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Mathematical Methods in Tomography

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The conference has been organized by *G.T. Herman* (Philadelphia), *A.K. Louis* (Berlin) and *F. Natterer* (Münster). The intention of this conference was to bring together researchers whose interest ranges from the theory of abstract Radon transforms to the diverse applications of tomography. The lively scientific atmosphere of the conference resulted in many stimulating discussions which certainly will contribute to further progress in the field.

Abstracts

J. BOMAN:

Helgason's support theorem for Radon transforms – a new proof and a generalization

We consider generalized Radon transforms of the form $R_\rho f(H) = \int_H f(\cdot)\rho(\cdot, H)ds$, for $H \in \mathcal{H}_n$, the manifold of all hyperplanes in $\mathbf{R}^n$. Here f is a function on $\mathbf{R}^n$ and ρ is a smooth, positive function of the manifold Z consisting of all pairs (x, H), $x \in H$, $H \in \mathcal{H}_n$. Consider $\mathbf{R}^n$ as sitting inside the real projective space $\mathbf{P}^n$, and let $\tilde{Z}$ be the manifold of all $(\tilde{x}, \tilde{H})$, $\tilde{x} \in \tilde{H}$, $\tilde{H}$ hyperplane in $\mathbf{P}^n$; thus $\tilde{Z}$ is a compactification of Z. We prove the following theorem, which generalizes the well-known support theorem of Helgason.

Theorem. Assume ρ is a positive real analytic function on Z that can be extended to a positive real analytic function on $\tilde{Z}$. Let K be a compact, convex subset of $\mathbf{R}^n$. Let f be continuous on $\mathbf{R}^n$,

$$f(x) = O(|x|^{-m}) \text{ as } |x| \to \infty \quad \text{for all } m,$$

and assume $R_\rho f(H) = 0$ for all H disjoint from K. Then $f = 0$ outside K.

The case when f is assumed to have compact support was considered earlier by myself and Todd Quinto (Duke Math. J., 1987).

Y. CENSOR:

Special-purpose optimization theory techniques for image reconstruction and other inversion problems

Some row-action algorithms which exploit special objective function and constraints structure have proven advantageous for solving huge and sparse feasibility or optimization problems. Recently developed block-iterative versions of such special-purpose methods enable parallel computation when the underlying problem is appropriately decomposed. This opens the door for parallel computation in image reconstruction problems of computerized tomography and in the inverse problem of radiation therapy treatment planning, all in their fully discretized modelling approach. Since there is more than one way of deriving block-iterative versions of any row-action method, the choice has to be made with reference to the underlying real-world problem.

R. CLACK:

Some aspects of three-dimensional tomography: The parallel and divergent X-ray transforms

For both the parallel and divergent x-ray transforms in three dimensions, the data obtained from projections taken over all directions heavily overdetermines the source distribution.

For the parallel case, this fact is an easy consequence of the Fourier central-slice theorem. Orlov's condition on the unit sphere can be used to verify that a specific subset of projection directions, Ω, measures all Fourier components and thus ensures that effective reconstruction algorithms are possible. For any such valid Ω, appropriate filters for both filtered-backprojection and backprojection-filtering can be found, and many interesting properties of these filters have been established.

For the divergent (or "cone-beam") case the situation is somewhat different. Although a sufficient subset of projection focal points, A, can be identified using Tuy's condition, there seems to be little hope of finding a filtered-backprojection algorithm (corresponding to the fan-beam formula in two-dimensions). The backprojection-filtering algorithm can be applied, but only for a few specific choices of A. Some general inversion formulas have appeared, but they are awkward to implement and the relationships between them have not been established.

There are comparatively few applications of the 3-D parallel transform, although the theory is quite well developed. On the other hand, there are only a few somewhat isolated results for the divergent transform, yet a large number of applications are appearing both in volume CT and SPECT imaging. The development of a comprehensive theory for the divergent transform has still to be established, and will undoubtedly be a source of active research in the near future.

D. COLTON:

New methods in diffraction tomography

We consider the inverse scattering problem of determining the (possibly complex) index of refraction of a nonhomogeneous medium from a knowledge of the far field patterns corresponding to time harmonic plane waves propagating in arbitrary directions. Two methods are given for solving this problem, both of which employ an averaging process to lower the number of unknown functions that one needs to compute. In method A, the inverse problem is reduced to an overdetermined Cauchy problem and in method B the inverse problem is reduced to an overdetermined interior impedance boundary value problem. Numerical examples are given for both methods. This represents joint work with Peter Monk.

M. DEFRISE:

Truly three-dimensional reconstruction in positron emission tomography

Traditional multi-ring PET scanners incorporate lead annular septa to shield the different detector rings. The three-dimensional (3D) image reconstruction problem is then decomposed in a set of independent 2D tomographic reconstructions. By removing the septa, the efficiency of the scanner may be improved significantly, but the 3D image reconstruction problem must then be solved globally.

We review truly 3D transform methods, and, in particular, the 3D filtered back-projection algorithm. The non-uniqueness of the reconstruction filter will be discussed, as well as various difficulties related to the numerical implementation of the algorithm. Finally, a novel algorithm for truly 3D reconstruction is proposed, which reduces the 3D problem to a set of independent 2D exponential Radon transform problems.

P. EGGERMONT:

Multiplicative iterative algorithms for convex optimization

We study multiplicative algorithms for the solution of convex minimization problems. The proto-type method is the EM algorithm for the maximum likelihood estimation for PET imaging, which can be written as

$$(1) \quad x_j^{n+1} = x_j^n - \omega_n x_j^n \, [\nabla \ell(x^n)]_j \,, \quad \forall j \,,$$

where $x \in \mathbf{R}^N$ denotes the unknown intensity field, and $\ell(x) = d(b|Ax) \equiv \sum_i b_i \log\{b_i/[Ax]_i\} + [Ax]_i - b_i$. For general convex functions $\ell(x)$ it pays to replace (1) by

$$(2) \quad x_j^{n+1} = \frac{x_j^n}{(1+\omega_n [\nabla \ell(x^n)]_j)} \,, \quad \forall j \,,$$

because then the same monotonicity results as for the EM algorithm can be proved. With minor modification, algorithm (1) also can be applied to monotone complementarity problems.

There are interesting connections between (1) and (2) and the system of differential equations of Volterra–Lotka type

$$(3) \quad \frac{dx_j}{dt} = -x_j \, [\nabla \ell(x)]_j \,, \quad \forall j \,,$$

for which $d(x^*|x(t))$ is a Lyapunov function provided x^* is any minimum of $\ell(x)$ over $x \geq 0$.

T. ELFVING:

Algorithms for norm–minimization over a closed convex set

We consider the two problems $\min \|f\|_2^2 \,, \quad If = w, \quad f \in C \subseteq H$ and $\min p\|f\|_2^2 + \|z - y\|_2^2 \,, \; If = Kz + u, \; f \in C \subseteq H$. Here $I : H \to \mathbf{R}^n$ is a bounded linear mapping, C a closed convex set in a Hilbert space H, z an unknown finite vector, K a given matrix and the smoothing parameter p a given positive number. Also y, u, w are given finite vectors. Based on the result $f = P_C(I^*\alpha)$, $z = y - pK^T\alpha$, (I^* is the dual of I and P_C denotes the orthogonal projection onto C) we describe numerical algorithms, of Newton type, for certain special 1-D cases (when $P_C(I^*\alpha)$ and its derivative w.r.t. α can be efficiently computed). We also outline a possible way to treat numerically certain 2-D discrete problems, e.g. $\min p\|x\|_2^2 + p\|A_1 x\|_2^2 + \|A_2 x - a_2\|_2^2$, $x \in \mathbf{R}_+^n$, where A_2 e.g. could be a discretization of the Radon transform and A_1 the discrete Laplacian.

A. FARIDANI:

Local tomography

Tomography produces the reconstruction of a function f from a large number of line integrals of f. Local tomography, as introduced initially, produced the reconstruction of the related function Λf, where Λ is the square root of $-\Delta$, the positive Laplace operator. The reconstruction of Λf is local, and Λf has the same smooth regions and boundaries as f. However,

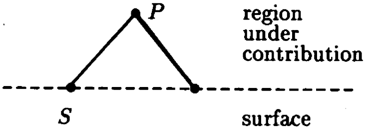
Λf is cupped in regions where f is constant. $\Lambda^{-1}f$, also amenable to local reconstruction, is smooth everywhere and contains a counter cup. A detailed study of the actions of Λ and Λ^{-1} is presented and several examples of what can be achieved with a linear combination. This represents joint work with E.L. Ritman and K.T. Smith.

V. FRIEDRICH:

Can backscattered light be used for a surface-near tomography?

The question above was the starting point for mathematical investigations on some models, which include attenuation as scattering as well. If we would detect only single scattered photons, the following problem arises: Find scattering λ and attenuation μ from the relations

$$\ell n\ g(P) - \int_S^P \mu(Q)ds - \int_P^D \mu(Q)ds$$



for different locations of light sources (at S) and detectors (D). First results about this problem have been reported.

Multiple scattering models need further investigations, wether the reduction of the Boltzmann equation for the distribution of photons can be reduced to a Helmholtz equation with sufficient accuracy for this problem.

R.J. GARDNER:

Sets uniquely determined by finitely many X-rays

There has been some interest in classifying measurable sets in $\mathbf{R}^2$ which are uniquely determined by their X-rays in a finite set S of directions. When S has 2 directions, this has been done by A. Kuba and A. Volčič, and by P.C. Fishburn, J.C. Lagarias, J.A. Reeds and L.A. Shepp, using "S-inscribable sets" and "S-additive sets", respectively. We present some results concerning the case when S has more than 2 directions.

D. GIRARD:

Fast stochastic techniques for computing optimal regularization parameters

When applying the method of regularization, the choice of the smoothing parameter, say τ, is very crucial. In the "white noise" case, the Generarized cross-validation (GCV) function and the Mallows' estimator are two well known functions for evaluating the goodness of a value of τ. Denoting $A_\tau y$ the predicted values in the data space (y is the data vector) for a given value of τ, computation of these functions requires the computation of $A_\tau y$ and of trace(A_τ).

In most of the large problems, we have at our disposal a fast algorithm to compute $A_\tau y$ for any given y, but the cost of computing $(1/n)$trace(A_τ) is very expensive. So I recently proposed to simply approximate this quantity by the average of m estimates, each one given by $w^T A_\tau w / w^T w$ with a certain pseudo-random data w.

The purpose of this talk is to show, both theoretically and practically, that, even with a small number of simulations (e.g. $m = 1$ or 10), these stochastic methods of choosing τ perform as well as the exact methods. The asymptotic optimality results which justify the use of the exact methods also hold for these stochastic versions with only $m = 1$.

F.B. GONZALEZ:

Range conditions for group-equivariant Radon transforms

We examine the range of the d-dimensional Radon transform $f \to \hat{f}$ on $\mathbf{C}^n$, where $d < n - 1$. This transform integrates functions on $\mathbf{C}^n$ over d-dimensional complex planes, and so maps functions on $\mathbf{C}^n$ to functions on $H(d, n)$, the affine Grassmannian manifold of d-planes on $\mathbf{C}^n$. Specifically, we consider the range $\mathcal{S}(\mathbf{C}^n)^\wedge$, where $\mathcal{S}(\mathbf{C}^n)$ is the space of rapidly decreasing functions on $\mathbf{C}^n = \mathbf{R}^{2n}$. When $d = n - 1$, the range is completely characterized by a variant of the Helgason moment conditions (Gelfand et al., *Generalized Functions, p. 123*). When $d < n - 1$, we prove that the range $\mathcal{S}(\mathbf{C}^n)^\wedge$ can be described as the space of rapidly descreasing functions φ on $H(d, n)$ satisfying a *single* fourth order partial differential equation $\Lambda_d \varphi = 0$. The operator Λ_d is invariant under the action of the complex motion group $G = U(n) \rtimes C$ on $H(d, n)$. This operator Λ_d arises from the infinitesimal left-regular action on $H(d, n)$ by an element U in the center of the universal enveloping algebra of G. The operator U can be written explicitly, and is the same for all d.

P. GRANGEAT:

(Co-authors: LE MASSON, P., MELENNEC, P., SIRE, P.)

3D cone beam reconstruction

We will give an overview on analytical methods for 3D cone beam reconstruction.

Then we will focus on the comparison between the cone beam backprojection algorithm (Feldkamp 1984) and the algorithm using the 3D Radon domain as rebinning space and computing the first derivative of the Radon transform (Grangeat 1987).

We have implemented it on two codes adapted to super computers. We have achieved reconstructions on a CRAY II computer from simulated data. We will compare the results and the computation times.

Finally we will present the application of cone beam reconstruction to the measurement of the mineral content of a lumbar vertebra.

E.L. GRINBERG:

Flat Radon transforms

The X-ray transform on a Riemannian manifold can be viewed as the operation of integration on 1-dimensional flat submanifolds. We will review some basic properties of this transform on symmetric spaces. Then we will examine its quasi-dual: the *maximal totally geodesic flat* transform. This transform has applications to isospectral deformation problems. We will give some injectivity and non-injectivity results for it and describe their isospectral consequences.

F.A. GRÜNBAUM:

Diffuse Tomography

I consider the situation resulting from using a source much weaker than X-rays. On top of absorption it is necessary to consider, and *image*, the scattering distribution in the object.

Some small numerical experiments based on an analytical approach give very encouraging results.

I will discuss "ghosts" as well as "positive results".

G.T. HERMAN:

Evaluation of reconstruction algorithms

An image reconstruction algorithm is supposed to present an image that contains medically relevant information that exists in a cross section of the human body. There is an enormous variety of such algorithms. The question arises: Given a specific medical problem, what is the relative merit of two image reconstruction algorithms in presenting images that are helpful for solving the problem? An approach to answering this question with a high degree of confidence is that of ROC analysis of human observer performance. The problem with ROC studies using human observers is their complexity (and, hence, cost). To overcome this problem, it has been suggested to replace the human observer by a numerical observer. An even simpler approach is by the use of distance metrics, such as the root mean squared distance, between the reconstructed images and the known originals. For any of these approaches, the evaluation should be done using a sample set that is large enough to provide us with a statistically significant result.

We concentrate in this paper on the numerical observer approach, and we reintroduce in this framework the notion of the Hotelling Trance Criterion, which has recently been proposed as an appropriate evaluator of imaging systems. We propose a definite strategy (based on linear abnormality-index functions that are optimal for the chosen figure of merit) for evaluating image reconstruction algorithms. We give details of two experimental studies that embody the espoused principles. Since ROC analysis of human observer performance is the ultimate yardstick for system assessment, one justifies a numerical observer approach by showing that it yields "similar" results to a human observer study. Also, since simple distance metrics are computationally less cumbersome than are numerical observer studies, one would like to replace the latter by the former, whenever it is likely to give "similar" results. We discuss approaches to assigning a numerical value to the "similarity" of the results produced by two different evaluators. We introduce a new concept, called rank-ordering nearness, which seems to provide us with a promising approach to experimentally determining the similarity of two evaluators of image reconstruction algorithms.

W.G. HAWKINS:

(Co-authors: P.K. LEICHNER and NAI-CHUEN YANG)

Theorems for the number of zeros of the radial Modulators of the 2D exponential Radon transform

Theorems and a transformation formula are developed for the 2D exponential Radon transform (ERT) whereby theorems for the number of nodes of radial modulators of the X-ray transform (no attenuation of internal sources) can be extended to the ERT. The results were applied to SPECT simulations with angular undersampling, and a spectral filter was shown to improve image quality in the region affected by angular aliasing, without altering interior regions that were not affected by angular aliasing.

A.N. IUSEM:

Algorithms for very large linearly constrained convex programming problems,with applications in image reconstruction

We present some algorithms, related to the EM Algorithm of Vardi, Shepp and Kaufman, useful for Positron Emission Tomography. Firstly we discuss a multiplicatively relaxed version of the EM Algorithm proposed by Tanaka and analyze its convergence properties. Such analysis provides also a proof of convergence of the unrelaxed (original) EM Algorithm, different from the known proofs by Csiszár and Tusnády, and Multhei. Next, we discuss a new algorithm for the same problem, obtained through dualization of a MART type algorithm for Burg's entropy. We establish its convergence by proving convergence of the dual sequence of Bregman's Convex Programming algorithm, which in turn results from the linear convergence rate of the primal sequence. The algorithm can be seen as an additively relaxed, sequential variant of the EM Algorithm with occasional zeroing of variables, and its convergence properties are more robust than those of the EM Algorithm. We consider also some implementation issues.

S.H. IZEN:

The practical inversion of a limited data x-ray transform in three dimensions

The three-dimensional density distribution of flowing gases is of interest to researchers in computational fluid dynamics and to designers of jet engines. The optical diagnostic technique of Diffuse Illumination Heterodyne Holographic Interferometry has been used to measure a restricted view x-ray transform of density distributions. The x-ray transform data is available in a parallel beam geometry where the viewing directions are restricted to a very narrow cone (central angle of about 10 degrees).

A reconstruction algorithm based on the projection-slice theorem and on the singular value decomposition has been developed and implemented. This algorithm has been tested on data obtained in the laboratory and has performed as expected, given the high degree of ill-posedness inherent in the sampling geometry.

To improve further the reconstructions, the use of constraints and *a priori* information is being incorporated into the algorithm.

L. KAUFMAN:

Implementing and accelerating methods for emission tomography

One of the main reasons that positron emission tomography (PET) for finding cancer tumors has not moved from the experimental laboratory into the clinical laboratory is that the medical community thinks that the computational costs are too high. However, the most commonly used algorithms for image reconstruction can be speeded up by using common sparse matrix techniques and well known methods in function optimization. Moreover, these algorithms are very well suited to parallel and vector machines and on these machines can be made economically feasible even on medically reasonable problems of 16000 variables.

Whether one uses least squares or maximum likelihood as a merit function to determine a metabolic map of the patient, computing the gradient of the merit function involves several matrix by vector multiplications with a large sparse probability matrix with millions of nonzero elements. With a grid that takes advantage of the spherical symmetry of the underlying physical system many of these nonzero elements are repeated and by taking advantage of this repetion and using the appropriate data structure there is no need to regenerate the matrix each gradient evaluation and moreover, super vector speeds can be obtained. With the cost of gradient evaluation reduced, work has begun on finding a good optimizer.

F. KEINERT:

Geophysical travel time inversion

We assume that the slowness of seismic waves in a half space $\{(x,z) : z \geq 0\}$ is of the form

$$n(x,z) = Y(az+b) + \Delta n(x,z) ,$$

where Δn is a small perturbation. By measuring the travel time perturbation due to Δn, this reduces to the mathematical problem:

Reconstruct a function $n(x,z)$, $z \geq 0$, from its integrals over circles centered on the line $z = -\frac{b}{a}$.

By a Fourier transform in the x variable, this reduces to a sequence of one-dimensional problems

$$\int_0^z \frac{2(z+\frac{b}{a})\cos\left[k\sqrt{(z+\frac{b}{a})^2-(p+\frac{b}{a})^2}\right]}{\sqrt{(z+\frac{b}{a})^2-(p+\frac{b}{a})^2}}\,\hat{n}_k(p)dp = \hat{t}_k(z) ,$$

where k is the wave number.

Following ideas of Paul Sacks, we propose to solve the integral equations by various techniques, depending on the size of k. We can then recover n by an inverse Fourier transform.

R. KRESS:

Numerical methods in inverse obstacle scattering

The inverse problem we consider is to reconstruct the shape of an obstacle from the knowledge of the far-field pattern for the scattering of incident time-harmonic acoustic waves. It occurs in a variety of applications such as remote sensing, ultrasound tomography and seismic imaging and is difficult to solve since it is nonlinear and improperly posed. After a review on some basic properties of the inverse problem, in par ticular on its ill-posedness and on uniqueness results, we describe two methods for the approximate solution. They belong to a new group of schemes which stabilize the inverse scattering problem by reformulating it as a nonlinear optimization problem and which do not require the solution to the forward scattering problem. A guideline of this survey will be to consider in detail only inverse scattering from an impenetrable sound-soft obstacle. But we note that the analysis can be extended to impenetrable scatterers with other boundary conditions and also to penetrable obstacles.

A. KUBA:

On the reconstruction of sets from two projections

The following problems are considered connected with the reconstruction of measurable plane sets/binary matrices:

1. The determination of the class of non–uniquely reconstructible sets/binary matrices.
2. An algorithm to construct the structure of the class of binary matrices with prescribed 1's and 0's.
3. Reconstruction of sets if the directions of the projections are arbitrary. If the directions are not known then the unique reconstruction is not possible. However, it is possible to decide whether the functions are the projections of a uniquely reconstructible set and also the correspondent directions are determinable.

R.M. LEWITT:

Application of radial basis functions to numerical inversion of the X-ray transform

We discuss an approach to numerical inversion of the X-ray transform, where the approach involves representation of the unknown function of tomography by a linear combination of the translates of a rotationally-symmetric basis function. We describe a family of basis functions that have compact support and rotational symmetry, that are effectively bandlimited, and that have convenient analytic expressions for their gradient, Laplacian, Fourier transform, and k-plane transform (where $k = 1$ is the x-ray transform and $k = n - 1$ is the Radon transform). When the solution to the reconstruction problem is obtained by iterative algorithms, the use of these basis functions restricts the solution to a set of functions whose smoothness may be controlled by adjusting the parameters of the basis functions.

A.K. LOUIS:

Wavelets and ill-posed problems

For variing scale the continuous wavelet transform converges, after a suitable normalization, to a derivative of the transformed function. The order of this derivative depends on the number of vanishing moments of the wavelet. Based on this result of Rieder, 1988, a selection of the wavelet according to the desired information is possible. The discrete wavelet transform inherit this property, and so we can separate in a so-called multi-scale analysis the smooth and the oscillating part of the solution. This is used for solving equations of the first kind with compact operators. The largest system to be solved is only half the size which is necessary in standard discretization, so the number of arithmetic operations to set up the systems of equations is not larger than in the standard method. The advantage is now that the condition of the system of linear equations is independent of the refinement step, hence we have derived a multigrid method for equations of the first kind.

As another application of the wavelet property we can automatically find the data in a CT scan which are useless due to metal implanted. A suitable interpolation of wavelet coefficients results in much better pictures which is demonstrated by reconstructions from real data of a patient with a metal hip link.

P. MAASS:

A singular value decomposition for the interior Radon transform

The interior Radon transform R_I arises in X-ray tomography when the emitted rays do not cover the whole object but are restricted to those rays intersecting the region of interest. Accordingly the adjoint operator R_I^* coincides with the usual backprojection only in the region of interest and differs otherwise.

Assume that the region of interest is the unit ball. Due to the invariance of the spherical harmonics under the action of R_I^* the problem of finding the generalized eigenfunctions of R_I^* reduces to the same problem for a family of integral operators T_ℓ for the radial parts. The construction of pairs of intertwining differential operators (D_1, D_2), i.e.

$$D_2 T_\ell = T_\ell D_1 \ ,$$

allows to compute the transforms of functions g satisfying $D_1 g = 0$. This results in a singular value decomposition for the interior Radon transform on $\mathbf{R}^2$ between weighted L_2-spaces. The Nullspace of R_I and the ill-posedness of the inverse problem given by $R_I f = g$ can be determined immediately.

W.R. MADYCH:

Wavelets, multiresolution analysis, and applications

We recall the basic properties of orthogonal wavelets and multiresolution analysis and stress the significance of the two scale functional equation

$$(*) \qquad \varphi(x) = \sum h_j \varphi(2x - j)\,.$$

After discussing several specific examples we show how (*) leads to efficient decomposition and reconstruction algorithms via the corresponding quadrature mirror filters. Finally, we indicate several extensions and applications

- multivariate cases
- biorthogonal bases and analogues of quadrature mirror filters
- understanding and extending multigrid methods
- tomography and related inverse problems.

We conclude with several historical comments.

D.E. McCLURE:

The Bayesian statistical approach to SPECT

This talk presents joint work of Stuart Geman and Donald McClure, together with Kevin Manbeck and John Mertus. The reconstruction problem in computed tomography is broadly concerned with inversion of a Radon transform or variants of it that explicity model physical effects (attenuation, scattering), sensor effects (imperfect collimation, noise), and sampling design (limited views). Statistical approaches invoke *optimization* principles to accomplish the inversion and do note use *inversion formulas*. The inversion problem for SPECT (single photon emission computed tomography) is formulated as a statistical estimation problem: estimate the nonhomogeneous intensity function of a two- or three-dimensional Poisson process from indirect observations. Previously this has been addressed using the principle of maximum likelihood; the likelihood method does not incorporate spatial information and needs to be regularized to be a *consistent* procedure for the non-parametric estimation problem. The Bayesian paradigm for image restoration provides a framework for carefully formulating regularization concepts. Spatial information about the unknown intensity function can be described by a Gibbs prior distribution and then standard methods from Bayesian decision theory lead to alternative reconstructions. The physical and mathematical models that form the foundation for the Bayesian statistical approach are described. The algorithms developed are illustrated with simulation experiments, with physical phantom experiments and patient data. The adaption of the approach to a real SPECT system uses *forms* of physical models derived from the basis physical principles, together with empirical calibration of these forms based on the results of simple physical phantom experiments.

R.G. MUKHOMETOV:

Uniqueness and stability in the problems of the integral geometry and tomography

In a bounded domain $M \subset \mathbf{R}^n$ the problem of integral geometry for a family of geodesics K, with respect to the metric $F(x,a)$, is considered. The boundary ∂M consist the part O, of which geodesics reflected one time, and the part D, where is consisted both the ends an every geodesic $k \in K$. It is considered too the problem with the multiple reflection on a plane, where the domain M is a sector of a disk, and the geodesics consist of straight segments only. The domain M can consist the hypersurface of the refraction. It is considered some problem of the emission tomography. For all problems the author obtained the estimates of the stabilities of which follow the uniqueness of the solutions.

F. NATTERER:

Tomography with unknown directions

Let f be a function in $\mathbf{R}^3$ with barycenter 0, and let $U_1, \dots, U_p \in SO(3)$. The problem is to recover f from the orthogonal projections $g_j = Pf_j$, $f_j(x) = f(U_j^{-1}x)$ on the plane $x_3 = 0$. If $U_1, \dots, U_p$ are known, then we have a problem of X-ray tomography. We consider the case of unknown $U_1, \dots, U_p$.

We describe first the method of Goncharov – Gelfand which makes use of the identity

$$Rg_j(\Theta', s) = Rf(U_j^{-1}\Theta, s)\,, \quad \Theta = \begin{pmatrix} \Theta' \\ 0 \end{pmatrix},$$

where R is the 2D and 3D Radon transform, respectively. Then we discuss the 2D case, using the Helgason consistency condition for the Radon transform. For uniformly distributed angles we suggest a probabilistic method for determining the angles.

H. OGAWA:

Radon transform and analog coding

An image reconstruction method is proposed which provides the correct original image even when projection data contain "errors" and "noises". The terminologies "error" and "noise" are used here for the so-called "impulsive noise" and the "random noise", respectively. The proposed method are able to detect and correct "errors" and is robust against "noises".

Let $g(x,\alpha)$ be the Radon transform of an image $f(x,y)$ and $u_k(x)$ a k-th polynomial. The redundant relation

(1) $$\int_0^{2\pi}\left(\int_{-1}^{1} g(x,\alpha)u_k(x)dx\right)e^{in\alpha}d\alpha = 0 \quad (k < |n|)$$

(Marr, 1974) is our starting point. If we put

(2) $$g_{k\ell} = \int_{-1}^{1} g\left(x, \frac{2\pi\ell}{2L}\right)u_k(x)dx$$

with $2L$ the total number of views, then it follows that

(3) $$\sum_{\ell=0}^{2L-1} W_n^\ell g_{k\ell} = 0 \quad (k < |n| < L)$$

with $W_n = \exp(i\pi n/L)$. Eq. (3) means that for each k the vector $g_k = (g_{k\ell})$ is a DFT code word. The DFT code is a typical example of analog code. Then, by using the DFT coding theory we can detect and correct "errors" contained in the projection data $\{g_{k\ell}\}$. Some experimental results are also illustrated.

L.R. OUDIN:

Determination of the specific density ρ of an aerosol through tomography

An aerosol is made of droplets of a diluted fluorescent product. Specific density and distribution of droplets diameter depend only from two coordinates (y,z) and not from x. The border of the aerosol is known. A light-sheet of green light ($\lambda = 0,504\mu m$) propagates in the plane $0yz$ along y and produces an orange light ($\lambda = 0,585\mu m$) which scatters across the aerosol towards the outlet border of the cloud. The function of emission lobe is known. The aerosol is diveded into N parallel equidistant slices (parallel to the light-sheet). An expression of the attenuation of the light-sheet averaged along z is known as a function of y. Thanks to these hypothesis, the conservation of energy provides flows across elementary windows in the different middle planes of the slices. The flow of light outgoing the cloud is obtained as a transfer function autoconvoluted N times by itself and once by the energy flow in the light-sheet plane. The transfer function is proportional to the specific density ρ is obtained as well as ρ by Fourier transform methods. The attenuation function of the light-sheet can therefore be more precisely known. The calculus of ρ is iterated once and ρ is more precisely reached.

V.P. PALAMODOV:

I. A new inversion formula for the X-ray transform in $\mathbf{R}^3$

An exact formula for reconstruction of a function with compact support will be given. This formula needs the X-ray transform data given only for rays which are tangent to the fixed surface. Related problems for the ray transform are considered.

II. A solution of the inverse scattering problem for one-dimensional acoustic equation

The inverse scattering problem for wave equation on the line (or half-line) with unknown velocity is treated. An explicit formula transforming the scattering amplitude to the velocity function will be given (avoiding any Fredholm equations).

Á. R. DE PIERRO:

EM algorithm and related methods

We present an overview of some iterative methods currently being used in computed tomography and related areas. One of the main features of these methods is that each iterate is generated by the preceding one in a multiplicative way preserving positivity. This is the case for example of the EM algorithm for maximum likelihood estimates in positron emission tomography. We discuss the main properties of this type of methods and analyse recent results, comparing with similar methods that use additive up-datings.

E.T. QUINTO:

Computed tomography and rockets

The speaker presents a problem in industrial non-destructive evaluation (NDE) and provides a solution using his algorithm for the exterior Radon transform. X-ray CT is effective for the NDE of rockets, but data through the center of a large rocket body are attenuated too much to be usable. However, the important defects are in the outer annulus of the rocket. To reconstruct the outer annulus, we use our algorithm for exterior datadata not intersecting the thick center. Numerical methods and reconstructions for fan beam data are given.

J. ROERDINK:

Cardiac magnetic resonance imaging by retrospective synchronization

In cardiac MRI one tries to reconstruct pictures of a cross section of the beating heart at several phases of the heart cycle.

Because of technical limitations, data needed for reconstruction of a single picture have to be collected during a large number of successive heart cycles. A major problem is the irregularity of the heart beat, which causes a blurring in the standard reconstructions.

We discuss a method to correct for these artifacts. The problem is approached mathematically by studying nonuniform sampling problems in certain Hilbert spaces.

P.C. SABATIER:

On the informative content of measurements

For an ill-posed (essentially underdetermined) problem, with given bounds for their misfit and their ugliness, the set of solutions is usually characterized by points which minimize a trade-off functional. It is shown that if chains of measurements interpolating each other are used, the minima evolution does not show new creations after a certain order of interpolation is obtained, provided the measurement functionals are smooth enough. Thus there is a chain with fewer measurements, which correspond to a "weak information" on the set of solutions, that cannot be dramatically modified by interpolating other measurements. Its appraisal can be done essentially by using the first and second variation of the measurement functionals. This remark suggests a program of evaluating the ill-posedness of scattering problems, defined after setting a given tolerance on misfit and ugliness, by appraising the number of measurements which are necessary to obtain the "weak information". Some tomographic images obtained on real data with few measurements illustrate the lecture.

W. TABBARA:

Diffraction tomography: facts and hopes for applications

Diffraction Tomography is a field of interest in our laboratory for more than ten years now. From the beginning we have used numerical simulation and experiment to assess the feasibility of this imaging technique. And as a consequence we focused very early on possible applications using electromagnetic and acoustic waves.

At first we were interested in reproducing qualitative imaging presenting well reconstructed contours and contrasts. Various parameters such as the number of views, the operating frequency and the sampling rate have been analysed in order to provide the user with a best choice of values for each application.

Furthermore we also studied different ways of improving the quality of the image by including some a priori information about the object. All this let us to build few prototypes of cameras based on this technique.

These last years and the present work are devoted to quantitative imaging. Here we are concerned with the retrieval of the physical parameters of the object such as its permittivity or attenuation. The difficulties are orders of magnitudes higher than in qualitative imaging. The link between the theory and the applications is stronger here and our investigations are going on in a number of directions with the hope to be able to find the best compromise for a large number of applications.

This presentation will describe the main issues we have addressed and show how we feel this method can be applied in some areas of interest.

A. VOLČIČ:

Tomography of measurable sets

We present two recent results on the tomography of measurable sets.

The first result is contained in a joint paper with G. Bianchi and is related to a uniqueness theorem by the author: A plane convex body K is uniquely determined by the X-rays taken at three non-collinear points not belonging to K.

Theorem: The mapping which assigns to K its three projections, is continuous and has continuous inverse.

The second result is due to U. Brehm and consists in two general methods to construct measurable sets, which are not uniquely determined by three and, respectively $n > 3$, points in generic positions. These examples accumulate near to the lines joining the point sources. For two sources an example is constructed of a set (union of 12 balls) which is not uniquely determined by the two corresponding projections, and stays away from the line joining the points.

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