varepsilon.)$ is absolutely continuous w.r. to the speed measure m,

(ii) $\frac{P_x(X,t,\varepsilon dy)}{dm(y)}$ is a unimodal function of y for all x.

FRANCESCO RUSSO *Density estimates for two-parameter diffusion: the case of generalized Hörmander conditions*

Under generalized Hörmander conditions introduced by D. Nualart and M. Sanz, the law of a two-parameter diffusion has a smooth density on the axes. We obtain an estimate of this density when the parameter is small. In this way, we extend the Varadhan estimate which is well-known for ordinary diffusions.

HERBERT SPOHN *Large scale structure of a stationary nonequilibrium measure*

We consider a reversible stochastic lattice gas subject to steady flux at the boundary. Of interest is the stationary measure in the limit of lattice constant $\varepsilon \to 0$. It is expected that density profiles become deterministic in the limit and converge to the steady solution of the appropriate nonlinear diffusion equation. Locally the distribution of particles should be given by the appropriate Gibbs measure with the local density. Jointly with G. Eynik and J. L. Lebowitz we prove the law of large numbers and local equilibrium. We use the technique of small entropy production of Guo, Papanicolaou, and Varadhan. This method requires the gradient condition, at least at present. Nontrivial examples are known only in one dimension. For this reason we are restricted to one dimension. An interesting unresolved question is the fluctuation field for the stationary measure.

A. S. USTUNEL *A class of distributions on Wiener space and some applications*

In this talk I have given a way of construction of a class of distributions on an abstract Wiener space (AWS) via the second quantization of a self-adjoint operator on the associated Cameron-Martin space.

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One obtains a class of test functions (which is an algebra) which is smaller than the test functions of Meyer-Watanabe and larger than the cylindrical test functions. The corresponding distributions are stable under the basic operations as Sobolev derivative or divergence (i.e. Skorohod integral). As an application I have proposed the solution of the following equation

$$-\Delta(\xi(x,\omega) - E[\xi(x,\omega)]) + \mathcal{L}\xi(x,\omega) = \xi(x,\omega)\dot{W}(x,\omega) .$$

where Δ is the Laplace operator on $L^2(\mathbf{R}^d)$ and $\mathcal{L}$ is the Ornstein-Uhlenbeck operator on the AWS (operating on $\omega \in AWS$). With the increasing dimension the solution of this equation becomes more and more singular.

MICHAEL VOGELIUS *Homogenization convergence of planar networks*

Motivated by a problem in impedance computed tomography we seek to determine limits of discrete voltage potentials in resistor networks. It is shown that any possible limit (as the distance between nodes converges to zero) satisfies a diffusion equation. For special geometries (e.g. an equilateral triangular network) it is possible to obtain quite specific information about the realizable diffusion matrices. The talk reports work in progress.

W. WEDIG *Simulation and analysis of mechanical systems with parameter fluctuations*

In most situations, noise sources like external turbulent forces enter into the equations of motion of elastic structures in terms of parameter fluctuations. A typical example gives an Euler-Bernoulli beam under axial forces or corresponding axial displacements $u(t)$. It is described by the following boundary value problem.

$$EIw_{xxxx}(x,t) + \beta w_t(x,t) + \mu w_{tt} - \frac{EA}{l}[u(t) + \frac{1}{2}\int_0^l w^2(x,t)dx]w_{xx}(x,t) = 0,$$

$$w(0,t) = w(l,t) = w_{xx}(0,t) = w_{xx}(l,t) = 0; \; w(x,t) = T(t)\sin\pi\frac{x}{l}.$$

Because of the simple boundary conditions the lateral beam vibration $w(x,t)$ can be separated by the first mode $\sin\pi\frac{x}{l}$ leading to the ordinary differential equation with the natural frequency $\omega_1^2 = \pi^4\frac{EI}{\mu l^4}$

$$\ddot{T}(t) + 2D\omega_1\dot{T} + \omega_1^2[1 + \frac{Al}{I\pi^2}u(t)]T(t) + \gamma T^3(t) = 0.$$

This is a bifurcation problem. For $\omega_1^2 > 0$, $D > 0$ the trivial solution $T(t) \equiv 0$ can be destabilized by increasing fluctuations of $u(t)$. To compute the bifurcation point, we assume white noise fluctuations $\dot{w}_t$ with the intesnity σ and transform the linearized T-equation by polar coordinates.

$$d\log A_t = -\omega_1[D - (D + \frac{1}{2}\sigma^2\cos^2\psi_t)\cos 2\psi_t]dt - \sigma\sqrt{\omega_1}\sin 2\psi_t dw_t,$$

$$d\psi_t = -\omega_1[1 + (D + \frac{1}{2}\sigma^2\cos^2\psi_t)\sin 2\psi_t]dt - \sigma\sqrt{\omega_1}\cos^2\psi_t dw_t.$$

The phase process ψ_t is independent of the growth behavior of the amplitude A_t. Furthermore, there exists an invariant measure $p(\psi)$ determined by the stationary Fokker-Planck equation. Integrated once, it leads to

$$\frac{1}{2}\sigma^2\cos^4\psi p'(\psi) + [1 + (D - \frac{1}{2}\sigma^2\cos^2\psi)\sin 2\psi]p(\psi) = C$$

where C is a constant of integration given by the normalization condition. The diffusion equation possesses two singularities at $\psi = \pm\frac{\pi}{2}$. We introduce a regularization applying backwards differences

$$p_n = \frac{2\Delta\psi C + \sigma^2\cos^4\psi_n p_{n-1}}{2\Delta\psi[1 + (D - \frac{1}{2}\sigma^2\cos^2\psi_n)\sin 2\psi_n] + \sigma^2\cos^4\psi_n}.$$

This recurrence formula allows a simple and fast computation of the invariant measure $p(\psi)$ in the range $-\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2}$ and therewith of the Lyapunov exponent

$$\lambda = -\omega_1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [D - (D + \frac{1}{2}\sigma^2 \cos^2 \psi) \cos 2\psi] p(\psi) d\psi$$

simply by evaluating the integral above.

RUTH J. WILLIAMS, W. A. ZHENG *On reflecting Brownian motion - a weak convergence approach*

Consider a d-dimensional domain D that has a finite Lebesgue measure and whose boundary has zero Lebesgue measure. We define a sequence of stationary diffusion processes with drifts that tend to infinity at the boundary in such a way as to keep the sample paths in D. We prove that this sequence is tight and any weak limit process is a continuous stationary Markow process in $\bar{D}$. When D is bounded and locally representable as the region lying on one side of the graph of a continuous function, we identify our process with the stationary reflecting Brownian motion defined by Fukushima using the Dirichlet form that is proportional to $\int_D |\nabla g|^2 dx$, $g \in H^1(D)$. Furthermore, under a mild condition on the boundary of D, which is easily satisfied when D is a Lipschitz domain, we show that our process has a Skorohod-like semimartingale representation.

J. ZABCZYK *Some applications of control theory to stochastic evolution equations*

We show first that if two Ornstein-Uhlenbeck processes have equivalent laws then the associated control systems generate the same sets of trajectories. From this we deduce explicit necessary conditions for the law equivalence. They give also sufficient conditions for the equivalence. Then we describe a solution to the deterministic exit problem for the first and second order gradient systems obtained in a joint paper with G. Da Prato and A. Pritchard. We show how this solution is related to the stochastic exit problem of Freidlin-Wentzel in infinite dimensional spaces.

MOSHE ZAKAI, D. NUALART *Generalized Brownian funtionals and the solution to a stochastic partial differential equation*

A class of generalized Wiener functionals, related to those of Hida and Watanabe, is introduced. These notions are applied to the derivation of a solution to the stochastic partial differential equation

$$\frac{\partial Y}{\partial t} = LY + \psi + Y \cdot \eta \tag{1}$$

where L is a second order partial differential operator in the d-space variables, η is a white noise operator in the $(d+1)$-space-time variables, $Y \cdot \eta$ denotes the Skorohod stochastic integral (which for integrands which are adapted with respect to the t-parameter coincides with a suitably defined Itô integral), ψ is a non-random function in the $(d+1)$ space-time parameters. The existence and uniqueness of a solution to equation 1 in the class of generalized Brownian functionals is established.

ZHIMING MA, S. ALBEVERIO *Singular Schrödinger operators associated with Feynman-Kac semigroups*

In this paper we discuss Schrödinger operators $H^\mu = -\frac{\Delta}{2} + \mu$ on $L^2(\mathbf{R}^d)$. Here $\mu = \mu^+ - \mu^-$ is a signed measure such that μ^+ and μ^- are both smooth in the sense of M. Fukushima. The class of smooth measures (S in notation) is quite large. It contains all Radon measures having no polar sets, consequently each non-negative locally integrable function f corresponds to a smooth measure $f dx$. In addition, there are smooth measures μ which are nowhere Radon in the sense that $\mu(G) = \infty$ for each non-empty open set G. By the theory of Dirichlet spaces we show that there always exists a representation of H^μ provided the corresponding perturbed form is bounded from below. In particular, H^μ exists always if μ^+ is smooth and μ^- is in the Kato class. Thus, there are Schrödinger operators with potentials which are singular on each neighborhood of an arbitrary point x of $\mathbf{R}^d$. We give also necessary and sufficient conditions for the existence of continuous Feynman-Kac semigroups given in terms of Brownian motion and the additive functionals associated with signed smooth measures. We improve also the KLMN theorem in our specific context. All the results are extended to the case of Schrödinger operators restricted to a domain $D \subset \mathbf{R}^d$ with mixed boundary conditions.

Berichterstatter: Ulrich Hornung und Peter Kotelenez

Tagungsteilnehmer

Prof. Dr. S. Albeverio
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Dr. M. Dozzi
Institut für
Mathematische Statistik
Universität Bern
Sidlerstr. 5

CH-3012 Bern

Prof. Dr. P. L. Chow
Department of Mathematics
Wayne State University

Detroit , MI 48202
USA

Prof. Dr. R. Figari
Dipartimento di Scienze Fisiche
Mostra d'Oltremare - Pad. 19

I-80125 Napoli

Dr. W. L. Craig
Dept. of Mathematics
Brown University
Box 1917

Providence , RI 02912
USA

Prof. Dr. J.-P. Fouque
Centre de Mathematiques Appliquees
Ecole Polytechnique
E.R. A. - C. N. R. S. 756

F-91128 Palaiseau Cedex

Prof. Dr. G. Da Prato
Scuola Normale Superiore
Piazza dei Cavalieri, 7

I-56100 Pisa

Prof. Dr. K. Golden
Department of Mathematics
Princeton University
Fine Hall
Washington Road

Princeton , NJ 08544
USA

Prof. Dr. G. Dell'Antonio
Dipartimento di Matematica
Universita degli Studi di Roma I
"La Sapienza"
Piazzale Aldo Moro, 2

I-00185 Roma

Prof. Dr. I. Ya. Goldsheid
Institute of Mathematics
Academy of Sciences of the USSR
- Ural Branch -
ul. Tukaeva 50

Ufa 450 057
USSR

Prof. Dr. L. G. Gorostiza
Centro de Investigacion y de Estud.
Avanzados d. Inst. Politecnico Nat.
Dept. de Matematicas
Apartado Postal 14-740

Mexico ,CP 07 000,D.F.
MEXICO

Prof. Dr. W. Kirsch
Institut f. Mathematik
der Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof. Dr. B. J. Grigelionis
Institute of Mathematics and
Cybernetics
Akademijos St. 4

232600 Vilnius
USSR

Prof. Dr. V.I. Klyatskin
Pacific Oceanological Institute
Far East Branch
USSR Academy of Sciences
7, Radio Street

Vladivostok , 690032
USSR

Prof. Dr. U. Hornung
SCHI
Postfach 1222

8014 Neubiberg

Prof. Dr. P. Kotelenez
Mathematisch Instituut
Rijksuniversiteit te Utrecht
P. O. Box 80.010

NL-3508 TA Utrecht

Prof. Dr. W. Jäger
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Prof. Dr. R. Manthey
Sektion Mathematik
Friedrich-Schiller-Universität
Jena
Universitätshochhaus, 17. OG.

DDR-6900 Jena

Dr. G. Jetschke
Sektion Mathematik
Friedrich-Schiller-Universität
Jena
Universitätshochhaus, 17. OG.

DDR-6900 Jena

Prof. Dr. Ma Zhiming
BiboS
Universität Bielefeld
Wellenberg 1

4800 Bielefeld 1

Prof. Dr. D. Nualart
Departament d'Estadistica
Facultat de Matematicas
Gran Via 585

E-08007 Barcelona

Dr. K. Oelschläger
Sonderforschungsbereich 123
"Stochastische math. Modelle"
Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Prof. Dr. G. C. Papanicolaou
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York , N. Y. 10012
USA

Prof. Dr. L. A. Pastur
Institute for Low Temperature,
Physics and Engineering
UKR.SSR Academy of Sciences
pr. Lenina 47

Kharkov 310164
USSR

Prof. Dr. R. G. Pinsky
Department of Mathematics
Technion
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof. Dr. U. Rösler
Institut für Mathematische
Stochastik
der Universität Göttingen
Lotzestr. 13

3400 Göttingen

Prof. Dr. H. Rost
Institut für Angewandte Mathematik
der Universität Heidelberg
Im Neuenheimer Feld 294

6900 Heidelberg 1

Dr. F. Russo
Dept. Reseaux
ENST
46, rue Barrault

F-75634 Paris Cedex

K. U. Schaumlöffel
Fachbereich 3
Mathematik und Informatik
der Universität Bremen
Bibliothekstr.1, PF 33 04 40

2800 Bremen 33

Dr. M. Scheutzow
Fachbereich Mathematik
der Universität Kaiserslautern
Erwin-Schrödinger-Straße
Postfach 3049

6750 Kaiserslautern

Prof. Dr. H. Spohn
Fachbereich Physik
Universität
Theresienstraße 37

8000 München 2

Prof. Dr. R. Williams
Dept. of Mathematics
University of California, San Diego

La Jolla , CA 92093
USA

Prof. Dr. A. S. Ustunel
2, Bd. Auguste Blanqui

F-75013 Paris

Prof. Dr. J. Zabczyk
Institute of Mathematics of the
Polish Academy of Sciences
ul. Sniadeckich 8

00-950 Warszawa
POLAND

Prof. Dr. M.S. Vogelius
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center

New Brunswick , NJ 08903
USA

Prof. Dr. M. Zakai
Dept. of Electrical Engineering
TECHNION
Israel Institute of Technology

Haifa 32000
ISRAEL

Prof. Dr. W. Wedig
Institut für Technische Mechanik
Fakultät für Maschinenbau
Universität Karlsruhe
Kaiserstraße 12

7500 Karlsruhe 1

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