MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 30/1990

Stochastic Image Models and Algorithms

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The conference was organized by R. Azencott (ENS 45 rue d'Ulm 75005 Paris, France) and D. Geman (University of Massachussets, Amherts, Massachussets 01003).

The main topics of the conference were the following

- Low Level Image analysis: reconstruction from emission tomography data, restoration, edge detection ...
- High Level Image analysis : deformable templates, tracking ...
- Parameter estimation.
- Simulation of Markov random fields; annealing.

There were theoretical talks alternating with concrete experiments dealing with "problems from the real world". However, as indicated by the title of the conference, all the participants were adopting a stochastic approach to image analysis. This common attitude made possible easy interactions between the participants, enabling some fruitful discussions. One formal dicussion, entitled "Stochastic methods in image analysis" has been organised, with a panel composed with professors R. Azencott, S. Geman, B. Ripley and F. Götze.

Abstracts of the talks :

D. E. McCLURE : **Aspects of Single Photon Emission Tomography.**

In the MRF Bayesian approach to reconstruction problems in single photon emission computed tomography, discrete lattice-based models for prior distributions are routinely used to model phenomena that are inherently continuous. In particular an eight-neighbor Φ-model has been used by S. Geman, D. McClure, et al., and the same neighborhood structure has been used by P. Green and others, with different forms of interactions between neighboring sites in the pixel lattice. Can such a lattice-based model embody essential properties of the continuum it models, such as isotropy? Numerical and analytical results developed S. Geman, B. Gidas, K. Manbeck and D. McClure are reported which show that the forms of models now widely used are indeed invariant under arbitrary rotations, in a continuum limit, independent of the values of parameters earlier thought to affect such invariances.

J.M. DINTEN : **Tomographic Reconstruction with a Limited Number of Projections: Regularization by Markov Random Fields.**

Tomographic reconstruction with a limited number of projections is an ill-posed problem. We propose a regularization by introduction of local geometric information by the use of Markov random fields.

We propose a hierarchical modelization which allows us to introduce some information about the distribution of material areas and the physical characteristics of the materials. The resulting algorithm leads to reconstructions which preserve sharp interfaces and reduces the effects due to noise and to the restricted number of projections.

We also propose an adaptation of our algorithm to axis-symmetric objects. We find the same performances: good restoration of the interfaces and an important noise reduction, especially around the axis.

Finally, we propose an adaptation of the cross validation criterion for the determination of the parameters of the model. We test it on a simple axis-symmetrical object.

J. KAY : **Some Algorithms for Edge Preserving Smoothing in Image Restoration.**

We consider the problem of restoring images that have been degraded by blur and noise while preserving the location of edges. We discuss two types of degradation, namely, those given by Gaussian and Poisson regression models, respectively. We apply three algorithms to simulated and real images, and compare their performance. We conclude that the OSL algorithm with a Geman-McClure prior provides restorations of a similar quality to those given by the EMS algrithm with a non-linear, edge preserving, smoother. The third algorithm includes edge-variables in the prior model and incorporates parameter estimation using pseudo-likelihood. The restorations are performed using ICM. The edge variables are estimated iteratively using local gradients. The only critical parameter which must be supplied is the percentage of edges to be allowed in the restoration. This algorithm also gives good quality results for piece-wise constant images; for other images it is useful for boundary detection. Finally we outline some ideas for the extension and applications of this algorithm.

D. GEMAN : **Reconstruction: Beware**

The linear image restoration problem is to recover an ideal brightness distribution X given the blurred and noisy observations $Y = KX + \eta$, where K and η represent the point spread function and measurement error respectively. This problem is typical of ill-conditioned (and ill-posed) inverse problems in low-level computer vision. In addition to the mathematical hazards, there is the rather unsettling fact that in general there is no correspondance between mathematical difficulty and biological difficulty, so that certain blurs (eg. motion blur) which are visually diabolical may in fact be relatively simple as mathematical inverse problems, and conversely. Consequently, in view of the (understandably) strong influence of visual evaluation, it is often difficult to assess the quality of competing approaches. This dillemma will be illustrated by experiments. Finally, a new approach for deblurring in advanced wich emphasizes the recovery of discontinuities and a new method are proposed for determining model parameters which is goal directed rather than data driven.

P. J. GREEN : **Emission Tomography and the EM algorithm.**

Single Photon Emission Computerized Tomography (SPECT) is a medical imaging technics for estimating the spatial distribution of a radioactive isotope introduced into the human body from the patterns of photons detected by a gamma camera. Physical considerations generate a large Poisson linear model, $y_t \equiv \text{Poisson}(\sum_s a_{st} x_s)$, in which $\{x_s\}$ must be estimated subject to $x_s \geq 0$. A Bayesian approach is adopted, using a pairwise interaction Gibbs distribution, $p(x) \propto \exp(-\beta \sum \phi(x_s - x_t))$, as a prior. A novel modification of the EM algorithm is devised, that gives a computationally efficient iterative method for maximum a posteriori reconstruction (IEEE-Trans. Med. Imaging, 1990). The method has been assessed on both real and synthetic data sets, and is seen to give very favorable results compared with standard approaches, whilst being faster than the stochastic algorithms of Geman and McClure.

S. GEMAN : **One Dimensional Recognition Problems.**

The dynamic programming solution to various calculations associated with Gibbs distributions is layed out, including the computation of most likely states, expected values, and partition functions. The computational feasibility of dynamic programming depends on the graph associated with the Gibbs distribution. In general, one dimensional models of low order (nearest or near-neighbor models) lend themselves to dynamic programming analysis, although some compromises, such as "pruned search", may be necessary. Various recognition tasks can be cast as the solution to optimization problems involving one dimensional Gibbs distributions, and thereby lend themselves to dynamic programming algorithms.

More specifically, R^2-valued 2nd-order Markov models are developped for the shapes of coronary arteries in angiograms and for handwritten numerals. These are examples of a general and well-known approach to shape modelling through "deformable templates". The particular formulation used here was developped by Ulf Grenander. The modelling step results in a collection of distributions, one for each generic shape. There may be, for example, ten models corresponding to the ten numerals $0, \ldots, 9$, or three models corresponding to the three major coronary arteries. Conditional data models are developed, which describe likely grey-level presentations of the shapes given a particular realization from a shape ensemble. A posterior distribution is derived which is approximately second order Markov. Dynamic programming is used to calculate the probability associated with the best fit of each model to the image. Good fits are candidate classifications.

Several refinements, such as a coarse-to-fine search strategy, are developed and the approach is illustrated with recognition experiments on courtesy numbers from bank checks and on digital angiogram from diagnostic procedures.

G. CH. PFLUG : **Random transformations of the plane.**

We study a class of Gaussian Random Processes with parameter space R^2 taking values in R^2 with independent components, expectation 0 and a covariance structure which depends only on the distance between two parameter points. These processes are caracterized by a rotation invariant spectral measure. In particular, we study the process with covariance $\sigma \exp(-\alpha(z_1 - z_2)^2)$. It is shown that this process has a version with infinitely often continuous differentiable paths. It is possible to give the Karhunen-Loeve representation of the process which can be used for the simulation of paths. The actual image that we observe is modelled as a randomly transformed template. Translation and rotation are considered as nuisance parameters. It is shown that in the sense of efficient estimate, the ignorance of nuisance parameter does not affect the efficiency of the estimation of the parameter in interest. The Fisher-Information matrix can be calculated.

M. PICCIONI : **Models of random deformations suggested by continuum mechanics.**

Deformation of templates should carry both informations about the structure of the template and capture variability of actual shapes. A way to obtain these features is to inject stochastic perturbations (more precisely, forces) into the equations of continuum mechanics, thinking to the template as, for example, an elastic body subject to forces. The mathematical model which emerges is a non-linear stochastic PDE, whose linearized version (the stochastic Navier equation) was proposed long ago by Ulf Grenander in its foundations of pattern recognition, and used (in a more simplified version) in the experiments which have been described by Yali Amit in this meeting. A number of open problems have been adressed.

Y. AMIT : **Deformable Templates for Structural Image Reconstruction.**

A smooth mapping of the unit square to R^2, which corresponds to the displacement field between one image $F(x)$ to another image $I(x)$ is found by minimizing the energy:

$$E[U(x)] = \frac{1}{2}\left\{\int_{I^2} \Delta^2 U(x).U(x)dx + \int_{I^2}[I(x) - F(x+U(x))]^2 dx\right\}.$$

If the approximate solution is sufficiently smooth, the topological structures such as maxima, minima, etc., of the function $F(x)$ considered as a surface are preserved and found in $I(x)$. This indicates a possible way of identifying topological features in images by matching them to standard template images characterized by these features.

Minimizing the above energy functional leads to the non-linear PDE

$$\Delta^2 U(x) = -(I(x) - F(x+U(x))).\nabla F(x+U(x)).$$

This PDE is solved in the fourier domain using a gradient descent method. First the minimization is in low frequencies, and then continued to higher and higher frequencies. This leads to far better solutions than solving for many frequencies at once. It is also computationaly faster.

If $I(x)$, $F(x)$ are sufficiently smooth the integral for calculating the fourier coefficients for the right-hand side can be well approximated by summing over a small set of equidistant points, thus significantly speeding up the calculation.

B. RIPLEY : **Deformable templates for images of nematodes and galaxies.**

Nematodes are unsegmented worms of about a mm long, whose species (out of 20000?) is recognised by the shape of their outline and internal features. It seems impossible to get clean enough images to use conventional filtering technics to segment the image. Rather, we have tried adapting deformable template ideas.

P. Mowforth (Turing Institute tech. report) recognized regions in an MRI scan of a brain by using a continuous 2D transformation to match the scan to a reference image, and then reading off 1 of 5 types of tissue from the reference. Chow, Grenander, and Keenan in HANDS (Div. Appl. Math. Brown report) studied the outlines of a human hand as a 1D template made up of *circa* 36 arcs. Each arc is deformed by scale change and a rotation; the deformation parameters form a Markov chain around the outline of the hand. There are also possibly random translations, scale change and rotations.

We used very similar ideas for the outlines of the heads and tails of nematodes. The prior process is simulated by an extended Gibbs sampler, changing a few arcs at a time. The posterior distribution is sampled by introducing a rejection sampling step on the change. Quite good fits are obtained in a few minutes on a Sun 4/370 workstation. Internal features are fixed relative to the outline, using another template and filtered images as adjunct data.

Galaxy classification is an important scientific problem. Galaxies are classified as elliptic/spiral/irregular with many subdivisions. Elliptical are E0-E11 depending on ellipticity. Spiral (S, or with bars SB) are classified a, b, c according to the tightness of the spirals, r, rs, s depending on the arm lengths. We modelled spiral galaxies by a central disc of random radius, a point process attaching bars/arms, and arms made up of a terminating Markov chain of fixed length segments joined by random angles. Once again we simulate the posterior.

The ultimate aim in both problems is *classification*. We propose to measure characteristics of the fitted sketches and use conventional multivariate techniques. It is important to know the uncertainty in the measured parameters, so we insist on sampling from the posterior rather than find the MAP point estimate (and MAP is not equiinvariant under nonlinear transformations).

Ref: B.D. Ripley and A.I. Sutherland (1990) Finding spiral structures in images of galaxies. *Phil. Trans. Roy. Soc. A.*

H. NOLTEMEIER : **Monotonous Bisectortrees.**

We present a new data structure called monotonous bisectortrees to represent large sets of geometric objects.

We prove that monotonous bisectortrees can be constructed in optimal height $O(\log n)$ and time $(O(n \log n))$ for every finite set $S \subset R^d$ and any L_p-metric $(1 \leq p \leq \infty)$. Generalisations to leaf oriented versions are proposed; some recent results are reported, which show how to handle large sets of convex closed sets in E^d efficiently.

C. GRAFFIGNE : **Road tracking algorithm.** We describe an algorithm of road detection. We are working on sattelite images (Spot images), and the problem which interest us particularly is to track large roads through these images. More precisely we suppose that a few points of a road are known (at least two), and that we know also the width of this road at these points. We wish to determine the position of this road across the image data. In order to do that, we part the image data in smaller windows (the image data is 512×512, and the window is 32×32, in our experiments), and work separately in these windows. First, in a window, we define road characteristics that will allow us to write down a probability distribution over the set of possible roads across the window.

Finally, we construct a window-tracking algorithm that will end up with the most likely road in the entire image, knowing that its start is given by the considered points. Results are given on a 512×512 Spot Image, and for several roads across the image.

B. GIDAS : **Parameter estimation for Markov random fields.**

A new method for estimating the parameters of MRF was presented. The method derives from the well-known fact of Classical Statistical Mechanics that the temperature is related to the "kinetic energy". The procedure provides an indirect way for solving the ML (maximum likelihood equations). In the case of a single positive parameter, θ, we proceed as follows: let Λ be a finite window in Z^d, and $U_\Lambda(x)$ the energy in Λ; let $\tilde{U}_\Lambda = U_\Lambda(\tilde{X})$ be an observed value. We interpret $\tilde{U}_\Lambda$ as the maximal energy of a microcanonical ensemble. For a sample x from this ensemble we set $V_\Lambda = \tilde{U}_\Lambda - U_\Lambda(x)$; V_Λ plays the role of a "kinetic energy" per unit volume. As $\Lambda \to Z^d$, the distribution of V_Λ is exponential, and the inverse of the mean, $< V_\Lambda >$, of V_Λ is approximately the ML estimator.

S-J. SHEU : **Probability problems arising in image analysis.**

We consider a R^d-valued stochastic process θ_n given by the following stochastic algorithm:

$$\theta_{n+1} = \theta_n + a_n b(\theta_n, X_{n+1}) + \alpha_n \sqrt{a_n} e_n,$$

$$P(X_{n+1} = x | F_n) = P_{\theta_n}(X_n, x),$$

$x \in S$, a finite discrete set; F_n = σ-algebra generated by $X_1, \dots, X_n, e_1, \dots, e_n$; e_{n+1} is independant to F_n and has distribution $N(0, I)$; a_n decreases to 0, $\alpha_n = \sqrt{\frac{\beta}{\log t_n}}$ if n is large, $t_n = \sum_{k=0}^n a_k$.

Assume $P_\theta(.,.)$ generates an ergodic Markov chain with unique invariant measure $\pi_\theta(.)$. Let $\bar{b}(\theta) = \sum b(\theta, x)\pi_\theta(x)$. Then, under some conditions,

$$E[f(\theta_n)] - \pi^{\alpha_n}(f) \to 0 \text{ as } n \to \infty,$$

π^ϵ is the unique invariant measure for the diffusion θ^ϵ defined by $d\theta^\epsilon = \bar{b}(\theta^\epsilon)dt + \epsilon dB(t)$.

P. J. GREEN : **Importance sampling and inference in Gibbs distributions**

Inference about parameter in Gibbs distributions, $p_\beta(x) = \exp(-\beta^T V(x))/Z(\beta)$, based on complete or noisy data, can be assisted by simple ideas in importance sampling. For example, if $L(\beta) = \log P_\beta(x)$, we have

$$L(\beta) - L(\beta_0) = -(\beta - \beta_0)V(x) - \log E_{\beta_0} \exp(-(\beta - \beta_0)V(x)),$$

and the expectation can be estimated by an empirical average from a simulation at β_0 alone. Also, for any f,

$$E_\beta(f(x)) = E_{\beta_0}(f(x)e^{-(\beta-\beta_0)V(x)})/E_{\beta_0}(e^{-(\beta-\beta_0)V(x)}).$$

This can be applied with $f(x) = V(x)$ or $I_{[V(x)\leq v]}$, to give the m.l.e. or "exact" confidence intervals, again calculated by simulation. In all of this, variances are large unless $|\beta-\beta_0|$ is small, but it may still be adequate to use a coarse grid of simulation values $\{\beta_0\}$. For these, and other purposes, Hastings' (Biometrica 1970) version of Metropolis's method is recommended, in preference to the Gibbs sampler, except in simple cases where the local characteristics are easily normalized. Barone and Frigessi's work suggest a possible general "proposal" distribution.

F. COMETS : **Consistensy of estimators for Markov Random Fields.**

We discuss a (well known in probability) large deviation inequality, together with some parametric estimation for MRF. This estimates states that space averages do not behave worse than under the worst ergodic MRF distribution, whatever the true (unknown) MRF may be. We give:

- a consistency result for a general class of estimators (direct observations).
- one for M.L.E. in the case of noisy data.
- an approach for optimality (weak, but seemingly relevant here).

B. CHALMOND : **3-D paths detection for sismic data.**

The 3-D original image is composed with a background on which some pixels are illuminated. From a blurred version of this image, we want to estimate a path family which links these illuminated points, by considering a prior information. The knowledge of the blur is then incorporated to the prior information, that is used to construct a 3-D Markov model on the set of admissible families. To do that, a general scheme for constructing Markov model is proposed. This original scheme is interactive and very flexible; it permits the determination of the unknown parameters and it controls the fit of the model to the prior information.

Then the family of paths is estimated searching a local maximum of the Gibbs distribution of the Markov random field. The proposed algorithm is related to the ICM algorithm.

O. CATONI : **Image restoration and stochastic smoothing of contour lines.**

We define a Bayesian model for a noisy image g coming from an original image f, $f, g : S \to [0, 2^n[\cap N$ where $S = [0, K[^2 \cap N^2$.

The prior distribution on f is

$$P(f) = \prod_{k=1}^{n} P([f(2^{-(n-k)}] \mid [f(2^{-(n-k+1)}])$$

with

$$P([f(2^{-(n-k)}] \mid [f(2^{-(n-k+1)}] \equiv \exp\Bigg(\frac{1}{T} \sum_{i \in S} (2I_{([f_i(2^{-(n-k)}] \geq 2[f_i(2^{-(n-k+1)})+1])} - 1) \\ \times (2([f_i(2^{-(n-k)}])_2 - 1)) \Bigg),$$

where $[x] \in N$, $[x] \leq x < [x]+1$, and $(m)_2 = m - 2[m]$. The noise follows the law

$$P(f|g) = \prod_k P([g(2^{-(n-k)}] \mid [f(2^{-(n-k)}], [g(2^{-(n-k+1)}]),$$

where

$$P([g(2^{-(n-k)}] \mid [f(2^{-(n-k)}][g(2^{-(n-k+1)}]) \equiv \exp\Bigg(\gamma \sum_{i \in S} (2I_{([f_i(2^{-(n-k)}] \geq 2[g_i(2^{-(n-k+1)})+1])} - 1 \\ \times (2([g_i(2^{-(n-k)}])_2 - 1)) \Bigg),$$

Restoration is done by minimizing successively

$$E_f(C([\hat{f}(2^{-(n-k)}], [f(2^{-(n-k)}]) \mid [g(2^{-(n-k)}], [f(2^{-(n-k+1)}]),$$

where

$$C(\hat{X}, X) = \sum_{i \in S} I_{(X_i \neq \hat{X}_i)}.$$

This is achieved by maximizing the marginal laws at each site, estimated with the help of a Gibbs sampler. We need n successive sampling implying very simple updating computations.

We perform a restoration which is qualitatively different from filtering in the sense that we use only the order relation on the grey levels and preserve discontinuities corresponding to smooth edges, although we do not need for that purpose to use an explicit edge process.

W. NAGEL : **Second moments (moment measures) in Integral and Stochastic Geometry.**

The definition of the second order surface measure as a stationary random closed set is given and illustrated. Several methods for its estimation are reviewed. One of them requires the measurement of section angles, alternative ones are based on pairs of intersection points.

F. HEITZ : **Motion estimation by using Markov Random Fields.**

A multimodel approach to the problem of velocity estimation in image sequences is proposed. The theoretical framework is based on global Bayesian decision associated with Markov Random Fields models. The presented approach addresses in parallel the problem of velocity estimation, of velocity segmentation and of occluded and occluding region differentiation. Results on real-world sequence are presented.

C-R. HWANG, A. FRIGESSI, L. YOUNES : **Rates of convergence and simulation algorithms.**

To sample from a Markov random field π and to calculate the space average of a certain function w.r.t. π both can be performed efficiently by simulating properly chosen Markov chains. Nevertheless the corresponding performances are measured by different criteria.

For sampling from π : we investigate the rate of weak convergence to π in terms of the second eigenvalue in absolute value of the transition matrix. A class of local updating dynamics that are reversible w.r.t. π is considered. We study the general algebraic structure and then the stochastic Ising model in depth. Our results include : Gibbs sampler is faster than locally updating twice by Metropolis algorithm; in the Ising case, Metropolis algorithm is the best at low temperature but the worst a high temperature; there are dynamics faster than the Gibbs sampler at high temperature.

For approximating the space averages : the asymptotic variance of the time average is used to measure the performance of the approximation. We obtain an optimal spectral structure theorem for reversible (w.r.t. π) stochastic matrices and a new simulation algorithm for MRF. We compute the minimum value for the second largest eigenvalue of all such matrices and characterize the class of matrices for which this minimum is attained. In fact they share a common right eigenvector that can be computed in terms of π. Furthermore, by iterating this procedure, we obtain a unique matrix which is minimal w.r.t. the lexicographic order of the eigenvalues. We give a probabilistic interpretation of the corresponding eigenvectors. Our results allow to devise a dynamic Monte-Carlo scheme which has an optimal worst case performance. Regarding the simulation of lattice based Gibbs distribution, we design a modified Gibbs sampler whose performances are better in terms of both weak convergence at low temperatures and asymptotic variance of time averages at all temperatures.

T-S. CHIANG : **The asymptotic behaviour of simulated annealing processes with absorption.**

An SAP with absorption is an inhomogeneous Markov chain X_t with transition rates of the following type:

$$Q_{ij}(t) = P_{ij}\exp(-U(i,j)/T(t)) = P_{ij}\lambda(t)^{U(i,j)}, \quad i \neq j$$

with $\sum_j Q_{ij}(t) < 0$ for at least one i.

We study the asymptotic behaviour of X_t and prove that under some regularity conditions, the following statements hold:

$$\lim_{t\to\infty} P(X_t = i)/\left\{\lambda(t)^{h(i)}[exp(-\delta\int_0^\infty \lambda^N(t) + O(\lambda^{N+1}(t))dt)]\right\} = \beta_i \in (0,\infty),$$

where $h(i)$ and N are non negative integers and $\delta > 0$.

This kind of asymptotic results is useful in estimating the expected time of hitting a state or a set of states for simulated annealing process without absorbing states.

Y-S CHOW : **Occupation times of annealing processes.**

Simulated annealing is a probabilistic optimization algorithm for finding the global minimum (or minima) of certain function on a finite set S. It is known that this algorithm converges weakly to a certain distribution $(\beta_i)_{i \in S}$ concentrated on the global minima set $\mathbf{S}$. In this talk we show that

$$\lim_n \frac{1}{n} \sum_{t=1}^{n} I_{\{X(t)=i\}} = \beta_i \text{ a.e. for } i \in \mathbf{S}.$$

That is, the fraction of the time spent at each ground state i converges a.e. to β_i.

I. ALTHOFER and K-U KOSCHNIK : **On the Convergence of "threshold accepting".**

Simulated Annealing (SA), a stochastic modification of local search, has become a very popular tool in combinatorial optimization since its introduction in 1982. Recently Dueck and Scheuer proposed another simple modification of local search which they called "threshold Accepting" (TA).

In contrast to SA, the TA algorithm has a deterministic rule of acceptence: the new configuration at time k is accepted if its value is at most $T(k)$ units worse than the value of the old configuration. The nonnegative thresholds $T(k)$ are the counterparts to the temperature parameters $c(k)$ in SA.

We have proved some results of the type: "if for some optimization problems the SA algorithm can achieve convergence to the global optima, then TA can achieve this too." The proofs are not constructive and make use of the fact that the transition matrices of SA (viewed as a Markov process) are convex combinations of the transition matrices of TA.

O. CATONI : **Exponential cooling schedule for simulated annealing algorithms.**

Simulated annealing algorithms are used to find approximate solutions of applied optimization problems. We study the Metropolis dynamic on a finite state space, using large deviation techniques. Introducing in this time-inhomogeneous Markovian framework Wentzell and Freidlin's decomposition of the space into cycles we get estimates for the law of the entry point and time into a set of trajectories having stayed in a prescribed other set. The argument is by induction and we use some tensor calculus on these quantities, showing that there are related composition rules for the estimates. The remaining part consists in applications. We give a complement to Hajek's theorem on necessary and sufficient conditions of convergence, an upper bound for the convergence rate, asymptotics of the law of the system when the cooling schedule assumes a variety of closed formulas. We show that the optimal temperature schedule for a given large horizon has inverse logarithmic decrease in the first part of its course, but presumably not when it approaches the horizon. Contrasted with this, cooling schedules with exponential decrease properly tuned to the horizon achieve the maximum convergence rate up to logarithmic equivalent.

G. WINKLER : **L^2 convergence for annealing.**

Under a suitable annealing scheme means in time over the annealing process converge to means in space w.r.t. the limits of its marginals. In general, we must cool slower than the usual limit theorem for the marginals.

Similar work has been done by H. Föllmer and N. Gantert.

C. JENNISSON : **Aggregation in simulated annealing.**

We consider convergence of simulated annealing and implications for annealing over a finite time period. The results of Hajek (1988, Maths of O.R.) suggest the main benefit of annealing is in escaping fairly shallow local minima and, when it works well, it does so by moving quite directly to the global minimum. Exact calculations for small problems demonstrate the need for large moves in the state space. Such large moves can be achieved by using a "cascade" approach, operating on successively finer scales. Initially, annealing is carried out on a sparse subset of the state space, then more points are introduced and links of the graph become shorter. The interpretation of the sparse subgraphs will depend on the problem being considered.

F. GOTZE : **Finite time behaviour of simulated annealing.**

The finite time behaviour of simulated annealing is investigated for neighborhood graphs with hight restrictions and restriction on the length of local improving paths resulting in an explicit estimate on the probability of reaching states with energy near to the absolute minimum. These results could be applied to energy functions used in image restoration yielding polynomial expected times to reach the minimum in the case of a unique stable minimum (in the sense of Peierls), and in the case of several stable minima polynomial expected times to reach states with energy below $U_{\min} + \alpha$ where $\alpha \geq (\log n^2)^{-\frac{1}{2}}(U_{\max} - U_{\min})$ and $U(x)$ is the energy function.

C. YANG : **Stochastic algorithms on a connected image world.**

Let S be an image lattice, and $\Lambda = \{0, \ldots, L-1\}$ be a set of grey levels. Relaxation plus annealing is often used to find the minimum of a real function H defined on Λ^S. When L is large, convergence of standard dynamics (Metropolis, Gibbs sampler) is very slow.

A connected image (at some level $L_d \in \Lambda$) is defined as a configuration in which every pixel with grey level l has at least one of his neighbours with grey level l' such that $|l - l'| \leq L_d$. Minimizing H only over the set of connected configurations does not in general harm the quality of the results. However, for this purpose, easy simulation rules can be developed that provide a much faster algorithm.

R. AZENCOTT : **Annealing: parallelization technics.**

After a description of some methods for accelerating sequential annealing algorithms, an overview of various parallelization technics is given. These technics have emerged from several discussions and seminars at the ENS (Paris) within a group of researchers with common interest in the subject.

One possible device is a massively parallel annealing for random fields, with one processor per site, each site being updated at some rate τ; results from A. Trouve and I. Gaudron are presented, that say that for fully parallel schemes ($\tau = 1$) the limiting distribution can be quite far from the limiting distribution of the sequential algorithm, whereas the performances appear to be very good for $\tau < 1$, even close to 1, as shown by theoretical results, and computer simulations on spin glasses.

Another technique is multiple search, where several processors all perform their own simulated annealing with possibly different temperatures, and that may or not interact with each other. Some conjectures from C. Graffigne concerning an algorithm inspired from Aarts and Laarhoven are given.

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