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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 51/1989

Wiener-Hopf Probleme, Toeplitz-Operatoren und Anwendungen

3.12. bis 9.12.1989

This conference was dedicated to a large variety of problems involving equations of Wiener-Hopf type and Toeplitz like operators. Many applications connected with such problems were discussed. One side of the spectrum concerned abstract results for general classes of pseudo-differential operators and of Toeplitz and Wiener-Hopf operators over higher dimensional domains. Here the main topics were Fredholm theorems and index formulas, and their relations to the topology of the underlying domains. On the other side of the spectrum were results of concrete character involving concrete equations from areas like diffraction theory, linear transport theory, mechanics, and mathematical physics. Methods of explicit factorization played an important role in the analysis of the concrete equations.

The conference had a very high level. Leading experts from different fields (and from different countries) gave well-prepared lectures on recent developments. For the first time a large group of mathematician from East-Germany and the USSR took part in this Wiener-Hopf conference. Also a group of young BRD mathematician presented their work here.

It was a general feeling among participants and organizers that the conference was successful and should have a successor in a couple of years.

The organizers wish to thank the staff of the Institute for creating an excellent atmosphere, its Director, Professor Barner, for admitting this meeting and Dr. Penzel for his assistance in administrative matters.

Tagungsleiter: I. Gohberg, M.A. Kaashoek, E. Meister

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H. BART:

Simultaneous Reduction to Special Forms of Pairs of Upper Triangular Toeplitz Matrices

(Joint work with G.Ph.A. Thijssen, Rotterdam)

The problem considered is the following. Given two upper triangular Toeplitz matrices A and Z, when does there exist an invertible matrix S such that $S^{-1}AS$ is upper triangular and $S^{-1}ZS$ is lower triangular? The motivation for considering simultaneous reduction to complementary triangular forms of pairs of matrices comes from systems theory. For upper triangular Toeplitz matrices, a complete answer is given. The argument involves two main steps. One is the simultaneous reduction of A and Z to "simple" forms. The other consists of a detailed analysis of a certain directed graph associated with A and Z. Along the way information is obtained about the structure of the similarity S. The results actually hold for a class of matrices strictly larger than that consisting of the upper triangular Toeplitz matrices. The simple forms mentioned above serve as simultaneous similarity invariants for the pair A, Z.

A. BÖTTCHER

Wiener-Hopf Determinants

Several types (or "levels") of exact and asymptotic formulas for rationally generated Toeplitz and Wiener-Hopf determinants are discussed, and in order to stimulate further research in this field, a few open problems are pointed out. These include the search for a way of deriving Gorodetski's formulas by the Bart/Gohberg/Kaashoek/van Schagen theory. It would also be interesting to find an Achiezer-Kac analogue of the Gorodetski formulas. Finally, it is suggested to combine results on Wiener-Hopf determinants with rational symbols and appropriate analyticity arguments to establish and prove a continual version of the Fisher-Hartwig conjecture. The author has in particular shown that if γ is a positive integer, then

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$$D_\tau\left[\frac{x^{2\gamma}}{(x^2+1)^\gamma}\right] \sim e^{-\gamma\tau}\left(\frac{\tau}{2}\right)^{\gamma^2} \frac{G(\gamma+1)^2}{G(2\gamma+1)} \quad \text{as} \quad \tau \to \infty,$$

where $G(z)$ is the Barnes G-function; it is proposed to prove that the same asymptotic formula holds for all complex γ such that $|\mathrm{Re}\ \gamma| < 1/2$.

J. BRÜNING

Elliptic Operators on Noncompact Manifolds

We outline some ideas of analysis for elliptic operators on arbitrary Riemannian manifolds. To each elliptic complex

$$0 \to C_0^\infty(E_0) \xrightarrow{d_0} \dots \xrightarrow{d_{N-1}} C_0^\infty(E_N) \to 0$$

we associate a single elliptic operator $D = d + d^t : \bigoplus_i C_0^\infty(E_{2i}) \to \bigoplus_i C_0^\infty(E_{2i+1})$. In general, D has infinitely many closed extensions. The complex, however, can be extended to a "Hilbert complex"

$$0 \to \mathcal{D}_0 \xrightarrow{D_0} \mathcal{D}_1 \to \dots \xrightarrow{D_{N-1}} \mathcal{D}_N \to 0 ,$$

where $\mathcal{D}_i : \mathcal{D}_i \to D_{i+1}$ is a closed extension of d_i in $C(L^2(E_i), L^2(E_{i+1}))$, only in a more restricted way: even though it is always possible, these extensions are unique in many interesting cases, thus defining a distinguished closed extension of D. We illustrate this by an example relevant to the analysis of singular algebraic curves. In a second example we show that these extensions D may have a calculable L^2 index without being Fredholm.

R. DUDUCHAVA

Wiener-Hopf equation with the transmission property

The classical Wiener-Hopf equation is considered

$$W_a\varphi(t) \equiv \varphi(t) - \int_0^\infty K(t-\tau)\varphi(\tau)d\tau = f(t), \quad 0 < t < \infty \qquad (1)$$

with $K \in L_1(\mathbb{R})$; the conditions on the kernel $K(t)$ are found which ensure the invariance of the function spaces of Hölder $C^s(\mathbb{R}^+)$, Zygmund $Z^s(\mathbb{R}^+)$ $(0 < s < \infty)$, Bessel potentials $H_p^r(\mathbb{R}^+)$ and Besov $B_{p,q}^r(\mathbb{R}^+)$ $(1 < p < \infty,\ 1 \le q \le \infty,\ \frac{1}{p} - 1 < r < \infty)$ under the operator W_a; the condition for the space $C^s(\mathbb{R}^+)(1 < s < \infty)$ is as follows

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$K \in L_1(\mathbb{R}) \cap C^{s-1}(\mathbb{R}^+)$. The factorization of the symbol is obtained and the solvability theorems of the equation are proved.

H. DYM

Hankel, Hamburger and maximal entropy

Let $h_0,\ldots,h_{2n}$ be a given set of pxp matrices with $h_j = h_j^*$. It is well known that there exists a pxp matrix valued function which is analytic with nonegative imaginary part in OUHP (the open upper half plane) with

$$\Phi(\lambda) = \frac{-h_0}{\lambda} \ldots - \frac{h_{2n}}{\lambda^{2n+1}} + \frac{o(1)}{\lambda^{2n+1}} \quad \text{as} \quad \lambda \uparrow \infty$$ (uniformly in each sector in OUHP) if and only if the block Hankel matrix H_n with ij entry equal to h_{i+j} for $i,j=0,\ldots,n$ is positive semidefinite. In this talk we use reproducing kernel Hilbert space methods to parametrize all the solutions to this interpolation problem when $H_n > 0$ and hence all solutions to the associated truncated Hamburger moment problem. This parametrization is used to characterize the solution to this moment problem which maximizes an entropy integral. A number of auxiliary results are established enroute. Full details and references to related work are reported on in December 1989 issue of Integral Equations and Operator Theory.

J. ELSCHNER

Spline approximation for Wiener-Hopf integral equations

We give a survey of recent results on stability and convergence of spline Galerkin and collocation methods for Wiener-Hopf integral equations. We also consider the corresponding iterated Galerkin solutions as well as the discrete and iterated versions of the collocation methods. The integral operators are not compact so that standard theories for the numerical analysis of second kind integral equations cannot be applied. Nevertheless, using graded meshes and modified spline spaces, one obtains the same optimal orders of convergence as in the case of second kind equations with smooth or weakly singular kernels on finite intervals. Our approach which relies heavily on the Wiener-Hopf factorization yields sharp stability results for the numerical methods under consideration.

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I. FELDMAN

On the finiteness of the discrete spectrum of a perturbed self-adjoint operator

If A and T are bounded self-adjoint operators in a Hilbert space such that

1) all the points of the interval (a,b) $(a < b < \infty)$ are regular points of the operator A,
2) the point $\lambda = a$ is not an eigenvalue of A,
3) the operator $(A-aI)^{-1}T$ is compact,

then the spectrum of the operator $A + T$ in the interval $(a,b-\epsilon)$ $(\epsilon > 0)$ consists of at most a finite number of eigenvalues.
This result enables us to obtain some sufficient conditions for the finiteness of the discrete spectrum of the linear pencil connected with the transport equation.

L.S. FRANK

Toeplitz operators and the theory of one parameter families of difference operators

A graded C^*-algebra of one parameter families of difference operators on the lattice $\mathbb{R}^n_h = h\mathbb{Z}^n \subset \mathbb{R}^n$, $h \in (0,h_0)$, along with the ellipticity concept for this class of operators, was introduced by the author in 1965. The investigation of boundary value problems for elliptic difference operators on the lattice $\mathbb{R}^n_{h,+} = h\mathbb{Z}^n_+$ in $\overline{\mathbb{R}^n_+}$ was started in 1966. It turned out that two characteristic numbers are to be taken into consideration in order to have a well posed boundary value problem: the winding number m for the principal symbol of the elliptic operator on the lattice $\mathbb{R}^n_{h,+}$ and the order $\varkappa$ of the plus-part in the canonical Wiener-Hopf factorization of the principal symbol.
The case $\varkappa = m$ was fully investigated in early seventies and the equivalence of the Shapiro Lopatinsky type coerciveness condition (introduced by the author in 1970) with the validity of a characteristic two-sided a priori estimate for the solutions of the coercive boundary value problems, was established. The case $m > \varkappa$ has many similarities with the situation in the theory of coercive singular perturbations (introduced and investigated by the author in

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1976 and later on with W.D. Wendt from the Bonn University) and is characterized by the presence of boundary layer type mesh-functions on the lattice $\mathbb{R}^n_{h,+}$, the small parameter being the mesh-size h. This case was recently fully investigated and it was again that an adequate Shapiro-Lopatinsky type coerciveness condition is equivalent with the validity of the corresponding two-sided a priori estimates for the solutions of the coercive boundary value problems, these estimates being valid uniformly with respect to h. When the symbols of the operators in the graded C*-algebra mentioned above do not depend on x and $h=1$, one gets back the classical Toeplitz operators on $\mathbb{Z}^n_+$ or the convolution operators on $\mathbb{Z}^n$.

I. GOHBERG

Generalized Toeplitz matrices and orthogonal functions

The Carathéodory-Toeplitz extension problem, the Szegö limit theorems, the analysis of orthogonal polynomials, the problem of maximum entropy are related in a natural way to finite and infinite Toeplitz matrices. A review of the main theorems was given. Nehari's problem of extension leads to a new class of matrices which behave like the Toeplitz ones. With these generalized Toeplitz matrices it is possible to define a system of orthogonal vector functions which is very similar to the system of Szegö orthogonal polynomials. The next generalization is connected with the four block problem (which was invented in control) and its generalizations. The main results were reviewed together with generalizations for nonstationary cases.

R. GORENFLO

Stability estimates of logarithmic convexity type for first kind Abel integral equations

Stability estimates of logarithmic convexity type and methods of their derivation are described for Abel's integral equation

$$\int_a^t u(t)(t-s)^{-\beta}\,ds = f(t) \quad , \quad 0 < \beta < 1,$$

where $-\infty \leq a < t < b \leq \infty$.

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The analysis is carried out in spaces $C[a,b]$ and in spaces $L_p(a,b)$, $1 \leq p \leq \infty$, the extra information being given in form of a bound for Lu with an unbounded linear operator L and of a bound for the error within which f is known. A distinction is made whether the perturbed right hand side is or is not in the range of the Abel operator. Results of Vesella, v. Wolfersdorf, Dang Dinh Ang and Rutman are cited.

G. HEINIG

Inverse problems for Hankel and Toeplitz matrices

The following problem is considered: Given vectors u_i, g_i $(i=1,\ldots r)$; ask for Hankel or Toeplitz matrices A satisfying $Au_i = g_i$. Special cases of this problem are inverse spectral problems for eigenvectors or Schmidt pairs. The problem is also motivated in connection with orthogonal polynomials and was treated in the literature for special cases of the g_i. We present a parametrization of the general solution for $r = 1$ and solvability and uniqueness conditions for $r = 2$. In this case the problem will be reduced to a Diophantine equation which can be solved by an $O(n^2)$ or even $O(n \log^2 n)$ complexity algorithm. Special attention is dedicated to Hermitian, regular and positive definite solutions of the problem.

A.E. HEINS

More on the bifurcated duct

We will solve the following problem. A duct contains a centered septum. The top boundary of the duct satisfies a Dirichlet boundary condition while the lower one satisfies a Neumann condition. The bottom of the septum satisifies a Dirichlet boundary condition and the top a Neumann condition. We solve the Helmholtz equation in this duct subject to the boundary conditions we have described and the propagation of the first and second modes in the larger duct and the lowest mode in the smaller ducts. This problem is formulated as a coupled pair of Wiener-Hopf integral equations and we will comment on the necessary complex analysis which is required. The reflection and transmission coefficients have very simple forms when the wave number is real.

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M.A. KAASHOEK

The Wiener-Hopf method for the transport equation: a finite dimensional version

The Wiener-Hopf method for solving half range problems in transport theory has a long history (see, e.g., [1] and the references therein). In this talk the main ideas and features of this method are illustrated on a model with a finite number of scattering directions. First the problem is reduced to a system of Wiener-Hopf integral equations with a rational symbol. The latter system is solved explicitly using a method of canonical factorization which is based on the notion of realization appearing in mathematical systems theory. For the isotropic case in an absorbing medium the solution is computed explicitly. The talk is based on joint work with I. Gohberg.

[1] Kaper, H.G., Lekkerkerker, C.G., Hejtmanek, J.: Spectral methods in linear transport theory. O.T. 5, Birkhäuser Verlag, 1982

G.S. LITVINCHUK

Singular integral equations with the linear fractional shift

The function $\alpha(t) = (t-\beta)(\bar{\beta}t-1)^{-1}$, $|\beta| \neq 1$ is the fractional linear a Karleman homeomorphism of the unit circle Π on itself, preserving ($|\beta| < 1$) or changing ($|\beta| > 1$) the orientation on Π, and $(W\varphi)(t) = \varphi(\alpha(t))$. Let's consider the operator

$K = (aI+bW)P_+ + (cI+dW)P_-$ in H^μ-space, $0 < \mu \leq 1$ or in L_p-space, $1 < p < \infty$, $P_\pm = \frac{1}{2}(I \pm S)$, $(S\varphi)(t) = (\pi i)^{-1} \int_\Pi \varphi(\tau)(\tau-t)^{-1} d\tau$, I is identity operator.

The defect numbers of operator K are calculated and the defect subspaces $\operatorname{Ker} K$ and $\operatorname{Coker} K$ are constructed.

M. LORENZ

Reflection of branching asymptotics for strongly hyperbolic problems

We pose a strongly hyperbolic problem

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$$A\,u = f \in C^\infty(\mathbb{R}^1_+ \times \mathbb{R}^n_+)$$

$$u|_{t=0} = g \in C^\infty(\mathbb{R}^n_+ \setminus \Gamma)$$

$$B\,u|_{x_1=0} = h \in C^\infty(\mathbb{R}^1_+ \times \mathbb{R}^{n-1})$$

where A is an NxN System of pseudodifferential operators of order 1. Compatibility conditions are satified at $t = x_n = 0$. Caustics do not arise. g has branching asymptotics along the closed C^∞ manifold Γ of codimension 1. The solution u has also branching asymptotics after reflection at the boundary or is smooth if some microlocal ellipticity condition is satisfied at the boundary. The talk is based on a joint paper with Yasser Smaid.

F. ALI MEHMETI

Interaction problems and microlocal analysis

Interaction is a unified formalism to describe evolution phenomena on families of media which have many different kinds of influence on one another (c.f. Math. Meth. Appl. Sci., 11, 665-685, 1989). It includes transmission problems on ramified spaces (c. f. G. Lumer, S. Nicaise, J.v. Below, B. Gramsch) as well as nonlocal conditions, systems with variable space dimensions etc. Interaction is specified by a closed subspace of the product of certain Sobolev spaces on the media. We describe the geometry of the singularities of solutions of a certain model problem: wave equations on two half-spaces of $\mathbb{R}^n$ which are connected by Fourier integral conditions on the interface. Reflexion and refraction of singularities is described in terms of the canonical relations of the operator of restriction to the interface and of the Fourier integral operator describing the interaction condition. This problem is a model of "active" interfaces with a modified law of reflection/refraction. For the classical case c.f. S. Hansen, V. Petkov, J. Cl. Normas.

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F. PENZEL

Sobolev Space methods for dual integral equations

We introduce Sobolev spaces of axialsymmetric funtions and give exlicit solutions for dual integral equations which come up in the problem of the electrified disk (E. Copson, 1947, Proc. Edinburgh Math. Soc.). We use a result of E. Stephan (1987, I.E.O.T.) , which proves bijectivity of the underlying boundary integral operator. In the case of radial symmetry the inverse operator is described as a product of two operators, which play the role of Bessel potential operators in the Wiener-Hopf theory for the half-space (E. Meister, F.-O. Speck, Proc. Conf. Dundee 1988 in: Ordinary and Partial Differential Equations, ed. Sleeman and Jarvis, 1989). Generalizations to other boundary conditions and to the non-radial case are pointed out.

S. PRÖSSDORF

On an integral equation arising from a cruciform crack problem

We consider the problem

$$(1)\quad \frac{1}{\pi}\int_{-1}^{1}\frac{u(y)}{y-x}dy + \frac{1}{\pi}\int_{-1}^{1}\frac{y(y^2-x^2)}{(y^2+x^2)^2}\,u(y)dy = f(x),\quad -1 < x < 1,$$

$$(2)\quad \int_{-1}^{1} u(y)dy = 0\ .$$

This problem arises in the study of a cruciform crack in an infinite isotropic elastic medium under constant load $f \equiv 1$ along its four branches. The problem (1), (2) is shown to have a unique solution $u \in L_2^{\tau,\sigma} := |x|^{\tau}(1-x^2)^{\sigma} L_2(-1,1)(-1/2 < \tau < 1/2,\ -1/2 < \sigma < 0)$ for all $f \in L_2^{\tau,\sigma}$ which is an odd function if f is even. The asymptotics of this solution for both $x \to \pm 1$ and $x \to 0$ are studied. We then propose a quadrature method for the numerical solution of (1), (2). Using localization techniques and the theory of Toeplitz operators, we prove the stability of this method. Numerical performances of this method are also presented. The talk is based on a joint paper with A. Rathsfeld.

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J. PRÜSS

Asymptotics of vector-valued Volterra equations

Es wird die Volterra Gleichung

$$(*) \qquad u(t) = f(t) + \int_0^t a(t-\tau)Au(\tau)d\tau \quad , \ t \geq 0$$

im Banachraum X betrachtet, wobei A ein abgeschlossener linearer Operator in X mit dichtem Definitionsbereich $D(A)$ sei. Die Resolvente $S(t)$ für $(*)$ existiere und sei gleichmäßig beschränkt, sowie Abel-ergodisch, d.h. $A - \lim_{t \to \infty} S(t) = P$ existiere. Es werden dann hinreichende Bedingungen für $\lim_{t \to \infty} S(t) = P$ angegeben; dabei kommen vektorwertige Taubersche Sätze zur Anwendung. Die Ergebnisse lassen sich auf die Gleichungen der linearen Viskoelastizitätstheorie anwenden.

A.D. RAWLINS

Matrix Wiener-Hopf Factorization Problems

We shall give the closed form solution to the problem of diffraction by a penetrable half plane. The asymetry of the approximate boundary conditions results in a matrix Wiener Hopf problem.

F. VAN SCHAGEN

Non-compact integral operators and exponential dichotomy

The talk concerns integral equations of the second kind on the half line with kernels of the type

$$k(t,s) = \begin{cases} C(t)U(t)\ Q\ U(s)^{-1}B(s) & 0 \leq s \leq t < \infty, \\ -C(t)U(t)(I-Q)U(s)^{-1}B(s) & 0 \leq t \leq s < \infty. \end{cases}$$

Here $B(t)$ and $C(t)$ are essentially bounded $n \times m$ and $m \times n$ matrix functions, respectively, and $U(t)$ is the solution of the differential equation $\dot{U}(t) = A(t)U(t)$, normalized by $U(0)=I$, where $A(t)$ is locally integrable. The symbol Q denotes a projection, which is an exponential dichotomy for $\dot{x}(t)=A(t)x(t)$ on

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$[0,\infty)$. Necessary and sufficient conditions for the unique solvability of the equations are given together with a formula for the resolvent kernel in terms of the given data. The analysis of this integral equation is based in the equivalence with time varying linear systems with initial conditions. The inversion involves to find an expoential dichotomy for the associate equation $\dot{x}(t) = (A(t) = (A(t) - B(t)C(t))x(t)$. This talk reports on joint work with I. Gohberg and M.A. Kaashoek.

E. SCHROHE

Boundary Value Problems on nOn-compact Manifolds

A system $(P,B_1,\ldots B_m)$ consisting of a differential operator P and m boundary operators $B_1,\ldots,B_m$ of orders m_j on a bounded domain X in $\mathbb{R}^n$ with smooth boundary extends to a Fredholm operator

$$(P,B_1,\ldots,B_m)H^m(X) \to H^0(X) \times \prod_1^m H^{m-m_j-\frac{1}{2}}(\partial x)$$

iff P is elliptic and the boundary operators satisfy the Lopatinsky-Shapiro conditions. This classical result extends to the case of the Boutet de Mouvel algebra over a compact manifold. However, these standard ellipticity conditions do not guarantee the Fredholm property on non-compact manifolds. Therefore a class of weighted symbols and Sobolev spaces together with a stronger notion of ellipticity was introduced in [ES]: On a family of asymptotically flat manifolds one has a symbolic calculus. In particular, the normal derivative is an operator in the class under appropriate assumptions on the Riemannian metric. This yields Fredholm results for differential boundary value problems. It also gives the ingredients for a Boutet de Mouvel type calculus on these manifolds, where pseudodifferential, Green, potential, and trace operators are in the above weighted symbol class [S].

[ES] : Erkip & Schrohe: Normal sovability for Boundary Value Problems on Asymptotically Flat Manifolds, preprint Mainz 1989.

[S] : Schrohe: A Boutet de Mouvel Type Calculus for Pseudo-differential Operators on Non-Compact Manifolds, in preparation.

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B. SILBERMANN

Banach algebras generated by two idempotents and Wiener-Hopf integral operators

About 20 years ago I. Gohberg and N. Krupnik developed a symbol calculus for one-dimensional singular integral operators with piecewise continuous coefficients. Their ideas were widely used in the following in order to study other classes of operators. I shall present another look at these things, which is based on Banach algebra techniques, especially on the local principle of Allan and on a certain invertibility criterion for elements belonging to a Banach algebra generated by two idempotents. It will be demonstrated that a symbol calculus for the closed subalgebra of $\mathfrak{L}(L^p(\mathbb{R}^+))$ $(1 < p < \infty)$ generated by all Wiener-Hopf and Hankel operators with generating functions from PCp can be obtained with help of such ideas.

F.-O. SPECK

A factorization procedure for two by two matrix functions on the circle with two rationally independent entries (joint work with S. Prößdorf)

The aim of this paper is the explicit canonical or standard factorization of matrix functions with Wiener algebra elements. It is based on the knowledge of how to factorize scalar functions and rational matrix functions. The present approach covers all regular 2x2 matrices where two entries are arbitrary and the remaining two are linear combinations of the former with rational coefficient functions.

F.S. TEIXEIRA

Diffraction by a rectangular wedge: Wiener-Hopf-Hankel formulation

The diffraction problem of an electromagnetic (or acoustic) wave by a right-angled wedge Ω_w with perfectly conducting (or soft) faces is treated in a Sobolev space setting. The problem is formulated as a Dirichlet boundary-value problem for the the Helmholtz equation

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in the extension Lipschitz domain $\mathbb{R}^2 \setminus \overline{\Omega}_w$. It is shown that this boundary-value problem is decomposable into two simpler ones whose (unique) solutions are explicitly determined by inverting, respectively, a Wiener-Hopf and a Hankel operator.

TRAN DUC VAN

Some applications of the theory of pseudodifferential operators with real analytic symbols to many-dimensional integral equations of first kind.

In this paper we apply the theory of pseudodifferential operators with real analytic symbols to establish well-defined solvability of some linear many-dimensional integral equations of first kind with symbol analytic in a domain $G \subset \mathbb{R}^N$ in the framework of the spaces $W_G^{\pm\infty}(\mathbb{R}^N)$. The space $W_G^{+\infty}(\mathbb{R}^N)$ is the space of entire functions whose Fourier transform is a hyperfunction supported by a compact set in G. Theorems of existence and uniqueness and methods of approximating the solutions are presented.

H. UPMEIER

Toeplitz Operators on Domains in $\mathbb{C}^n$

For every bounded domain Ω in $\mathbb{C}^n$ one can define the Bergman space $H^2(\Omega)$ of holomorphic functions and Toeplitz operators $T_f\varphi := P(f\varphi)$ acting on $H^2(\Omega)$. Here P is the Bergman projection and f is a continuous function on the closure $\overline{\Omega}$ called the symbol of T_f. It is shown that the C*-algebra $\mathcal{T}(\Omega)$ generated by these operators reflects the boundary geometry of Ω in form of a composition series $I_1 \subset \ldots \subset I_r$ of closed ideals generalizing the compact operators. Taking subquotients I_{k+1} / I_k of these ideals one obtains new index theorems even for Toeplitz C*-algebras not of type I (where the index is not an integer).

N.L. VASILEVSKI

Toeplitz operators associated with the Siegel domains and group convolution operators on the Cauchy-Riemann standard manifolds

Let D be a Siegel domain (of type I or II), let B be its Shilov

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boundary and let $H_2(D)$ be the corresponding Hardy space. Denote by P the orthogonal projector of $L_2(B)$ onto $H_2(D)$.
The algebras generated by all Toeplitz operators $T_A = PA|_{H_2(D)}$ with presymbols A from some operator algebra are investigated.
In the case of Siegel domain of type II the new explicit description of the space $H_2(D)$ and of the projector $P: L_2(B) \to H_2(D)$ are given.
The two-sided group convolution operators associated with the Cauchy-Riemann standard manifold and algebras generated by them are studied.

R. VREUGDENHIL

The spectral measure of a selfadjoint Wiener-Hopf operator with a rational matrix symbol

Explicit formulas for the spectral measures (resolutions of the identity) of selfadjoint convolution and Wiener-Hopf operators with rational matrix symbol are constructed. The formulas are given in terms of a realization of the symbol. The construction is based on general formulas for the resolvents of convolution and Wiener-Hopf operators.

H. WIDOM

Asymptotics of the eigenvalues of non-selfadjoint Toeplitz matrices

If f is a bounded real-valued function on the unit circle then the $n \times n$ Toeplitz matrix $T_n(f)$ is self-adjoint. Szegö's classical theorem states that if $\mu_{n,f}$ is the measure giving each eigenvalue n^{-1} times its multiplicity, then $\mu_{n,f}$ converges weakly as $n \to \infty$ to the measure μ_f which assigns each set E the normalized Lebesgue measure of $f^{-1}(E)$. The analogue of this in the nonself-adjoint case is not generally true. But for all known counterexamples the function f extends analytically to an annulus $1 < |t| < r$ or $r < |t| < 1$ and we raise the question of whether in all other cases $\mu_{n,f}$ does converge weakly to μ_f. Here we assume f is continuous

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and piecewise C^∞ but not C^∞ and are almost, but not quite, able to prove the assertion. We determine the asymptotics of the Toeplitz determinants $D_n(f)$ for such function, with ind(f) arbitrary, and we are able to deduce that $\mu_{n,f} \to \mu_f$ weakly in case f has one singular point, and in certain other cases. It is also shown that if f is continuous one-one and piecewise smooth then the conjecture is true if the support of a limiting measure either lies strictly inside the curve described by f or is a subset of this curve.

L. VON WOLFERSDORF

On a class of nonlinear singular integral equations

Nonlinear singular integral equations with the Cauchy or Hilbert singular operator are considered in the literature for a long time. But in the most papers they were dealt with as perturbations of linear equations with a sufficiently small parameter before an additional nonlinear term. Inspired by some applications in hydrodynamics we have been developed a method for treating strongly nonlinear singular integral equations without such smallness assuptions on the nonlinearity. The method consists in reducing the equations to equivalent fixed point equations to which the Schauder fixed point theorem in the space of continuous functions is applied. In the lecture an overview about this method and its results is given.

Berichterstatter: F. Penzel

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Tagungsteilnehmer

F. Ali-Mehmeti
Fachbereich Mathematik
der Universität Mainz
Postfach 3980
Saarstr. 21

6500 Mainz

Prof. Dr. H. Bart
Econometrisch Instituut
Erasmus Universiteit
Postbus 1738

NL-3000 DR Rotterdam

Prof. Dr. A. Böttcher
Sektion Mathematik
Technische Universität
Karl-Marx-Stadt
PSF 964

DDR-9010 Karl-Marx-Stadt

Prof. Dr. J. Brüning
Mathematisches Institut
der Universität Augsburg
Universitätsstr. 8

8900 Augsburg

Dr. M. Costabel
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. R. V. Duduchava
Institute of Mathematics
Georgian Academy of Sciences
ul. Z. Rukhadze Str. 1

Tbilisi 380093
USSR

Prof. Dr. H. Dym
Dept. of Mathematics
The Weizmann Institute of Science
P. O. Box 26

Rehovot 76 100
ISRAEL

Dr. J. Elschner
Akademie der Wissenschaften
Institut für Mathematik
Postfach 1304
Mohrenstr. 39

DDR-1086 Berlin

Prof. Dr. I. A. Feldman
Institute of Mathematics and
Computer Science AN MSSR
str. Ja Grosu 5

Kishinev 277028
USSR

Prof. Dr. L. S. Frank
Mathematisch Instituut
Katholieke Universiteit Nijmegen
Toernooiveld

NL-6525 ED Nijmegen

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Prof. Dr. I.C. Gohberg
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof. Dr. R. Gorenflo
Institut für Mathematik III
der Freien Universität Berlin
Arnimallee 2-6

1000 Berlin 33

Prof. Dr. B. Gramsch
Fachbereich Mathematik
der Universität Mainz
Postfach 3980
Saarstr. 21

6500 Mainz

Prof. Dr. G. Heinig
Sektion Mathematik
Technische Universität
Karl-Marx-Stadt
PSF 964

DDR-9010 Karl-Marx-Stadt

Prof. Dr. A. E. Heins
Dept. of Mathematics
University of Michigan
345 West Engineering Bldg.

Ann Arbor , MI 48109-1092
USA

Prof. Dr. R. A. Hurd
R. R. I

Cardinal, Ontario K0E 1E0
Canada

Prof. Dr. M.A. Kaashoek
Faculteit Wiskunde en Informatica
Vrije Universiteit
De Boelelaan 1081

NL-1081 HV Amsterdam

Prof. Dr. G. S. Litvinchuk
ul. Sov. armii, 3
Otdelenie MGJ AN Ukr.SSR

270100 Odessa
USSR

Prof. Dr. M. Lorenz
Sektion Mathematik
Technische Universität
Karl-Marx-Stadt
PSF 964

DDR-9010 Karl-Marx-Stadt

Dr. E. Lüneburg
DLR NE-HF
Institut für Nachrichtentechnik
DFVLR Oberpfaffenhofen

8031 Wessling/Obb.

Prof. Dr. H. H. Martens
Institutt for Matematikk
Universitetet i Trondheim
Norges Tekniske Hogskole

N-7034 Trondheim NTH

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Prof. Dr. E. Meister
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. A. D. Rawlins
Dept. of Mathematics
Brunel University
Kingston Lane

GB- Uxbridge, Middlesex , UB8 3PH

Prof. Dr. R. Mennicken
Fakultät für Mathematik
der Universität Regensburg
Universitätsstr. 31
Postfach 397

8400 Regensburg

Dr. G.A. Santana
Do Vostrebovaniya
Odessa 270 063
USSR

Dr. F. Penzel
Fachbereich Mathematik
der TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. A. F. dos Santos
Departamento de Matematicas
Instituto Superior Tecnico
Avenida Rovisco Pais

P-1096 Lisboa Codex

Prof. Dr. S. Prößdorf
Akademie der Wissenschaften
Institut für Mathematik
Postfach 1304
Mohrenstr. 39

DDR-1086 Berlin

Prof. Dr. F. van Schagen
Faculteit Wiskunde en Informatica
Vrije Universiteit
De Boelelaan 1081

NL-1081 HV Amsterdam

Prof. Dr. J. Prüss
c/o Mathematisches Institut
Universität Zürich
Rämistr. 74

CH-8001 Zürich

Dr. E. Schrohe
Fachbereich Mathematik
der Universität Mainz
Postfach 3980
Saarstr. 21

6500 Mainz

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Prof. Dr. B. Silbermann
Sektion Mathematik
Technische Universität
Karl-Marx-Stadt
PSF 964

DDR-9010 Karl-Marx-Stadt

Prof. Dr. H. Upmeier
Department of Mathematics
University of Kansas

Lawrence , KS 66045-2142
USA

Prof. Dr. F.-O. Speck
Mathematisches Institut A
der Universität Stuttgart
Pfaffenwaldring 57

7000 Stuttgart 80

Prof. Dr. N. L. Vasilevski
Dept. of Mathematics
Odessa State University
Petra Velikogo 2

270057 Odessa
USSR

Prof. Dr. E. P. Stephan
Mathematisches Institut der
Universität Hannover
Welfengarten 1

3000 Hannover

R. Vreugdenhil
Mathematisch Instituut
Vrije Universiteit
De Boelelaan 1081

NL-1081 HV Amsterdam

Prof. Dr. F. S. Teixeira
Departamento de Matematica
Instituto Superior Tecnico
Av. Rovisco Pais

P-1096 Lisboa - Codex

Prof. Dr. H. Widom
Dept. of Mathematics
University of California

Santa Cruz , CA 95064
USA

Prof. Dr. Tran Duc Van
c/o Professor Gorenflo
Institut für Mathematik III
Freie Universität Berlin
Arnimallee 2 - 6

1000 Berlin 33

Prof. Dr. L. von Wolfersdorf
Sektion Mathematik
Bergakademie Freiberg
Akademiestr. 6

DDR-9200 Freiberg

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