MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 31/1990

Konvexgeometrie
22.07. bis 28.07.1990

Die Tagung fand unter der Leitung von Herrn R. Schneider (Freiburg i. Br.) und Herrn J.M. Wills (Siegen) statt. Sie hatte 50 Teilnehmer, von denen 43 Vorträge hielten. Angesichts des unverändert starken Andrangs zu dieser Tagung waren das noch vertretbare Obergrenzen.

Gegenüber den vorherigen Tagungen zum selben Thema waren dieses Mal eine größere Zahl von Vertretern der Geometrie der Banachräume beteiligt. Die bemerkenswerten Beiträge aus diesem Gebiet haben in den letzten Jahren auch der endlichdimensionalen Konvexgeometrie wesentliche neue Gesichtspunkte erschlossen.

Die Vorträge behandelten sowohl Entwicklungen der klassischen Konvexgeometrie als auch neuartige Problemstellungen und verschiedene Anwendungen. Besonders hervorzuheben sind die Lösungen und Teillösungen, die zu einigen bekannten, längere Zeit offenen Problemen vorgetragen werden konnten. Zu nennen sind: affine plank problem, minimale Seitenzahlen von Polytopen ohne dreieckige 2-Seiten, McMullen's g-conjecture ohne Methoden der algebraischen Geometrie, Busemann-Petty-Problem. Behandelte Themen waren ferner klassische und neuere Ungleichungen, konvexe Polytope, kombinatorische Konvexgeometrie, Packungs- und Überdeckungsprobleme (insbesondere in hohen Dimensionen), ferner Fragen der Integralgeometrie, Optimierung und computational geometry, jeweils mit Bezügen zur Konvexität, und andere. Insgesamt waren die Beiträge sehr vielseitig und zeugten von der ungebrochenen mathematischen Vitalität der Konvexgeometrie.

Vortragsauszüge

K. BALL:

The plank problem for symmetric bodies

It is shown that if C is a symmetric convex body in $\mathbb{R}^d$ and $H_1,\dots,H_n$ are n hyperplanes, there is a copy $x + \frac{1}{n+1} C$ of C, inside C, whose interior does not meet any H_i. The result solves the affine plank problem of Bang (for symmetric bodies) and has applications to Diophantine approximation.

I. BÁRÁNY:

On the integer convex hull

Given a convex set $K \subset R^d$ we write K_I for the convex hull of the integer points in K. Let $P \subset R^d$ be a rational polyhedron of size φ. It is known that P_I has at most $O(\varphi^{d-1})$ vertices. Here we give an example showing that P_I can have as many as $\Omega(\varphi^{d-1})$ vertices. The construction (which is a joint work with R. Howe and L. Lovász) uses the Dirichlet unit theorem.

The same question arises when K is a smooth convex body. For instance, let d=2 and $K=rB^2$, the disk of radius r in the plane. In a joint work with A. Balog we show that K_I has const.$r^{2/3}$ vertices. This result can be extended to smooth convex bodies in the plane.

M.M. BAYER:

Weakly Neighborly Polytopes

A polytope is weakly neighborly if every set of k+1 vertices is contained in a face of dimension at most 2k. Weakly neighborly polytopes are "equidecomposable", that is, all triangulations (with no new vertices) have the same number of simplices of each dimension. Furthermore, these numbers can be computed from the face lattice of the original polytope, using the (generalized) h-vector. Examples of weakly neighborly polytopes are even-dimensional neighborly polytopes, the product of two simplices, and Lawrence polytopes. The pyramid over any weakly neighborly polytope and any subpolytope of a weakly neighborly polytope are weakly neighborly. A study of weakly neighborly polytopes may give insight into the h-vectors of nonsimplicial polytopes.

K. BEZDEK:

Hadwiger-Levi's covering problem

The following conjecture was published by Hadwiger (1957) and also by Gohberg and Markus (1960): Any convex body of $\mathbf{E}^d$ can be covered by 2^d smaller homothetic bodies. The planar case was proved by several authors but the first proof is due to Levi (1955). The problem is open for $d \geq 3$. The best 3-dimensional result was obtained by Lassak (1984) (P.S. Soltan and V.P. Soltan (1986)) who proved the conjecture for centrally symmetric convex bodies (centrally symmetric convex polytopes) of $\mathbf{E}^3$. We generalize this result as follows:

Theorem 1. Let $\mathbf{P}$ be a convex polyhedron of $\mathbf{E}^3$ with affine symmetry. Then $\mathbf{P}$ can be covered by 8 smaller homothetic copies.

The 21 pages proof of the above theorem is based on the following separation technique.

Theorem 2. The light-sources (i.e. affine subspaces) $L_1, L_2 \ldots, L_n \subset \mathbf{E}^d \setminus \mathbf{K}$, $d \geq 1$, $0 \leq \dim L_1 = \ldots = \dim L_n = \ell \leq d-1$ illuminate the boundary of the closed convex set $\mathbf{K} \subset \mathbf{E}^d$, $0 \in \operatorname{int} \mathbf{K}$ if and only if any face of the polar set $\mathbf{K}^* = \{X \in \mathbf{E}^d \mid \langle \overrightarrow{OX}, \overrightarrow{OY} \rangle \leq 1$ for all $Y \in \mathbf{K}\}$ which is disjoint from 0 can be strictly separated from 0 by a hyperplane of $\mathbf{E}^d$ which contains at least one of the affine subspaces $(0 \notin) \hat{L}_1, \hat{L}_2, \ldots, \hat{L}_n$ of $\mathbf{E}^d$ of dimension $d-\ell-1$ where

$$\hat{L}_i = \bigcap_{Q \in L_i} \{H_Q \mid H_Q = \{X \in \mathbf{E}^d \mid \langle \overrightarrow{OX}, \overrightarrow{OQ} \rangle = 1\}\}, \quad 1 \leq i \leq n.$$

Th. 2 made it possible to find short proofs for most known results too.

T. BISZTRICZKY:

Line segment illuminators

Let S be a domain (compact, simply connected set) in the plane. A (line) segment $L \subset S$ illuminates S if for any $p \in S$, there is a $q \in L$ such that $pq = \operatorname{conv}\{p, q\} \subset S$.

In a joint work with A. Bezdek and K. Bezdek, we obtained the following results (among others) about illuminating S by a segment of S:

1. If any two points of S are illuminated by a segment of S parallel to a given line α then S contains a segment illuminator parallel to α.

2. If any three points of S with a smooth boundary are illuminated by a translate of a segment T then S is illuminated by a segment of S which is a translate of T.
3. If any three points of an 0-symmetric S with boundary a simple polygon are illuminated by a segment of S through 0 then S contains a 0-symmetric illuminator which is either a segment or a parallelogram.

R. BLIND:

Convex polytopes without triangular 2-faces

There is a conjecture of Kupitz (1980): Is it true, that a convex polytope with less vertices than the cube has always a triangular 2-face?

This is proved by

Theorem 1: Let P be a d-polytope without triangular 2-faces, and with $f_j(P)$ j-faces ($j=0,\dots d-1$). Let C^d be the d-cube.
Then $f_j(P) \geq f_j(C^d)$, and equality is attained for some j if and only if P is combinatorially equivalent to the d-cube.

The main tool in the proof is the fact that such polytopes have pairs of disjoint facets according to

Theorem 2: Let P be a d-polytope without triangular 2-faces. Let F be a facet of P. Then there exists a facet F' disjoint from F.

Another consequence of Theorem 2 is

Theorem 3: Let P be a d-polytope without triangular 2-faces and with 2d+1 facets. Then P is combinatorially equivalent to the (d-2)-fold d-prism over the pentagon.

J. BOKOWSKI:

On the final polynomial method in convexity with extensions

Many well known theorems about convex polytopes have been extended to the more general case of matroid polytopes. The rapid growth of oriented matroid theory with its many independent sources still benefits from the final polynomial method with its origin in convexity as far as the natural realizability question is concerned.

The talk reports about new applications and classifications.

J.B./B. Sturmfels, Computational synthetic geometry, Springer '89.
J.B., On the generation of oriented matroids with prescribed topes.
J.B./W. Kollewe, On representing contexts in line arrangments.
J.B./J. Richter, On the finding of final polynomials, Europ. J. Comb. '90.
J.B./J. Richter, On the classification of non-real oriented matroids.
J.B./J. Richter, A new Sylvester-Gallai configuration...
J.B./J. Richter/W. Schindler, On the distribution of order types.
J.B./A. Guedes de Oliveira, On the orientability of matroids.

J. BOURGAIN:

On the Busemann-Petty problem

(no abstract)

U. BREHM:

X-ray problems with point sources

For each finite set of points (sources) $X \subseteq \mathbb{R}^d$ (no three collinear), families of bounded Borel sets (bounded away from the sources) having the same X-ray images from the points in X are constructed in a universal way.
We can even prescribe the sets in some neighborhood U of any given countable bounded set Y, which is bounded away from the lines connecting the sources.
Finally it is shown that for a bounded Borel set there exists a pair of sets each consisting of 12 homothetic images of the given set, having the same X-ray image from two point sources, which implies that there is a pair of star-shaped polygons having the same X-ray image from two point sources.

G. EWALD:

On the Alexandrov-Fenchel equality

We modify a conjecture of R. Schneider on equality in the Alexandrov-Fenchel inequality for $V(K,L,K_1,\ldots,K_{n-2})$ in case $\dim (K_1+\ldots+K_{n-2}) = n-1$ and prove this modification for polytopes.

G. FEJES TÓTH:

Packing convex bodies in E^d

Blichfeldt's density function method was used to derive upper bounds for the packing density of convex bodies in E^d. Among others, the following theorem was proved:

Let K be a convex body in E^d with insphere radius r. For some real numbers $0 < \rho \leq r$ and $1 < \lambda \leq \sqrt{3}$, consider the outer parallel set $(K_{-\rho})_{\lambda\rho}$ of radius $\lambda\rho$ of the inner parallel set of radius ρ of K. Denote by $M(d,\phi)$ the maximum number of spherical caps of angular diameter ϕ forming a packing on the d-dimensional spherical space S^d. If congruent copies of K form a packing in E^d, then their (upper) density is at most

$$\frac{V(K)[M(d-1,\arccos(1-2/\lambda^2))+1]}{V\left((K_{-\rho})_{\lambda\rho}\right)}.$$

This bound is particularly strong for cylinders of large height.

W.J. FIREY:

Worn Stones; a Second Look

A convex stone is tumbling at random inside a closed convex surface, whose inner surface is coated with abrasive material. What can be said about the successive shapes of the stone? It is assumed that the stone rolls freely within the abrasive surface. By idealization, a partial differential equation, with initial data, is developed which is fully non-linear (à la Monge-Ampère) and governs this process. Some properties of solutions will be given.

R.J. GARDNER:

Cutting corners

The talk centers around the following problem of L. Fejes Tôth. If P_0 is a convex polyhedron, define a sequence (P_n) by letting P_n be the convex hull of the midpoints of the edges of P_{n-1}. What can be said about the boundary structure of $\lim_n P_n$? Some results are given which represent joint work with M. Kallay. The problem and similar ones have connections to certain areas of applicable mathematics, and these will also be discussed.

P. GOODEY (joint work with W. WEIL):

Crofton Formulas and Radon Transforms

We investigate the Radon transforms $R_{ij} : C(L_i^d) \to C(L_j^d)$

when applied to projection functions of convex bodies. In the case of higher rank Grassmannians (i.e. $i,j \neq 1,d-1$) these are not necessarily injective operators.
If $i < j$, Hadwiger's characterization of intrinsic volumes gives

$$[R_{ij}V_i(K|\cdot)](L) = V_i(K|L)$$

where $L \in \mathcal{L}_j^d$.
For $i > j$, we obtain the Crofton-type formula

$$[R_{ij}V_i(K|\cdot)](L) = \int_{\mathcal{E}_{d-i+j}^d} V_j(K \cap E|L)\mu_{d-i+j}(dE)$$

for centrally symmetric bodies K.
We will also discuss the injectivity properties of these Radon transforms when restricted to projection functions of convex bodies.

J.E. GOODMAN:

<u>Common Tangents (and Common Transversals)</u>

Two years ago M. Sharir posed the problem of finding the combinatorial complexity of the space of hyperplane transversals to a suitably separated family of compact convex sets in $\mathbb{R}^d$. The solution to the corresponding problem in the plane had been found shortly before by H. Edelsbrunner and M. Sharir, and Sharir wondered whether their result could be extended to higher dimensions.
In joint work with S. Cappell, J. Pach, R. Pollack, M. Sharir, and R. Wenger, we have succeeded in solving this problem, and in fact in (asymptotically) extending the Edelsbrunner-Sharir upper bound.
It turns out that the following purely geometric result plays a key role in the solution:
<u>Theorem</u>. If $A = \{a_1,\ldots,a_k\}$, $1 \leq k \leq d$, is a separated family of compact and strictly convex sets in $\mathbb{R}^d$, and $\mathcal{T}_{d-1}(A)$ is the space of oriented hyperplanes which support all the members of A and whose normal vectors point away from each a_i, then $\mathcal{T}_{d-1}(A)$ is a topological sphere of dimension $d-k$.
This theorem is intuitively appealing, but its proof has turned out to be surprisingly elusive. In the proposed talk, we discuss the theorem and its proof.

P. GRITZMANN:

Minkowski addition of polytopes: Computational complexity and applications

The talk which is based on joint work with Bernd Sturmfels deals with a problem from computational convexity and indicates an application to computer algebra.

We determine the complexity of computing the Minkowski sum of k convex polytopes in $\mathbb{R}^d$, which are presented either in terms of vertices or in terms of facets. In particular, if the dimension d is fixed, we obtain a polynomial time algorithm for adding k polytopes with up to n vertices.

This result can be used in the context of Buchberger's Gröbner bases algorithm for polynomial ideals. Applying our computational results to Newton polytopes we show that the following problem can be solved in polynomial time for any finite set of polynomials $F \subset K[x_1,\dots,x_d]$, where d is fixed: Does there exist a term order τ such that F is a Gröbner basis for its ideal with respect to τ?

P. GRUBER:

Endomorphisms of the lattice of convex bodies

Let $\mathcal{C}$ be the space of all compact convex subsets of $\mathbb{E}^d$. $\mathcal{C}$ endowed with the ordinary intersection $\wedge$ and the convex hull of the union $\vee$ is a lattice. An endomorphism of $\langle \mathcal{C}, \wedge, \vee \rangle$ is a mapping $E : \mathcal{C} \to \mathcal{C}$ such that $E(C \wedge D) = E(C) \wedge E(D)$ and $E(C \vee D) = E(C) \vee E(D)$ for $C, D \in \mathcal{C}$.

Theorem. A mapping $E : \mathcal{C} \to \mathcal{C}$ is an endomorphism of the lattice $\langle \mathcal{C}, \wedge, \vee \rangle$ precisely in the following cases:

(i) There is a fixed convex body $D \in \mathcal{C}$ such that $E(C) = D$ for each $C \in \mathcal{C}$.

(ii) $d = 1$ and there is a strictly monotone function $\varphi : \mathbb{R}\ (= \mathbb{E}^1) \to \mathbb{R}$ such that

(a) $E(\emptyset) = \emptyset$,

(b) $E(x)$ is a (compact) interval (possibly a point) contained in the interval $\varphi(x-0) \vee \varphi(x+0)$ for each $x \in \mathbb{R}$, and

(c) $E(x \vee y) = E(x) \vee E(y)$ for each interval $x \vee y \subset \mathbb{R}$.

(iii) $d > 1$ and there is a non-singular affine transformation $a : \mathbf{E}^d \to \mathbf{E}^d$ such that $E(C) = a(C) (=\{a(x) : x \in C\})$ for each $C \in \mathcal{C}$.

G. KALAI:

Characterization of f-vectors for simplicial spheres

Let $f = (f_0, f_1, \ldots, f_{d-1})$ be a vector of non-negative integers. Put $f_{-1} = 1$ and define $h = h[f] = (h_0, h_1, \ldots, h_d)$ by the relation

$$\sum_{k=0}^{d} h_k x^{d-k} = \sum_{k=0}^{d} f_{k-1}(x-1)^{d-k}.$$

The following theorem was conjectured by P. McMullen around 1970 and proved by Billera and Lee (Sufficiency part) and Stanley (Necessity part):

"g-Theorem". $f = (f_o, f_1, \ldots, f_{d-1})$ is the f-vector of a simplicial d-polytope iff $h[f] = (h_0, \ldots, h_d)$ satisfies the following relations

(i) $h_i = h_{d-i}$

(ii) $1 = h_0 \leq h_1 \leq h_2 \leq \ldots \leq h_{[\frac{d}{2}]}$

(iii) Put $h_k - h_{k-1} = \binom{a_k}{k} + \binom{a_{k-1}}{k-1} + \ldots + \binom{a_i}{i}$ where $a_k > a_{k-1} > \ldots > a_i \geq i > 0$,

then $h_{k-1} - h_{k-2} \geq \binom{a_k - 1}{k-1} + \binom{a_{k-1} - 1}{k-2} + \ldots + \binom{a_i - 1}{i-1}$.

Our main result is:

Theorem 2. Conditions (i),(ii) and (iii) hold for the f-vectors of arbitrary simplicial (d-1)-dimensional spheres.

The proof relies on the method of algebraic shifting which was introduced by me a few years ago and, in particular, on a refinement of SARKARIA's results on shifting and embeddability and on my results on the relation between shifting, homology and Cohen-Macaulayness.

J. KINCSES:

Intersection properties of convex bodies

A convex body has the (n,k) intersection property if any n translates of K intersect provided any k of them intersect and it has the k I.P. if it has the (n,k) I.P. for each $n > k$. The classification problem of convex bodies was solved so far only when $k = 2$ or 3. The main aim of this talk is to give the complete classification of convex bodies with 4 and 5 intersection properties.

P. KLEINSCHMIDT:

Curves in polytopes

We present curves in the interior of a convex polytope which come from the action of a one-parameter torus group. The curves induce a strange line-geometry and a metric in the polytope. They have a similar behavior as the trajectories generated by Karmarkar's algorithm in the respect that they converge to an LP optimum.

D.G. LARMAN (joint work with T.C. HU and V.L. KLEE):

Optimisation and a measure of asymmetry

We define a continuous function φ to be δ-mid point convex on a domain in a finite dimensional normed space E if

$$\varphi(\tfrac{1}{2}x+\tfrac{1}{2}y) \leq \tfrac{1}{2}\varphi(x) + \tfrac{1}{2}\varphi(y) \qquad \text{if } \|x-y\| \geq \delta.$$

This definition allows 'blips' of nonconvexity in the function, e.g. $\varphi(t) = \eta t^2 + \cos t$ is convex on $\mathbb{R}$ if $\eta \geq \frac{1}{2}$ and δ-convex on $\mathbb{R}$ if $\eta \geq \frac{1}{2}\,\frac{\sin^2 \delta/4}{(\delta/4)^2}$.

To facilitate the discussion we introduce a notion of asymmetry $p(C)$ on every convex body C in E. If ext C is the set of extreme points of C and $c \in C \setminus \text{ext } C$ we define $\xi(C,c) = \inf\{\|c-p\|; p \in \text{ext } C\}$, $p(C,c) = \frac{1}{2}$ diameter $(C \cap 2c-C)$. Let $\rho(C,c) = \xi(C,c)/p(C,c)$; $\rho(C) = \sup\{\rho(C,c); c \in C \setminus \text{ext } C\}$; $\rho(E) = \sup\{\rho(C); C \subset E\}$. Then, if E is d-dimensional, $\rho(E) \leq d$ and this bound is sharp. In R^d, the simplex T^d has $\rho(T^d) = \sqrt{d+1}$, d even, $\rho(T^d) = \sqrt{d}$, d odd. Probably $\rho(R^d) = \sqrt{d+1}$, d even, $\rho(R^d) = \sqrt{d}$, d odd, and this is true for $d = 2$. However, only (essentially) the upper bound $\rho(R^d) \leq \sqrt{5d}$ is known.

For the function φ the following theorems hold which are obvious analogues to known results about convex functions:

Theorem. If φ is a mid-point δ convex function that attains a local δ-minimum at a point q then q is the global minimum point.

<u>Theorem</u>. Suppose that φ is a mid-point δ convex function defined on a convex body C in a finite dimensional normed space E. Then, for each value α of φ there are points q and $x \in C$ such that $\varphi(q) \geq \alpha$, x is an extreme point of C and $\|q-x\| \leq \delta\rho(C)$.

M. LASSAK:

<u>Approximation of convex bodies, reduced bodies</u>

A convex body R of Euclidean d-space E^d is called reduced if there is no convex body properly contained in R of thickness $\Delta(R)$ of R. Here are a few from the presented properties of reduced bodies in E^2. The width of a reduced body $R \subset E^2$ in direction ℓ is $\Delta(R)$ if and only if at least one of the supporting lines of R perpendicular to ℓ is the left or the right supporting line of R. Through every boundary point of R a supporting line passes such that the width in the perpendicular direction is $\Delta(R)$. The diameter of R is not greater than $\sqrt{2}\Delta(R)$ and the perimeter is at most $(2 + \frac{1}{2}\pi)\Delta(R)$. The diameter of every reduced polygon V is not greater than $\frac{2}{3}\sqrt{3}\Delta(V)$ and the perimeter is at most $2\sqrt{3}\Delta(V)$. All the estimates are the best possible.

K. LEICHTWEISS:

<u>On various definitions of an affine surface area</u>

Starting from Minkowski's definition

(1) $$O(K) := \lim_{\rho \to 0} \frac{1}{\rho}(V(K_\rho) - V(K))$$

of the euclidean surface area O(K) of an arbitrary convex body K in $\mathbb{R}^n$ W. BLASCHKE 1923 (for n = 3) resp. H. BUSEMANN 1945 proposed to define an equiaffinely invariant surface area of K by

(2) $$O_{aff}(K) := \lim_{\delta \to 0} c_n \frac{1}{\delta^{\frac{2}{n+1}}} (V(K) - V(K_{[\delta]}))$$

resp.

(3) $$\left(O'_{aff}(K)\right)^{\frac{n+1}{n}} := \inf_E \left(\lim_{\rho \to 0} \frac{1}{\rho}(V(K^E_\rho) - V(K))\right)$$

($K_\rho := K + \rho B$ = outer parallel body of K, $K_{[\delta]}$ = floating body of K and $K_\rho^E := K + \rho E$ = outer parallel body of K relative to an ellipsoid E with center 0 and volume V(B), c_n = constant depending only on the dimension n.)

Later K. LEICHTWEISS 1986 and C. SCHÜTT, E. WERNER 1989 resp. C. PETTY 1974 and E. LUTWAK 1988 made improvements of the definitions (2) resp. (3). It is the aim of the talk to discuss these improvements and to compare them with each other.

J. LINDENSTRAUSS:

Covering a set with diameter 1 in R^n by balls of the same diameter

Let c_n be the smallest number such that any set K of diameter 1 in R^n can be covered by c_n balls of diameter 1. It is proved that

$$\left(\sqrt{\tfrac{9}{8}} - \varepsilon\right)^n \le c_n \le \left(\sqrt{\tfrac{3}{2}} + \varepsilon\right)^n \qquad \text{for } n \ge n(\varepsilon).$$

The lower estimate is easy and can also be slightly improved. The main point is in the upper estimate. The question whether it can be improved can be reduced to considering subsets of diameter 1 of the sphere of radius $\sqrt{\frac{3}{8}}$.

This work is a joint work with J. Bourgain.

E. LUTWAK:

Inequalities for mixed projection bodies

Mixed projection bodies are related to ordinary projection bodies in the same way that mixed volumes are related to ordinary volume. It turns out that one can prove analogues for mixed projection bodies of the classical Brunn-Minkowski and Alexandrov-Fenchel inequalities. As a consequence of these inequalities one has results such as:

Theorem. If C is a class of convex bodies in $\mathbb{R}^n$ and

$$V(\Pi_1(K,Q)) = V(\Pi_1(L,Q)), \qquad \text{for all } Q \in C$$

or

$$V(\Pi_1(Q,K)) = V(\Pi_1(Q,L)), \qquad \text{for all } Q \in C$$

then K and L are translates.

Here V denotes n-dimensional volume and $\Pi_1(C,D)$ is the mixed projection body whose support function, for $u \in S^{n-1}$, is given by $h(\Pi_1(C,D),u) = v(C^u,\dots,C^u,D^u)$, where C^u, D^u denote the images of the orthogonal projections of C,D onto the hyperplane orthogonal to u, and v denotes the (n-1)-dimensional mixed volume. This result holds if the volume V is replaced by any quermass-integral W_i.

P. MANI-LEVITSKA:

Dvoretzky's Theorem revisited

A strong version of A. Dvoretzky's Theorem on almost spherical sections can be derived from the following relative of Knaster's Conjecture: Let $f : S^{n-1} \to \mathbb{R}$ be a continuous function on the Euclidean sphere, and consider a set $K \subset S^{n-1}$ with n points, such that the symmetry group $\Gamma(K) = \{ r \in O(n) : rK = K \}$ is generated by $\{\sigma_{e_n}, \sigma_{e_{n-1}}, \sigma_{e_{n-2}}\}$, where σ_p, $p \in S^{n-1}$, is the reflection $\sigma_p(x) = x-2<x,p>p$. Then there is a rotation $r \in O(n)$ such that f, restricted to rK, is a constant map. Our proof uses simple algebraic topology, with one element still not clean: that a certain decomposition of the quotient space $O(n)/\Gamma(K)$ is properly cellular.

H. MARTINI (joint work with E. MAKAI jr.):

A new characterization of convex plates of constant width

Let $D \subset \mathbb{R}^n$ $(n \geq 2)$ be a convex body of diameter 1. We say that D has the property (P) if any n mutually perpendicular chords, having a common point, have total length ≥ 1.
It is shown that a convex plate $D \subset \mathbb{R}^2$ of diameter 1 has property (P) if and only if it is of constant width 1. Moreover, if D has constant width 1, in property (P) we have strict inequality for non-degenerate chords. In higher dimensions, there is shown only one implication: Let a convex body $D \subset \mathbb{R}^n$ $(n \geq 2)$ of diameter 1 satisfy the property (P) (for non-degenerate chords). Then D is of constant width.

P. McMULLEN:

On certain euclidean tilings

The "projection method" for constructing quasiperiodic tilings is shown to result from applying duality and general sections to certain tilings of euclidean space, which generalize Voronoi tilings. The "projection" should then be regarded as mapping the lower dimensional tiling back into the "broken surface" in the original tiling.

M. MEYER:

On sections of convex bodies

By definition, a convex centrally symmetric body K in $\mathbb{R}^d$ verifies Buseman-Petty property B.P.p (resp. B.P.p.c) if for any other centrally symmetric convex body L such that $|L \cap H| \geq |K \cap H|$ for every hyperplane H through 0, we have $|L| \geq |K|$ (resp. $|L| \geq c|K|$), where $|\cdot|$ denotes volume in $\mathbb{R}^{d-1}$ or $\mathbb{R}^d$. The following results are stated and proved.

1) The cross polytope $K = \{(x_i)_{i=1}^{i=d}, \sum_{i=1}^{i=d} |x_i| \leq 1\}$ verifies B.P.p.

2) For some constant $c > 0$, independent on d, the polar bodies of zonoids verify B.P.p.c.

V. MILMAN:

Some facts of the asymptotic theory of convex bodies

A proof of the following theorem will be outlined:

There is a universal constant C such that for every $n \in \mathbb{N}$ and any convex (centrally symmetric) compact body $K \subset \mathbb{R}^n$ there are two linear maps $u_1, u_2 \in SL_n$ such that, if

$$T = \operatorname{conv}(K \cup u_1 K)$$

then

$$P = T \cap u_2 T$$

C-isomorphic to some ellipsoid $\mathcal{E}$ (i.e. $d(P,\mathcal{E}) = \inf\{R/r \,|\, P \subset R\mathcal{E}, r\mathcal{E} \subset P\} \leq C$). We will derive this fact from the inverse Brunn-Minkowski inequality (of the author -85). We also recall and discuss this inequality.

J. PACH:

On a problem of Hadwiger and Kneser

Let $\{c_o, \ldots, c_n\}, \{c_o', \ldots, c_n'\}$ be two point sets in the plane satisfying

(*) $\qquad |c_i - c_j| \leq |c_i' - c_j'| \qquad$ for all i and j.

Vasilis Capoyleas and I have proved that under these circumstances the perimeter of the convex hull of $\{c_o,\ldots,c_n\}$ cannot exceed the perimeter of the convex hull of $\{c'_o,\ldots,c'_n\}$.
This settles a weak version of a longstanding conjecture of H. Hadwiger and M. Kneser, which states that the above assumptions imply

$$\text{Area}\left(\bigcup_{i=0}^{n} C_i\right) \leq \text{Area}\left(\bigcup_{i=0}^{n} C'_i\right),$$

where C_i and C'_i denote the unit disks around c_i and c'_i, respectively.
We can also prove a similar theorem in the maximum norm: If $\{c_0,\ldots,c_n\},\{c'_0,\ldots,c'_n\}$ are two point sets in the plane satisfying (*) in l_∞ (the maximum norm), then

$$\text{Per}_\infty \text{ conv}\{c_0,\ldots,c_n\} \leq \text{Per}_\infty \text{ conv}\{c'_0,\ldots,c'_n\},$$

where Per_∞ stands for the perimeter measured in l_∞.

A. PAJOR:

Volume of symmetric polytopes with few faces

We outline the proof of the following result (a joint work with K. Ball). Let $u_1,\ldots,u_n$ be vectors in $\mathbb{R}^k$, $k \leq n$, $1 \leq p < \infty$ and $r = \left(\sum_{i=1}^{n} |u_i|^p/k\right)^{1/p}$ then the volume of the symmetric polytope P whose boundary functionals are $\pm u_1,\pm u_2,\ldots,\pm u_n$ is bounded from below as $\text{vol}(P)^{1/k} \geq 1/r\sqrt{p}$. This contains the estimate by Vaaler and Gluskin.

M.A. PERLES (in collaboration with N. PRABHU):

Gaps in the vertex numbers of simple d-polytopes

For $d \geq 2$ define: $V_d = \{f_0(P) : P \text{ a simple d-polytope}\}$
$(= \{f_{d-1}(P) : P \text{ a simplicial d-polytope}\})$.
For d even, V_d contains all sufficiently large positive integers. For d odd, V_d consists of even numbers only, and contains all sufficiently large even integers. Note also that if $v \in V_d$, then $v + d - 1 \in V_d$ (truncation at a vertex). Thus V_d is determined by the smallest members of its intersections with the various residue classes modulo d-1.
Define g(d) to be the largest integer not in V_d (d even), or the largest even integer not in V_d (d odd). The following

theorem describes the asymptotic behavior of $g(d)$ as $d\to\infty$.

Theorem.

$$g(d) = \begin{cases} d\sqrt{d} + O(d^{5/4}) & \text{for } d \text{ odd} \\ d\sqrt{2d} + O(d^{5/4}) & \text{for } d \text{ even.} \end{cases}$$

S. REISNER (joint work with M. MEYER):

Characterization of affinely rotation invariant log-concave measures

The following is proved in analogy with a similar characterization of ellipsoids: Let μ be a finite, symmetric and non-degenerate log-concave measure on $\mathbb{R}^n$. The following are equivalent:

1) There exists a regular affine transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ so that the measure $T\mu$ is rotation invariant.
2) For every direction $u \in S^{n-1}$, all the centroids, with respect to the mass distribution induced by μ, of hyperplanes orthogonal to u lie on a straight line.

J.R. SANGWINE-YAGER:

Stability for a cap body inequality

If K is a convex body in $\mathbb{E}^d$ with inradius r and $\varepsilon > W_{d-2}-rW_{d-1}$, we construct a cap body C close to K in terms of the Hausdorff metric. This Hausdorff distance is bounded when r and the circumradius of K are bounded. To prove the result we solve the following problem for $i = 1$: Let $\chi_1,\dots,\chi_{i+1} \in \mathrm{bd}K$ determine an i-flat H. Find a small set $\omega \subset S^{d-1}$ such that $S_i(K,\omega) \geq cV_i(K \cap H)$, where c depends on i,d, the χ_j. In conclusion we show that $W_1^2-W_0W_2 \geq c(K)(W_1-rW_2)$. For $d = 3$ this yields a weak stability result for cap bodies.

R. SCHNEIDER:

Stability estimates for some geometric inequalities

In joint work with Helmut Groemer, inequalities of the following type are established. Let $W_0,W_1,\dots,W_d$ $(= \kappa_d)$ be the quermassintegrals of a convex body K in $\mathbb{R}^d$ whose inradius ρ and circumradius R satisfy $\rho_0 \leq \rho \leq R \leq R_0$, where $\rho_0,R_0 > 0$ are given. Then

$$W_j^{d-i} - \kappa_d^{j-i}W_i^{d-j} \geq c(d,\rho_0,R_0)\,\delta(K,B_K)^{\frac{d+3}{2}}$$

for $0 \leq i < j < d$, with a constant $c(d,\rho_0,R_0)$ independent of K. Here δ denotes the Hausdorff metric and B_K is the ball which has the same Steiner point and mean width as K. A special case gives a Bonnesen-type improvement of the isoperimetric inequality.

C. SCHÜTT:

The floating body of polytopes

Let K be a convex body. The convex floating body K_δ of K is the intersection of all halfspaces whose defining hyperplanes cut off a set of volume δ from the set K.
Blaschke showed that

$$\lim_{\delta \to 0} c_n \frac{\mathrm{vol}_n(K)-\mathrm{vol}_n(K_\delta)}{\delta^{\frac{2}{n+1}}} = \int_{\partial K} K(x)^{\frac{1}{n+1}} \, d\mu(x)$$

provided that $n = 3$, δ small and ∂K is in the class C^∞. This result was generalized to arbitrary dimensions by Leichtweiß. In a paper by Schütt and Werner this formula was established for all convex bodies where $K(x)$ denotes now the generalized Gauss Kronecker curvature.
For polytopes the above integral is 0 and thus the formula does not provide any information on the volume of $K \setminus K_\delta$. In a paper by Bárány and Larman we have for polytopes P

$$\mathrm{vol}_n(P) - \mathrm{vol}_n(P_\delta) \sim C(P)\,\delta\left(\log \frac{1}{\delta}\right)^{n-1}$$

We were able to determine the coefficient $C(P)$.
Theorem. Let P be a polytope in $\mathbb{R}^n$ with nonempty interior. Then

$$\lim_{\delta \to 0} \frac{\mathrm{vol}_n(P)-\mathrm{vol}_n(P_\delta)}{\delta\left(\log \frac{1}{\delta}\right)^{n-1}} = \frac{\psi_n(P)}{n!\,n^{n-1}}$$

where $\psi_n(P)$ is the number of flags of the polytope P, i.e.
$\psi_n(P) = \#\{(f_0,\ldots,f_{n-1}) \mid f_i \text{ i-dimensional face of } P,\ f_i \subseteq f_{i+1}\}$.

G.C. SHEPHARD:

Duality

Let F be a partially ordered set and $\delta : F \to F$ be a bijection that reverses the ordering, that is $x < y \iff \delta(y) < \delta(x)$. Then δ is called a self-duality, and F is said to be self-dual. In this case $\delta^2 = \alpha$ is an automorphism of F and we write D(F) and

A(F) for the group of self-dualities and automorphisms of F, and the group of automorphisms of F. For any such δ define the rank of δ as the smallest r such that δ^r is the identity; if no such r exists the rank is infinity. For a given self-dual F we define the rank of F as the smallest r such that F possesses a self-duality of rank r. In this lecture these ideas were applied to tilings of the plane and to convex polyhedra (polytopes in E^3) with F as the lattice of faces. It is remarkable that before 1988 no examples of tilings or polyhedra were published with rank greater than 2.

Several results were stated, including the following:

Theorem 1. The rank of any self-dual tiling of the plane is either 2, 4 or ∞.

Theorem 2. For every positive integer k there exists a convex self-dual polyhedron Q_k such that rank of $Q_k = 2^k$.

Theorem 3. If P is a self-dual, centrally symmetric polyhedron then rank P is either 2 or 4.

This work was done jointly with Jonathan Ashley, Branko Grünbaum and Walter Stiernquist.

A. VOLČIČ :

Inscribed polygons and tomography of convex bodies

The following result generalizes a theorem due to O. Giering (1962):

Theorem. Let K be a plane convex body, l a line s.t. $K \cap l = \emptyset$, $p_1 \dots p_n$ points on l. Suppose moreover that Q is a polygon inscribed in K, such that the edges are contained in lines through the p_i's. Then if E is a measurable set having the same projections as K at the p_i's, $Q \subset E$ a.e.

Kramer and Németh (1972) proved that given a strictly convex and smooth convex body K and three directions, there are exactly two inscribed triangles with edges in those directions.

Using an idea of Armsen (1977), we generalize that result to triangles having edges on lines through any three points $p_1, p_2, p_3 \notin K$, and study the existence of inscribed convex n-gons. In particular, we show that for some choice of five directions, no of the 22 types of convex bounded pentagons can be inscribed in a disc.

B. WEISSBACH:

Körper fester Breite und Überdeckungsaussagen

Im Zusammenhang mit Überdeckungsaussagen für beschränkte Mengen stößt man oft naturgemäß auf Körper konstanter Breite. Andererseits kann man gelegentlich allgemeine Vermutungen zumindest für diese besonderen konvexen Körper bestätigen.
Vorgestellt werden einerseits Abschätzungen für Volumen und Oberfläche konvexer Körper mit Mittelpunkt, in die man jede Menge mit gegebenem Durchmesser durch eine Bewegung einlagern kann.
Zum anderen wird gezeigt, daß Körper fester Breite in euklidischen Räumen genügend hoher Dimension d durch weniger als 2^d Translate ihres offenen Kerns überdeckt werden können.

J.A. WIEACKER:

Integral geometry in Minkowski space

We show that several results of euclidean integral geometry concerning intersections of surfaces have analogues in a d-dimensional Minkowski space (i.e. a d-dimensional real normed space). To do this we have to replace the Hausdorff measures $\mu^1,\dots,\mu^{d-1}$ by surface areas $\nu^1,\dots,\nu^{d-1}$ which have been previously considered by Holmes and Thompson (Pacific J. of Math. 85(1979),77-110) in a different context. In particular, it turns out that ν^{d-1} is induced by a measure generalizing the usual (d-1)-dimensional integralgeometric measure of euclidean geometry, and similar results hold for $\nu^1,\dots,\nu^{d-2}$ if the metric of the space is a hypermetric.

J.M. WILLS (joint work with M. HENK):

Successive-minima-type inequalities

For a centrally symmetric convex body K in Euclidean d-space E^d, its volume V(K) and its successive minima (with respect to the lattice $\mathbb{Z}^d$) $\lambda_1(K) \leq \dots \leq \lambda_d(K)$ Minkowski proved

$$(d!)^{-1}2^d \leq \lambda_1(K)\dots\lambda_d(K)V(K) \leq 2^d$$

which improves his fundamental theorem in geometry of numbers:

$$(\lambda_1(K))^d V(K) \leq 2^d.$$

Inequalities of same type (for higher successive minima) were proved by Hlawka et al. We show that inequalities of this type hold for various convexity functions (K symmetric or asymmetric):

(1) for Minkowski's quermassintegrals and the λ_i,

(2) for the lattice point enumerator and the λ_i,

(3) for series of in- and outradii generated by sections and projections,

(4) for the covering minima introduced by Kannan and Lovász in the theory of integer programming.

T. ZAMFIRESCU:

Long geodesics on convex surfaces

It is unknown whether a convex surface has a geodesic of length at least 1. We show this to be true for most convex surfaces. Moreover we prove the existence of geodesics of arbitrary lengths without self-intersections, on most convex surfaces. A sketch of proof will be presented.

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