MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 13/1990

Maßtheorie

18.3. bis 24.3.1990

Nach einer Pause von fast sieben Jahren wurde wieder eine Arbeitstagung "Maßtheorie" in Oberwolfach veranstaltet. Diese Tagung stand unter der Leitung von S. Graf (Passau), D. Kölzow (Erlangen) und D. Maharam-Stone (Boston). An ihr nahmen 50 Mathematiker aus 16 Ländern teil. Es wurden 48 Vorträge gehalten. Außerdem fand eine "Problem Session" statt.

Wie bisher ist geplant die Proceedings der Tagung als Lecture Notes in Mathematics des Springer-Verlages zu veröffentlichen.

Dem Direktor des Mathematischen Forschungsinstituts, Herrn Professor Barner, und seinen Mitarbeitern sei an dieser Stelle für die große Unterstützung gedankt, die den erfolgreichen Verlauf der Tagung ermöglichte.

Vortragsauszüge

1. Meßbarkeitsprobleme

W. Filter

Measurability in the dual of a Riesz space

Let L be a Riesz space with separating order continuous dual $L_n^\sim$. Fix a weak order unit ω of the extended order continuous dual $\Gamma(L)$, let $C(\omega)$ denote the set of all components of ω, and let $R \subseteq C(\omega) \cap L_n^\sim$ satisfy
(i) $g, h \in R \Longrightarrow g \vee h, g \wedge h \in R$,
(ii) $g, h \in R, h \leq g \Longrightarrow g - h \in R$, and
(iii) $R \ni g_n \downarrow g \Longrightarrow g \in R$.
For all $\xi \in \Gamma(L)^+$ and $\delta > 0$ there is a greatest $\xi_\delta \in C(\omega)$ with $\xi\xi_\delta \geq \delta\xi_\delta$. Set $\widetilde{R} := \{\xi \in \Gamma(L)^+ \mid \xi_\delta \wedge g \in R \text{ for all } g \in R, \delta > 0\}$ and $M_R := \widetilde{R} - \widetilde{R}$. Then M_R is a Riesz subspace and a unital subalgebra of $\Gamma(L)$, and the elements of M_R are called *R-measurable*. Moreover, if R satisfies a Hahn-type decomposition property, then a Radon-Nikodym theorem holds for the elements of L (with densities in M_R) which is valid – in contrast to the classical measure theoretic situation – without any further assumptions. The proofs use representation theory of Riesz spaces.

E. Grzegorek

Hereditary measurable sets

If $\mathcal{A}$ is a σ-field, then we denote by $I(\mathcal{A})$ the family of all $A \in \mathcal{A}$ such that $X \in \mathcal{A}$ holds for every $X \in \mathcal{P}(A)$. The following proposition is known:

Proposition. *Let $\mathcal{A}$ be a σ-field on S such that $\mathcal{A} \setminus I(\mathcal{A}) \in$ CCC and $\mathcal{P}(X) \setminus I(\mathcal{A}) \notin$ CCC for every $X \in \mathcal{P}(S) \setminus I(\mathcal{A})$. Then $I(\mathcal{A} \cap X) = I(\mathcal{A}) \cap X$ for every $X \in \mathcal{P}(S)$.*

On the other hand we have the following results:

Theorem 1. (ZFC) *There is a σ-field $\mathcal{A}$ on the real line $\mathbf{R}$ such that $[\mathbf{R}]^{\leq\aleph_0} \subseteq \mathcal{A}$, there is a nontrivial nonatomic finite measure on $\mathcal{A}$, and there is $X \in \mathcal{P}(\mathbf{R}) \setminus I(\mathcal{A})$ with $I(\mathcal{A} \cap X) = \mathcal{P}(X)$ and $I(\mathcal{A}) \cap X = [X]^{\leq\aleph_0}$.*

Theorem 2. (ZFC) *Let $\mathcal{B}$ be a σ-field on S such that $\mathcal{B} \setminus \{\emptyset\} \subseteq [S]^{>\aleph_0}$, and let $\mathcal{A}$ be the σ-field generated by $\mathcal{B}$ and $[S]^{\leq\aleph_0}$. Then $I(\mathcal{A}) = [S]^{\leq\aleph_0}$.*

Here $\mathcal{P}(S)$ denotes the power set of S, $[S]^{\leq\aleph_0} := \{X \in \mathcal{P}(S) \mid \text{card}\, X \leq \aleph_0\}$, and $[S]^{>\aleph_0} := \{X \in \mathcal{P}(S) \mid \text{card}\, X > \aleph_0\}$.

G. KOUMOULLIS

A generalization of Borel measurable functions

R. W. Hansell's theorem that a disjoint Borel additive family in a complete metric space is σ-discretely decomposable is generalized to Čech-complete (nonmetrizable) spaces, replacing discreteness by a weaker concept of scatteredness and assuming that the cardinal of the family is less than the least two-valued measurable cardinal. Moreover, the result holds for point-finite H-Borel additive families. (An *H-Borel set* is a member of the σ-algebra generated by the family of all H-sets or resolvable sets.) Then we show that most of the consequences of Hansell's theorem in the theory of Borel measurable functions continue to hold for H-Borel measurable functions on nonmetrizable spaces. Finally, using Baire category in spaces of Radon measures in some cases, we find connections between measurability properties of functions and the concept of (σ-)fragmentability.

A. H. STONE

The measurability of nonsingular transformations

An example is given, on the assumption of the continuum hypothesis, of an involution (a bijection of period 2) f of the unit interval onto itself, such that f (and f^{-1}) takes null sets to null sets, but f takes a measurable set to a nonmeasurable one. Question: Is the continuum hypothesis needed here?

2. Maßfortsetzungen

D. BIERLEIN

Measure extensions and measurable neighbours

Let $(\Omega, \mathcal{A}, p)$ be a probability space, let $(f_t)_{t\in T}$ be a family of real functions, where T is any index set, and define $f_T := (f_t)_{t\in T}$, $\mathcal{B}_T := \mathcal{B}(\mathbf{R}^T)$, and $\mathcal{A}_T := \sigma(\mathcal{A} \cup f_T^{-1}(\mathcal{B}_T))$. We consider the set $\mathcal{F}$ of all measure extensions of p to $\mathcal{A}_T$. According to a result of 1982, there is a one-to-one correspondence between $\mathcal{F}$ and the set of all probability measures μ on $\mathcal{A} * \mathcal{B}_T$ satisfying
(i) $\mu(A \times \mathbf{R}^T) = p(A)$ for all $A \in \mathcal{A}$, and
(ii) $\mu^*(h_T(\Omega)) = 1$, where $h_T : \Omega \to \Omega \times \mathbf{R}^T : x \mapsto (x, f_T(x))$.
By adding further conditions to (i) and (ii), we define $\mathcal{F}_{gl}$ and $\mathcal{F}_{\sigma l}$ as the two subsets of $\mathcal{F}$ corresponding to the *globally* resp. *σ-locally* $\mathcal{A}$-measurable neighbours $g_T : \Omega \to \mathbf{R}^T$ of f_T. Using a σ-locally $\mathcal{A}$-measurable neighbour g_T of f_T, we extend

p first to $\mathcal{A} * \mathcal{B}_T$ and then to $\mathcal{A}_T$. In this way we obtain $\mathcal{F}_{gl} \subseteq \mathcal{F}_{\sigma l} = \operatorname{ex}\mathcal{F}$. The identity $\mathcal{F}_{\sigma l} = \operatorname{ex}\mathcal{F}$ was first proven by W. Stich (1989) who used a different method involving a result of Haupt and Pauc on σ-ideals of p_*-null sets. Examples show that each of the following situations may occur: $\emptyset \neq \mathcal{F}_{gl} \neq \operatorname{ex}\mathcal{F}$, $\emptyset = \mathcal{F}_{gl} \neq \operatorname{ex}\mathcal{F} = \mathcal{F}$, and $\emptyset = \mathcal{F}_{gl} \neq \operatorname{ex}\mathcal{F} \neq \mathcal{F}$.

W. HACKENBROCH

Conditionally independent common extensions of measures

Let $(\mu_i)_{i\in I}$ be a finite family of probability measures on σ-algebras $\mathcal{A}_i$ of subsets of some set Ω, respectively, and let $\mathcal{B} \subseteq \bigcap_{i\in I} \mathcal{A}_i$ be a fixed σ-algebra on which all μ_i agree (i. e. $(\mu_i)_{i\in I}$ is *$\mathcal{B}$-consistent*).

THEOREM. *Assume that for all but one $i \in I$ there exist regular versions of the conditional expectations $E^{\mathcal{B}}_{\mu_i}$. Then there exists a (unique) common extension μ of the μ_i which is $\mathcal{B}$-conditionally multiplicative (i. e. $E^{\mathcal{B}}_{\mu}\chi_{\cap A_i} = \prod E^{\mathcal{B}}_{\mu}\chi_{A_i}, A_i \in \mathcal{A}_i$) iff $(\mathcal{A}_i)$ is $(\mathcal{B},(\mu_i))$-conditionally independent (i. e. $\bigcap A_i = \emptyset \Longrightarrow \prod E^{\mathcal{B}}_{\mu_i}\chi_{A_i} = 0$, $A_i \in \mathcal{A}_i$). This latter condition is also implied whenever each $\mathcal{B}$-consistent family $(\mu_i, \mu'_j)_{i\neq j\in I}$ with $\mu'_j \ll \mu_j, i \neq j \in I$, admits a common extension ($\mathcal{B}$-conditionally multiplicative or not).*

This generalizes a classical result of Marczewski for $\mathcal{B} = \{\emptyset, \Omega\}$ and a recent result of Bartfai and Rudas for $\mathcal{B} = \bigcap_{i\in I} \mathcal{A}_i$.

J. LEMBCKE (with H. WEBER)

Decomposition of group-valued measures with respect to their regular extendability

Let $\mathcal{R}$ be a subalgebra of some Boolean algebra $\mathcal{P}$, $\mathcal{K} \subseteq \mathcal{P}$, G a complete Hausdorff Abelian group, and $\mu : \mathcal{R} \to G$ an s-bounded finitely additive measure. Under certain assumptions on the regularity of μ and on $\mathcal{K}$, μ has a unique decomposition $\mu = \mu_r + \mu_d$ with s-bounded measures μ_r and μ_d such that

(i) μ_r admits an extension to an s-bounded $\mathcal{K}$-regular measure on the algebra $\mathcal{Q}$ generated in $\mathcal{P}$ by $\mathcal{R} \cup \mathcal{K}$, and

(ii) for every s-bounded $\mathcal{K}$-regular measure λ on $\mathcal{Q}$ with $\lambda|_{\mathcal{R}} \ll \mu_d$, one has $\lambda|_{\mathcal{R}} = 0$.

Moreover, μ_r and μ_d can be explicitly calculated by means of μ, and there is even an s-bounded $\mathcal{K}$-regular extension ν_r of μ_r to $\mathcal{Q}$ such that $\mathcal{R}$ is dense in $\mathcal{Q}$ with respect to the topology defined by ν_r on $\mathcal{Q}$. The proof makes use of a Lebesgue decomposition theorem with respect to FN-topologies by T. Traynor and an extension theorem for regular s-bounded measures by Z. Lipecki.

Z. LIPECKI

Extensions of positive additive set functions from an algebra to a larger one: A short survey

The following topics are discussed:

1. Peano-Jordan and Lebesgue completions.
2. Adding new null sets.
3. Adding a single set.
4. Extension to 2^{Ω}.
5. Extreme extensions.
6. Maximal extensions.
7. Tight extensions.
8. Hahn-Banach type extensions.
9. Other dominated extensions.
10. Common extensions.
11. Stochastically independent extensions.
12. Simultaneous extensions.

K. D. SCHMIDT (with G. WALDSCHAKS)

Common extensions of order bounded vector measures

Let $\mathcal{M}$ and $\mathcal{N}$ be algebras of subsets of some set Ω, let G be a Dedekind complete Riesz space, and let $\mu : \mathcal{M} \to G$ and $\nu : \mathcal{N} \to G$ be order bounded vector measures which agree on $\mathcal{M} \cap \mathcal{N}$. We give a general condition on $\mathcal{M}$ and $\mathcal{N}$ in terms of the (finite) partitions of Ω in these algebras which implies that μ and ν have an order bounded common extension $\varphi : 2^{\Omega} \to G$. The result unifies and extends two results obtained by Z. Lipecki (1986) in the case $G = \mathbf{R}$.

R. M. SHORTT (with K. P. S. BHASKARA RAO)

Extensions of finitely additive measures

Let $\mathcal{A}, \mathcal{B}, \mathcal{F}$ be fields of subsets of some set Ω with $\mathcal{A} \subseteq \mathcal{F}$ and $\mathcal{B} \subseteq \mathcal{F}$, let G be an Abelian group, and let $\mu : \mathcal{A} \to G$ and $\nu : \mathcal{B} \to G$ be charges (i. e. finitely additive set functions) which agree on $\mathcal{A} \cap \mathcal{B}$ (i. e. they are *consistent*). We ask when there is a *common extension* ρ on $\mathcal{F}$ for μ and ν (i. e. when there exists a charge $\rho : \mathcal{F} \to G$ such that $\rho|_{\mathcal{A}} = \mu$ and $\rho|_{\mathcal{B}} = \nu$). It is known from A. Basile and K. P. S. Bhaskara Rao (1988) that this is true whenever G is compact or a direct summand of such a group. This includes the case of divisible groups, in particular $G = \mathbf{R}$. We ask whether such a ρ exists for *all* Abelian groups. We have proved this in case either

(1) $\mathcal{A}$ or $\mathcal{B}$ is finite, or
(2) $\mathcal{A}$ and $\mathcal{B}$ are *independent* (i. e. $\emptyset \neq A \in \mathcal{A}, \emptyset \neq B \in \mathcal{B} \Longrightarrow A \cap B \neq \emptyset$).
For a free Abelian group of cardinal c there is an example where such a ρ *does not* exist. This derives from an observation of G. M. Bergman (1990). The main open question is whether such a ρ exists for $G = \mathbf{Z}$.

3. Produktmaße, Desintegration und Liftings

R. A. Johnson (with W. Wilczynski)

Finite products of Borel measures

Suppose μ, ν, λ are finite, countably additive, nonnegative, nonzero measures on the Borel sets of X, Y, Z, respectively. Define, if possible, $\nu\mu$ on the Borel sets of $X \times Y$ by $(\nu\mu)(M) := \int \nu(M_x)\, d\mu$, where $M_x := \{y \in Y \mid (x, y) \in M\}$. This is possible if $\nu(M_x)$ is μ-measurable as a function in x for each $M \in \mathcal{B}(X \times Y)$. It is easy to see that $\nu\mu$ extends the usual direct product measure $\mu \times \nu$ defined on $\mathcal{B}(X) \times \mathcal{B}(Y)$ and that a one-sided Fubini theorem holds for $\nu\mu$. To what extent does associativity hold for such products? If $\lambda(\nu\mu)$ is defined, then so is $(\lambda\nu)\mu$ and these two are equal. But if $(\lambda\nu)\mu$ is defined, we do not know if $\lambda(\nu\mu)$ is always defined. Moreover, even if $\lambda\nu$, $\lambda\mu$, and $\nu\mu$ are defined, we do not know if $(\lambda\nu)\mu$ is defined.

D. Ramachandran

A note on perfect measures

Ever since Kolmogorov introduced the trinity $(\Omega, \mathcal{A}, P)$ as the model for probability theory, there have been several attempts at defining "nice" classes of probability spaces with just enough structure to be useful in applications. P on $(\Omega, \mathcal{A})$ is called *compact* if there exists $\mathcal{K} \subseteq \mathcal{A}$ such that
(i) $\{K_n\} \subseteq \mathcal{K}, \bigcap_{n=1}^{\infty} K_n = \emptyset \Longrightarrow \bigcap_{n=1}^{m} K_n = \emptyset$ for some $m \geq 1$, and
(ii) for each $A \in \mathcal{A}$, $P(A) = \sup\{P(K) \mid K \in \mathcal{K}, K \subseteq A\}$.
P is called *perfect* if $P|_{\mathcal{A}_0}$ is compact for all countably generated $\mathcal{A}_0 \subseteq \mathcal{A}$. The notion of Doob's regular conditional probability is introduced and the question of V. V. Sazonov whether there is a perfect probability space which does not admit Doob's regular conditional probability is answered by using an example due to K. Musial. In the process, compact probability spaces are characterized as disintegrable probability spaces. Some useful consequences are derived and some open questions are stated.

W. STRAUSS

On strongly lifting compact spaces

From a paper of A. G. Babiker, G. Hello, and W. Strauss it is known that, for any strongly lifting compact space, each Baire measure on that space has the property that any lifting is almost strong. The converse is an open problem. For Banach spaces under their weak topology the converse holds, but it is unknown whether it holds for dual Banach spaces under their weak* topology.

4. Geometrische Maßtheorie

R. J. GARDNER (with M. LACZKOVICH)

Failure of the cancellation law

In 1929, A. Tarski showed the equivalence of the following two statements:

(1) *There is a finitely additive, G-invariant measure* $\mu : \mathcal{P}(X) \to [0,\infty]$ *such that* $\mu(E) = 1$.

(2) *E is not G-paradoxical (i. e.* $E \not\sim_G 2E$*).*

(Here $E \subseteq X$, $\mathcal{P}(X)$ is the power set of X, G is a group of bijections acting on X, $\sim_G$ means *equidecomposable* with respect to G, and nA means n copies of A). The proof of this uses the cancellation law, $nA \sim_G nB \Longrightarrow A \sim_G B$, which was proved by Kőnig and Válko in 1925. Stan Wagon asked if, when $\mathcal{A} \subseteq \mathcal{P}(X)$ is a proper subalgebra, the cancellation law holds when all the sets concerned are to be in $\mathcal{A}$; this would give the corresponding version of Tarski's theorem where $\mathcal{P}(X)$ is replaced by $\mathcal{A}$. But we show by an example that the cancellation law fails, even if we have the following simple situation: $X \subseteq \mathbf{R}$, $\mathcal{A}$ is the algebra generated by intervals intersected with X and with endpoints in $\mathbf{R} \setminus X$, and G is a group of isometries of $\mathbf{R}$.

P. MATTILA

Singular integrals and rectifiability of measures in the plane

Let μ be a locally finite Borel measure in the complex plane $\mathbf{C}$. It is well-known that for sufficiently regular one-dimensional μ, e. g. an L^2-measure on a rectifiable curve, the Cauchy principal value $C_\mu(z) := \lim_{\varepsilon\to 0} \int_{\mathbf{C}\setminus B(z,\varepsilon)} (\zeta - z)^{-1}\, d\mu(\zeta) \in \mathbf{C}$ exists for μ-almost all $z \in \mathbf{C}$. To the converse direction one can prove:

THEOREM. *If, for* μ*-almost all* $z \in \mathbf{C}$, $\liminf_{\varepsilon\to 0} \varepsilon^{-1}\mu(B(z,\varepsilon)) > 0$ *and* $C_\mu(z)$ *exists, then* μ *is rectifiable in the sense that there are rectifiable curves* $\Gamma_1, \Gamma_2, \ldots$ *such that* $\mu(\mathbf{C} \setminus \bigcup_{i=1}^{\infty} \Gamma_i) = 0$.

L. MEJLBRO

Positivity principles in geometrical measure theory

In connection with the work on the still unfinished book of J. P. R. Christensen, L. Mejlbro, D. Preiss, J. Tišer, *Uniqueness of Radon Measures and Vitali Relations in Infinite Dimensional Spaces*, Preiss and Tišer have produced the following results:

THEOREM 1. *Any signed measure on a separable Hilbert space that is nonnegative on all "small balls" is nonnegative if and only if the dimension of the Hilbert space is finite.*

THEOREM 2. *Any signed measure on a separable Hilbert space that is nonnegative on all "large balls" is nonnegative if and only if the dimension of the Hilbert space is infinite.*

THEOREM 3. *There exists a separable Banach space X and a truly signed measure on X which is nonnegative on all balls.*

W. F. PFEFFER

The Gauss-Green theorem

I define a well behaved averaging process such that the divergence of any continuous vector field differentiable outside a set of σ-finite codimension one Hausdorff measure is averageable and the Gauss-Green formula holds. Domains of integration are bounded sets of finite perimeter (BV sets), and the integral is invariant with respect to lipeomorphisms.

5. Maße und Integrale auf unendlich-dimensionalen Räumen

I. DOBRAKOV

Feynman type integrals as multilinear integrals

We give two equivalent definitions of Feynman type integrals as multilinear integrals. One of them is based on the finiteness of the multiple L^1-gauge, while the second is based on the existence of certain iterated integrals. The equivalence of the two definitions is one of the deepest results in the theory of multilinear integration. The theory of multilinear integration developed by the author is now available. In particular, we have the validity of the Lebesgue dominated convergence theorem.

G. W. JOHNSON (with G. KALLIANPUR)

Homogeneous chaos, p-forms, scaling, and the Feynman integral

Let $C_0(\mathbf{R}_+)$ be the space of continuous functions x on $\mathbf{R}_+$ such that $x(0) = 0$, and let P_1 be Wiener measure on $C_0(\mathbf{R}_+)$. The scaled Wiener measures $P_\sigma := P_1 \circ \sigma^{-1}, \sigma > 0$, corresponding to Wiener processes with variance σ^2, are concentrated on a continuum $\Omega_\sigma, \sigma > 0$, of disjoint subsets of $C_0(\mathbf{R}_+)$. This introduces certain measure-theoretic subtleties which, however, cause little difficulty when a fixed scaling is involved. When a problem involves various scalings, as with the analytic Feynman integral, attention must be paid to these subtleties. Hu and Meyer recently gave a formula for the Feynman integral of a function f in terms of the decomposition of f in Wiener chaos. This formula came out of earlier work of Meyer and Yan on the Hida calculus and involves the problem of extending in a "natural" way the multiple Wiener-Itô integral from Ω_1 to Ω_σ. The solution of this problem can be obtained by the lifting of p-forms on the white noise space $L^2(\mathbf{R}_+)$ to random variables on $C_0(\mathbf{R}_+)$.

I. KLUVANEK

Integration structures

An integration structure on a space Ω is determined by a vector space $\mathcal{L}$ of scalar valued functions on Ω, a seminorm $q : \mathcal{L} \to \mathbf{R}_+$, and a linear map $\iota : \mathcal{L} \to E$, where E is a Banach space, satisfying the following conditions:

(C) $|\iota(f)| \leq q(f)$ for every $f \in \mathcal{L}$, and

(BL) if $f_j \in \mathcal{L}$ for $j = 1, 2, \ldots$, $\sum_{j=1}^{\infty} q(f_j) < \infty$, and f is a function on Ω such that $f(\omega) = \sum_{j=1}^{\infty} f_j(\omega)$ for every $\omega \in \Omega$ for which $\sum_{j=1}^{\infty} |f_j(\omega)| < \infty$, then $f \in \mathcal{L}$ and $q(f) \leq \sum_{j=1}^{\infty} q(f_j)$.

A few examples of integration structures, indicating the extent of the generalization of the classical integration theory, are given. In particular, a definition of $\int_a^b f\, dg$, where g is a function having *infinite variation* on every sub-interval of (a, b) in the vein of Paley-Wiener-Zygmund, is indicated.

M. SION

Measures on infinite dimensional spaces

Given an index set I and a range space V, the problem is to construct a measure P on some $\Omega \subseteq V^I$ having specified finite dimensional projections, i. e. images of P under projections $\pi_j : \Omega \ni \omega \mapsto \omega|_j \in V^j$ for finite $j \subseteq I$. A canonical construction of an outer measure P^* on V^I provides conditions on Ω and the system of finite dimensional distributions to obtain the desired P. This includes results of Wiener, Minlos, Sazonov, and leads to the study of concepts of nuclearity: Nuclear spaces and nuclear cylinder measures.

6. Rekonstruktionsprobleme

H. G. KELLERER

Uniqueness in bounded moment problems

Let $(\Omega, \mathcal{A}, \mu)$ be a σ-finite measure space and let $\mathcal{K}$ be a closed subspace of $\mathcal{L}^1(\mu)$ with supp $\mathcal{K} =_\mu \Omega$. The following uniqueness problem is treated: Which sets $A \in \mathcal{A}$ are determined by the integrals $\int f\chi_A \, d\mu$, $f \in \mathcal{K}$? More precisely, χ_A is compared with functions $0 \leq g \leq 1$ and their integrals $\int fg \, d\mu$. Results of Shepp (1986) and Kemperman (1988) show that the condition $A =_\mu \{f \geq 0\}$ for some $f \in \mathcal{K}$ is sufficient but not necessary for uniqueness. To get a complete characterization of all $\mathcal{K}$-determined sets it is not enough to consider $\mathcal{K}$-separated sets of order γ, where γ is a countable ordinal, but one has to enlarge $\mathcal{K}$ to some hull $\tilde{\mathcal{K}}$ by extending the usual weak convergence to limits not in $\mathcal{L}^1(\mu)$.

MAIN RESULT: *A is $\mathcal{K}$-determined iff there exists a representation $A =_\mu \{f > 0\}$ and $\overline{A} =_\mu \{f < 0\}$ for some $f \in \tilde{\mathcal{K}}$.*

A. VOLČIČ

Rearrangement of measurable sets

Extending some results from the discrete case due to Ryser in a joint work with A. Kuba, we prove the following result:

THEOREM. *If $T(x,y) := x - y + \int_y^\infty R_G(t)\,dt - \int_0^x S_G(t)\,dt$, where G is the normalized rearrangement of a measurable set F of finite measure, then all sets having the same projections as G contain $I_1(G) := \bigcup [0,x] \times [0,y]$ and are distinct from $I_0(G) := \bigcup [x,\infty) \times [y,\infty)$, where in both cases the union is taken over all (x,y) such that $T(x,y) = 0$.*

The complement of $I_1(G) \cup I_0(G)$ can be covered by countably many sets having the same projections as G, and also by countably many complements of such sets. Rearranging back $I_1(G)$ and $I_0(G)$, one gets results for the original set F.

7. Stochastische Probleme

R. BECKER

Continuity and separability of random functions

We study processes which have a given separating set S (in Doob's theory). The set of their laws is an extremal set of measures. With an *almost sure* continuity assumption, we show that the law of each process is determined by its projection on S. With only an assumption of continuity *in probability*, the projection on S of the law of the process has a property of extremality.

A. IWANIK

Baire category theorems for stochastic operators

Let $(\Omega, \mathcal{A}, \mu)$ be a standard probability space. An operator $T \in \mathcal{L}(L^1(\mu))$ is called *stochastic*, $T \in \mathcal{S}$, if $Tf \geq 0$ and $\|Tf\| = \|f\|$ for every $f \geq 0$. Every $T \in \mathcal{S}$ extends naturally to arbitrary nonnegative measurable functions. If $Tg \leq g$ for some $g > 0$, then we say that T admits the subinvariant measure $d\lambda = g\,d\mu$. By $\mathcal{S}_{\leq\lambda}$ we denote the family of all $T \in \mathcal{S}$ for which λ is subinvariant, where λ is any σ-finite measure equivalent to μ. An operator $T \in \mathcal{S}_{\leq\lambda}$ is called *mixing*, $T \in \mathrm{Mix}\,\mathcal{S}_{\leq\lambda}$, if $\int_B T^n \chi_A \, d\mu \to \infty$ whenever $\lambda(A) + \lambda(B) < \infty$. We investigate the Baire category of $\mathrm{Mix}\,\mathcal{S}_{\leq\lambda}$ with respect to the three natural operator topologies in $\mathcal{L}(L^1(\mu))$. The set $\mathrm{Mix}\,\mathcal{S}_{\leq\lambda}$ turns out to be residual with respect to the operator norm topology and the strong operator topology, while it is meager for the weak operator topology. The latter result extends the classical theorems of Rokhlin and Sachdeva for transformations.

NGUYEN DUC TUAN

Classifying the infinitely divisible probability measures via the generalized selfdecomposable structure

A special subclassification of the infinitely divisible probability measures is discussed which is based on the fractional order (G, β)-selfdecomposable structure, where $G = \{G(t) \mid t > 0 \text{ and } \lim_{t\to 0} G(t)x = 0 \text{ for each } x \in X\}$ is a strongly continuous one-parameter multiplicative group of bounded linear operators on a Banach space X and $\beta \geq 0$. First-type representations of α-order, $0 < \alpha \leq \infty$, (G, β)-selfdecomposable probability measures are solved. Their second-type representations are improved and perfected, and relations between the first- and second-type representations are established. Some related problems are discussed.

L. SUCHESTON (with L. SZABO)

Almost everywhere operator convergence theorems and martingales

Because of a general principle reducing multiparameter convergence to the one-parameter case, it is important to know when a multiparameter martingale can be represented by iterated conditional expectations depending on index sets of lower dimension. This problem is considered for block martingales.

H. VON WEIZSÄCKER (with G. WINKLER)

A comment on stochastic integration

The following standard results of semimartingale theory are shown to be valid (with *easy* extensions of the proofs) without the "usual conditions" on the underlying filtration:

THEOREM 1. (following an idea of Stroock and Varadhan) *The space of right-continuous adapted processes satisfying* $\|\sup_{t\geq 0}|X_t|\|_p < \infty$ *is complete for this seminorm.*

THEOREM 2. *Every admissible measure on the predictable sets is induced by a right-continuous a. s. increasing predictable process.*

THEOREM 3. *If* X *is a right-continuous semimartingale, then it is also one for every equivalent probability measure.*

(From the authors' text *Stochastic Integrals: An Introduction*, Vieweg, to appear)

8. Funktionalanalytische Probleme

R. FRANKIEWICZ

The Enflo-Rosenthal theorem on L^p-spaces

The following results can be proved:

THEOREM 1. *Let* μ *be a finite measure and* $1 < p < \infty$. *If* $\omega_2 \leq \dim L^p(\mu) := \min\{\operatorname{card} A \mid A \subseteq L^p(\mu) \text{ and } \operatorname{span} A = L^p(\mu)\}$, *then* $L^p(\mu)$ *cannot be embedded into a Banach space with unconditional base.*

THEOREM 2. *If* ZFC *is consistent, then* ZFC *and* Theorem 1 *with* ω_2 *replaced by* ω_1 *is consistent.*

THEOREM 3. *It is consistent that the measure algebra* LM/Δ, *where* LM *consists of all Lebesgue measurable subsets of* $[0,1]$ *and* Δ *is an ideal of sets of measure zero, can be embedded into the quotient of the power set of* ω *modulo the ideal of all finite sets when* CH *is false.*

K. Musial

A characterization of weak compactness in the space of Bochner integrable functions

Let $(\Omega, \mathcal{A}, \mu)$ be a finite measure space and let X be a Banach space. Denote by $S(\mu, X)$ the collection of all X-valued simple functions, and let τ be the topology on $L^1(\mu, X)$ generated by $S(\mu, X^*) \subseteq L^1(\mu, X)^*$. Let Π denote the family of all finite $\mathcal{A}$-partitions of Ω ordered by refinement and, for $f \in L^1(\mu, X)$ and $\pi \in \Pi$, let f_π denote the conditional expectation of f with respect to $\sigma(\pi)$. Let $K \subseteq L^1(\mu, X)$ be a set satisfying the following conditions:

(i) K is bounded and μ-uniformly integrable, and

(ii) for each $E \in \mathcal{A}$, the set $\{\int_E f\, d\mu \mid f \in K\}$ is relatively weakly compact.

Proposition. *If X has RNP, then the following are equivalent:*

(1) *K is relatively weakly compact.*

(2) *For each $f \in L^1(\mu, X)$, the net (f_π) converges weakly quasi-uniformly on $\overline{K}^\tau$.*

(3) *Each $u \in L^1(\mu, X)^*$ is τ-continuous on $\overline{K}^\tau$.*

The previous result is due to F. G. J. Wiid who gave a nonstandard proof of it. I present a standard proof and variants of the result.

9. Integraldarstellungen

B. Anger (with C. Portenier)

Radon integrals and Riesz representation

As an example of an abstract Riemann type approach to integration and Riesz representation through function cones, we introduce Radon measures on arbitrary Hausdorff spaces X from the functional analytic point of view and prove representation theorems of Riesz type by means of Radon integrals.

A *Radon integral* is a linear functional $\mu : \mathcal{S}(X) \to (-\infty, +\infty]$, defined on the function cone $\mathcal{S}(X)$ of all lower semicontinuous functions from X to $(-\infty, +\infty]$ which are positive outside some compact set, which is regular (i. e. $\mu(s) = \mu_*(s) := \sup_{-s \le t \in \mathcal{S}(X)} -\mu(t)$ holds for all $s \in \mathcal{S}(X)$). Suppose that $\mathcal{T}$ is a lattice cone of lower semicontinuous functions on X and that $\tau : \mathcal{T} \to (-\infty, +\infty]$ is a regular linear functional which is tight (i. e. $\sup_{K \in \mathcal{K}(X)} \tau^*(t\chi_{\overline{K}}) = 0$ holds for all $t \in \mathcal{T}_-$, where $\mathcal{K}(X)$ is the collection of all compact subsets of X). If $\mathcal{T}_\emptyset$ denotes the lattice of upper envelopes of families of functions in $\mathcal{T}$, then the following holds:

EXTENSION THEOREM. *If $\mathcal{S}(X) = \mathcal{T}_\emptyset$, then τ_* is the only extension of τ to a Radon integral.*

REPRESENTATION THEOREM. *If $\mathcal{S}_-(X) \subseteq \mathcal{T}_\emptyset$, then there exists a unique Radon integral on $\mathcal{S}(X)$ representing τ.*

COROLLARY. *If $\mathcal{T} \subseteq \mathcal{C}(X)$ is a linearly separating Stonean vector lattice, then there exists a bijection between the tight positive linear forms on $\mathcal{T}$ and those Radon integrals which essentially integrate $\mathcal{T}$.*

C. PORTENIER

The Prokhorov-Sazonov theorem as a representation theorem

Using the *Corollary* of B. Anger's abstract, I give a new proof of the theorem of Prokhorov-Sazonov:

THEOREM. *If $\mathcal{H}$ is a Hilbert space, there is a bijection between bounded Radon integrals μ on $\mathcal{H}'_\sigma$ and positive definite functions Φ on $\mathcal{H}$ which are continuous with respect to the topology generated by the seminorms $\varphi \mapsto (\varphi|G\varphi)^{1/2}$, where G is a positive nuclear operator in $\mathcal{H}$. This is given by $\Phi(\varphi) = \int e^{i(\varphi|\cdot)}\, d\mu$ for $\varphi \in \mathcal{H}$.*

The main step is to construct μ from Φ using the *Representation Theorem.* One defines a linear form η on the algebra $\mathcal{A}$ of all trigonometric polynomials on $\mathcal{H}'_\sigma$ by $\eta(\sum c_h e^{i(\varphi_h|\cdot)}) := \sum c_h \Phi(\varphi_h)$. Using Bochner's theorem in the finite dimensional case, one proves first that η is positive and can be extended to a positive linear form τ on the uniform closure of the real part of $\mathcal{A}$. This is a vector lattice satisfying the conditions one needs, and all amounts to prove that τ is tight, which follows from Minlos' lemma.

The same method can be used to prove Minlos' theorem.

G. WINKLER

Simplexes of measures with closed extreme boundary and presentability of Hausdorff spaces

We consider closed bounded Choquet simplexes of tight measures on a Hausdorff space. If the extreme boundary is closed, then each element is the barycenter of a uniquely determined tight probability measure on the extreme points. (μ is the *barycenter* of p if $\mu(B) = \int \nu(B)\, dp(\nu)$ for every Borel set B of the underlying Hausdorff space.) Consequently, for Hausdorff spaces tight exchangeable measures can uniquely be decomposed into products of tight measures by means of a tight measure on the products. In the terminology of P. Ressel (which mimics the terminology introduced by Hewitt and Savage) this means that Hausdorff spaces are *Radon presentable*.

10. Methoden der Nonstandard-Analysis

W. A. J. LUXEMBURG

Nonstandard discrete measures

The main purpose of the talk is to explain the role the nonstandard real line or *hyperreal line* plays in analysis, in particular measure theory. The following is a brief example:

Let $[0,1]$ be the unit interval of $\mathbf{R}$, the real line, and let ${}^*[0,1]$ be its extension in ${}^*\mathbf{R}$, a hyperreal number system. Let ω_0 be an infinitely large natural number and let $\omega := \omega_0!$. Define the hyperfinite grid $\Omega := \{\frac{k}{\omega} \mid k = 0, \ldots, \omega\}$ for the unit interval. From the definition of ω it follows that Ω contains all the rationals. Furthermore, for every $x \in [0,1]$ there exists a unique $k \in [0,\omega]$ such that $\frac{k}{\omega} \leq x < \frac{k+1}{\omega}$ and so the standard part of Ω in all of $[0,1]$. Let $F(\Omega)$ be the space of all the internal ${}^*\mathbf{R}$-valued functions defined on Ω and let $M(\Omega)$ be the subspace of all $\mu \in F(\Omega)$ such that, for all $f \in C[0,1]$, $\mu(f) := \sum_{k=0}^{\omega} \mu(\frac{k}{\omega}) {}^*f(\frac{k}{\omega})$ is finite. Then it can be shown that the mapping $\Phi : \mu \mapsto \mathrm{st}(\mu)$ of $M(\Omega)$ into the algebraic dual of $C[0,1]$ is onto. Let $M_1(\Omega)$ be the family of all $\mu \in M(\Omega)$ with $\sum_{k=0}^{\omega} |\mu(\frac{k}{\omega})|$ finite. Then the image of $M_1(\Omega)$ under Φ is the Banach dual of $C[0,1]$. In particular, if $\mu(\frac{k}{\omega}) = \frac{1}{\omega}$ for all k, then $\Phi(\mu)$ is the Lebesgue integral.

By means of this method one obtains a hyperfinite representation theory of integrals which has a wide range of applications in measure theroy, probability theory, and the theory of stochastic processes.

D. ZIMMERMANN

Uniqueness of ergodic decompositions via nonstandard methods

Let $\mathcal{P}$ be a set of transition probabilities on a measurable space $(\Omega, \mathcal{F})$ and let $C(\mathcal{P})$ be the set of its invariant probability measures. It is shown that if there exists a representing measure for $p \in C(\mathcal{P})$ on $(\mathrm{ex}C(\mathcal{P}), \sigma(\{q \to q(F) \mid F \in \mathcal{F}\}))$, then it is uniquely determined. To prove this, we use nonstandard methods to identify the representing measure with the weak limit of a net which is independent of the representing measure.

11. Fraktale Begriffsbildungen und Methoden

C. Bandt

Hausdorff measures and interior distance on fractals

The study of shortest paths in fractals is motivated by physical problems concerning transport in disordered media. For a class of finitely ramified self-similar sets we show that an interior metric can only be defined by means of a certain Hausdorff measure. Conditions for the existence of such a metric are given. An example shows that, even for simple deterministic fractals, transport properties can heavily depend on the direction.

C. D. Cutler

Dimension distributions and exact-dimensionality of ergodic transformations

Let $T : K \to K$ be a measurable mapping, where K is a compact subset of $\mathbf{R}^d$, and suppose μ is an ergodic, invariant measure with respect to T. Associated with μ are various definitions of dimension (e. g. Hausdorff dimension, pointwise dimension, correlation dimension). We discuss the relations between these notions of dimension and present conditions under which μ can be regarded as *exact-dimensional*. The relation of this to the study of dynamical systems and attractors is discussed.

G. A. Edgar

Fractal dimension of self-affine sets

The computation of the Hausdorff dimension and the packing dimension is by now more or less understood for self-*similar* fractals. I consider the problem of computing the dimension of self-*affine* fractals. Some explicit examples in two-dimensional Euclidean space are discussed.

D. Kölzow

On Hausdorff decompositions of measures

Based on a decomposition theorem for vector lattices with respect to a chain of bands, a unified approach to the decomposition theorems of Lebesgue, Caratheodory, and Rogers-Taylor is given. In addition, a new decomposition, based on exact absolute continuity with respect to Hausdorff measures, is presented (a joint work with B. Bongiorno). By an example, it is shown that the diffuse measures in the sense of Rogers and Taylor form a strictly smaller class than those of the new decomposition. Finally, an application to vector analysis is sketched.

R. D. MAULDIN

Continua which admit a metric under which it has σ-finite H^1-measure

Let X be a metrizable continuum. For each $x \in X$, set $K_0(x) := X$ and, for each ordinal α, let $K_{\alpha+1}(x)$ be the maximal subcontinuum of $K_\alpha(x)$ which contains only countably many local separating points of $K_\alpha(x)$ and $K_\alpha(x) := \bigcap_{\beta<\alpha} K_\beta(x)$ if α is a limit ordinal.

THEOREM. (1) *For each $\alpha < \omega_1$, there are only countably many $x \in X$ such that $K_\alpha(x)$ is nondegenerate.*
(2) *There exists an $\alpha < \omega_1$ such that $K_\alpha(x) = \{x\}$ for all $x \in X$.*

CONJECTURE. *If* (1) *and* (2) *hold, then X possesses a metric under which X has σ-finite linear Hausdorff measure.*

P. SINGER

A Fourier transform associated with fractal Brownian motion

The following fractional-integrated Fourier transform is studied:

$$\mathcal{F}_\alpha\varphi(t) := c_{\underline{\alpha}} \int_{\mathbf{R}^d} \frac{\exp(it\cdot x) - 1}{\|x\|^{\underline{\alpha}}}\varphi(x)\,dx,$$

where $\underline{\alpha} := (\alpha + d)/2$ with $0 < \alpha < 2$ and $\varphi : \mathbf{R}^d \to \mathbf{C}$. In the case $\alpha = d = 1$ resp. $d = 1$, Wiener resp. Molchan and Ciesielski dealt with such a transform. We study $\mathcal{F}_\alpha$ on $L^1(\|x\|^{(1-d)/2}\,dx)$ resp. $L^2(dx)$, establish relations to ordinary Fourier transform, and prove an inversion formula. We apply this to fractional Brownian motion to show SDE for this process, a Fourier representation by white noise of the dual process, and a factorization of the covariance operator $f \mapsto \int_{\mathbf{R}^d} \Gamma_\alpha(.,s)f(s)a(s)\,ds$, where Γ_α is the covariance of fractional Brownian motion and a a weight function. Furthermore, we use $\mathcal{F}_\alpha$ to prove an inversion formula of Helgason type for a fractional Radon-transform of Noda type associated to fractional Brownian motion.

S. J. TAYLOR

Using measures to define fractals

If a fractal is to be more than a "pretty picture" one needs defining properties. Hausdorff measure is based on economical covering of a set $E \subseteq \mathbf{R}^d$, while packing measure uses efficient packing of disjoint balls with centres in E. Using the power functions $\varphi(s) = s^\alpha$, either of these measures defines a dimensional index. It is proposed that a set should be called a *fractal* if these two indices coincide. Additional regularity conditions then yield desirable properties and theorems.

S. C. WILLIAMS

Multiplicative processes

Random measures are generated through a multiplicative cascade. The Hausdorff dimension of the support is calculated along with conditions for the non-triviality of the measures.

P. ZAJDLER

A local definition of multifractals

There are fractals which have a certain Hausdorff dimension but locally are not of this dimension. Therefore, the Hausdorff dimension in a point x with respect to a set E is defined to be the infimum of the Hausdorff dimensions of the intersections of E with open neighbourhoods of x. This local dimension is also given by the $\alpha = \alpha_0$ where the (convex) upper α-density of x with respect to E (introduced by Wallin) jumps from 0 to ∞. The inequality between Hausdorff and topological dimension also holds for these Hausdorff dimensions in a point. Therefore, a set for which the local Hausdorff dimension is always greater than the topological dimension and which has at least two different Hausdorff dimensions could be called a *multifractal.*

12. Nicht-kommutative und nicht-additive Maßtheorie

P. DE LUCIA

A non-commutative version of the Lebesgue decomposition theorem

Let $(L, \leq, 0, 1, ')$ be an orthomodular lattice, let G be a topological commutative group, and let $a(L,G)$ denote the set of all additive functions from L to G. If M is an ideal of L, we say that $\mu \in a(L,G)$ is *M-continuous* (resp. *M-singular*) if $M \subseteq N(\mu)$ (resp. if there exists some $c \in L$ such that $c \in M$ and $c' \in N(\mu)$). An ideal M of L is a *p-ideal* if $x \wedge (x' \vee c) \in M$ holds for all $c \in M$ and $x \in L$. We can prove the following:

THEOREM. *Let $\mu \in a(L,G)$, let κ be an infinite cardinal, and let M be a κ-orthocomplete p-ideal of L such that $M \setminus N(\mu)$ satisfies the κ-chain condition. Then μ is uniquely representable as $\mu = \eta + \xi$ with $\eta, \xi \in a(L,G)$ such that η is M-continuous and ξ is M-singular.*

E. PAP

Non-additive measure theory and applications to non-linear partial differential equations

We investigate $\perp$-decomposable measures, where $\perp$ is a t-conorm. Among other results we have analogues of classical measure theory theorems: Lebesgue decomposition, Saks decomposition, Darboux property, compact range theorem, extension theorems. The important tool for that is the topological connection with submeasures and corresponding results of L. Drewnowski and I. Dobrakov. We investigate also the regularity of t-conorm decomposable measures and pay special attention to *sup*-decomposable measures and the non-uniqueness of their extension. This fact causes some difficulties in the construction of an integral analogous to the Lebesgue integral. Using results of V. P. Maslov and his coworkers, we apply the obtained results to some non-linear partial differential equations and a generalized Bellman equation which gives a close link with algorithms for parallel processing on computers of the fifth generation.

13. Verschiedenes

A. JOVANOVIĆ

More details on Rudin-Keisler order

In the Rudin-Keisler order of types of ultrafilters, normality conditions characterize the minimal types. In the set M_K of all ultrafilters over some index K the normal ultrafilters are the minimal ones. Weakly normal ultrafilters are the minimal ones in the layer of uniform ultrafilters. The key is the existence of minimal unbounded functions modulo ultrafilter leading to generalized normality: An ultrafilter D over K is λ (*weakly*) *normal* if there is an $f \in \prod_D K$ such that $g <_D f$ implies $g_D =$ const. ($g_D <_D$ const. $< K$ for weak normality). A λ weakly normal D which is uniform contains information about minimal elements in the λ^{th} layer. Thus the weak normality trace of an ultrafilter gives all information about minimal elements below the ultrafilter in all ultrafilter norms. λ weak normality is preserved by

CCC extensions. The Rudin-Keisler order expands to a hierarchy of real valued measures, where the minimality of measures is determined by the same sort of normality conditions.

E. NOVAK

Average errors in numerical analysis

Let $S(f) := \int_{[0,1]^d} f(x)\,dx$, where f is a function from $F := \{ f : [0,1]^d \to \mathbf{R} \mid \|f^{(\alpha)}\|_\infty \leq 1$ for all partial derivatives of order $k \}$. We approximate $S(f)$ by sums $\sum_{i=1}^n a_i f(x_i)$. The worst case error of optimal methods tends to 0 as $n^{-k/d}$. Therefore, we consider the average error. For any measure on F, the optimal methods converge as $n^{-k/d-1/2}$ on the average. We next study the average error of quadrature formulas for a class of monotone functions. We use a natural measure and prove that adaptive methods are much better than nonadaptive ones. We also study the approximate solution of $f(x) = 0$ and prove that bisection, which is optimal in the worst case, fails to be optimal on the average for the natural measure and also for the Brownian bridge. (Some of these results have been obtained in a joint work with S. Graf.)

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