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MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 8/1990

FUNKTIONENTHEORIE

11.2. bis 17.2.1990

Die diesjährige Tagung über **Funktionentheorie** stand unter der Leitung der Herren Gehring (Ann Arbor), Mues (Hannover) und Pommerenke (Berlin).

Schwerpunkte waren die beiden Gebiete *Analytische dynamische Systeme* sowie *Meromorphe Funktionen und gewöhnliche Differentialgleichungen*, wobei hier natürlich auch (Iteration ganzer Funktionen) Wechselbeziehungen auftraten. Eine Vielzahl von Vorträgen über aktuelle Probleme und Resultate, zum Teil auch aus anderen Bereichen der Funktionentheorie, sowie lebhafte Diskussionen erzeugten insgesamt eine gelungene Tagung.

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Vortragsauszüge

L.V. AHLFORS:
Jordan algebra and power-series in several dimensions

The talk deals with power-series of the form $\sum p_n z^n$ where p_n and z belong to a commutative but non-associative algebra A^+. The question arises whether there is a reasonable theory of analytic continuation for such series.

A. BAERNSTEIN II:
Some results related to Landau's constant

Landau proved in 1925 the existence of an absolute constant L for which $|f'(0)| \leq \frac{1}{L} d(f(\Delta))$ for every function f holomorphic in the unit disk Δ. Here $d(\Omega)$ is the radius of the largest disk contained in the plane domain Ω. It is known that the largest possible L, call it L_0, satisfies $\frac{1}{2} < L_0 \leq .54\ldots$ The upper bound is conjectured to be the exact value of L_0. It comes from consideration of the universal covering map f_ω of Δ onto $\mathbb{C} \setminus L_0$, where L_0 is the lattice containing 0, 1, and $e^{\pi i/3}$, with $f_\omega(0) = \frac{1}{3}(1 + e^{\pi i/3})$.

Proposition 1: $\sup_{z \in \Delta}(1 - |z|^2)|f'_\omega(z)| = |f'_\omega(0)|$.

This proposition shows that one cannot disprove the conjecture $L_0 = .54\ldots$ merely by composing f_ω with a Möbius transformation. It is a special case of Proposition 2, which asserts that cusp forms invariant under the full modular group achieve their maxima at the points of 3-fold symmetry. To make further progress on the conjecture, one can try to prove such a result for "cusp forms" on $H \times T$, where T is the Teichmüller space of the n-times punctured torus.

I.N. BAKER:
Boundaries which arise in the iteration of entire functions

Let f be a transcendental entire function and G be an unbounded invariant component of the set $N(f)$ where the iterates f^n form a normal family. Then either (i) ∞ belongs to the impression of every prime end of G or

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(ii) $f^n(z) \to \infty$ locally uniformly in G. If case (ii) occurs, it is possible for ∂G to be a Jordan curve. In case (i) all prime ends of the first type have impression $\{\infty\}$, so for almost all prime ends of G the impression is an unbounded continuum. An example is described where there are no prime ends of first type, so many of the second and third types.

W. BERGWEILER:

Periodic points in the iteration of entire functions

Let f be an entire transcendental function and denote the n-th iterate of f by f_n. A complex number z_0 is called a periodic point of f if $f_n(z_0) = z_0$ for some $n \geq 1$. In this case, n is called a period of z_0 and the smallest n with this property is called the primitive period of z_0. A periodic point z_0 of period n is called repelling, if $|f_n'(z_0)| > 1$. I.N. Baker (see W.K. Hayman, *Research problems in function theory*, London 1967, Problem 2.20) conjectured that if $n \geq 2$, then f has infinitely many periodic points of primitive period n. We prove the following more general result.

Theorem. If $n \geq 2$, then f has infinitely many repelling periodic points of primitive period n.

R. BRÜCK:

Generalizations of Walsh's equiconvergence theorem by the application of summability methods

Let f be holomorphic in the disk $D_R = \{z \in \mathbb{C} : |z| < R\}$ $(R > 1)$, let $L_n(\cdot; f)$ be the Lagrange polynomial interpolating f in the $(n+1)$-th roots of unity, and let $S_n(\cdot; f)$ be the n-th partial sum of the power series expansion of f about 0. Then the well known equiconvergence theorem of Walsh (1932) states that the difference $L_n(z; f) - S_n(z; f)$ tends to zero as $n \to \infty$ for all $z \in D_{R^2}$. In recent years this result has been generalized in various ways.

We shall generalize Walsh's result by applying certain summability methods. The progress of our results consists in the fact that the disk D_{R^2} of equiconvergence is enlarged to regions of equisummability. For example, we prove that the difference $L_n(z; f) - S_n(z; f)$ is summable B (B being

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Borel's method) to zero for all $z \in \mathcal{E}_{L,S}$, where

$$\mathcal{E}_{L,S} = \bigcap_{k=1}^{\infty} \bigcap_{c \notin S_f} (c^{1+k} H),$$

S_f is the Mittag-Leffler star of f, and H is the half plane $\{z \in \mathbb{C}\colon \operatorname{Re} z < 1\}$. $\mathcal{E}_{L,S}$ is a region star-shaped with respect to 0 and containing the disk D_{R^2}. In particular, if $S_f = \mathbb{C} \setminus [R, \infty)$, then $\mathcal{E}_{L,S}$ is the half plane $\{z \in \mathbb{C}\colon \operatorname{Re} z < R^2\}$.

D. DRASIN:

Gross' star theorem or relation of maximum modulus to characteristic (Joint work with Bao Qin Li and Chong Ji Dai)

If f is meromorphic in the plane and $0 \leq \Phi(x)$ increases with $\int dx/\Phi(x) < \infty$, then we show that

$$\log M(r,f)\,[T(r)\Phi(\log T(r)) \log \Phi(\log T(r))]^{-1} \to 0$$

as $r \to \infty$ avoiding a set of zero logarithmic density. Using conformal mapping of strip-like domains and approximation of subharmonic/δ-subharmonic functions by $\log|f(z)|$ with $f(z)$ meromorphic we show this estimate is sharp. If f is entire, the factor $\log \Phi(\log T(r))$ can be removed.

C.J. EARLE:

Seeking the infinite Nielsen kernel

Let X be a Riemann surface whose universal covering surface is the unit disk and whose "ideal boundary" bd X is a nonempty union of disjoint simple closed curves C. We assume that the fundamental group $\pi_1(X)$ is not cyclic. Each boundary curve C is then freely homotopic to a unique simple closed Poincaré geodesic C' in X. The Nielsen kernel $K(X)$ is the Riemann surface obtained by removing from X all the closed annuli bounded to the simple closed curves C', so we can iterate this procedure and form $K^2(X) = K(K(X))$, $K^3(X) = K(K^2(X))$, etc. Lipman Bers asked for a description of the "infinite Nielsen kernel" obtained by intersecting all the subregions $K^n(X)$ of X. We can find it in some cases.

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F.W. GEHRING:

Isolation in discrete Möbius groups (Joint work with G.J. Martin)

Let M denote the group of Möbius transformations acting on the extended plane $\overline{\mathbb{C}}$ and for each f, g in M let

$$d(f,g) = \sup\,(q(f(z), g(z)) : z \in \overline{\mathbb{C}}),$$

where q denotes the chordal metric in $\overline{\mathbb{C}}$. A subgroup G of M is then discrete if it is isolated with respect to this metric, i.e., if there exists a positive constand $d = d(G)$ such that

$$d(f,g) \geq d$$

for each pair f, g in G.

This lecture is concerned with both quantifying the above inequality and studying the isolation in discrete groups in terms of two other metrics. One result is that G is discrete iff it is conjugate to a subgroup H of M where

$$d(f,g) \geq d_0 \geq 2(\sqrt{2} - 1) = .828\ldots$$

for each pair f, g in H.

A consequence of some of these results is a new lower bound for the volume of a complete hyperbolic 3-manifold.

W.K. HAYMAN:

Automorphic and normal functions

(Joint work with R. Aulaskari and P. Lappan)

Suppose that f is an automorphic function with respect to a Fuchsian group Γ. Let F be a fundamental region and denote by $f^*(z)$ the spherical derivative of $f(z)$. Suppose that

$$\int\int_F (1 - |z|^2)^{p-2} f^*(z)^p \, |dz|^2 < \infty.$$

If $p = 2$, then the image of F has finite area and from this Pommerenke (1974) deduced that f is normal. The result extends to the case $p > 2$, in a slightly strengthened form. It breaks down completely if $p < 2$, even when Γ is the identity and F the whole unit disk.

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A. HINKKANEN:

Schwarzian derivatives and zeros of solutions of second order linear differential equations

Let A be a linear combination of functions of the form $P \exp(Q)$ where P and Q are polynomials. Making use of univalence criteria involving Schwarzian derivatives, John Rossi and the speaker have proved that then any nontrivial solution of $y'' + Ay = 0$ has at most one zero in suitable angular domains that depend on A. Earlier results only guarantee that such domains contain an unspecified (not effectively computable) finite number of zeros, that number possibly also depending on the solution considered.

P.W. JONES:

The traveling salesman meets complex analysis

We will discuss the relationship between the classical Traveling Salesman Problem (TSP) in $\mathbb{R}^2$ and universal covering maps. The TSP is closely related to L^2 estimates for Lipschitz domains. We give a simple necessary and sufficient condition for a planar set to be a subset of a rectifiable curve. Applications to conformal mapping and harmonic measure are given.

H. KRIETE:

The relaxed Newton method for polynomials (Joint work with F. v. Haeseler)

We study the relaxed Newton method $N_{\lambda,h}(z) = z - h\,\dfrac{p_\lambda(z)}{p'_\lambda(z)}$, $0 < h \leq 1$, for polynomials p_λ of degree 3, parametrized by $p_\lambda(z) = z^3 + (\lambda - 1)z - \lambda$, $\lambda \in \mathbb{C}$. Let Γ be the union of the real interval $\left[\frac{1}{4}, 1\right]$ and the curve $\sigma(t) = 2\cos\left(\dfrac{\pi - t}{3}\right) e^{it}$, $0 \leq t < 2\pi$. Γ is the set of those parameters $\lambda \in \mathbb{C}$ such that two critical values of p_λ are lying on the same ray $\{re^{i\alpha} : r > 0\}$.

Using the qc-surgery we show for every $\lambda \in \Gamma$ and every $h_0 \in]0,1]$ the existence of an $h \in]0, h_0[$ such that $N_{\lambda,h}$ has an additional attractor, i.e., an attractor distinct from the roots of the polynomial p_λ.

For $m \geq 2$ let $\Lambda_{m,h}$ denote the set of the parameters such that $N_{\lambda,h}$ has an additional attractor of order $\leq m$. In 1989 M. Flexor and P. Sentenac

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established $\bigcup_{m\geq 2} \Lambda_{m,h} \to \Gamma$ for $h \to 0$. We show $\Lambda_{m,h} \to \{1\}$ for $h \to 0$ and fixed $m \geq 2$.

R. KÜHNAU:

Über ein Modul-Kapazitätsproblem von Herrn D. Gaier

Es sei Γ ein 0 enthaltendes Kontinuum innerhalb der Einheitskreisscheibe. Wann ist die euklidische Kapazität unter allen Γ einer fest vorgegebenen hyperbolischen Kapazität minimal? Dieses Problem von D. Gaier wird hier für den Fall gelöst, daß die vorgegebene hyperbolische Kapazität $\leq$ einer gewissen numerischen Konstante ist: Die Extremalkontinuen Γ sind von 0 ausgehende Strecken.

Beweis: 1. Mischung aus den Methoden von Randvariation und innerer Variation, 2. umfangreiche Diskussion des entstehenden quadratischen Differentials. Letzteres enthält — das ist eine wesentliche Komplikation — zusätzlich gewisse Funktionswerte der Extremalabbildung im Innern.

I. LAINE:

Meromorphic solutions of Malmquist and non-Malmquist differential equations

This talk refers to some recent results due to the speaker jointly with He Yuzan and G. Gundersen. Recall first the Steinmetz refinement of the classical Malmquist theorem for binomial differential equations $(y')^n = R(z,y)$. Steinmetz's result (1978), proved for a birational $R(z,y)$, was partially extended to the general admissible case by v.Rieth in 1986. Now, we are able to tive a complete admissible generalization, thanks to a partial generalization to algebroid functions of the recent Hayman-Miles result about the characteristics of derivatives of meromorphic functions. Secondly, we consider $w' = \sum_{k=0}^{n} P_k(z)w^k$, $n \geq 3$, $P_k(z)$: s polynomials, giving an upper bound for the number of meromorphic (i.e. rational) solutions. Thirdly, we consider $\Omega(z,w) = \sum_{k=0}^{n} A_k(z)w^k$ with small coefficients except for one suitable A_q, having a few zeros and poles. Then one can prove that no global meromorphic solutions appear. A number of counterexamples shows that this last result is more or less optimal.

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J.K. LANGLEY:

Oscillation results for higher order linear differential equations

In recent years progress has been made concerning the zeros of solutions of second order linear differential equations with entire coefficients. Some results have been obtained by considering the product of two solutions of such an equation: this product itself satisfies a relatively simple differential equation. We consider analogous methods and results for higher order differential equations.

G.J. MARTIN:

Iteration theory and discrete groups

Recursive enumeration of a sequence of words in a free subgroup of $SL_2\mathbb{C}$, leads to an iteration scheme in the complex plane. If this scheme is bounded, then the free group is not discrete.

This leads to new geometric information about the structure of the space of free groups in $SL_2\mathbb{C}$.

H.-G. MEIER:

The relaxed Newton method for rational functions: The limiting case

Starting with a short introduction to phase portraits with respect to the differential equation $\dot{\varphi} = -\frac{R}{R'} \circ \varphi$ (Newton flow) we consider the relaxed Newton method $N_{R,h}(z) := z - h\frac{R(z)}{R'(z)}$ for a rational function R. Interpreting $N_{R,h}$ as Euler discretisation of a Newton flow, the question on the relation of these systems for small h will be discussed. So it will be shown, in which sense the Julia set $J(N_{R,h})$ approximates the stable manifolds of the Newton flow for small h. Here, following ideas from Smale, the theory of schlicht functions turns out to be useful.

J.B. MILES:

On the growth of solutions of $f'' + gf' + hf = 0$

Suppose g and h are entire functions with the order of h less than the order

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of g. If the order of g does not exceed $1/2$, it is shown that every (necessarily entire) nonconstant solution f of the differential equation $f'' + gf' + hf = 0$ has infinite order.

This result (due to Hellerstein, Miles, and Rossi) extends previous work of Ozawa and of Gundersen.

F. PRZYTYCKI:

Perron-Frobenius-Ruelle operator, singular telescopes and coding for iterations of holomorphic maps on $\hat{\mathbb{C}}$

For f a rational map on the Riemann sphere, φ a continuous function on the Julia set, the Perron-Frobenius-Ruelle operator defined by $\mathbb{P}_\varphi : C(J) \to C(J)$, $\mathbb{P}_\varphi(h)(x) = \sum_{y \in f^{-1}(x)} h(y) \exp \varphi(y)$.

Theorem 1. If φ is Hölder continuous, then for every Hölder h the iterates $\mathbb{P}^n_{\varphi - P(f,\varphi)}(h)$ are equicontinuous on J and converge ($P(f,\varphi)$ denotes pressure).

Theorem 2. There exists unique f-invariant Gibbs measure for f, φ.

Theorems 1 and 2 were first proved by M. Denker and M. Urbański [1]. We prove them using "telescopes"-control of distortion on backward trajectories of components of f^{-n} (disc), [2].

We prove also dynamical version of Beurling's theorem where coding replaces Riemann mapping: the exceptional set of non-convergent branches and each set of branches convergent to one point are "thin", [3].

References:

[1] M. Denker, M. Urbański, *Ergodic theory of equilibrium dates for rational maps.* Preprint Göttingen 1989.

[2] F. Przytycki, *On the Perron-Frobenius-Ruelle operator...* To appear in Bol. Soc. Braz. Math.

[3] F. Przytycki and J. Skrzypczak, *Convergence and pre-images of limit points for coding trees...* Preprint.

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M. REINDERS:

Eindeutigkeitssätze für meromorphe Funktionen, die vier Werte teilen

f und g seien meromorphe Funktionen, die vier Werte $a_1, \ldots, a_n$ teilen.

Satz 1. Nimmt f die Werte a_1, a_2 und g die Werte a_3, a_4 nur mehrfach an, dann ist $f = L \circ \hat{f} \circ h$ und $g = L \circ \hat{g} \circ h$ mit einer ganzen Funktion h und einer Möbiustransformation L. $\hat{f}$ und $\hat{g}$ sind dabei die Funktionen aus dem Beispiel von Gundersen, die die Werte 0, 1, ∞ und $-1/8$ DM teilen.

Satz 2. Gilt ($f(z) = a_\nu$ einfach $\Longrightarrow$ $g(z) = a_\nu$ mindestens dreifach) und ($g(z) = a_\nu$ einfach $\Longrightarrow$ $f(z) = a_\nu$ mindestens dreifach), dann ist $f = L \circ F \circ h$ und $g = L \circ G \circ h$ mit einer ganzen Funktion h und einer Möbiustransformation L. F und G sind dabei zwei elliptische Funktionen, die die Werte 0, 1, ∞ und -1 DM teilen.

P.J. RIPPON:

Compositions of analytic self-mappings of a convex domain

If $\mathcal{F}$ is a family of analytic self-mappings of a convex domain, then under what conditions on $\mathcal{F}$ does the sequence

$$\varphi_n = f_1 \circ f_2 \circ \cdots \circ f_n, \quad f_n \in \mathcal{F}, \quad n = 1, 2, \ldots, \qquad (\star)$$

always converge in D to a constant?

An easy sufficient conditions is that, for some compact K in D, $f : D \to K$, for $f \in \mathcal{F}$, but this seems very restrictive. However, an example shows that K cannot be enlarged, even to a Stolz angle.

A pair of conditions sufficient for the convergence of $(\star)$ is:

(I) $|f'| \le 1$ in D, for all $f \in \mathcal{F}$, and

(II) $\sup_{\mathcal{F}} |f'(a)| < 1$, for some $a \in D$.

By considering $\mathcal{F} = \{f_b(z) = e^{bz} : |b| \le 1/e\}$, with $D = \{|z| < e\}$ and $a = 0$, we recover an old result of Thron that

$$e^{b_1 e^{b_2 e^{b_3 \cdots}}} \text{ is convergent if } |b_n| \le \tfrac{1}{e},\ n = 1, 2, \ldots$$

These and other similar results appear in joint work with I.N. Baker.

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S. ROHDE:

Conformal welding and quasicircles

Let $C \subset \mathbb{C}$ be a quasicircle and let f, g map the unit disc conformally onto the interior, exterior domains of C. The welding $\varphi = g^{-1} \circ f$ is a quasisymmetric mapping of the unit circle $\mathbf{T}$ onto itself. It is known that φ is singular if C has no tangent. We will show that under the stronger geometric assumption

$$\inf_{w_1, w_2 \in C} \sup_{w \in \langle w_1, w_2 \rangle} \frac{|w_1 - w| + |w_2 - w|}{|w_1 - w_2|} > 1$$

($\langle w_1, w_2 \rangle$ is the smaller arc between w_1, w_2 on C) there is a set $E \subset \mathbf{T}$ with $\dim E < 1$ and $\dim \varphi(\mathbf{T} - E) < 1$.

Quasisymmetric maps with this property have recently been constructed by P. Tukia.

J.F. ROSSI:

p-subharmonic functions and quasiregular mappings

In this talk we will study the growth of entire p-harmonic (p-subharmonic) functions in $\mathbb{R}^n$ along asymptotic paths, i.e. weak continuous solutions (subsolutions) of

$$(\star) \qquad \operatorname{div}\left(|\nabla u|^{p-2} \nabla u\right) = 0$$

for $p \in (1, \infty)$. We generalize results of Barth, Brannan and Hayman which established, in the classical case, $p = n = 2$, the existence of an asymptotic path along which a harmonic function grows rapidly. Due to the non-linearity of $(\star)$ our methods differ substantially from theirs. Furthermore there is a natural extension of our results to maps quasiregular in $\mathbb{R}^n$.

G. SCHMIEDER:

Zu den Grundlagen der komplexen Approximationstheorie

Die komplexe Approximationstheorie gründet auf dem Satz von Mergelyan, dem Fusionslemma von A. Roth und dem Lemma von Nersesyan. Es werden starke Zusammenhänge und Wechselbeziehungen zwischen den

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letztgenannten Lemmata aufgezeigt und hieraus Erweiterungen beider hergeleitet. Unter einer bestimmten topologischen Zusatzannahme kann die bestmögliche Fassung für beide Aussagen erhalten werden.

W. SCHWICK:

Eine Abschätzung von $m\left(r, \frac{f'}{f}\right)$ für Familien meromorpher Funktionen

Drasin bewies in der Arbeit *Normal families and the Nevanlinna theory*, daß man bei der Abschätzung von $m\left(r, \frac{f'}{f}\right)$ für Familien analytischer Funktionen, die nicht normal in 0 sind, den Fehlerterm $\log^+ \log^+ \dfrac{1}{|f(0)|}$ eliminieren kann. Dies ist für meromorphe Familien falsch, wie das Beispiel $g_j(z) = \dfrac{e^{j(z-1)}}{z + \frac{1}{j}}$ zeigt. Stattdessen kann man folgenden Satz beweisen:

Satz. F sei eine Familie meromorpher Funktionen in $|z| < 1$, die nicht normal in 0 ist, und $(f_j)_{j \in \mathbb{N}}$ die Folge in F, die man mit der Zalcmanschen Charakterisierung der Nichtnormalität in 0 erhält. Gilt dann $f_j(0) \neq \infty$ für alle $j \in \mathbb{N}$, so existiert eine Konstante $C \in \mathbb{R}^+$ mit

$$m\left(r, \frac{f_j'}{f_j}\right) < C\left[\log^+ T(r, f_j) + \log^+ \frac{1}{R-r} + \log^+ \log^+ \frac{1}{a_j} + 1\right]$$

für $\frac{1}{2} < r < R < 1$, und es gilt

$$\frac{1}{a_i} \leq \max_{|z| \leq 1/j} \frac{|f_j'(z)|}{1 + |f_j(z)|^2}.$$

Auf die Voraussetzung $f_j(0) \neq \infty$ kann nicht verzichtet werden, wie das Beispiel $S_j(z) = \dfrac{1}{(j-z)^j}$ zeigt.

D.F. SHEA:

Estimates for sets where a meromorphic function is large

For f meromorphic and of finite order λ we consider the sets $E(r, c)$ on $|z| = r$ where $\log |f(z)| > cT(r, f)$. Here T is the usual Nevanlinna charachteristic. We seek estimates for $\sigma_c(f) = \limsup_{r \to \infty} \text{meas } E(r, c)$ in terms

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of λ and the deficiency $\delta(\infty, f)$. We discuss a new proof of the known estimates for σ_c when $c = 0$ or $c > 0$. Our proof also gives sharp information when $c < 0$ and $\lambda \leq 1$.

M. SHISHIKURA:

Holomorphic surgery of polynomials and tuning

Holomorphic surgery is a method to create a new complex dynamical system from given system. Using the surgery, we try to explain the appearance of small copies of the Mandelbrot set in the Mandelbrot set itself or in other families. Here the Mandelbrot set is the set of parameters c such that the maps $z \to z^2 + c$ have connected Julia set.

K. STREBEL:

A geometric characterization of Teichmüller differentials

A Teichmüller mapping $f : R \to R'$ of two Riemann surfaces is linked with a pair of holomorphic quadratic differentials φ on R and ψ on R'. Teichmüller's theorem says that to every homeomorphism g of two compact surfaces with punctures there exists exactly one Teichmüller mapping homotopic to it.

Now the idea is to characterize the two Teichmüller differentials φ and ψ associated with the homeomorphism g directly, without going via f and Teichmüller's theorem.

This goal can be achieved using the notion of "height" of a simple closed loop, determined by a quadratic differential φ, and the mapping by heights g_H. This mapping sends the differentials on R to those on R', with the same heights on corresponding loops. The necessary and sufficient condition is that the mapping by heights takes φ into ψ and $-\varphi$ into (some positive constant times) $-\psi$ (joint work with A. Marden, to appear).

It is then indicated that this property can be used to give a new proof of the Teichmüller mapping theorem.

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SH. STRELITZ:

Upper bounds for the logarithmic derivative of a meromorphic function and existence problems concerning linear differential equations

Let $f(z)$ be a meromorphic function in the domain $D = \{z : |z| < R \leq \infty\}$ with the characteristic $T(r, f) \to \infty$ as $r \to R$. G. Gundersen (1988) found upper bounds for the logarithmic derivative $f'(z)/f(z)$ as $D = \mathbb{C}$ in terms of the characteristic $T(cr, f)$ with a given constant $c > 1$ as $z \in \mathbb{C}$ outside a system of disks covering the poles and zeroes of $f(z)$ having a 'small measure'. The constant $c > 1$ is often a palpable shortcoming in applications. We show that the constant c can be made equal 1 as $z \in D$ and is outside a sequence of annuli with center at $z = 0$ of 'small measure' thus establishing common upper bounds for the logarithmic derivative $f'(z)/f(z)$ in D as $R = \infty$ as well as $R < \infty$. We apply these results solving problems on the existence of solutions possessing in a certain sense maximal growth of linear differential equations with integral coefficients.

M. URBAŃSKI:

Sullivans's conformal measures for rational maps

Given a rational map $T : \overline{\mathbb{C}} \to \overline{\mathbb{C}}$ of degree $d \geq 2$ and a real number $t \geq 0$ we say that a Borel probability measure m on $\overline{\mathbb{C}}$ is t-conformal for T iff $m(J(T)) = 1$ ($J(T)$ is the Julia set of T) and

$$m(T(A)) = \int_A |T'|^t \, dm \tag{1}$$

for any Borel set $A \subset J(T)$ such that $T|_A$ is injective. If $F \subset J(T)$ is a finite set we say that m is t-conformal (mod F) iff (1) is satisfied for all sets as before which moreover does not intersect F. If we do not want to indicate a particular set F we use the name conformal measure (mod fin). Let

$$(E(t) = \{t \geq 0 : \text{ a } t\text{-conformal measure exists}\},$$
$$E_0(T) = \{t \geq 0 : \text{ a } t\text{-conformal measure (mod fin) exists}\}$$

and let

$$\delta(T) = \inf\,(E(T)), \quad \delta_0(T) = \inf\; E_0(T).$$

If

$$dD(J(T)) = \sum \{HD(\mu) : \mu \text{ is ergodic } T\text{-invarinat at positive entropy}\},$$

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then (see M. Denker, Urbański)

$$dD(J(T)) = \delta_0(T) \leq \delta(T).$$

The quantity $dD(J(T))$ is called to be the dynamical dimension at the set $J(T)$. We give an efficient criterion for the equality $\delta_0(T) = \delta(T)$ to hold. In an expansive case, that is if $J(T)$ contains no critical points at T we show that

$$dD(J(T)) = \delta_0(T) = \delta(T) = HD(J(T)).$$

Berichterstatter: E. Mues

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