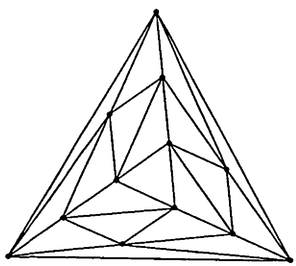
MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 24/1990

Graphentheorie

3.6. bis 9.6.1990

Die Tagung fand unter der Leitung von Herrn G. Ringel (Santa Cruz, Kalifornien) statt.
Die 39 Teilnehmer aus der großen Anzahl von Graphentheoretikern kamen aus den europäischen Ländern Dänemark, Deutschland, England, Italien, Niederlande und Ungarn, sowie aus den Ländern Japan, Kanada, Rußland Südafrika und Vereinigte Staaten von Amerika.
In den Vorträgen wurde über neue Forschungsergebnisse berichtet, die etwa in den Teilgebieten Ramseytheorie, Zufallsgraphen, Knotentheorie, Gruppen und Graphen, Färbungsprobleme, Unendliche Graphen, geometrische Graphentheorie oder Einbettungsprobleme einzuordnen sind. In der Figur ist der Graph des Ikosaeders so gezeichnet, daß alle endlichen Flächen von gleichem Flächeninhalt sind.



Außer abendlichen Arbeitsgruppen fand am Dienstag eine Problemsitzung statt. Am Donnerstag wurde die Nachmittagssitzung dem siebzigsten Geburtstag von Gerhard Ringel gewidmet und ihm anschließend von Frau Förstner-Henn die Festschrift "Topics in Combinatorics and Graph Theory" feierlich überreicht.

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Vortragsauszüge

DAN ARCHDEACON:

Self-dual polyhedra

A 2-dimensional polyhedron is a graph embedded on a surface such that the union of any two faces is simply-connected. Any such embedding has a natural geometric dual which reverses the role of vertices and faces. A polyhedron is self-dual if there is a map isomorphism between the primal and dual. For example, the tetrahedron is self-dual. I survey recent results on self-dual polyhedra. Included are classification theorems when the surface is the sphere or the projective plane, constructions for involuntary self-duality maps on more general surfaces, and constructions of self-dual embeddings for sprcific classes such as complete bipartite graphs.

JØRGEN BANG-JENSEN

Local tournaments, a generalization of tournaments

In this talk we introduce a new class of directed graphs called local tournaments. These are defined to be those digraphs for which the set of in-neighbours (respectively, out-neighbours) of any vertex induces a tournament. This class contains the tournaments, but is much more general. In fact, the underlying graphs of local tournaments are precisely the proper circular-arc graphs. We show that many classic theorems for tournaments have natural analogues for local tournaments. For example, any connected local tournament has a hamiltonian path and any strong local tournament has a hamiltonian cycle. We consider connectivity properties, domination orientability, orientability as local tournament (i.e. recognition of proper circular-arc graphs) and algorithmic aspects of local tournaments. Some of the results on connectivity are new even for tournaments. Finally we give several examples of tournament results that do not extend to local tournaments.

CLAUDE BERGE

A generalization of Vizing's Theorem

We study different generalizations of the famous Theorem of Vizing about edge-coloring graphs or multigraphs. In particular:

1) We present a short proof of the following: Let G be a multigraph of maximum degree D and of multiplicity p; for each vertex x, t(x) denotes the maximum multiplicity of an edge issuing from x.
If the set of all vertices x with degree d(x)=D and t(x)=p is independent (or empty), then the edges of G can be colored with D+p-1 colors (results in collaboration with J. C. Fournier).

2) We survey different approaches of two conjectures which generalizes Vizing's Theorem (in particular some partial results are obtained with A.J.W. Hilton).

RAINER BODENDIEK

On forbidden graphs

The spindle-surface S_2 arises from a torus by contracting two different meridians to a single point, each. Since the well-known partial ordering relation $>_i$, i=2,3,4, does not preserve the embeddability with respect to S_2, we cannot follow the finiteness of the minimal basis $M_i(S_2)$ from Bodendiek/Wagner or Robertson/Seymour. Therefore, this talk deals with a new finiteness proof for S_2.

BÉLA BOLLOBÁS

Cycles through specified vertices

Extending Dirac's Theorem, Katchalski conjectured that the circumference of a graph of order n and minimal degree d is at least $\lceil x \rceil$, where $x=n/(\lceil n/d \rceil -1)$. Alon came very close to proving this, when he showed that the circumference is at least $\lfloor x \rfloor$. The full conjecture was proved, independently, by Egawa & Miyamoto, and Bollobás & Häggkvist.

The main aim of the talk was to give a proof of the following extension of this result.

Theorem 1. Let G be a graph of order n with $W \subset V(G)$, $|W|=w$, such that $d(x) \geq d$ for every $x \in W$. Suppose that $s=\lceil w/(\lceil n/d \rceil -1)\rceil \geq 3$. Then G contains an (s,W)-cycle, i.e. a cycle containing at least s vertices of W.

The proof is surprisingly simple; it is, in fact, much simpler than the two proofs mentioned above concerning the case $W=V(G)$.

For $(2,W)$-cycles one has the following trivial observation.

Theorem 2. If $n \leqq (d-1)w+1$ then there is a $(2,W)$-cycle.

As almost always with results of this type, it is very easy to give an Ore-type variant of the results above, although the precise formulation is not too appetizing, and so is omitted here.

J. A. BONDY

Triangle-free subgraphs of powers of cycles

Let $t(m,n)=ex(K_3,C_n^m)/mn$ be the proportion of edges of C_n^m (the m-th power of an n-cycle) in a largest triangle-free subgraph, where $n \geq 2m+1 \geq 3$. Turán's theorem says that $t(m,2m+1)=(m+1)/(2m+1)$. Chung and Trotter (1984) proved that, for $n>>m$, $.586 \approx 2-\sqrt{2} \leq t(m,n) \leq (5+\sqrt{3})/11 \approx .612$.

Using two families of graceful graphs due to Abrham (1984), S. C. Locke and I proved that, for all n and many values of m, $t(m,n) \leq .60008$. We describe the construction giving the lower bound of Chung and Trotter and the proof of the upper bound of Locke and myself.

FAN R. K. CHUNG

Quasi-random classes of hypergraphs

We investigate the relations among a number of different graph properties for k-uniform-hypergraphs, which are shared by random hypergraphs. Various graph properties form equivalence classes which in turn constitute into a natural hierarchy. The analogue for binary functions on k-tuples and for hypergraphs with small density are also considered. Several classes are related to communication complexity and expander graphs.

WALTER DEUBER

The hypergraph of ultrafilters on the natural numbers

The Stone-Cech compactification and its closed subsemigroups may be used for studying Ramsey type problems. This is done here for partition regular matrices which were introduced by Richard Rado.

REINHARD DIESTEL

The structure of rayless k-connected graphs

We prove that every rayless k-connected graph has a tree-decomposition into finite k-connected factors, where the decomposition tree is rayless. (If any two adjacent factors overlap in at least k vertices, then the converse is also true.) As a corollary we obtain that every rayless k-connected graph contains a finite k-connected subgraph.

YOSHIMI EGAWA

Removable edges in 3-connected graphs

For a graph G and a vertex x of G, we define a graph G_x as follows: If x has degree 2, then, letting u and v denote the neighbors of x, we set $V(G_x)=V(G)-\{x\}$, $E(G_x)=(E(G)-\{xu,xv\})\cup\{uv\}$, if u and v are not adjacent in G, $E(G_x)=E(G)-\{xu,xv\}$, if u and v are adjacent in G, that is to say, G_x is obtained from G by "suppressing" x; if the degree of x in G is not 2, then we simply let $G_x=G$. For an edge e=xy of a graph G, we define $G \ominus e$ as $((G-e)_x)_y$, which is the same as $((G-e)_y)_x$.
An edge e of a 3-connected graph G is called removable if $G \ominus e$ is still 3-connected. In this talk, we discuss the following theorem:

Theorem. Let $G \neq K_4$ be a 3-connected graph. Then G has at least $(3|V(G)|+18)/7$ removable edges.

RUDOLF HALIN

Tree-partitions of infinite graphs

If G is a graph and π a partition of V(G), then G/π is the graph with vertex set π in which $P,Q \in \pi$ are adjacent iff there is at least one edge in G joining vertices of P and Q. A connected graph is called a pseudo-tree (quasi-tree) if a π with finite (and inducing connected subgraphs of G) classes exists such that G/π is a tree. The pseudo-trees are characterized (among the connected graphs) by forbidden configurations, and it is shown that every pseudo-tree which is not a quasi-tree must contain a subdivision of a specific configuration composed of infinitely many $K_{2,\omega}$'s, which e.g. contains the ω-regular tree.

FRANK HARARY

Some problems on sum graphs

The sum graph $G^+(S)$ of a set S of positive integers is the graph (V,E) where V=S, and $ij \in E$ if and only if $i+j \in S$. Then a sum graph is isomorphic to the sum graph of some S. We showed that any given graph H can be made into a sum graph by adding a sufficient number of isolated nodes; the smallest such number we call the sum number S(H). M. Ellingham proved my conjecture that every tree has sum number 1. We established that the sum number $S(K_n)=2n-3$ and we showed that the families of product graphs and of sum graphs are coextensive. Currently we are working on sum digraphs in nonabelian groups, modulo sum graphs, and other generalizations.

HEIKO HARBORTH

Crossing problems in drawings of graphs

For drawings D(G) of graphs G in the plane (two edges have at most one point in common, either an endpoint or a crossing) the following problems are discussed: (1) The number of nonisomorphic drawings. (2) The minimum number cr(G) of crossings. (3) The maximum number CR(G) of crossings. (4) m-fold crossings in $D(K_{2m})$. (5) Edges in $D(K_n)$ with maximum number of crossings. (6)/(7) The minimum number $h_s(n)$/maximum number $H_s(n)$ of edges in $D(K_n)$ with at most s crossings. - Recent results are: (I) $H_1(n)=\binom{n}{2}$ for $n\leq 6$, $H_1(7)=18$, $H_1(8)=20$, $H_1(9)=22$, $0\leq H_1(n)-2n+2-[\frac{n-1}{2}]\leq 9$ for $n\geq 8$. (II) $CR(Q_3)=34$ for the cube graph Q_3.

NORA HARTSFIELD

Self-dual embeddings of Cayley graphs for certain nonabelian groups

Self-dual embeddings are presented for Cayley graphs for alternating, symmetric, and metacyclic groups. Some open questions are discussed.

ANDREAS HUCK

A sufficient condition for graphs to be weakly k-linked

We consider graphs, which are finite, undirected, without loops, and in which multiple edges are possible. For each natural number k let g(k) be the smallest number n, so that the following holds:

Let G be an n-edge-connected graph and let $s_1,\ldots,s_k,t_1,\ldots,t_k$ be vertices of G. Then for every $i=1,\ldots,k$ there exists a path P_i from s_i to t_i, so that $P_1,\ldots,P_k$ are pairwise edge-disjoint.

We prove $g(k)\leq k+1$ if k odd, and $g(k)\leq k+2$ if k even.

A. A. IVANOV

On a graph having J_4 as the automorphism group

Let J_4 be the fourth Janko's sporadic simple group. It is known, that J_4 contains a maximal 2-local subgroup $H\cong 2^{10}.L_5(2)$. Let $\Gamma=\Gamma(J_4)$ be a graph whose vertices are the subgroups of J_4

which are conjugate to E. Two vertices are adjacent if their intersection is a subgroup of order 2^3. The graph Γ has valency 15 and it contains a collection P of Petersen subgraphs possessing the following property: For any vertex x of Γ the incidence system having the vertices adjacent to x as points and the subgraphs from P passing through x as blocks, is isomorphic to the projective space PG(3,2).

Theorem. Let $\phi:\tilde{\Gamma}\to\Gamma$ be a covering of the graph $\Gamma=\Gamma(J_4)$. Suppose that for any vertex $\tilde{x}$ of $\tilde{\Gamma}$ the restriction of ϕ on the subgraph $\tilde{\Gamma}$ induced by the vertices which are at distance at most 2 from $\tilde{x}$, is an isomorphism. Then ϕ is an isomorphism.

BRAD JACKSON

Edge labelings of trees

Two kinds of edge labelings of trees are discussed. For a tree T with p edges we label the edges 1,2,...,p. If the sum of the labels at each vertex is different we say that the labeling is antimagic and if in addition the sum at each vertex is different modulo p+1 we say that the labeling is edge-graceful. It has been conjectured (Hartsfield, Ringel, 1988) that every tree different from K_2 is antimagic and (Lee, 1986) that every tree with an even number of edges is edge-graceful. It is shown that every tree with at most one vertex of degree two is antimagic and those with an even number of edges are also edge-graceful. Techniques for labeling trees with more than one vertex of degree two are also discussed.

FRANCOIS JAEGER

Combinatorial aspects of knot theory and relations between links, graphs and matroids

A link is a finite collection of disjoint simple closed curves in R^3. Links can be represented by plane projections called diagrams. A diagram can be coded as a plane graph with signed edges. We show that the Kauffman polynomial (a recently discovered link invariant) yields, via this coding, an invariant of planar matroids with signed elements, and we discuss problems concerning the interpretation of this invariant.

DIETER JUNGNICKEL

Group invariant conference matrices and strongly regular Cayley graphs

Let A be a conference matrix of size 2m+2, and let H be a group of order 2m+2. We call A an H-invariant matrix if it may be indexed by the elements of H in such a way that one has $a_{gh}=a_{g+k,h+k}$ for all $g,h,k \in H$. We conjecture that no such matrix can exist, and obtain the following partial result:

Theorem. Assume the existence of an H-invariant conference matrix A of order 2m+2. Then m is divisible by 4, 2m+1 is a perfect square, and H is non-abelian.

In fact, this result is obtained by constructing from A a strongly regular graph on $2m+2=4s^2+4s+2$ vertices with parameters $a=s(2s+1)$, $c=s^2-1$ and $d=s^2$ admitting H as a regular automorphism group. Using a theorem of Bridges and Mena on strongly regular Cayley graphs, H cannot be abelian. We note that an SRG with the parameters given above always exists if 2s+1 is a prime power; the smallest case is just the Petersen graph. It seems to be an open problem if any such graph can be a Cayley graph, i.e. admit a (necessarily non-abelian) regular group. The motivation for studying group invariant conference matrices lies in applications of our theorem in the theory of relative difference sets (equivalently, divisible designs with a Singer group).

Reference: D. Jungnickel:"On automorphism groups of divisible designs, II: Group invariant generalized conference matrices", Archiv Math. 54(1990), 200-208.

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ALEXANDER KELMANS

More about graph planarity

The graph planarity problem for an arbitrary graph can be easily reduced to the problem for 3-connected graphs. We give some strengthenings of the Kuratowski and the Whitney planarity criteria for 3-connected graphs. We also give a planarity criterion for 3-connected graphs in terms of non-separating cycles. We show that the problem for 3-connected graphs can also be easily reduced to the problem for 3-connected graphs without essential 3-cuts or triangles. It turns out that the above mentioned criteria can be strengthened for this class of graphs. Here are some of the results.
1. A 3-connected graph distinct from K_5 is nonplanar iff it contains a cycle with three overlapping chords-edges.
2. A 3-connected graph is planar iff it has a semi-dual graph. (A semi-duality between two graphs G and F is a one-to-one map e:EG→EF such that if C is a cycle of G then e(C) is a cocycle of F.) 3. Let G be a 3-connected graph. Then G is planar iff each edge of G belongs to exactly two non-separating cycles. 4. If G is a nonplanar 3-connected graph without essential 3-cuts or triangles then each edge of G belongs to at least three non-separating cycles.

ROLF H. MÖHRING

Graph problems related to gate matrix layout

We consider the complexity status of graph problems occurring in linear VLSI layout architectures such as gate matrix layout, folding of programmable logic arrays, and Weinberger arrays. These include a variety of mostly independently investigated graph problems such as augmentation of a given graph to an interval graph with small clique size, node search of graphs, matching problems with side constraints, and other. New results presented include: NP-completeness of gate matrix layout on chordal graphs, and efficient algorithms for trees, cographs, Halin graphs, and certain chordal graphs.

ORTRUD R. OELLERMANN

Steiner distance stable graphs

Let G be a connected graph and S a nonempty set of vertices of G. Then the Steiner distance $d_G(S)$ of S is the smallest number of edges in a connected subgraph of G that contains S. Let k,l,s and m be nonnegative integers with $m \geq s \geq 2$ and k and l not both 0. Then a connected graph G is said to be k-vertex l-edge (s,m)-Steiner distance stable, if for every set S of s vertices of G with $d_G(S)=m$, and every set A consisting of at most k vertices of G-S and at most l edges of G, $d_{G-A}(S)=d_G(S)$. It is shown that if G is k-vertex l-edge (s,m)-Steiner distance stable, then G is k-vertex l-edge (s,m+1)-Steiner distance stable. Further, a k-vertex l-edge (s,m)-Steiner distance stable graph is shown to be k-vertex l-edge (s-1,m)-Steiner distance stable graph for $s \geq 3$. It is then shown that the converse of neither of these two results holds.

If G is a connected graph and S an independent set of s vertices of G such that $d_G(S)=m$, then S is called an I(s,m)-set. A connected graph is k-vertex l-edge I(s,m)-Steiner distance stable if for every I(s,m)-set S and every set A of at most k vertices of G-S and l-edges of G, $d_{G-A}(S)=m$. It is shown that a k-vertex l-edge I(3,m)-Steiner distance stable graph, $m \geq 4$, is k-vertex l-edge I(3,m+1)-Steiner distance stable.

M. D. PLUMMER

Matching extension in the plane

A graph is bicritical if G-u-v has a perfect matching for all pairs of points u and v in V(G). The 3-connected bicritical graphs (called bricks) currently are the "atoms" in a certain decomposition theory for graphs in terms of their maximum matchings. If $1 \leq n \leq p/2$, graph G is n-extendable if every matching of size n in G is a subset of a perfect matching in G. In particular, if G is 2-extendable then G is either bipartite or bicritical.

Here we confine ourselves to planar graphs. No planar graph is 3-extendable, but many are 1-extendable. We focus on 2-extendable planar graphs and present several results about this class.

ERICH PRISNER

On the clique graph function

Facing a function f from a set Γ of graphs into itself (where isomorphic graphs are considered as identical), I am mainly interested in two sorts of questions. The first one is the question whether the set of all iterated f-graphs of a given graph is finite or infinite. The second contains inverse problems, for example, how many graphs H of Γ obey f(H)=G, given a graph G. In a larger project, these problems are investigated for almost all known graph-valued functions. In this talk I shall speak on the clique graph function only. The clique graph C(G) of a graph G is the intersection graph of the set of all cliques (= maximal complete subgraphs) of G. Results on this questions from the literature as well as own research are presented.

HANS JÜRGEN PRÖMEL

Coloring K_{l+1}-free graphs in linear expected time

We present a linear expected time algorithm to color all graphs which do not contain a clique of size l+1 as a subgraph with a minimal number of colors. This extends a result of Dyer and Frieze (J. Algorithms 10(1989), 451-489) for l-colorable graphs. For the proof we develop a new method which allows us to precisely estimate the number of graphs with certain structural properties. This is joint work with A. Steger.

GERHARD RINGEL

In 1936 Klaus Wagner proved that each planar graph is rectifyable, i.e. each edge can be represented by a straight line segment. This talk is a short report about additional geometric properties which have been studied by graph theorists in the last decade: 1) Coin graphs, where each vertex is represented by a circle and an adjacency is represented by two tangent circles. 2) Visibility graphs, where each vertex is represented by a horizontal line segment and each edge by a vertical line segment. 3) Planar graphs, where the vertices are lattice points. 4) Planar graphs, where the edges have integer length. 5) Planar graphs, where all the finite regions have the same area, we call equiareal graphs. For instance, the icosahedron graph is equiareal. There exist planar graphs which are not equiareal, but we conjecture that each cubic planar graph is equiareal.

HORST SACHS

Counting perfect matchings in plane lattice graphs

This is partly joint work with Khaled al-Khnaifes from Damascus who investigated relations between graph theory and linear algebra in his doctoral dissertation (1988).
In some parts of chemistry and physics, tesselations of the plane and the number of perfect matchings (corresponding to Kekulê patterns and dimer coverings, respectively) which are contained in finite sections of the corresponding lattice graphs play an important role. Using a well-known result of P. W. Kasteleyn (1961) and a graph algorithm for calculating determinants, the speaker describes a simple algorithm which allows these numbers to be efficiently calculated for certain classes of tesselations.

ALEXANDER SCHRIJVER

The uniqueness of minimal graphs on surfaces

Let G be a graph embedded on a surface S. For any closed curve C on S, let $\phi_G(C)$ denote the minimal number of intersections of D with G, where D ranges over all closed curves freely homotopic to C.
It is not difficult to see that ϕ_G is invariant under taking the dual graph and under ΔY-transformation. We call G mini-

mal if $\phi_{G'} \neq \phi_G$ for each proper minor G' of G. (That is, $\phi_{G'}(C) < \phi_G(C)$ for at least one C.)
We show that if G and G' are minimal graphs with $\phi_G = \phi_{G'}$, then G' can be obtained from G by the following operations: Shifting the graph over the surface, taking the dual graph, and ΔY-transformation and its converse (under the condition that G is cellularly embedded).

PAUL D. SEYMOUR

Excluded minors of infinite graphs

Let κ be a cardinal, finite or infinite. When does a graph G admit a "tree-decomposition" into pieces of cardinality $<\kappa$, in which the tree has no 1-way infinite path? For G and κ finite, there is such a decomposition if and only if G has no large square grid minor. For $\kappa = \aleph_0$, there is such a decomposition if and only G has no 1-way infinite path (this is a result of Halin). For $\kappa \geq \aleph_1$, there is such a decomposition if and only if G has no T_κ-minor (T_κ is the regular tree of valency κ). The result is a consequence of the analysis of a pursuit-evasion game played on G. (Joint work with N. Robertson and R- Thomas.)

VERA T. SÓS

Quasirandomness and Szemerédi-partition

Chung, Graham and Wilson proved that certain graph properties shared by random graphs are equivalent. With Simonovits we proved that also the following property belongs to that class: G_n has a Szemerédi-partition with a.a. densities $1/2+o(1)$.

The k-spectrum of graphs

The k-spectrum of a graph is the set of those integers e for which there is an induced subgraph with k vertices and e edges. The following questions are investigated: 1) Characterisation-problem: Which $S \subseteq \{0, \ldots, \binom{k}{2}\}$ can be the k-spectrum of some G_n? 2) Reconstruction-problem: Which k-spectrum (resp.

k-spectrum with multiplicities) belongs to a unique G_n?

GOTTFRIED TINHOFER

Graceful trees and perfect matchings

A graph $G=(V,E)$ on n vertices and m edges is a graceful graph if there exists a graceful numbering of G, i.e. a bijective mapping $l:V\to\{0,...,n-1\}$ such that the mapping $g:E\to\{1,...,n-1\}$ defined by $g(\langle u,v\rangle)=|l(u)-l(v)|$ is a bijection, too. From the definition it follows $|E|=n-1$.
It is conjectured that every tree is a graceful graph. Various classes of graceful trees are known, the best known and maybe most important class being the class of caterpillars.
In this talk the following two results on graceful graphs will be presented and discussed: 1) The graceful tree conjecture is valid iff it is valid for the class of trees which have eigenvalue 0. 2) There is a simple gracefulness preserving operation, called vertex addition, which replaces a gracefully numbered graph on n vertices by one on n+1 vertices. This operation will be defined and it will be shown that every graceful numbered graph on n vertices is constructable by a sequence of n-1 vertex additions starting with the single vertex with label 0.

ZSOLT TUZA

Some open problems on colorings and coverings of graphs

We present conjectures and open problems concerning: 1) the minimum cardinality of an edge set sharing an edge with all triangles (=complete subgraphs on 3 vertices) of an undirected graph or with all cyclic/transitive triangles of a directed graph, 2) the structure of edge-intersection graphs of triangles in a graph, 3) the minimum number of mutually edge-disjoint triangles in agraph of n vertices and m edges ($m>n^2/4$), 4) the number of steps needed to decide whether a given graph is perfect (or contains an induced cycle of odd length ≥ 5), 5) the minimum number of perfect subgraphs that cover the edge set of a graph on n vertices, 6) the minimum number of induced matchings that cover the edge set of a k-regular (bipartite) graph, and 7) a simple Ramsey-type question on 3-term arithmetic progressions.

BERND VOIGT

Finding minimally weighted subgraphs

Given finite graphs H and G we consider the problem to decide whether G contains a weak, i.e., not necessarily induced, H-subgraph. More generally, if additionally a weight-function on the edges of the graph G is given, we are looking for a minimally weighted H-subgraph, where of course the weight of a subgraph is the sum of the weights of its edges. Let us denote by WSG(H;G) the minimal time complexity of an algorithm solving this problem, where G is the input of the algorithm and H is not considered as being part of the input, but rather belongs to the algorithm itself.
Theorem. Let H have tree-width w-1. Then $WSG(H;G)=O(n^w)$, where n is the number of vertices of G.
As corllaries from the algorithm we may deduce the following results:
Corollary 1. Let T be a tree, then WSG(T;G)=O(e), where e is the number of edges of G.
Notice that already finding just an edge of minimal weight requires time O(e). However, the implicit constants of Corollary 1 grow exponentially in the size of the tree T, and the same holds with respect to the next corollary.
Corollary 2. Let C be a cycle, then WSG(C;G)=O(n*e).
This is joint work with Jürgen Plehn from Bonn.

LUTZ VOLKMANN

Class 1 conditions of simple graphs

We consider finite, undirected and simple graphs G=(V(G),E(G)). We denote by $\Delta(G)$ the maximum degree, $\delta(G)$ the minimum degree,

$\chi'(G)$ the chromatic index, and $k(G)$ the number of vertices of maximum degree of G, respectively. If $\Delta(G)=\chi'(G)$, then G is class 1. A graph which satisfies $|E(G)|>|\lfloor V(G)|/2\rfloor\Delta(G)$, is called overfull. - Our main theorems give new sufficient conditions for graphs to be class 1.

Theorem 1. A graph G with 2n vertices is class 2, if $\delta(G)\geq n+k(G)-2$.

Theorem 2. A graph G with 2n+1 vertices, which is not overfull, is class 1 if $\delta(G)\geq n+k(G)+\lfloor\Delta(G)k(G)/(2n+1)\rfloor$.

HEINZ-JÜRGEN VOSS

Long cycles with many diagonals

A well-known theorem of G. A. Dirac states that if G is a graph with minimum degree $\delta(G)\geq r\geq 3$ then G has a cycle of length $\geq r+1$. Related to this result is an observation of J. Czipszer that G has a cycle with $\geq r-2$ diagonals. Involving the girth and the connectivity number of G one can prove some further theorems (see [1]). One of these theorems is the new result proved in [2]: Let k,r,s,t be integers with $k\geq 2$, $s\geq 1$, $r\geq 3$ and $t\geq t_0(s,r)$, where $t_0(s,r)$ is an appropriate constant. Let G be a cyclically k-vertex-connected graph with $\delta(G)\geq r$ and girth $g(G)\geq t$. Then each s-tuple of edges, $s\leq k-1$, lying on a cycle of length $>s$ is on a cycle with $\geq(r-1)^{t/12-5s-15}$ diagonals.

[1] H.-J. Voss, Cycles and Bridges in Graphs. VEB Deutscher Verlag der Wissenschaften, 1990.

[2] H.-J. Voss, Cycles with many diagonals in cyclically k-vertex-connected graphs (to appear in "Wagner-Festschrift").

WOLFGANG WOESS

Harmonic functions with finite energy on infinite graphs

Let G(X,E) be an infinite graph, connected, locally finite, with bounded vertex degrees. The energy of a function $g:X\to R$ is $D(g)=\Sigma_{[x,y]\in E}(g(y)-g(x))^2$. We say that g is harmonic if $g(x)=(1/\deg(x))\cdot\Sigma_{y:[x,y]\in E}g(y)$. We are interested in the space HD(G) of harmonic functions with finite energy on G. In particular, we want to know for which classes of graphs HD(G) consists only of the constant functions, or contains nonconstant functions, respectively. This is of interest in the study of electric currents in (infinite) graphs, see the work by Flanders, Zemanian, Yamasaki and Doyle-Snell.

Theorem 1. (Soardi-W.) If G is a vertex-transitive and has polynomial growth then HD(G)={constants}.

Theorem 2. (Thomassen) If G is the Cartesian product of two infinite graphs then HD(G)={constants}.

Theorem 3. (Soardi-W.) If G has more than one end and satisfies a strong isoperimetric inequality then HD(G) contains nonconstant functions.

Theorem 4. (Cartwright-W.) If G is uniformly embedded in the hyperbolic disc, satisfies a strong isoperimetric inequality and has at least two limit points, then HD(G) contains nonconstant functions.

Arbeitsgemeinschaften

über: Enden in Graphen und Gruppen; Beziehungen zwischen dem "accessibility problem" der Gruppentheorie und Fragen über unendliche Graphen.
Teilnehmer: Halin, Woess, Seymour, Diestel.

über: Kanten überdeckende Knotenmengen für Hypergraphen.
Teilnehmer: Erdös, Tuza.

über: Kantenfärbungen von Graphen.
Teilnehmer: Aigner, Zuza.

über: Bedingte chromatische Zahlen.
Teilnehmer: Harary, Tuza.

Ungelöste Probleme

J. A. BONDY:

1. Conjecture: Let G be a simple 2-connected graph on n vertices. Then the edges of G can be covered by at most $(2n-1)/3$ cycles.

2. Conjecture: Let G be a simple 2-edge-connected graph on n vertices. Then the edges of G can be double covered (exactly) by at most $n-1$ cycles.

F. CHUNG:

What is the maximum number of edges in a C_{2n}-free subgraph of an n-cube? The conjecture is $(2^{1-n}+o(1))|E(Q_n)|$.

R. GRAHAM:

Is there a family of graphs G(n) on n vertices, $n\to\infty$, with
(i) All 3-vertex graphs occur as induced subgraphs of G(n) (asymptotically) equally often, and
(ii) Some 4-vertex graph does not occur at all as an induced subgraph of G(n)?

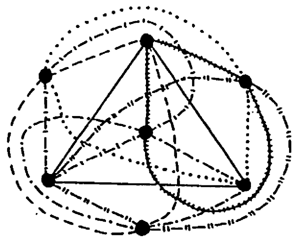
F. HARARY:

Let $\alpha(G)$ be the domination number of G and $\alpha_\iota(G)$ be the independent domination number. "What is the difference between α and α_ι of a hypercube" is the title of a note with M. Livingston submitted to the American Math. Monthly. Except for $n=5$ where we found for Q_n that $7=\alpha<\alpha_\iota=8$, we saw for $n\leq 8$ that $\alpha=\alpha_\iota$. Also for two infinite subfamilies of Q_n clustered around $n=2^k$ it was proved in the coding theory literature that $\alpha=\alpha_\iota$. The question: Is there a value of $n\neq 5$ with $\alpha<\alpha_\iota$?

H. HARBORTH:

1. Does there exist a drawing of K_9 in the plane (two edges intersect at most once, and two adjacent edges don't intersect) such that the edge set can be partitioned into triangles where at each vertex of each triangle as many of the

remaining edges are inside as are outside of the triangle? For K_7 an example is known.



2. Does a drawing of the cycle graph C_{10} exist such that every edge has exactly 6 crossings?

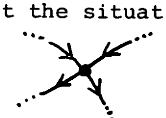
3. Does a planar graph exist which cannot be drawn in the plane with straight line edges of integer length?

A. HUCK:

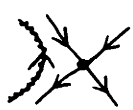
Conjecture: Let $n \geq 5$ be an odd number and G be an n-regular n-edge-connected graph (finite, undirected, without loops, multiple edges possible). Then there exists a perfect matching M E(G) so that G-M is (n-1)-edge-connected. (This conjecture is not true if n=3; counterexample: Petersen graph.)

F. JAEGER:

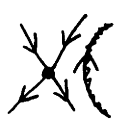
Problem: Let G be a 4-regular connected finite plane graph (loops and multiple edges are allowed) which is directed in such a way that the situation near each vertex is as shown below:

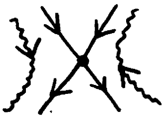
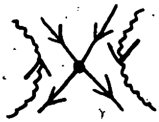


Let us fix a special edge e. An eulerian undirected cycle C of G is conect if it does not cross itself and , when we travel along C starting from e in its specified direction, the first passage through a vertex is never of the following types:



or



We denote by t(C) the number of vertices where C behaves like this:  or 

One can show using methods from Knot Theory that the number of directed eulerian circuits of G equals $\Sigma 2^{t(C)}$ (sum over all conect eulerian undirected cycles C).
Find a bijective proof.

O. R. OELLERMANN:
Let S be a nonempty subset of the vertices of a connected graph G on $p \geq 2$ vertices. Then the Steiner distance $d(S)$ of S is the minimum number of edges in a connected subgraph of G that contains S. For any integer n, $2 \leq n \leq p$, the n-eccentricity of v is defined as $e_n(v)=\max\{d(S)\,|\,v \in S,\ S \subseteq V(G)$ and $|S|=n\}$. Further, the n-radius of G is given by $\mathrm{rad}_n G = \min\{e_n(v)\,|\,v \in V(G)\}$ and the n-diameter of G is defined by $\mathrm{diam}_n G = \max\{e_n(v)\,|\,v \in V(G)\}$. It is known that for n=3 and 4, $\mathrm{diam}_n G \leq 2(n+1)/(2n-1)\,\mathrm{rad}_n G$ and it is conjectured that this inequality holds for all n, $3 \leq n \leq ?$.

H. Sachs:
An n-packing B is a finite collection of unit balls in n-dimensional space where any two balls of B are allowed to touch but not to overlap (i.e., the interiors of any two balls of B are disjoint). Let χ_n denote the minimum number of colours that suffice for colouring the balls in any n-packing B such that any two balls of B which touch must have different colours. It is known that $\chi_2=4$ and $5 \leq \chi_3 \leq 10$. What is χ_n? In particular, what is χ_3?

Z. TUZA:
1. (The dual of Turán's problem) Let F be an arbitrary (fixed) graph. Define sat(n,F) as the minimum number of edges in an F-saturated graph G on n vertices, i.e. a graph G that does not contain F as a subgraph, but adding any new edge to G a subgraph isomorphic to F occurs. - Conjecture:

For every F there is a constant $c=c(F)$ such that $sat(n,F) = cn+o(n)$ as $n\to\infty$.

2. (Hamiltonian circuits in digraphs) A directed graph K is semi-complete if it contains no pair of independent vertices. An arc xy of K is called a double arc if yx also is an arc in K, and is a single arc if yx is not an arc in K. The digraph is strong if for each ordered pair x,y of vertices there is a directed path from x to y. - Conjecture: Let K be a semi-complete digraph and t a natural number. If for every set Y of less than t vertices the subgraph of K induced by the vertices not belonging to Y is strong and contains at least one single arc, then K has a Hamiltonian circuit that contains at least t single arcs.

Berichterstatter: H. Harborth

Tagungsteilnehmer

Prof.Dr. Martin Aigner
Institut für Mathematik II
der Freien Universität Berlin
Arnimallee 3

1000 Berlin 33

Prof.Dr. Bela Bollobas
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Prof.Dr. Dan Archdeacon
Dept. of Mathematics
University of Vermont
16, Colchester Ave.

Burlington , VT 05405
USA

Prof.Dr. J. Adrian Bondy
Department of Combinatorics and
Optimization
University of Waterloo

Waterloo, Ontario N2L 3G1
CANADA

Prof.Dr. Jorgen Bang-Jensen
Dept. of Computer Science
University of Copenhagen
Universitetsparken 1

DK-2100 Copenhagen

Dr. Fan R. K. Chung
Bell Communications Research Center
435, South Street

Morristown , NJ 07960-1910
USA

Prof.Dr. Claude Berge
Ecole des Hautes Etudes en Sciences
Sociales, Centre d'Analyse et de
Mathematiques Sociales
54, Boulevard Raspail

F-75270 Paris Cedex 06

Prof.Dr. Walter Deuber
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Rainer Bodendiek
Mathematisches Institut
Pädagogische Hochschule Kiel
Olshausenstr. 75

2300 Kiel

Reinhard Diestel
Bodenstedtstr. 9

3000 Hannover 1

Prof.Dr. Yoshimi Egawa
Dept. of Applied Mathematics
Science University of Tokyo
1-3 Kagurazaka, Shinjuku-ku

Tokyo 162
JAPAN

Prof.Dr. Heiko Harborth
Bienroder Weg 47

3300 Braunschweig

Prof.Dr. Paul Erdös
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15
P. O. Box 127

H-1053 Budapest

Prof.Dr. Nora A. Hartsfield
Dept. of Mathematics
Western Washington University

Bellingham , WA 98 225
USA

Prof.Dr. Ronald L. Graham
AT & T
Bell Laboratories
600 Mountain Avenue

Murray Hill , NJ 07974-2070
USA

Andreas Huck
Institut für Mathematik
der Universität Hannover
Welfengarten 1

3000 Hannover 1

Prof.Dr. Rudolf Halin
Mathematisches Seminar
der Universität Hamburg
Bundesstr. 55

2000 Hamburg 13

Prof.Dr. Alexander A. Ivanov
Institute for System Studies
Academy of Science of the USSR
9, Prospect 60 Let Oktyabrya

117 312 Moscow
USSR

Prof.Dr. Frank Harary
Dept. of Computer Science
New Mexico State University
Box 3CU

Las Cruces , NM 88003
USA

Prof.Dr. Bradley W. Jackson
Dept. of Mathematics
University of California

Santa Cruz , CA 95064
USA

Prof.Dr. Francois Jaeger
Laboratoire de Structures Discretes
Institut IMAG
Universite Joseph Fourier
Boite Postale 53x

F-38041 Grenoble Cedex

Prof.Dr. Michael D. Plummer
Dept. of Mathematics
Vanderbilt University
P.O. Box 6213 - B

Nashville , TN 37235
USA

Prof.Dr. Dieter Jungnickel
Mathematisches Institut
der Universität Giessen
Arndtstr. 2

6300 Gießen

Dr. Erich Prisner
Mathematisches Seminar
der Universität Hamburg
Bundesstr. 55

2000 Hamburg 13

Prof.Dr. Alexander K. Kelmans
Profsouznaya 130-3-33

117321 Moscow
USSR

Prof.Dr. Hans Jürgen Prömel
Institut für Diskrete Mathematik
Universität Bonn
Nassestr. 2

5300 Bonn

Prof.Dr. Rolf Hermann Möhring
Fachbereich Mathematik - FB 3
Technische Universität Berlin
Straße des 17. Juni 136

1000 Berlin 12

Prof.Dr. Gerhard Ringel
Dept. of Mathematics
University of California

Santa Cruz , CA 95064
USA

Prof.Dr. Ortrud Oellermann
Dept. of Mathematics and Computer
Science
University of Natal
King George V Avenue

Durban 4001
SOUTH AFRICA

Prof.Dr. Horst Sachs
Technische Hochschule Ilmenau
Postfach 327

O - 6300 Ilmenau

Prof.Dr. Alexander Schrijver
Center for Mathematics and
Computer Science
P. O. Box 4079

NL-1009 AB Amsterdam

Dr. Bernd Voigt
Fakultät für Mathematik
der Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Paul D. Seymour
Bellcore
445, South Street

Morristown , NJ 07960-1910
USA

Prof.Dr. Lutz Volkmann
Lehrstuhl II für Mathematik
(für Ingenieure)
der RWTH Aachen
Templergraben 55

5100 Aachen

Prof.Dr. Vera Sos
Mathematical Institute of the
Hungarian Academy of Sciences
Realtanoda u. 13 - 15
P. O. Box 127

H-1053 Budapest

Prof.Dr. Heinz-Jürgen Voss
Sektion Mathematik
Pädagogische Hochschule Dresden
Wigardstr. 17, PSF 365

O -8060 Dresden

Prof.Dr. Gottfried Tinhofer
Mathematisches Institut
der TU München
PF 20 24 20, Arcisstr. 21

8000 München 2

Prof.Dr. Wolfgang Woess
Dipartimento di Matematica
Universita di Milano
Via C. Saldini, 50

I-20133 Milano

Prof.Dr. Zsolt Tuza
Computer and Automation Institute
Hungarian Academy of Sciences
P. O. Box 63

H-1502 Budapest