MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Problems in Nonlinear Elasticity

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This conference was organized by J. E. Marsden (Berkeley) and K. Kirchgässner (Stuttgart). The main topics treated were

- Nonlinear stability, bifurcation, and rotating systems with relative equilibria.
- Reduction of higher–dimensional systems to lower–dimensional ones such as rods, shells and coupled structures. Homogenization methods.
- Existence questions in hyper– and visco–elasticity. Phase transitions in crystals. Well–posedness in the case of Coulomb friction.
- Gauge theories, Yang–Mills models of dislocations.

Invited lectures where given by S. Antman, M. Golubitsky, T. Healey, R. James, P. Krishnaprasad, E. Kröner, H. LeDret, A. Mielke, S. Müller, F. Murat, O. Oleinik, J. Simo, I. Volovich, C. Wang.

Abstracts

S.S. Antman (College Park):
Dynamical Problems and Dissipative Mechanisms

Consider the large motion of an elastic rod with a tip mass. Typically the motion is governed by a quasilinear hyperbolic system, which can admit shocks. An inkling of the complications that can arise is given by studying problems in which the initial state is a higher buckled mode of the rod. The goals of this lecture are to illuminate complicated snapping motions of such rods, to indicate the concepts that might be useful in a complete resolution of the problem, and to emphasize the role of dissipative motions. The main steps in this lecture are:

i) A brief account is given of the work of Antman & Malek–Madani on large shearing motions of nonlinearly viscoelastic bodies. The shock structure for this problem with very general constitutive functions differs significantly from that found for dissipative mechanisms inspired by gas dynamics.

ii) The ODE $\ddot{w}+\varphi'(w) = 0$ is analyzed for potentials with $\varphi(0) < \infty$ and $\varphi'(0) = -\infty$. This equation has a "shock" when $w = 0$. The shock structure can be resolved (in a surprising way) by adding a dissipative term of the form $\nu g(w, \dot{w})$ and studying the

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nonuniform limit as $\nu \to 0$. Here $g(w, \dot{w})\dot{w} > 0$ for $\dot{w} \neq 0$ and there is a ψ such that $g(w, \dot{w}) \leq \psi'(w)\dot{w}$ for $w < 1$, $\dot{w} < 0$.

iii) Using comparable restrictions, Antman & Seidman have proved the global solvability of initial boundary–value problems for $w_{tt} = \sigma(w_s, w_{st})_s$. The essential difficulty is to get a pointwise positive lower bound for the strain w_s.

iv) The motion of a tip mass on a nonlinearly viscoelastic spring with small mass is governed by $\varepsilon \rho w_{tt} = \sigma(w_s, w_{st})_s$, $w(0,t) = 0$, and $m w_{tt}(1,t) + \sigma(w_s(1,t), w_{st}(1,t)) = 0$. It is shown that the leading term (with $\varepsilon = 0$) typically is not governed by an ODE for the position $w(1,t)$ of the tip mass. A rigorous asymptotic analysis is based on step (iii) and on novel a priori estimates.

v) The corresponding leading term for the rod problem considered in the beginning leads to an ODE for the position $\mathbf{r}(t)$ of the end mass of the form $\ddot{\mathbf{r}}(t) + \mathbf{f}(\mathbf{r}(t))$, where $\mathbf{f}$ is multivalued. It appears that the motion is such that an amplitude of a solution can jump at the folds of a corresponding multivalued surface in ($\mathbf{r}$,amplitude)–space. These jumps are presumed to describe snap–buckling and shocks.

J. BAILLIEUL (BOSTON):
Global Effects of Constrained Relative Motions in Rotational Mechanics

How do constraints on the relative motions of components in a rotating elastic, mixed, or multibody structure affect the global dynamics? We answer this question in the context of some specific systems that have been studied in the recent literature by Bloch [1987], Krishnaprasad and Marsden [1987], and Baillieul and Levi [1987]. It is noted that various simplifying assumptions in modeling the dynamics of elastic beams imply that the beam is rigid in certain ways. In a number of cases, such assumptions predict features in both the equilibrium and dynamic behavior which are qualitatively different from what is seen if the assumptions are relaxed. Our remarks are focussed on several planar beam models which differ from one another in terms of various strains being constrained to be zero. We describe strikingly dissimilar features in the steady–state rotations of planar body–beam systems which depend on which of the proposed beam models is involved.

A. BLOCH (COLUMBUS):
Stabilization and Control of Rigid Body Dynamics by Internal Torques

(Joint work with P.S. Krishnaprasad & J.E. Marsden & G. Sanchez de Alvarez)
We discuss the problem of stabilizing the angular momentum equations for the rigid body by means of internally driven rotors. Such a system is a model for certain kinds of satellites. Since the torques are internal, angular momentum is preserved for this system, but we show moreover that for certain feedback laws this system is Hamiltonian (more precisely Lie–Poisson on the Lie algebra $so(3)$) despite the presence of torques. We are then able to use the energy–Casimir method for stability, to show that the body may be stabilized about its intermediate (unstable) axis, by sufficiently large feedback in a

single rotor about the major or minor axis.

We also present an extension of a result of Montgomery which enables us to give a formula for the attitude drift that occurs for the rigid body–rotor system when it is perturbed a small amount from a stable equilibrium. This drift is a phase shift in the sense of Berry and Harnay. We also give a method for compensating for this drift.

D. CIORANESCU (PARIS):
Exact Boundary Controllability in Perforated Domains

(Joint work with P. Donato and E. Zuazua)
We consider the wave equation in a perturbed domain Ω_ε of $I\!R^3$ ($\varepsilon \to 0$):

$$y_\varepsilon'' - \Delta y_\varepsilon = 0 \text{ in } \Omega_\varepsilon \times (0,T),\ y_\varepsilon = v_\varepsilon \text{ on } \partial\Omega_\varepsilon \times (0,T),\ y_\varepsilon(0) = y_\varepsilon^0,\ y_\varepsilon'(0) = y_\varepsilon^1 \text{ in } \Omega_\varepsilon.$$

We would like to answer the following questions: may we construct some controls depending continuosly on ε? Is there a uniform exact controllability time?

A special kind of pertubated domains are periodically perforated domains with small holes. More precisely Ω_ε is obtained by removing from Ω (bounded set) a set of periodically distributed balls of radius r_ε.

We construct here an exact control v_ε (i.e. such that for $T > T_0$, $y(T) = y'(T) = 0$) where T_0 depends only on a diameter of Ω. We show that if $r_\varepsilon = \varepsilon^3$, $\tilde{y}_\varepsilon \rightharpoonup y$ ($\tilde{}$ denotes the extension by zero on the whole of Ω) in $L^\infty(0,T;L^2(\Omega))$ weakly$*$ where y is solution of

$$y'' - \Delta y + \mu y = \mu F \text{ in } \Omega \times (0,T),\ y = V \text{ on } \partial\Omega \times (0,T),\ y(0) = y^0,\ y'(0) = y^1$$

where F and V are some well defined functions. Moreover $y(T) = y'(T) = 0$. Here μ is a positive constant. In the limit problem we have an internal control μF and a boundary Dirichlet control V. If $r_\varepsilon \ll \varepsilon^3$, $\mu \equiv 0$ so that, in the limit, we have a problem controlled only by a boundary control.

The result extend to any dimension of the space.

G. FRANCFORT (PARIS):
Effective Behavior of Mixtures of Isotropic Elastic Materials with Essentially Constant Shear Moduli

(Joint work with L. Tartar)
The effective behavior of mixtures of isotropic elastic materials is investigated in the framework of homogenization theory. Specifically it is assumed that a sequence of elastic tensors of the form

$$A^\varepsilon_{ijkh}(x) = \lambda^\varepsilon(x)\delta_{ij}\delta_{kh} + \mu^\varepsilon(x)(\delta_{ik}\delta_{jh} + \delta_{ih}\delta_{jk})$$

with $0 < \alpha \le N\lambda^\varepsilon(x) + 2\mu^\varepsilon(x) \le \beta < \infty$ (N space dimension), $0 < \alpha \le \mu^\varepsilon(x) \le \beta < \infty$, and $\mu^\varepsilon(x) \to \mu(x)$ almost everywhere, converges in the sense of homogenization (H–convergence) to an elasticity tensor $A(x)$. It is shown that $A(x)$ is isotropic with Lamé "constants" $\lambda(x)$ and $\mu(x)$, where $\mu(x)$ is the a.e. limit of $\mu^\epsilon(x)$ and $\lambda(x)$ is given by

$$\frac{1}{\lambda(x)+2\mu(x)} = \text{weak* } L_\infty\text{–limit of } \frac{1}{\lambda^\epsilon(x)+2\mu^\epsilon(x)}.$$

The above result allows for a trivial proof of Hashin and Shtrikman's bounds on the bulk modulus of (isotropic) mixtures of two isotropic elastic materials at fixed volume fraction of each material. Appropriate generalizations are also obtained.

J. GAWINECKI (WARZAW):
The Initial–Value Problem in Nonlinear Hyperelasticity

(Joint work with A. Piskorek and D.D. Hung)
We consider the initial value problem for the nonlinear partial differential equations describing the motion of inhomogeneous and anisotropic hyperelastic medium in three–dimensional space, of the following form:

$$(*) \qquad \rho\partial_t^2 u_j = \partial_k p_{jk} + \rho f_j,\ p_{jk} = (\delta_{jl} + \partial_l u_j)\frac{\partial\sigma}{\partial e_{kl}}, \quad j,k,l = 1,2,3,$$

where p_{jk} are the components of Piola Kirchhoff's first stress tensor, σ the stored energy function, which is a function of the point (x_1, x_2, x_3) and Green–St.Venant strain tensor e, and belongs to the class C^∞ with respect to x and e.

Our aim is to prove the local (in time) existence and the uniqueness of smooth solution to the problem $(*)$ with initial values $u(0,x) = u^0(x)$ and $\partial_t u(0,x) = u^1(x)$. In order to do so, at first we transform the problem to an equivalent initial value problem for a quasilinear symmetric hyperbolic system of first order, using the modified Sommerfeld's method. Next applying the methods of S. Klainerman and A. Majda we get the local (in time) existence and uniqueness theorems for the initial value problem.

M. GOLUBITSKY (HOUSTON):
Boundary Conditions of Symmetry Constraints

Reaction diffusion equations with Neumann boundary conditions (NBC) undergo "non-generic" bifurcations. This bifurcation can be understood by embedding NBC in periodic boundary conditions (PBC). This observation provides an explanation for the existence of anamolous Tailor vortices appearing in the Couette–Tailor experiment.

The source of non–genericity is that the PDE can be extended to a larger domain on which it has greater symmetries. In joint work with M. Field and I.N. Stewart we find solutions to reaction diffusion equations on a hemisphere satisfying NBC on the equator by first extending the equations to the 2–sphere and using known results about bifurcation with $O(3)$–symmetry.

T. HEALEY (ITHACA) & H. KIELHÖFER (AUGSBURG):
Symmetry and Nodal Properties in Global Bifurcation Analysis of Quasi–Linear Elliptic Equations

We present a generalization of a result due to Crandall and Rabinowitz concerning global separatation and unboundedness of bifurcation solution branches of nonlinear Sturm–Liouville problems. Specifically we consider quasi–linear equations of the form

$$(*)\qquad a_{ij}(\nabla u, u)u_{x_i x_j} + b_i(\nabla u, u)u_{x_i} + f(\nabla u, u, \lambda) = 0,$$

on rectangular domains Ω in $\mathbb{R}^2$. Motivated by nonlinear diffusion problems and problems of anti–plane shear in nonlinear elasticity, we impose orthotropic material symmetry for $(*)$. We consider a broad class of homogeneous boundary conditions corresponding to any combination of Dirichlet and nonlinear Neumann (zero–flux) conditions of the form $q_i(\nabla u, u)n_i = 0$ along the straight edges of Ω.

The basic difficulty in extending the work of Crandall and Rabinowitz to the problem at hand is the lack of anything like Sturm–Liouville's theory (insuring nodal properties) for elliptic PDEs. However, our problem has hidden symmetry. We first extend the domain to all of $\mathbb{R}^2$ and work in a space of doubly periodic functions, which yields a problem equivariant under the action of a representation of $O(2) \times O(2) \times \mathbb{Z}_2$. We show that the exploitation of equivariance yields simple bifurcation problems in various fixed–point subspaces of the function space, the solutions of which satisfy the given boundary conditions. Hence, we can readily establish the existence of global solution branches, each having a distinct symmetry and, in particular, a minimal nodal pattern identical to that of the eigenfunction of the linearized problem. That is, symmetry fixes the location of certain nodal lines. However it does not rule out the "birth" of other nodal sets along solution branches "far" from the trivial solution, which, in turn, implies that global solution branches may be connected and bounded. To show that this is not the case, we apply a subtle maximum principle (including a boundary lemma at corners). Hence, we conclude that each of the solution branches with symmetry is unbounded, globally separated from the others, and intersects the trivial solution only once.

G. HERRMAN (STANFORD):
On Conservation Laws for Dissipative Systems

(Joint work with T. Hohein and N. Chien)
The purpose of this contribution was, to advance and illustrate a procedure for constructing conservation laws (i.e. divergence–free expressions) for dissipative systems described by partial differential equations. Such expressions are most useful for a variety of reasons. Whereas for non–dissipative systems, which come from a variational principle, Noether's first theorem is available to establish conservation laws in a systematic fashion, no corresponding methodology appears to have existed and exploited for dissipative systems, since these might not be related to a variational principle and thus Noether's procedure becomes inapplicable.

In a non–dissipative system let L be a Lagrangian and $E(L) = 0$ the associated Euler–Lagrange differential equation. An associated conservation law may be expressed as $\mathrm{Div}\, P = \partial_i P^i = Q\, E(L) = 0$ where Q is the "characteristic" of the consrevation law. Now let a dissipative system be given by $\Delta(u) = 0$. We still set $f\Delta(u) = \partial_i P^i$ where now f is not pre–determined, but is to be formed from the above equation. Since $f\Delta(u)$ is required to be divergence–free, it has to be a null–Lagrangian. We set $\mathcal{L} = f\Delta(u)$ and require $\delta\mathcal{L} \equiv 0$. For example, if $\Delta(u) = u_t - uu_x = 0$, $\delta\mathcal{L} = 0$ leads to $f_t - uf_x = 0$ and to $P^x = -c[tu^{n+2}/(n+2) + xu^{n+1}/(n+1)]$ and $P^t = c[tu^{n+1}/(n+1) + xu^n/n]$ where n is any real number except -2, -1, or 0.

W. Hrusa (Pittsburgh):
On Smooth Solutions to Initial Value Problems in One–Dimensioanl Nonlinear Thermoelasticity

We consider various initial and initial–boundary value problems for one–dimensional nonlinear thermo–elastic bodies. We discuss results concerning global (in time) existence of smooth solutions for initial data that are sufficiently close to equilibrium. We also discuss results concerning the formation of singularities in finite time for initial data with steep gradients.

R.D. James (Minneapolis):
Domain Structures of Magnetic Materials

(Joint work with D. Kinderlehrer)
We consider the case of a rigid magnet and examine the theory of *micromagnetics* due to W.F. Brown with exchange energy omitted. This leads to the problem

$$\min_{\substack{|m|=1 \\ m\in L^\infty}} \int_\Omega \varphi(m(x))\, dx + \frac{1}{2}\int_{R^3} |\nabla u|^2 dx \quad \text{subject to} \quad \mathrm{div}(-\nabla u + m\chi_\Omega) = 0.$$

For a uniaxial material $\varphi(\pm m_1) < \varphi(m)\ \forall m \neq \pm m_1$ while for a cubic material $\varphi(\pm m_1) = \varphi(\pm m_2) = \varphi(\pm m_3) < \varphi(m)\ \forall m \neq \pm m_i$, where $\{m_i\}$ is an orthonormal basis. We show that the minimum is not attained in the uniaxial case and that it is attained in the cubic case. Minimizing sequences in the uniaxial case consist of laminar or columnar domains which become finer and finer as the energy is reduced. However, all minimizing sequences in the uniaxial case are essentially the same insofar as macroscopic properties are concerned in that they all have the same Young measure. In the cubic case any minimizer may have large domains in the interior of Ω but must exhibit domain refinement near $\partial\Omega$ if the normal is not $\perp$ to m_1 or m_2 or m_3.

In the second half of the talk we give a new theory of magnetostriction and apply the theory to $Tb_xDy_{1-x}Fe_2$. We show that some minimizing sequences match domain patterns observerd by Donald Long. Further confirmation awaits a study of arbitrary minimizing sequences.

B. Kawohl (Heidelberg):
An Optimal Design Problem

(Joint work with J. Stara and G. Wittum)
Let Ω be the cross–section of a cylindrical bar. Given two different elastic materials and their proportions, one tries to find the distribution of materials in Ω which renders optimal torsional rigidity. This variational problem has no classical solution unless Ω is a circle (or ball in $\mathbb{R}^n$). In the latter case one puts the "stronger" material in an annulus and the "softer" material inside the annulus. There are, however, generalized solutions which display homogenized regions. It is shown, that there is uniqueness for the generalized solution (open problem posed by Murat and Tartar). Moreover, the talk contains numerical results for the case that Ω is in a square.

H. Kielhöfer (Augsburg): see T. Healey

P.S. Krishnaprasad (College Park):
Geometric Phases and Optimal Control [Kinematic Drifts in the Presence of Flexible Attachments]

Relative motion in a system of coupled rigid bodies can yield global reorientation (or phase shift). We give formulas to compute such phase shifts and interpret them in geometric terms. The theory of connections in principal bundles provides the appropriate framework. A natural mechanical connection known to Smale (and Kummer and others) plays an important role. Detailed knowledge of curvature in examples provides sufficient information to plan relative motion with prescribed holonomy.

The problem of optimal relative motions with prescribed holonomy is of great interest from the point of view of control theory. In the present context, solution to the optimal control problem induce a singular / sub–Riemannian geometry on the configuration space (Brockett[1981], Strichartz[1986], and others). *Infinitesimal versions* of optimal control problem lead to certain interesting normal forms (for instance geodesics of a singular Riemann metric on a nilpotent group). We show explicit integrability of the geodesic equations in a 4–dim nilpotent algebra using elliptic functions.

A very interesting mechanical problem is to understand the possibility of *secular drifts* (kinematic drifts) induced in a rigid body by the presence of resonant modes in *flexible attachments*. We take up a model problem of a rigid body carrying 2 driven linear oscillators. We compute explicit drift rates at 1:2 resonance and show that the associated small deformation optimal control problem corresponds to the nilpotent normal form (planar case):

$$(*) \qquad \dot{\xi}_1 = u_1,\ \dot{\xi}_2 = u_2,\ \dot{z} = \xi_1^2 u_2 - \xi_2^2 u_1;$$

$$\min_{u_1,u_2} \int_0^T (u_1^2 + u_2^2)dt \quad \text{subject to } (*),\ \xi_i(0),\ \xi_i(T) \text{ fixed,}$$
$$\text{and } z(T) - z(0) = \textstyle\int_0^T (\xi_1^2\dot{\xi}_2 - \xi_2^2\dot{\xi}_1).dt$$

This is the normal form on a 4–dim nilpotent group alluded to in the end of the second paragraph and is solvable by elliptic functions.

For the 3–dim rigid body more complicated normal forms appear.

E. KRÖNER (STUTTGART):

Field Theory of Defects in Ordered Structures

Defects in ordered structures, such as solid and liquid crystals, spin and (certain) polymer structures, can be understood as singularities of stress and srtain fields (in a general sense). They surround themselves with these fields, move around and interact through the fields. We distinguish between a "classical" and "quantum" approach. The classical approach is obtained from the quantum approach (not considered here) by a limiting process which yields the so–called "continuized crystal" (in the illustrative example of a crystalline medium). The continuized crystal preserves the two essential characteristics of the crystal: (i) crystallographic directions exist at each point and (ii) length measurement is possible by counting lattice steps along atomic rows. The analogy between the universe filled with moving elementary particles and the crystal world with moving elementary defects is emphasized. For illustration we take the Bravais crystal.

Once the ordered structure is specified, e.g. by an order parameter field, it is possible to find the type of possible elementary defects. In Bravais lattices these are the point defects vacancy, self–intensistial and slip–fault (newly introduced), the line defect dislocation (not the disclination) and the (not yet classified) surface defect. Since point defects and dislocations mutually convert, a theory which considers only dislocations is not closed, but must be combined with the theory of point defects. This is done in this lecture in the language of differential geometry of affinely connected spaces. This geometry has just the functional degrees of freedom to describe the Bravais crystal with its elementary point and line defects.

It has been shown by Kondo in 1952 that dislocations are the discrete version of the notion of torsion (the antisymmetric part of the connexion Γ) introduced into differential geometry by Cartan in 1922. It is proposed to understand the elementary point defects as defects of the crystal metric, because they obviously disturb the counting of lattice steps along atomic rows. Thus the distance between two points depends on the density and types of point defects in the space between them. Hence a metric tensor g is introduced which describes the densities of the point defects. Vacancy and self–intensistial are antidefects which can mutually annihilate. They are discribed by the isotropic part of g, wheras the deviator of g gives the density of the slip defects. Thus the differential geometry specified by the connexion Γ and the independent metric g, hence the "affine connexion" (g, Γ) combines the geometric theory of dislocations and point defects in Bravais crystals.

Accepting the standpoint of an internal observer who finds the defects, but not the strains resulting from outside deformation of the crystal, it is possible, also in the fully non–linear theory, to understand the equilibrum conditions for forces and moments as Bianchi identities of a so–called stress space, in which Beltrami's stress function tensor plays the role of a metric (subjected to some gauge condition) and the two stress tensors (force and moment) are Einstein and torsion tensor, respectively. This stress space has to be united with the geometrical space, now called strain space, in a manner which is not yet understood, The great problem in the development of a dynamical theory is that any motion of a defects is always dissipative. This is also the obstacle in the application of gauge concepts to defect theory. It is clear, however, that only the existence of dislocations allows the new freedom of mobility which makes a crystal plastic, because translations forbidden in elastic media become possible. Such translations through the Burger's vector do not change the internal state of the crystal, so that the potential energy is invariant with respect to these local translations. Therefore, as far as the gauge concept is applicable, dislocation theory is a translational gauge theory.

M. LANZA DE CRISTOFORIS (PADOVA):
Large Deformation of Structures in Fluids

(Joint work with S. Antman)
We consider the large deformation of a nonlinearly elastic panel produced by the action of a cavitating steady, irrotational 2–dimensional flow of an incompressible inviscid fluid. The elastic body is modelled by a rod which can suffer stretching, bending and shearing. The deformed shape of the panel and two parameters, namely the prescribed velocity V and pressure P at infinity unique determine the cavitational flow. The flow determines a pressure field "p" on the boundary of the panel. We substitute p onto the equilibrum equations and obtain a system of functional equations in the unknown configuration of the body. We then construct an equivalent system which can be transformed into a fixed–point form. We then study the fixed–point problem by means of a continuation method and deduce an alternative theorem for the behaviour of the branch of solutions containing the undeformed reference configuration. Similar strategies in different mathematical settings have been adopted to study other fluid–solid interaction problems. Our basic objective is the development of effective mathematical tools to treat nonlinear fluid–solid interactions when the solid suffers large deformation and study the behaviour of nonlinearly elastic structures under a class of natural loading of nonlocal nature.

H. LE DRET (PARIS):
Elastodynamics for Structures with Junctions

This lecture is concerned with the dynamics of elastic structures that comprise parts (3d, plates, rods) coupled through junctions. Our main problem is the modeling of such structures starting from three–dimensional elasticity, using a limiting process as the thickness of the thin parts goes to 0. This modeling is achieved in the following fashion: First, rescale thin parts so as to define domains that do not depend on the thickness. This rescaling must be performed in such a way that the junction regions are counted twice, once in each rescaled domain to which they pertain. Thus, the rescaled solutions, now defined on separate domains, must satisfy a set of compatibility conditions in the rescaled images of the junction regions. These relations in turn yield the junction conditions to be satisfied by the solutions of the limit model. After rescaling, the limiting process is fairly straight forward. Existence of a weakly convergent subsequence (in the appropriate topology) of rescaled solutions is obtained via energy estimates. Then, as explained above, we pass to the limit in the compatibility conditions to obtain the limit junction conditions. By use of appropriate test–functions, the limit equations are then identified, as well as the limit initial data. The process is completed by showing that the limit model is well- posed.

In the case of a two–plate structure, the resulting model is a coupled 2d–2d model. Classical dynamical plate equations are coupled through the common edge of the two plates. The motion of the junction is rigid. The angle between the two plates stays constant during the motion.

As the method we propose is quite general, we have chosen for the sake of simplicity to exemplify it in the case of the wave equation in a two–dimensional L–shaped domain.

M. Levi (Boston):

A Geometric Integration Formula on $SO(3)$ and its Applications in Mechanics

In this talk I derive a geometrical formula for the curve $X(t)$ in $SO(3)$ given by the matrix ODE $\dot{X} = A(t)X$. $A = -A^T$ is a real 3×3 matrix:

$$X(T) = P_{\mathcal{S}} \exp\left(\frac{A(0)}{|A(0)|}\int_0^T |A(t)|dt\right) P_{\mathcal{B}}^{-1},$$

where $|A|^2 = \mathrm{tr}(AA^T)$, $\mathcal{S}$ and $\mathcal{B}$ are two curves on S^2 defined below, and $P_{\mathcal{C}} \in SO(3)$, $\mathcal{C} = \mathcal{S}$ or $\mathcal{B}$, is the matrix of parallel transport along $\mathcal{C}$, extended to the whole of $I\!R^3$.

The curve $\mathcal{S}$ (for space) is traced out by the normalized angular velocity $\mathcal{S} = \omega/|\omega|$ where $\omega(t) \times a = A(t)a$ for all $a \in I\!R^3$, while the curve $\mathcal{B}$ (for body) is defined by its geodesic curvature $k_{\mathcal{B}}(t) = k_{\mathcal{S}}(t) - |\omega|/|\dot{\mathcal{S}}|$ and by the condition of tangency to $\mathcal{S}$ at $\mathcal{S}(0)$.

Here is one application:

If Σ is a convex rigid surface rolling along the xy–plane so that the point of contact traverses a closed path on Σ exactly once, and there is no sliding, then the new position of Σ is the result of a parallel translation along the xy–plane followed by the rotation around the z–axis through the angle $\int\int_D K dS + \int_0^T \omega_z/v\, dt (\mathrm{mod}) 2\pi$, where D is the region enclosed by the path of the contact point, ω_z is the z–component of the angular velocity, v is speed of the point of contact and K is the Gaussian curvature of Σ. As a particular case, this gives the "Berry phase" of the turn of a free rigid body as discussed in:

[1] R. Montgomery, "By how much does a rigid body rotate?"
[2] M. Levi, "A geometric phase for a free rigid body".
[3] P. Krishnaprasad, A. Bloch, "A geometric phase for a free rigid body with a rotor".

J.H. MADDOCKS (COLLEGE PARK):
Hamiltonian Dynamics of a Finite Rigid Body in a Central Force Field

(Joint work with P.S. Krishnaprasad & L.S. Wang)
Study of the relative equilibria of earth–pointing orbits of a satellite in an inverse–square gravitational field is a classic problem of rigid–body mechanics. In the usual approach the finite–extent of the satellite is accounted for by an expansion of the fact, that the characteristic dimension of the satellite is much smaller than the radius of the orbit. Then all but the lowest order non–zero terms are discarded from the force and moment balance equations. For the force equation the zeroth order term is retained, whilst in the moment balance the second order must be kept. The solutions to these approximate equations comprise circular orbits for which the orbit of the centre of mass of the satellite and the centre of the gravitational field are coplanar. There are actually 24 families of solutions, parameterized by the orbit radius, depending upon allowable orientation.

In this work I show that the governing equations can be written as a 9–dimensional degenerate noncanonical Hamiltonian system with a Casimir function or first–integral that is conserved independent of the form of the Hamiltonian. The Hamiltonian structure persists for any level of approximation of the gravitational potential, including the exact equations, and a consistent hierarchy of models can therefore be constructed. The first provides detailed stability and instability information. The other can be used to prove existence of solutions. If the body has three planes of symmetry then the exact equations have 24 families of solutions in which the centre of mass orbit and the centre of the potential are coplanar. This is in complete agreement with the predictions of the approximate models. However it is shown that for an asymmetric body there are non–great circle orbits, in which the plane of the centre of mass orbit and the centre of potential are offset. The size of the offset is necessarily tiny, but there is numerical evidence that there can be large deviations from the allowable orientations as predicted by the approximate models. Thus the validity of the classic analysis for asymmetric bodies is called into question.

A. MIELKE (STUTTGART):
A Hamiltonian Approach to Saint–Venant's Problem for Nonlinearly Elastic Beams

We consider the elastostatic deformations of an infinitely long cylindrical body with the restriction of uniformly bounded strains. It is shown, if the bound on the strains is sufficiently small, that all the possible deformations lie on a twelve–dimensional manifold. The solutions on this manifold can be described by a differential equation having exactly the form of the rod equations of Kirchhoff [1859] and Antman [1972]. Thus we obtain a rigorous method to derive a fully nonlinear rod model from 3–d elasticity.

A natural question arising in this context is whether, starting with a hyperelastic 3–d material, we obtain a hyperelastic rod model. This question can be answered affirmative by using the Hamiltonian approach. Therefore the 3–d problem, having a variational structure, is understood as a Hamiltonian system. This system can be reduced in a natural way onto the 12–dimensional center manifold. The reduced Hamiltonian system then generates an associated reduced variational problem.

R. MONTGOMERY (BERKELEY):
Gauge Theory and the Falling Cat

A deformable body can re–orient itself through a series of shape changes; for example, a cat, dropped with zero angular momentum, is able to land on her feet. We show how to translate this problem in mechanics & control, namely how to achieve the re–orientation, into the following problem in gauge theory & geometry: find a loop with a given holonomy. The total space of the principal bundle is the configuration space Q, consisting of all inertial configurations of the deformable body. The structure group is the rotation group $SO(3)$. The base space is the space $S = Q/SO(3)$ of shapes. The re–orientation is the holonomy of a special connection on this bundle, namely the connection whose horizontal vectors are *zero angular momentum* deformations. This is also the connection defined by declaring 'horizontal' to mean orthogonal (w.r.t. the kinetic energy metric) to vertical, where 'vertical' means infinitesimal rigid rotations. These facts were observed by Guichardet in around 1984, and later by Wilczek & Shapere. In this talk we added new entries to this dictionary between mechanics & control theory on the one hand, and Riemmannian geometry & gauge theory on the other. Notably we partially solved (in the sense of reduction to an ODE) the problem of optimal control and optimal feedback control for the deformable body trying to achieve a desired re–orientation. As cost function we use the length of the loop in the shape space, where length is measured by the naturally induced metric on S from that of Q.

S. MÜLLER (PITTSBURGH):
Singular Pertubation Techniques for Fine Phase Mixtures

Mathematical models for phase transitions based on minimization of the free energy typically lead to ill–behaved variational problems. Singular pertubation techniques can be used to obtain a better behaved limiting problem.

Consider for example for $v : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ the problem:

$$\text{Minimize} \quad \int_\Omega f(v)dx \quad \text{subject to} \quad \frac{1}{|\Omega|}\int_\Omega v\,dx = m,$$

where $f(\alpha) = f(\beta) = 0$ and $f > 0$ else. The minima are very degenerate as every function v, with $v = \alpha$ on Ω_α, $v = \beta$ on $\Omega - \Omega_\alpha$ with $\text{meas}\Omega_\alpha = (\beta - m)/(\beta - \alpha)\text{meas}\Omega$ yields a minimum. As shown by various authors a selection criterium for these minimizer can be obtained by considering minimizers of $\int_\Omega f(v) + \varepsilon^2|\nabla v|^2 dx$. As $\varepsilon \to 0$ one obtains solutions of the original problem with the additional property that $\text{area}(\partial\Omega_\alpha \cap \Omega)$ is minimal.

In the talk some of the difficulties of adapting this approach to solid–solid phase transitions (à la Ball–James) in elastic crystals are described and model problems are discussed. Specifically it is shown that minimizers of

$$I^\varepsilon(u) = \int_0^1 \varepsilon^2 u_{xx}^2 + (u_x^2 - 1)^2 + u^2 dx \quad + \text{ periodic bd. conditions}$$

are (nearly) periodic. Moreover evidence is presented that minimizers of

$$\int_0^1 \int_0^1 (u_y^2 - 1)^2 + u_x^2 + \varepsilon^2 u_{yy}^2 + \varepsilon^2 u_{xx}^2 dx\, dy \quad \text{with } u(0,y) = 0 \text{ for } y \in (0,1)$$

develop a self–similar structure as $\varepsilon \to 0$.

FRANCOIS MURAT (PARIS):
Corrector For the Homogenization of the Wave Equation

(Joint work with S. Brahim–Otsmane and G.A. Francfort)
We consider the wave equation

$$(*) \begin{cases} \rho^\varepsilon \frac{\partial^2 u^\varepsilon}{\partial t^2} - \operatorname{div}(A^\varepsilon \nabla u^\varepsilon) = 0 \in \Omega \times (0,T), \\ u^\varepsilon = 0 \text{ on } \partial\omega \times (0,T),\ u^\varepsilon(0) = a^\varepsilon,\ \frac{\partial u^\varepsilon}{\partial t}(0) = b^\varepsilon \text{ on } \Omega. \end{cases}$$

Assuming that $\rho^\varepsilon(x) \geq \lambda_1 > 0$, $\rho^\varepsilon \rightharpoonup \overline{\rho}\, L^\infty(\Omega)\, w*$, $\|A^\varepsilon\|_{L^\infty} \leq C$, $A^\varepsilon(x) \geq \lambda_2 I$, A^ε converges to A^0 in the sense of homogenization, $a^\varepsilon \rightharpoonup a^0\ H_0^1(\Omega)\, w$ and $\rho^\varepsilon b^\varepsilon \rightharpoonup \overline{\rho} b^0\ L^2(\Omega)\, w$, we pass to the limit in $(*)$, obtaining the limit (homogenized) wave equation

$$(**) \begin{cases} \overline{\rho} \frac{\partial^2 u}{\partial t^2} - \operatorname{div}(A^0 \nabla u) = 0 \text{ in } \Omega \times (0,T), \\ u = 0 \text{ on } \partial\Omega \times (0,T),\ u(0) = a^0,\ \frac{\partial u}{\partial t}(0) = b^0 \text{ in } \Omega. \end{cases}$$

Note that the energy E^ε of $(*)$ defined by

$$E^\varepsilon = \frac{1}{2} \int_\Omega \left[\rho^\varepsilon \left| \frac{\partial u^\varepsilon}{\partial t} \right|^2 + (A^\varepsilon \nabla u^\varepsilon) \cdot \nabla u^\varepsilon \right] dx = \frac{1}{2} \int_\Omega \left[\rho^\varepsilon \left| b^\varepsilon \right|^2 + (A^\varepsilon \nabla a^\varepsilon) \cdot \nabla a^\varepsilon \right] dx$$

does not converge in general to the energy E^0 of $(**)$. This rules out the possibility to obtain a corrector result for u^ε. We thus partition u^ε in a sum of two terms: $u^\varepsilon = \tilde{u}^\varepsilon + v^\varepsilon$. The first term $\tilde{u}^\varepsilon$ solves the same wave equation as u^ε but with initial conditions $\tilde{a}^\varepsilon$ and $\tilde{b}^\varepsilon$ which are designed in a manner such that the corresponding energy $\tilde{E}^\varepsilon$ converges to E^0. A corrector result is thus obtained for $\tilde{u}^\varepsilon$, namely: $\frac{\partial(\tilde{u}^\varepsilon - u)}{\partial t} \to 0$ strongly in $C^0([0,T]; L^2(\Omega))$ and $\nabla \tilde{u}^\varepsilon - P^\varepsilon \nabla \tilde{u} \to 0$ strongly in $C^0([0,T]; (L^1(\Omega))^N)$ (and in $C^0([0,T]; (L^2(\Omega))^N)$ if u is sufficiently smooth).

As far as v^ε is concerned, we prove that v^ε tends to zero weakly$*$ in the energetic space. This convergence is strong if and only if $a^\varepsilon - \tilde{a}^\varepsilon$ and $b^\varepsilon - \tilde{b}^\varepsilon$ tend strongly to 0 in $H_0^1(\Omega)$ and $L^2(\Omega)$, respectively. If this is not the case, v^ε is a pertubation which tends weakly$*$ to zero but permeates all times if its energy is considered.

Y. NUTKU (ANKARA):
Bi–Hamiltonian Systems of Biology

The Kermack–McKendrick model of epidemics is given by a dynamical system which admits bi–Hamiltonian structure. Earlier (Phys. Lett A **145** (1990) 27) Lotka–Volterra equations were shown to admit a similar structure. I argued that generalized Poisson brackets need not result from the choice of problems with appropriate symmetries. They exist already in interesting models everywhere!

O. OLEINIK (MOSCOW):
Boundary–Value Problems for the Elasticity System in Unbounded Domains. Korn's Inequalities

Korn's inequalities are proved for starshaped domains with asymptotically sharp constants and also for domains which depend on parameters, like frames, lattices, towers, shells, junctions, deformed cylinders and others. See: 1) C.R. Acad Sci Paris, **308**, 1989. 2) Russian Math Survey, **44** No.6, 1989. 3) Russian Math Survey, **45** No.4, 1990.

The boundary–value problems in unbounded domains are considered in Russian Math Survey, **43** No. 5, 1988. Here, in Oberwolfach, I proved some theorems for elasticity. Two of them I formulate below. (*First publication*).

Theorem 1: Suppose that $u = (u_1, \dots, u_n)$ is a solution of the following boundary–value problem for the elasticity system in the domain $\Omega = K \setminus G$, where K is a cone and G is bounded such that Ω is an unbounded domain with the Lipschitz boundary,

$$\begin{aligned} \sum_{i,h,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}^{kh}(x)\frac{\partial u_h}{\partial x_j}\right) &= f_k, \quad k=1,\dots,n \quad \text{in } \Omega, \\ \sigma_k(u) = \sum_{i,h,j=1}^{n} a_{ij}^{kh}(x)\frac{\partial u_h}{\partial x_j}\nu_i &= 0, \quad k=1,\dots,n \quad \text{on } \partial\Omega \end{aligned}$$

where $a_{ij}^{kh}(x) = a_{ji}^{hk}(x) = a_{kj}^{ih}(x)$ and $\lambda_1|\eta|^2 \le \sum_1^n a_{ij}^{kh}(x)\eta_i^k\eta_j^h \le \lambda_2|\eta|^2$ for all η with $\eta_k^i = \eta_i^k$.

Then there exists a constant skew symmetric matrix A and a constant vector B such that

$$\int_\Omega |\nabla(u - Ax - B)|^2 dx \le C \int_\Omega |f|^2|x|^2 dx,$$

where the constant C does not depend on u. (This is a stability theorem.)

Theorem 2 (First Korn's inequality): Let Ω be a bounded domain and $u = (u_1, \dots, u_n) \in H^1(\Omega)^n$. Suppose that $\partial\Omega = \cup_{j=1}^N S_j$ and $u_1 = \dots = u_{j-1} = u_{j+1} = \dots = u_n = 0$ on S_j for $j = 1, \dots, N$. Then

$$\int_\Omega |\nabla u|^2 dx \le \frac{1}{2}E(u,\Omega) = \frac{1}{2}\int_\Omega \sum_{i,j=1}^{n}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 dx.$$

I am very grateful to the administration of the Institute in Oberwolfach for the wonderful conditions to work during the Conference.

N. OWEN (SHEFFIELD):
Some A–Priori Estimates in Nonlinear Elasticity

(Joint work with P. Baumann & D. Phillips)
We give several a priori estimates for smooth solutions to the elliptic system

$$(*)\qquad \Delta u + v_y H'(d)_x - v_x H'(d)_y = 0, \quad \Delta v - u_y H'(d)_x + u_x H'(d)_y = 0, \quad (x,y)\in\Omega,$$

which arises in nonlinear elasticity. Here, $\Omega \subset \mathbb{R}^2$ is open and bounded, $d = u_x v_y - u_y v_x$ and $H \in C^3(0,\infty)$ is convex, non–negative and behaves like d^{-s}, for some $s > 0$, as $d \to 0+$. Equation $(*)$ is the Euler–Lagrange equation for the energy $I(\chi) = \int_\Omega \sigma(D\chi(x,y))\,d(x,y)$, where $\chi = (u,v)$ is the deformation and $\sigma(F) = \frac{1}{2}|F|^2 + H(\det F)$ is the stored energy function for a compressible neo–Hookean material. We define a smooth solution of $(*)$ to be a diffeomorphism χ such that $\chi \in C^3(\Omega)\cap C^1(\overline{\Omega})$ with $\det D\chi > 0$.

To obtain estimates in the principal stretches $0 < v_1 \le v_2$ (i.e. eigenvalues of $[D\chi(D\chi)^T]^{1/2}$) we show that d and $z := \frac{1}{2}|D\chi|^2 + f(d)$, where $f(d) = dH'(d) - H(d)$, satisfy super– and sub–elliptic estimates. By the Maximum Principle, it follows that $\inf_\Omega \ge \inf_{\partial\Omega} d$ and $\sup_\Omega z \le \sup_{\partial\Omega} z$ which, in turn, implies $\sup_\Omega(1/v_1 + v_2) \le c(\inf_{\partial\Omega} v_1, \sup_{\partial\Omega} v_2)$.

A density argument using the Calderon–Zygmund inequality implies that

$$\|d^{-s}\|_{L^p(\Omega')} \le c_1(1 + I(\chi) + \|\,|D\chi|^2\|_{L^p(\Omega)})$$

for any $\Omega' \subset\subset \Omega$, for any $p \in (1,\infty)$, and $c_1 = c_1(\Omega', p)$.

Finally, using an application of the Aleksandrov Maximum Principle to local estimates for elliptic equations due to Trudinger, we deduce the interior estimate

$$\sup_{\Omega'}\left(\frac{1}{v_1} + v_2\right) \le c_2(\Omega', I(\chi), \|D\chi\|_{L^q(\Omega)}),$$

for any $\Omega' \subset\subset \Omega$ and $q = \max(16, 12 + 16/s)$.

D. PHILLIPS (WEST LAFAYETTE):
Equilibrium Equations for Elasticity

(Joint work with P. Baumann & N. Owen)
We consider a multiple integral of the form $W(U) = \int_\Omega (F(DU) + H(\det DU))dx$ with $U : \Omega \subset \mathbb{R}^n \to \mathbb{R}^n$, where F is smooth, quasiconvex, and $|P|^q \le F(P) \le c(1 + |P|^q)$ for $q > n$. H is convex, nonegative and smooth for $d > 0$, $H(d) = \infty$ for $d \le 0$ and $H(d) = d^{-s}$ as $d \to 0+$ for some $s > 0$.

Such an integral is well–defined for variations of the form $U_\varepsilon(x) \equiv U(x + \varepsilon\varphi(x))$ when $\varphi \in C_c^1(\Omega; \mathbb{R}^n)$. This gives an equilibrium equation system which we analyze. We show that a $C^{1,\beta}$–equilibrium U has $\det DU > 0$ in Ω. If F is strictly elliptic a consequence is that U is of class $C^{2,\beta}$. We also give an example showing this is the best possible one can do. We construct an equilibrium of class C^1 with $\det DU = 0$ at some point in Ω.

P. PODIO–GUIDUGLI (ROM):
Equilibrum Phase Mixtures of Elastic Materials

(Joint work with G. Vergara Caffarelli)
A homogenous elastic body in a pressure chamber may assume a variety of equilibrum configurations. As noticed by Varley&Day (ARMA 22, 1966) the simplest of those are the stationary points of the *enthalpy*

$$\widehat{\gamma}(\pi, F) = \pi \det F + \widehat{\sigma}(F), \text{ i.e., the solutions of } \pi \widehat{F}^{*}(F) + \widehat{S}(F) = 0$$

(here π is pressure; F is deformation gradient, and $\widehat{F}^{*}(F) := (\det F)F^{-T}$ is the cofactor of F; $\widehat{\sigma}$ is stored energy p.u.v., and $\widehat{S}(F) := \partial_F \widehat{\sigma}(F)$ is Piola stress).

In this paper a full description of the stationary set is provided, under the following circumstances:

i. $\exists$ a local diffeomorphiom $F \leftrightarrow (\alpha, A)$, with $\alpha \in \mathbb{R}^{++}$ and $A \in V$, an eight–dimensional manifold in the space of tensors with positive determinant;

ii. $\exists$ a mapping $(\alpha, A) \mapsto \widetilde{\sigma}(\alpha, A) \in \mathbb{R}^{+}$ such that $\widetilde{\sigma}(\alpha(F), A(F)) = \widehat{\sigma}(F)$ for each F and that $\widetilde{\sigma}(\cdot, A)$ is strictly convex for each A.

It is shown that the stationary set consists of three components (*equilibrum phases*); thus, equilibrum phase *mixtures* may in principle be generated in a pressure chamber. Remarkably, one of the phases admits interpretation as a Bain transformation, i.e., the deformation occurring at austenite $\rightarrow$ martensite changes in structure.

A. RAOULT (PARIS):
Elastodynamics for a 3D–2D Junction

The mathematical study of the stabilization of space structures, of structures which are commonly used nowadays in the industry like robots, asks for precise modeling of their time–dependent behavior.

The asymptotic method provides a rigorous framework in which models for lower–dimensional structures can be derived from genuine three–dimensional elasticity. This setting allows for proving convergence results.

In this talk, we use the method for deriving a limit time–dependent junction model between a three–dimensional linearly elastic body and a linearly elastic plate (the plate is partially inserted in the body).

Technical dificulties arise from the fact that by letting the thickness of the plate go to zero, one has to consider a three–dimensional body whose shape changes as ε decreases. This difficulty is solved by associating with the sequence of the displacement vector fields of the structure two sequences one on the whole of the parallelepiped O associated with the three–dimensional body and the other one on the cylindrical body Ω of thickness 2 which is obtained by dilatation of the plate. As a consequence, test–functions in the variational formulation of the system of elasticity are now pairs linked by junction conditions. After identifying pairs which are limits of such sequences, one can prove the convergence of the displacements.

The limit problem consists in

i) a coupled time–dependent problem posed on the whole of the parallelipiped minus a slit (i.e. minus the middle surface of the inserted plate) and the middle surface of the plate. The problem can be interpreted as the three–dimensional system of elasticity in the cube, the plate equation with a coupling term in the force term (which can be seen as the action of the cube on the plate) and continuity conditions in the displacements.

ii) a static problem posed over the middle surface of the plate for the horizontal components of the displacement.

We emphasize the fact that compatibility conditions on the initial data allow us to prove convergence results in $L^2(0,T;H^1(O)\times H^1(\Omega))$.

T. RATIU (BERKELEY):
Linearization of Hamiltonian Systems

It is shown how to linearize a Hamiltonian system along a given solution. The technique involved are symplectic connections and Lie–Poisson srtuctures. The example of the free rigid body was treated in detail. The equations which are linearized, are Hamiltonian on a fixed tangent space with Hamiltonian function given by the second variation corrected by connection terms.

MICHAEL RENARDY (BLACKSBURG):
Ill–Posedness at the Boundary for Sliding Contact Problems

It is shown that sliding contact problems may become ill–posed due to failure of the complementing condition. Two such cases are considered:

1. A neo–Hookian elastic solid sliding under the influence of Coulomb friction
2. A viscoelastic fluid with a memory slip law at the boundary.

The second example may be related to the phenomenom of melt fracture in the extrusion of molten polymers.

J. SAINT JEAN PAULIN (METZ):
Asymptotic Study of Large Space Structures

(Joint work with D. Cioranescu)
The large space structures studied here are made of identical cells periodically distributed. The period ε is small compared with the global dimensions of the structure. The thickness of the material $\varepsilon\delta$ is small compared with the period (that is δ is another small parameter). And in the case of a gridwork the thickness of the grid $e=\varepsilon\eta$ is also much smaller than ε. The aim is to obtain the assymptotical mathematical behavior of the structure when all the parameters ε, δ, and η converge to zero. Several types of mathematical problems can be considered such thermo or elasticity problems or eigenvalue problems.We can consider Dirichlet or Neumann or Fourier boundary conditions on the boundary of the holes. The limit coefficients are explicit algebraic expressions of the coefficients of the material.

As an example, if we consider the problem

$$-\frac{\partial}{\partial x_j}\left(a_{ijkh}\frac{\partial u_k^{\epsilon\delta}}{\partial x_h}\right) = F_i^\epsilon \text{ in the body,}$$
$$a_{ijkh}\frac{\partial u_k^{\epsilon\delta}}{n_3} = G_i^{\epsilon\pm} \text{ on top and bottom surface,}$$
clamped on the exterior lateral boundary, free on the holes,

it can be proved (after appropriate normalization) that the limit problem for the deflection u_3 is

$$\frac{E}{6}\left[\frac{\partial^4 u_3}{\partial z_1^4}+\frac{\partial^4 u_3}{\partial z_2^4}\right]+\frac{4\mu}{3}\frac{\partial^4 u_3}{\partial z_1^2\partial z_2^2} = \mathcal{F}_3 \text{ in the cross section,}$$
$$u_3 = 0,\ \partial u/\partial n = 0 \text{ on the lateral boundary.}$$

For the lateral displacement $u_\alpha = -z_3\partial u_3/\partial n + w_\alpha$ we find

$$-E\frac{\partial^2 w_\alpha}{\partial x_\alpha^2} = \mathcal{G}_\alpha \text{ in the cros section}$$
$$w_\alpha n_\alpha = 0 \text{ on the boundary (no summation in } \alpha).$$

Here E is the Young modulus of the material and μ the Lamé constant. Moreover we can derive error estimates in terms of ε and δ.

E. SANCHEZ–PALENCIA (PARIS):
Thin Shell Theory as Limit of 3–d Elasticity

It is known that the nature (shape and boundary conditions) of the medium surface of a shell plays an important role on the structure of the solutions in shell theory. Roughly speaking there are two kinds of medium surfaces: surfaces admitting pure bendings and not admitting them. Here a pure bending means a deformation keeping constant the first fundamental form of the surface. As a flatened portion of an elastic body is much more rigid with respect to membrane tensions than with respect to flexions, the natural trend of the thin shell is to move in the subspace (or submanifold, in the non–linear case) of pure bendings. If this is not possible, i.e. if the surface is geometrically rigid, the displacement is very small, and the corresponding stress state is governed by the membrane theory, the flexions being negligible in this case. We obtain these two asymptotic behaviors (according to the nature of the surface) directly from the tree–dimensional elasticity, without using a shell theory. The method is an asymptotic 2–scale method with different scales for the surface parameters and for the normal variable.

J. SCHEURLE (HAMBURG):
Numerical Results for a "String Model"

A model for the planar motion of an elastic string or planar shear motion of a three dimensional incompressible elastic body is considered. This model can be formulated as a four dimensional hyperbolic conservation law with one space and the time variable. Numerical solutions for the corresponding Riemann problem are shown. It turns ot that the occurence of contact discontinuities rather than shocks or simple regions is the dominant feature of these solutions. So the material appears to behave more or less linearly, although the constitutive function is nonlinear. This suggegsts, that the existing theory for weak solutions of hyperbolic conservation laws might not always be appropriate for such problems in elasticity.

F. SCHURICHT (LEIPZIG):
Bifurcation Problems for Variational Inequalities

As a simple example, we consider an elastic rod, fixed at its ends, compressed by an axial force and constrained by obstacles. Then we are led to eigenvalue variational inequalities:

$$(u, \lambda) \in K \times I\!R : \langle a'(u) - \lambda b'(u), v - u \rangle \leq 0 \; \forall v \in K \qquad (VI)_N,$$

where we assume that K is a closed convex cone in a Hilbert space.

Now, we are looking for bifurcations of $(VI)_N$ from the origin. One can easy prove, that a bifurcation value of $(VI)_N$ is also an eigenvalue of the following "linearization":

$$(u, \lambda) \in K \times I\!R : \langle a''(0)u - \lambda b''(0)u, v - u \rangle \leq 0 \; \forall v \in K \qquad (VI)_L.$$

But, the inversion is only valid for the greatest eigenvalue of $(VI)_L$ in general. For higher bifurcation values, only estimates were given up to now.

Here we prove a general bifurcation result of the following quality: if $\lambda > 0$ is a "minimax" eigenvalue of $(VI)_L$, then λ is also a bifurcation value; where a minimax eigenvalue is obtained by a minimax principle as in the Ljusternik–Schnirelman theory.

M. SEREDYNSKA (WARSAW):
Lie–Poisson Equations in Elasticity

We consider the dynamics of an affinely-rigid body as a constrained continuum with only homogenous deformations allowed. Hyperelastic constitutive equations satisfying objectivity are assumed. Hamiltonian formulation of the problem and semidirect product reduction with respect to left orthogonal transformations yield equations of motion in the Lie–Poisson form. The dynamical variables of the reduced system are the right affine momentum and Green deformation tensor.

Equations of motion of a free affinely rigid body with kinetic energy expressed by material metric tensor due to the left symmetry of the group acting on itself are written down as an Euler system for affine momenta.

J.C. SIMO (STANFORD):
Stability of Relative Equilibriain Hamiltonian Systems: Application to Nonlinear Elasticity

A general approach to the rigorous nonlinear stability analysis of relative equilibria in

Hamiltonian systems is discussed in detail. The proposed methodology, referred to as the *reduced energy momentum method*, constitutes a substantial extension of results in Arnold [1966,68], Smale[1970] and others. In particular, the method employs information associated only with the configuration space and not the full phase space, enforces automatically the constraint of conservation of total angular momentum without introducing Lagrange multipliers, and does not require explicit knowledge of conserved quantities in the reduced space (Casimirs).

A new block–diagonalization procedure for second variation is introduced which, for rotating structures, leads to an explicit decoupling of the "internal" (deformation) and rotational modes in the stability analysis. The stability conditions associated with the rotational modes are explicit and reduce (for the case in which the configuration space is isomorphic to the symmetry group) to the criterion proposed by Arnold [1960].

I.V. Volovich (Moscow):
Gauge Theory of Dislocations and Disclinations and Conservation Laws for Non–Linear Equations

Internal dynamics of defects in gauge theory of dislocations and disclinations is considered. In two dimensions it is equivavalent to the string theory with dynamical torsion [1]. A solution of equations of motion is presented.

A general theory of conserved local and non–local currents for non–linear equations is described [2]. Examples including non–lagrangian equations are considered. A reation between Yang–Mills fields and chiral (director) fields is discussed.

[1] M.O. Katanaev and I.V. Volovich, Annals of Physics **197** (1990) 1–39.

[2] V.S. Vladimirov and I.V. Volovich, Teoreticheskaya i Matematicheskaya Fizika **62** No.1 (1985) 3–29.

W. von Wahl (Bayreuth):
Boundary Value Problems for curl and div and related questions

We study first the precise range of the operators curl and div under zero boundary conditions. This is done by considering stationary Maxwell's equations. The main point is e.g. to construct a solution of $\operatorname{curl} u = \gamma$ in Ω, $u|_{\partial\Omega} = 0$ which fulfills $\|\nabla u\|_p \leq c\|\gamma\|_q$ (for suitable γ). Secondly the question is treated whether it is possible to obtain an estimate (for any vector field u)

$$(*) \qquad \|\nabla u\|_p \leq c(\|\operatorname{curl} u\|_p + \|\operatorname{div} u\|p), \quad p > 1.$$

If $u|_{\partial\Omega} = 0$ it is easy to show $(*)$. If however $\nu \times u = 0$ or $(\nu, u) = 0$ on $\partial\Omega$ (ν = outer normal) the result can be completely described in terms of Betti–numbers of Ω.

C.C. WANG (HOUSTON):
Problems of Bending, Torsion, Expansion, and Eversion for Compressible Isotropic Elastic Bodies

The problems of bending, torsion, expansion, and eversion for incompressible isotropic elastic material bodies were solved by Rivlin, Ericksen, and others in the late forties and early fifties. In this talk I present a class of solutions for compressible materials. These solutions are found by patching together universal solutions for elastic membranes. Thus reducing the problems to a nonlinear ODE which generally depends on the material. Comparison of this ODE with the governing equations for 1–dimensional elasticity will be made also.

Reported by: A. Mielke, Stuttgart.

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