MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 29/1990

Variationsrechnung

08.07. bis 14.07.1990

Die Tagung fand unter der Leitung von Herrn R.Hardt (Houston), Herrn J.Jost (Bochum) und Herrn F.Tomi (Heidelberg) statt. Die Teilnehmer kamen aus der Bundesrepublik Deutschland, Italien, den USA und anderen Ländern und vertraten einen breiten Themenkreis der Variationsrechnung. Schwerpunkte der Vorträge stellten Variationsprobleme aus der Differentialgeometrie dar, insbesondere Ergebnis aus der Theorie der Minimalflächen, der harmonischen Abbildungen, der Yang-Mills Felder, der hamiltonschen Systeme sowie allgemeiner elliptischer Variationsprobleme.

Die Ergebnisse wurden in interessanter und verständlicher Weise vorgetragen. Sicherlich gab es auf der Tagung viele Anregungen.

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Vortragsauszüge

A. Ambrosetti:

Singular Hamiltonian Systems

The existence of periodic solutions of conservative systems

$$\begin{cases} \ddot{q} + V'(q) = 0 \\ \frac{1}{2}|\dot{q}|^2 + V(q) = h \end{cases}$$

($h < 0$, given) is discussed in the case when V behaves like $-\frac{1}{|q|^\alpha}, 0 < \alpha < 2$. The results apply to gravitational field (i.e. when $V(q) = -\frac{1}{|q|} + W(q)$) as well as to a class of N-body systems. .

The proof rely on a variational principle consisting in looking for critical points of

$$f(u) = \frac{1}{2}\int_0^1 |\dot{u}|^2 \cdot \frac{1}{2}\int_0^1 V'(u)u$$

on the manifold $M_n = \{u \in H^{1,2}(S^1, \mathbb{R} - \{0\}) : \int_0^1 V(u) + \frac{1}{2}V'(u)u = h\}$.

G. Anzellotti:

Functionals Depending on Curvatures

I consider the problem of the existence of minimizers for functionals of the type

$$\mathcal{F} = \int_M f(\text{curvature of } M) d\mathcal{H}^n$$

where M is a n-dimensional submanifold of the Euclidean (n+1)-dimensional space. The key idea is to consider the rectifiable current $|[G]|$ associated to the graph $G = \{(x, \nu(x)) \mid x \in M\}$ of the Gauss map $\nu : M \to S^n$. The key remark is that the area of G is bounded by the sum of the area of M and of the L^1 norm of the curvatures (exterior powers of tangential gradient of ν) of M. It follows that if M_j is a minimizing sequence for $\mathcal{F}(M)$, under suitable coerciveness assumption for the functional, and with suitable boundary or side conditions, the currents $|[G_j]|$ have a subsequence that converges weakly to a rectifiable current Σ in $\mathbb{R}^{n+1} \times S^n$ and this Σ is a good candidate to be a minimizer. The properties of the currents Σ obtained as above, and the functionals defined on these currents are investigated.

All this has been obtained in a joint work with Raul Serapioni and Italo Tamanini of Trento University.

G. Buttazzo:

Relaxed Formulation for a Class of Shape Optimization Problems

A shape optimization problem can be considered as a minimum problem of the form

$$\min\{\Phi(A) : A \in \mathcal{A}\} \tag{1}$$

where Φ is a functional to be minimized over a class $\mathcal{A}$ of admissible domains. Problems of this kind arise in various questions of mechanics and structural engineering, where the volume constraint $|A| \leq k$ seems also a reasonable condition to impose.
In several situations it is possible to prove that problem (1) does not admit any solution, so that the "relaxed" problem

$$\min\{\overline{\Phi}(\mu) : \mu \in \mathcal{M}_0\} \tag{2}$$

is introduced, where $\mathcal{M}_0$ is a suitable class of Borel measures. We prove that the relaxed problem has always a solution and we find some necessary conditions of optimality.

G. Dal Maso:

Integral Representation of Functionals Defined on $BV(\Omega)$

Let Ω be a bounded open subset of $\mathbb{R}^n$ with smooth boundary, let $\mathcal{B}(\Omega)$ be the σ-field of all Borel subsets of Ω, and let $F : BV(\Omega) \times \mathcal{B}(\Omega) \to [0, +\infty)$ be a functional such that
(a) for every $u \in BV(\Omega)$ the set function $F(u, \cdot)$ is a Borel measure on Ω,
(b) for every open subset A of Ω the function $F(\cdot, A)$ is convex and $L^1(\Omega)$-lower semicontinuous on $BV(\Omega)$,
(c) there exists a positive constant c such that

$$\int_B |Du| \leq F(u, B) \leq c[|B| + \int_B |Du|]$$

for every $u \in BV(\Omega)$ and for every $B \in \mathcal{B}(\Omega)$.

Then there exists two Borel functions $f, h : \Omega \times \mathbb{R}^n \to [0, +\infty)$ and a non-negative Radon measure μ on Ω such that for every $(u, B) \in BV(\Omega) \times \mathcal{B}(\Omega)$ we have the integral reprsentation

$$F(u, B) = \int_B f(x, \nabla_\mu u) d\mu + \int_B h(x, \frac{Du}{|Du|}) |D^s_\mu u|,$$

where $Du = (\nabla_\mu u) d\mu + D^s_\mu u$ is the Lebesgue-Nilrodym decomposition of the vector measure Du and $\frac{Du}{|Du|}$ is the Radon- Nilrodym derivative of the measure Du with respect to its variation $|Du|$. The function $f(x, z)$ is convex with respect to z , while $h(x, z)$ is positively homogeneous of degree one with respect to z. Moreover we have $|z| \leq f(x, z) \leq c(1 + |z|)$ and $|z| \leq h(x, z) \leq c|z|$.
This integral representation result, proved in collaboration with G.Bouchilti, has several applications to Γ-convergence and relaxation problems for functionals with linear growth.

U. Dierkes:

Singular Variational Problems

We consider minimizers of the potential energy functional

$$E_\alpha = \int_\Omega u^\alpha \sqrt{1 + |Du|^2} + \frac{1}{1+\alpha} \int_{\partial\Omega} |u^{1+\alpha} - \phi^{1+\alpha}| d\mathcal{H}_{n-1}$$

in $BV^+_{1+\alpha} := \{u \in L_{1+\alpha} : u \geq 0, u^{1+\alpha} \in BV(\Omega)\}, \alpha > 0$, where $\Omega \subset \mathbb{R}^n$ denotes some domain with Lipschitz boundary and $\phi \in L_{1+\alpha}(\partial\Omega)$ is a prescribed function.

Theorem 1: There exists a minimum $u \in BV^+_{1+\alpha}(\Omega)$ of E_α. If $n \leq 6$, then each minimum $u \in C^0(\Omega) \cap C^\omega(\{u > 0\})$. In general it is true that $|\{u = 0\}| > 0$ in particular, if $\alpha = 1$ then $|\{u = 1\}| \geq \frac{|\Omega|}{\mathcal{H}_{n-1}(\partial\Omega)} - \sup_{\partial\Omega} \phi$.

Theorem 2: Suppose that $\partial\Omega$ is mean convex near $x_0 \in \partial\Omega$ and that ϕ is continuous at x_0. Then $\lim_{x \to x_0} u(x) = \phi(x_0)$. Furthermore, if in addition, $\partial\Omega \in C^3$, $0 < \phi \in C^{1,\alpha}$ and $u \in C^2(\Omega_\varepsilon) \cap C^0(\bar{\Omega}_\varepsilon)$, where $\Omega_\varepsilon := \{x \in \Omega : dist(x, \partial\Omega) < \varepsilon\}$, then $u \in C^{0,1}(\bar{\Omega}_\varepsilon)$.

Theorem 3: Let $\alpha > 0$, $n \geq 2$ be arbitrary. To each $R > 0$ there exists $r \in (0, R)$, $\delta > 0$, $w \in C^{0,\frac{1}{2}}(T_{R,r})$, where $T_{R,r} = B_R(o) - \overline{B_r(o)}$, $w \notin C^{0,\frac{1}{2}+\varepsilon}$ for any $\varepsilon > 0$, which minimizes

$$\int_{T_{R,r}} u^\alpha \sqrt{1 + |Du|^\alpha} + \frac{1}{1+\alpha} \int_{\partial T_{R,r}} |u^{1+\alpha} - w^{1+\alpha}| d\mathcal{H}_{n-1}$$

where

$$w = \begin{cases} \delta & \text{on } \partial B_R(o), \\ 0 & \text{on } \partial B_r(o). \end{cases}$$

The cones $C_n^\alpha := \{x_{n+1} = \sqrt{\frac{\alpha}{n-1}}[x_1^2 + \ldots + x_n^2]^{\frac{1}{2}}\}$ are stationary for E_α. Concerning the minimizing properties of C_n^α we have:

Theorem 4: Suppose $\alpha + n \geq 7$ where $\alpha \geq 2,\ n \geq 3$ or $\alpha + n \geq 8$ where $\alpha \geq 1,\ n \geq 2$. Then C_n^α minimize E_α locally in $BV_{1+\alpha}^+$. If $\alpha = 1,\ n \leq 6$ or $n = 2,\ \alpha \leq 5$ then C_n^α do not minimize E_α.

Let $M \subset \mathbb{R}^n \times \mathbb{R}^+$ be an n-dimensional manifold and let $\mathcal{E}_\alpha = \int_M x_{n+1}^\alpha d\mathcal{H}_n$ denote its "α-energy". It turns out that the second variation is given by

$$\delta^2 \mathcal{E}_\alpha(m) = \int_M x_{n+1}^\alpha \{|\nabla \xi|^2 - \alpha x_{n+1}^{-2} \nu_{n+1}^2 \xi^2 - |A|^2 \xi^2\} d\mathcal{H}_n$$

C_n^α is called stable, if $\delta^2 \mathcal{E}_\alpha(C_n^\alpha) \geq 0$ for all ξ with compact support in $C_n^\alpha - \{o\}$.

Theorem 5: C_n^α are stable, if $\alpha + n \geq 4 + \sqrt{8}$. If $\alpha + n < 4 + \sqrt{8}$ then there is no stable cone in $\mathbb{R}^n \times \mathbb{R}^+$ with singularity at zero.

Remark: Theorems 4 and 5 imply that for $\alpha \in [2 + \sqrt{8}, 5]$ the two-dimensional cones C_2^α are stable but not minimizing.

F. Duzaar:

Integral Currents with Prescribed Mean Curvature Form

Suppose we are given an oriented (n-1)-dimensional compact submanifold $\Gamma \subset \mathbb{R}^{n+k}$, $k \geq 1$, without boundary and a vector-valued n-form $\Omega : \mathbb{R}^{n+k} \to \wedge^n(\mathbb{R}^{n+k}, \mathbb{R}^{n+k})$ of class C^1 having the property that for all simple n-vectors $v_1 \wedge \cdots \wedge v_n$

$$\Omega(x, v_1 \wedge \cdots \wedge v_n) \perp Span[v_1, \cdots, v_n].$$

Assuming that the associated scalar mean curnature form $\omega : \mathbb{R}^{n+k} \to \wedge^{n+1}\mathbb{R}^{n+k}$ being defeined by the equation

$$\omega(x, v_0 \wedge \cdots \wedge v_n) = v_0 \cdot \Omega(x, v_1 \wedge \cdots \wedge v_n), \ \forall v_1 \wedge \cdots \wedge v_n$$

is closed we can prove the following

Theorem.

(A) Suppose that ω satisfies that

$$|\omega|_\infty < n \cdot \sqrt[n]{\frac{\alpha_{n+1}}{2A_\Gamma}}$$

where A_Γ denotes the area of an area minimizing current subject to the boundary $|[\Gamma]|$. Then there exists an integer multiplicity rectifiable current $T = \tau(M,\theta,\xi)$ with compact support and boundary $|[\Gamma]|$ which is a solution of the "mean curvature equation"

$$(*) \qquad \int_M (div_M X + X \cdot \Omega(\cdot,\xi))\theta d\mathcal{H}^n = 0$$

for all $x \in C_0^1(\mathbb{R}^{n+k})$, spt$X \cap \Gamma = \emptyset$.

(B) Let $\mathcal{R} := \{x \in \mathbb{R}^{n+k} \setminus \Gamma$, spt$T \cap B_\rho(x)$ is for some ball $B_\rho(x)$ an oriented n-dimensional submanifold of class C^2 with mean curvature vector $\Omega(\cdot,\xi)\}$. Then $\mathcal{R}$ is open and dense in spt$T \setminus \Gamma$. in the codimension one case k=1 we have Sing$(T) = \emptyset$ for $n \leq 6$, Sing(T) distrete for n = 7, $\mathcal{H}$-dim(Sing(T)) $\leq$ n - 7 for n $\geq$ 8.

References:

[1] F.Duzaar, M.Fuch: On the existence of integral currents with prescribed mean curvature vector, Manus. Math. 67, 41-67 (1990)

[2] F.Duzaar, M.Fuch: On integral currents with constant mean curvature, to appear on Rend. Sem. Mat. Univ. Padova 1991

J. Eells:

Exponential Harmonicity

Preliminary report on work by J. Eells and L. Lemaire.
Consider the functional $\mathbf{E}(\phi) = \int e^{|d\phi|^2}$ of maps $\phi : M \to N$ between Riemannian manifolds. Formally its Euler- Lagrange operator is

$$(1) \qquad \mathcal{T}(\phi) = div(e^{|d\phi|^2} d\phi) = e^{|d\phi|^2}(\tau(\phi) + d\phi \cdot \nabla |d\phi|^2),$$

where $\tau(\phi)$ is the standaed tension field of ϕ ($\equiv div(\phi)$).
Does every minimum of $\mathbf{E}$ in $\cap_{p\geq 1} \mathcal{L}_1^p(M,N)$ satisfy $\tau(\phi) = 0$ weakly? Yes, in cases dim $M = 1$ and dim $N = 1$.

Prop.: If M is a circle, then the smooth solutions of $\mathcal{T}(\phi) = 0$ coincide with those of $\tau(\phi) = 0$.

The operator $d\phi \cdot \nabla|d\phi|^2$ has a life of its own:

Prop.: If dim $N \geq 3$ and $\phi : M \to N$ is a harmonic morphim, then $\mathcal{T}(\phi) = 0$ iff the fibres of ϕ are minimal submanifolds -- that being a characterization of solutions of

$$(2) \qquad d\phi \cdot \nabla|d\phi|^2 = 0.$$

In case $N = \mathbb{R}$ and M a domain of $\mathbb{R}^m$, G.Arònsson has made a lovelý study of solutions of (2), characterizing it as the Euler-Lagrange equation of the functional

$$\lim_{p\to\infty} \sqrt[p]{\int |d\phi|^{2p}}.$$

Gu Chaohao:

Some Problems in Physics Related to Extremal Surfaces

Three kinds of problems in physics which we related to the extremal surfaces in Euclidean space and Minkowski space-time are presented.

(1) Potential flows of incompressable ideal fluid:

It is proved that minimal surfaces can be expressed as

$$x = \Phi_u, y = \Phi_v, z = \Phi_w$$

where Φ is a harmonic function of 1st degree and (u, v, w) is the direction of the normal.
Consider (u, v, w) as the space-coordinates and (x, y, z) as the flow velocity at the point (u, v, w). We obtain exact solutions of stationary three dimensional potential flows.
Dual Plateau problem is reduce to solving the Dirichlet problem for a linear elliptic PDG on sphere which is solvable.

(2) The motion of strings in Minkowski space-time $\mathbb{R}^{1+n}$:

The world surface of a moving string in a non space-like extremal surface. The Cauchy problem for the motion is solved explicitly and globally provided the initial data are C^2 and space-like exact for a discret set of points.
The explicit expression of the position of a string at time t can be

$$\tilde{x} = f_1(\frac{t+s}{2}) + f_2(\frac{t-s}{2}), \quad \tilde{x} = (x_1, \dots, x_n).$$

Where $f_1(s_1)$ and $f_2(s_2)$ are equations of two curves in $\mathbb{R}^n$ and s_1, s_2 are there arc lengths respectively.

(3) The Born-Imfeld equation for nonlinear electrodynamics

The equation is

$$(1+\varphi_x^2)\varphi_{tt} - 2\varphi_x\varphi_t\varphi_{xt} - (1-\varphi_t^2)\varphi_{xx} = 0 \quad (\varphi_t = E, \varphi_x = -H)$$

This is just the equation of extremal surface in $\mathbb{R}^{1+2}$ if we identify $\varphi = y$. If $E+1 > H$ the equation in hyperbolic, the Cauchy problem can be solved globally without singularities.
If $E+1-H$ changes its sign, from the theory of extremal surface of mixed type, the Cauchy problem with C^2 initial data does not admit a solution in general, even locally.

Matthias Günther:

Isometric Embeddings of Riemannian Manifolds

In 1956 J.Nash proved, that every Riemannian manifold (M,g) of class $C^s (s \geq 3$ or $s = \infty)$ possesses an isometric embedding $u \in C^s$ in an euclidian space $\mathbb{R}^q$ with a suitable (high) value of q. One of the main steps in Nash's proof is the solution of the associated perturbation problem, i.e. the determination of an isometric embedding $u+v$ of $(M, g+f)$ if an isometric embedding u of (M,g) is known and f is small in some sense. In solving this perturbation problem a serious difficulty arises,namely the so-called loss of differentiability. To overcome this difficulty, Nash invented a complicated iteration procedure, which is nowadays known as Moser-Nash technique or hard implicit function theorem.
In our talk we give a very simple method to handle the perturbation problem with the help of the Banach fixed point theorem. Further we give a result concerning a minimal value of q.

Ch. Hamburger:

Regularity for an Elliptic System

We prove global regularity for the vector-valued differential $m-$forms $w = (w^1, \dots, w^N)$ defined on a Riemannian manifold with boundary M, which are solutions of the system

(*) $$\delta(\rho(|w|^2)w) = 0 \quad dw = 0 \text{ in } M,$$

subject to the Neumann boundary condition

$$(N) \qquad \omega^{\perp} = 0 \quad \text{in } \partial M.$$

$(*)$ and (N) arise as Euler equations and natural boundary condition for cohomological minima of the functional $\int_M f(w)vg$, where $f(w) = k(|w|^2)$ with $k' = \rho$.

<u>Theorem.</u> Let M be of class $C^{1,1}$ (which implies that the metric g is Lipschitz). Suppose that

$$f''(w) \cdot (\xi, \xi) \geq |w|^{p-2}|\xi|^2$$
$$|f''(w) - f''(y)| \leq c(|w|^2 + |y|^2)^{\frac{p-2-\alpha}{2}}|w - y|^2$$

for $p > 1, \alpha > 0$.

Then any weak solution $w \in L^p_{\text{loc}}$ of the system $(*)$ with boundary condition (N) is globally Hölder continuous in M.

The theorem is a generalization of the interior regularity results of K. Uhlenbeck (1977) and M. Giaquinta — G. Modica (1976).

We prove boundary regularity by freezing the metric at a boundary point and by approximating w by a solution $\mathcal{X}$ of $(*)$ with respect to the frozen metric.We then extend $\mathcal{X}$ by reflection a arc the boundary and we apply the interior estimates to the extended from $\mathcal{X}$. By virtue of the Lipschitz continuity of the metric, we can control the norm $\int(\mathcal{X} - w)^2$, this obtaining the required boundary estimates for w.

Robert M. Hardt:

Some New Harmonic Maps from B^3 to S^2

Here we discuss the question of the existence of a harmonic map from a spatial domain to S^2 which has prescribed Dirichlet boundary data <u>and</u> prescribed singularities. There are several related and partial results. B. Chen and R. Hardt show this is possible if $\int |\nabla|^2 dx$ is replaced by $\int |\nabla u|^p dx$ for $2 < p < 3$. Given the singular set, L. Mon (Comm. P.D.E. 1989) constructs a bridge-like domain connecting the singularities and a suitable harmonic map on this domain. C. Poon obtains for any $a \in \bar{B}^3$, a harmonic map u_a which is smooth away from a and which equals the identity on S^2. R. Hardt, F.H. Lin, and C. Poon solve the prescribed singularity problem in case the boundary data is axially-symmetric, nonconstant, and nonsurjective and the singular set lies on the axis. The latter paper involves work on a relaxed energy functional studied by F. Bethuel, H. Brezis, J. Coron, M. Giaquinta,G. Modica and J. Soucek (Manuscripta Math. 1989).

Hu Hesheng:

On the Lump Phenomena of Yang-Mills Fields

We give a nonexistence theorem of the lump phenomena for Yang-Mills fields in 4 dimensional Minkowski space-time $\mathbb{R}^{3+1}$.

Definition. If there exists positive constants R, ε, t_0 such that

$$\int_{|x|<R} T_{00}(t,x)d^3x > \varepsilon$$

holds true for every $t > t_0$, it is called that the field admits lump phenomena.

Here T_{00} is the energy density of the Yang-Mills field and $|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$.

Theorem. For any compact group, there are no lumps for a pure Yang-Mills field in $\mathbb{R}^{3+1}$ whose energy satisfies

$$\int_{\mathbb{R}^3} \frac{T_{00}(x,t)}{r^\lambda} d^3x < A \quad (A \text{ constant})$$

where $0 \leq \lambda < \frac{1}{3}, r = |x|$.

The theorem weakens the finite energy condition (which corresponds $\lambda = 0$ here) in Weder's paper.

The energy condition in the above statement cannot be removed.We show this by given an exact solution of the pure Yang-Mills fields admitting lump phenomena.

Ex. Take Gauge group $SU(2)$ and potential in the following form

$$b_0 = 0$$

$$b_1 = b_2 = b_3 = k \operatorname{sinlemn}(kt)$$

where k is a constant sinlemn(kt) is an elliptic function.

$$\int_{B(R)} T_{00} dV = 2\pi R^3 k^4 > 0$$

this is an nonstatic lump solution of the $SU(2)$ pure Yang-Mills field.

The above result holds true not only for the Yang-Mills fields but also for the Gauge fields with energy-momentum tensor obeying the conservation law. Some lump solutions which satisfy the conservation law but not satisfy the Yang-Mills equations are given.

Gerhard Huisken:

Interior Estimates for Hypersurfaces

Mean curvature evolution of smooth hypersurfaces has been studied under various global assumptions. Here we prove regularity estimates for geometric quantities of any order which are interior both in space and time. In particular, we obtain local gradient and curvature estimates in regions where the hypersurface can be represented as a graph.

A major application of these estimates is the unexpected result that the mean curvature flow admits a smooth solution for all time in the class of entire graphs over Euclidean space, assuming only local Lipschitz continuity of the initial surface without growth conditions near infinity.
All of this represents recent joint work with Klaus Ecker (Melbourne).

Norbert Jakobowsky:

Interior and Boundary Regularity of Weak Solutions to the Equation of Prescribed Mean Curvature

Weak solutions to the system $\Delta x = 2H(x)x_u \wedge x_v$ in a domain $\Omega \subset \mathbb{R}^2$ are proved to belong to $C^{2,\mu}(\Omega, \mathbb{R}^3), \mu \in (0,1)$, if $H(x) = H_1(x) + H_2(x), H_i \in C^{0,1}(\mathbb{R}^3, \mathbb{R}) \cap L^\infty, \nabla H_i \in L^\infty (i = 1,2,)$, $\sup_{\mathbb{R}^3} |x||\nabla H_1| < \infty$, $\sup_{|x|>K} |xH_2(x),| < 1$. Considering the real valued function $|x-a|^2$ for some $a \in \mathbb{R}^3$, using the special structure of the system, the Courant-Lebesgue lemma, and the isoperimetric inequality, first continuity is obtained a.e. in Ω, then throughout Ω. Finally, well known results imply $C^{2,\mu}$ regularity.

Boundary regularity of weak solutions to the corresponding Dirichlet problem can be obtained by using the result of partial regularity. If the boundary data are assumed to be bounded and continuous, weak solutions are proved to be bounded and continuous resp.

J. Jost:

Group Actions, Gauge Transformations, and the Calculus of Variations

This is a joint work with Xiao-Wei Peng.

Let $\mathcal{M}$ be a Banach manifold with "weak" Riemannian metric $< \cdot, \cdot >$, $\mathcal{G}$ a Banach-Lie group acting isometrically on $\mathcal{M}$.
Let θ be a Banach space, $\Phi : \mathcal{M} \to \theta$ smooth.
Suppose

$$0 \to \text{ Lie}(\mathcal{G}) \xrightarrow{i_p} T_p\mathcal{M} \xrightarrow{j_p} \theta \to 0$$

is an elliptic complex, where $i :$ Lie$(\mathcal{G}) \to T\mathcal{M}$ is the differential of the action of $\mathcal{G}$, i_p its evalution at $p \in \mathcal{M}$; likewise $j_p : T_p\mathcal{M} \to \theta$ is the differential of Φ.

Examples:

1) M a compact manifold, $\mathcal{M} = \{$ metrics on $M\}$, $\mathcal{G} = \{$ diffeomorphisms of $M\}$, Φ giving some curvature condition like Einstein or Kähler-Einstein.

2) M a compact Riemannian manifold, E a vector bundle over M with structure group G, $\mathcal{M} = \{G-$connections on $E\}$, $\mathcal{G} = \{$ Gauge transformations$\}$, Φ again a curvature condition like antiselfduality or flatness.

w.l.o.g. $\Phi(p) = 0$. One is interested in $(\mathcal{M}_0 = \Phi^{-1}(0))$

$$\mathcal{M}_0/\mathcal{G}$$

and the induced metric on this space.

For $p, q \in \mathcal{M}$, let

$$E(p,q) := \inf_{g \in \mathcal{G}} \text{ dist}^2(p, gq).$$

For q close to p, the implicit function theorem implies the existence of a unique minimizing g. In the above examples, the metric is an L^2-metric

1): $$(h, h') = \int_M tr(g^{-1}hg^{-1}h')\,\text{dvol}_g \quad \text{for } g \in \mathcal{M}$$

2): $$(A, B) = \int_M tr(A \wedge *B) \quad \text{w.l.o.g. } G \text{ semisimple, compact.}$$

The Euler-Lagrange eqs. of E then determine g.

For $\alpha, \beta \in T_p\mathcal{M}$, tangential to $\mathcal{M}_0$, orthogonal to $\mathcal{G}-$orbit:

$$< \alpha, \beta >= \frac{\partial^2}{\partial s \partial t} E(p, p + t\alpha + s\beta)_{|s=t=0}.$$

Thus, in order to compute the metric in the above examples, one has to vary the Euler-Lagrange eqs. of E and compute the corresponding variation of the solution.

This method enables us to give a general formula for the curvature of the induced metric on $\mathcal{M}_0/\mathcal{G}$, including as special cases results of Tromba, Wolpert, Groisser-Parker, Zograf-Takhtadzhyan.
Applications to symplectic geometry are given. In particular, one considers the moduli space of stable vector bundles over a Riemannian surface Σ. This moduli space has a symplectic structure depending only on the differentiable structure of Σ, but not on its complex structure. A complex structure on Σ, however, in turn determines a complex strucutre on this moduli space, and then also a metric.
Our method can compute the affect of variations of the complex structure of Σ on the metric of the moduli space of stable vector bundles.

S. Luckhaus:

Stability of Minimizing $p-$Harmonic Maps

The problem is to show that a weakly convergent sequence of minimizing $p-$harmonic maps with values in an arbitrary compact Riemannian manifold $N \subset \mathbb{R}^k$, is converging strongly to a minimizer. This is achieved using as a tool the following lemma to change the boundary data with small cost in the energy.

<u>Lemma.</u> If u, v are in $H_p^1(\partial M, N)$ then for small λ, with constants depending on M, p only, one can find $w \in H_p^1(\partial M \times (0,\lambda), N)$ with $w(\cdot,0) = u, w(\cdot,\lambda) = v$ and

$$\int |\nabla w|^p = c\lambda \int [|\nabla u|^p + |\nabla v|^p + |u-v|^p] =: c\lambda K^p$$

and

$$\text{ess. sup dist}\,(w,N) \leq cK^{1-\frac{1}{2p}}\Big(\int |u-v|^p\Big)^{\frac{1}{2p^2}} \lambda^{-\frac{n}{p}}$$

A consequence is the following theorem

<u>Theorem.</u> If u_ν are converging weakly in $H_p^1(M,N)$ to u, and if for $v \in H_p^1(M,N)$ such that $v - u_\nu|_{\partial M} = 0$ otherwise arbitrary, $\int |\nabla u_\nu|^p \leq \int |\nabla v|^p$ holds. Then $\int |\nabla u_\nu|^p$ converges to $\int |\nabla u|^p$. And u also minimizes $\int |\nabla u|^p$ among all functions in $H_p^1(M,N)$ with the same boundary values.
Similar functionals and problems with constraints can also be treated.

L. Modica:

Singular Perturbations and Calculus of Variations

Consider the minimization problem

$$(*) \qquad \min_{u \in H} F(u)$$

where H is a function space and F is a functional defined on H. When existence, or uniqueness, or regularity of solutions of $(*)$ fails to be true, it is often useful to add to F a singular perturbation $\varepsilon S(u)$, to solve the problem

$$(*_\varepsilon) \qquad \min_{u \in H} [\varepsilon S(u) + F(u)],$$

and to study the asymptotic behavior as $\varepsilon \to 0^+$ of the solutions u_ε of $(*_\varepsilon)$. In fact, it frequently happens that the limit point u_0 of u_ε has the right properties which were lacking for the solutions of $(*)$.

Example 1. In the gradient theory of phase transitions one has

$$F(u) = \int_\Omega W(u)dx, H = \{u \in L^1(\Omega; \mathbb{R}^n) : \int_\Omega u dx = m\}$$

with $m \in \mathbb{R}^n$ given, $W : \mathbb{R}^n \to \mathbb{R}$ continuous, $W \geq 0$, $W(u) = 0$ if and only if $u \in \{\alpha_1, \ldots, \alpha_i\}$ with $\alpha_i \in \mathbb{R}^n$ $(1 \leq i \leq k)$. Solutions of $(*)$ are given by piecewise constant functions on partitions of Ω in k subsets with prescribed volumes, without any regularity of the interfaces of the partition.
By taking $S(u) = \int_\Omega |\nabla u|^2 dx$, the limit point u_0 of u_ε has the same form as before, but in addition the interfaces satisfy a variational principle and so they are smooth.

Example 2. In the theory of elastic solid materials by I. Fonseca a similar phenomenon occurs. One has here

$$F(u) = \int_\Omega W(\nabla u)dx, H = \{u : \Omega \subset \mathbb{R}^3 \to \mathbb{R}^3 : \int_\Omega u dx = m, \int_\Omega \nabla u dx = \theta A + (1-\theta)B\}$$

where ∇u is matrix, $W \geq 0$, $W(M) = 0$ if and only if $M = RA$ or $M = RB$ where A, B are fixed matrices, $A - B = a \otimes n$, R is any rotation, $m \in \mathbb{R}^3$, $\theta \in]0,1[$ are given. It is easy to find that the solutions of $(*)$ are piecewise affine functions loyered orthogonally to n. The singular perturbation $S(u) = \sum_{i,j} \int_\Omega |\nabla \frac{\partial u_i}{\partial x_j}|$ selects among them only those ones satisfying a minimum principle.

Example 3. The functional

$$F(u) = \int\int_Q ((u_x^2 - 1)^2 + u_y^4) dx dy \quad Q =]0,1[\times]0,1[\subset \mathbb{R}^2$$

has no minimum point for suitable boundary values. A singular perturbation method allows to construct a sequence of minimizers of perturbed problems which represents a generalized solution of $(*)$ in the sense of the Young measures.

I. Nikolaev:

Smoothness of Convex Surfaces and Generalized Solutions of the Monge-Ampère Equation on the Base of Differential Properties of Quasiconformal Mappings

This is a joint work with Prof. S.Z. Šefel.Considered is the equation $Z''_{xx} \cdot Z''_{yy} - Z^2_{xy} = f(x, y, z, \nabla z) \geq m > 0$ $(*)$. Here $Z(x, y), x^2 + y^2 \leq r^2$ is considered to be a generalized solution. We prove that if $f(x, y, z, \nabla Z)$ is bilatorally bounded by positive constante m and r then $Z(x, y) \in W^2_\ell \cap C^{1,\alpha}, \ell \in [f, +\infty)$, $\alpha \in (0,1)$ and $\ell \to \infty$, $\alpha \to 1$ as $M/m \to 1$. Apriori estimates are given also. To state a second result we introduce a concept of functions has $m, \alpha)-$ approximate differential $(m = 0, 1, 2 \ldots, \alpha \in (0,1))$. We define it for a null point (a general case is defined similary):
$\varphi(x), |x| \leq r, x \in \mathbb{R}^4$ has a $(m, \alpha)-$approximate differenital if there exists a polynomial $P_m(x)$ such that $|\varphi(x) - P_m(x)| \leq C|x|^{m+\alpha}, |x| \leq r$. Our second result states that if at some point $f(x, y, z, z'_x, z'_y)$ has a $(m, \alpha)-$approximate differential then at the very same point the solution of $(*)$ has $(m + 2, \alpha)-$approximate differential. We also give corresponding apriori estimates.
To prove these both theorems we consider a mapping Φ which one can defined using the commutative diagramm:

$$\begin{array}{ccccc} & & (x,q) & & \\ & \nearrow & & & \\ (x,y) & & \uparrow & & (q = z'_y, p = z'_x). \\ & \searrow & & & \\ & & (p,y) & & \end{array}$$

The mapping Φ turns out to be a quaisconformal one whose coefficient is estimated by the right-hand side of $(*)$. Besides well known properties of quasiconformal mappings, we had to prove the theorem stating that any "almost conformal mapping φ", that is $K_\varphi(x)1 \leq C|x|^{m+\alpha}$ has $(m + 1, \alpha)-$approximate differential.

J. Pitts:

Recent Results in Minimal Surfaces and Applications

Let M be a smooth, compact, oriented, $(k+1)-$dimensional Riemannian manifold $(k \geq 2)$, and let B_k^2 denote the space of $k-$dimensional flat chain boundaries modulo two in M. Since B_k^2 is a $K(\mathbb{Z}_2, 1)$ space, it follows $H_\ell^k(B_k^2, \mathbb{Z}_2) = \mathbb{Z}_2$ for all $\ell = 1, 2, \ldots$. Representatives in each homology class may be explicitly constructed. A minimum/maximum construction in a Lusternik-Schnielman setting may be carried out in each homology class to yield stationary $k-$dimensional varifolds on M.

If $2 \leq k \leq 6$, then the varifolds are supported on smooth, compact embedded minimal hypersurfaces in M. Under various fairly general conditions, one may show that M supports many or infinitely many distinct minimal surface. When $k = 2$, these methods may be used to find new minimal surfaces of computable genus (depending on the topology of M). Here are also some new existence results when $k \geq 1$, in which case the minimal surfaces are regular except possibly for a compact singular set of Hausdorff dimension at most $k - 1$.

H. Rosenberg:

Surfaces of Constant Curvature

We discuss existence and unicity results for compact embedded surfaces in $\mathbb{R}^3$ of constant Gaussian or mean curvature ($K-$surfaces and $H-$surfaces).

Conjecture. Every Jordan curve without inflection points in $\mathbb{R}^3$ bounds a $K-$surface.

Theorem. Let C_1, C_2 be convex curves in parallel planes. Then they bound a $K-$surface and $H-$surface; both connected (Hoffman, Rosenberg, Spruck).
The main tool is the technique of Cafferelli, Nirenberg, Spruch where they prove a graph over a convex planar curve extends to a graph that is a $K-$surface.

Theorem (Sa Earp, Brits-Meeks, Rosenberg): Let C be a convex planar curve and M a compact embedded $H-$surface, transverse to the plane of C along C, $\partial M = C$. Then M is contained in a halfspace determined by C. In particular M inherits the symmetries of C.

Theorem (Rosenberg, Sa Earp): Assume M is complete embedded $H-$surface, $\partial M = C =$ planar convex curve, $M \subset$ halfspace determined by C and M vertically cylindrically bounded. Then M inherits the symmetries of C; in particular, if C is a circle then $M =$ Deleauney.
We obtain conditions which ensure M is indeed in a halfspace.

F. Sauvigny:

Minimal Surfaces Solving a Semi-Free Boundary Value Problem with a Simple Projection onto a Plane

Let $S := \{(s(t), z) \in R^3 | t \in R, z \in R\}$ be a supporting surface which is a cylinder over the plane embedded curve Σ given by $s(t) : R \to R^2 \in C^{2+\alpha}$. We take a Jordan arc $\Gamma := \{(x, y, h(x, y)) | (x, y) \in \underline{\Gamma}\}$ which is a graph over the plane convex arc $\underline{\Gamma}$ and meets S only at its end points $(s(t_{1/2}), h(s(t_{1/2}))) \in S$ with $t_1 < t_2$. The curves $\underline{\Gamma}$ and Σ bound a plane domain G with the convex hull G_0. We now fix the conditions

C1: The straight line $L(t) := \{p \in R^2 | (p - s(t)) \cdot s'(t) = 0\}$ only meets $\Sigma \cup \underline{\Gamma}$ in the point $s(t)$ for all $t \in (-\infty, t_1) \cup (t_2, +\infty)$.

C2: The lines $L(t), t \in [t_1, t_2]$ yield a simple foliation of $\overline{G_0}$.

Theorem. Let $\{\Gamma, S\}$ be a configuration as above with C1 and C2. Then each parametric minimal surface $X(u, v) := (x(u, v), y(u, v), z(u, v)) : B \to \mathbb{R}^3$ solving the partially free boundary value problem for this configuration is a graph defined over the plane domain G.

Proof. We show by an index method that the plane map $f(u, v) := (x(u, v), y(u, v)) : B \to \mathbb{R}^2$ is a diffeomorphism, taking the free boundary condition into account.

Remark. We do not need a convexity condition on the free boundary.

Applications:

1. We obtain a solution of the minimal surface equation in the — not necessarily convex — domain G with mixed boundary conditions.
2. We achieve a uniqeness result for the semi-free boundary value problem for a parametric minimal surfaces spanned into such configurations.

R. Schumann:

Regularity for Signorini's Problem in Linear Elasticity

In 1981 Kinderlehrer proved that the solution of the variational inequality corresponding to Signorini's problem of plane elasticity belongs to the Hölder space $C^{1+\alpha}$ for some $\alpha \in (0, 1)$. Here the result is that $C^{1+\alpha}$-regularity is true for all space dimensions $n \geq 2$. By means of a pseudodifferential operator the problem is reduced to a variational inequality on the boundary of the domain which can be treated by scalar methods.

L. Simon:

Asymptotic Behavior of Geometric Extrema near Singular Points

Suppose u is an energy minimizing map between compact Riemannian manifolds M, N (of dimension n and m respectively), and let x_0 be a singular point for u such that u has a "cylindrial" tangent map $\varphi : \mathbb{R}^n \to N$ at x_0; that is, sing φ is a line $\underline{\ell}$ and φ is invariant under translation in direction $\underline{\ell}$. Thus, modulo a rotation of $\mathbb{R}^n$ taking the line $\underline{\ell}$ to the x^n coordinate axis, we have

$$\varphi(x^1, \ldots, x^{n-1}, x^n) = \varphi_0(x^1, \ldots, x^{n-1})$$

where $\varphi_0 : \mathbb{R}^{n-1} \to N$ is minimizing, non-constant, and homogenous of degree zero. The set of all such φ is denoted by $\mathcal{T}$. Notice that in case $n = 4$ and $m = 2$ we have that φ_0 is a conformal map of S^2 to N; denote its topological degree by $k (\geq 1)$ and let $\mathcal{T}_1$ be the set of φ in $\mathcal{T}$ with $k = 1$.

Theorem. If $n = 4$, if N is diffeomorphic to S^2, and if $\varphi \in \mathcal{T}_1$ is a tangent map for u at a singular point x_0, then $\exists \, \sigma > 0$ such that $B_\sigma(x_0) \cap$ sing u is an embedded $C^{1,\alpha}$arc for some $\alpha = \alpha(M, N) \in (0,1)$.

An unsatisfactory feature of this theorem is that it requires $\varphi \in \mathcal{T}_1$ rather than $\varphi \in \mathcal{T}$. However if N is metrically close to S^2, then a theorem of Brezis, Coron and Lieb, and a remark of F. H. Lin gives $\mathcal{T}_1 = \mathcal{T}$, and in this case we can assert that sing u is a finite union of closed embedded Jordan arcs which are $C^{1,\alpha}$ in their interior, such a picture was previously obtained, with $C^{0,\alpha}$ arcs, by Hardt and Lin.

N. Smale:

On the Yamabe Problem for Non-Compact Manifolds

In this talk, we will discuss some recent results on the Yamabe problem for non-compact manifolds. In particular, we consider the question: for what submanifolds $\Lambda^k \subseteq S^n$, does there exists on $S^n \backslash \Lambda^k$, a complete, conformally flat metric of constant positive scalar curvature? It follows from a theorem of Schoen and Yau, that for such a metric to exists, we must have $k \leq \frac{n-2}{2}$. On the other hand very little is known on the existence question. Schoen has proven that one can take $\Lambda = \{p_1, \ldots, p_k\}$, $k > 1$, or $\Lambda =$ limit set of certain Kleinian groups. Also there are explicit solutions for Λ a round subsphere. In recent work (jointly with Rafe Mazzes) we have studied perturbations of the known solutions on $S^n \backslash S^k$, and have proven: If Λ is any sufficiently small $C^{3,\alpha}$ ($\alpha \in (0,1)$) perturbation of a round $S^k \subseteq S^n$, then $S^n \backslash \Lambda$ admits an infinite dimensional family of complete conformally flat metrics of constant positive scalar curvature. The theorem is proven by studying the linearized equations about the trivial solutions and using the implicit function theorem. The main difficulties are that the linearized operator has serious degeneracies, and infinite dimensional null space. These difficulties are overcome by using techniques of micro-local analysis and the theory of conic operators.

M. Struwe:

Periodic Solutions of Hamiltonian Systems on Almost Every Energy Surface

Let H be a Hamiltonian of class C^2 on $\mathbb{R}^{2n}$ and suppose $S_1 = H^{-1}(\{1\})$ is a (C^2-) compact energy surface. Then for almost every β in a suitable neighborhood of 1 we establish the existence of a periodic solution to the Hamiltonian system

$$\dot{x} = J\nabla H(x), J = \begin{pmatrix} 0 & id \\ -id & 0 \end{pmatrix},$$

on the energy surface $S_\beta = H^{-1}(\{\beta\})$.
The proof is based on a refinement of ideas by Viterbo, resp. Hofer and Zehnder.

B. White:

Existence of Embedded Minimal Disks

Let F be a smooth positive function on the unit sphere $\partial\mathbb{B} \subset \mathbb{R}^3$. Then F defines a functional (also denoted F) on compact surfaces in $\mathbb{R}^3$ by the formula

$$F(M) = \int_{x \in M} F(\nu(x))dA(x)$$

where $\nu(x)$ is the unit normal to M at x and $dA(x)$ in the surface area element. I prove:

Theorem. Let C be a smooth closed embedded curve in $\partial\mathbb{B}$. Then among all smooth embedded disks D with boundary C, there exists one for which $F(D)$ is a minimum.

In particular, in case $F \equiv 1$ this shows existence of a smooth embedded area minimizing disk.

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