MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 35/1990

Anwendungen der Infinitesimalmathematik

19.08. bis 25.08.1990

Die Tagung fand unter der Leitung von Herrn S. Albeverio (Bochum), Herrn D. Laugwitz (Darmstadt) und Herrn W. A. J. Luxemburg (Pasadena) statt. Im Mittelpunkt des Interesses standen Fragen über Anwendungen der Infinitesimalmathematik.

Auf folgende Gebiete wurde die Infinitesimalmathematik angewandt: Mathematische Physik, Quantenelektrodynamik, Dynamische Systeme, Wirtschaftswissenschaften, Lie Algebren, Funktionalanalysis, Distributionen, Maßtheorie, Stochastik, Logik. In einigen Vorträgen wurden mit Hilfe der Infinitesimalmathematik neue Resultate in der Standardanalysis erzielt. Andere Vorträge brachten interessante und neue Einblicke in die Modelle der Infinitesimalmathematik und in die neuen mathematischen Strukturen, die durch die Infinitesimalmathematik entstanden sind (Loeb–Räume, Nonstandardhüllen von Banach–Räumen).

Die Veranstalter haben es sehr bedauert, daß so viele Interessenten an der Tagung keine Einladung bekommen konnten.

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

S. Albeverio:

Remarks on hyperfinite walks, Dirichlet forms, fields

We discuss the relation between Dirichlet forms, symmetric Markov semigroups and associated processes on topological spaces, via the construction and study of hyperfinite Dirichlet forms. We mention a structure theorem for such forms and discuss briefly some open problems, especially towards extension of the theory of infinite dimensional (standard) Dirichlet forms. We shortly discuss a new result by Fan Ruzong on construction of strong Markov processes associated with Dirichlet forms on metric spaces, obtained by techniques of non-standard analysis. In the second part of the lecture we mention three areas of the theory of quantum fields and strings on manifolds where methods of non-standard analysis combined with those of differential geometry and stochastic analysis could turn out to be very fruitful.

L. Arkeryd:

On the Carleman equation with measure valued initial data

We start from the result by Oberguggenenberger that the Cauchy Problem for the Carleman Equation

$$(\partial_t + \partial_x)u = v^2 - u^2, (\partial_t + \partial_x)v = u^2 - v^2, x \in \mathbf{R}, t > 0,$$

has solutions in the sense of the Colombeau distributions, if the initial values are measures with finite mass. Using NSA techniques we show that those solutions are Young measure solutions in general, and in the case of no singular continuous component of the initial values, that the solutions are in $L^\infty(\mathbf{R})$ for $t > 0$.

I. van den Berg:

Extended use of Internal Set Theory

The three axiom schemes of Internal Set Theory are the principles of Transfer, Idealization and Standardization. They are stated of carefully delimitated types of formulas and sets: Transfer is limited to standard formulas, Idealization to internal formulas and Standardization to standard reference sets. However, the axiom schemes may be thought of as summarizing certain ways of reasoning. Indeed, some conditions may be altered or relaxed. Thus we may "transfer" or "idealize" some kinds of external properties, and "standardize" or "saturate" on internal or external sets. We give applications concerning ordinary analysis. They include ameliorations of the nonstandard proofs of the theorems of Du Bois–Reymond and Borel–Ritt and the theorem of existence of rivers, and new developments within the theory of regular variation.

M. Capinski:

Nonstandard densities of measures on function spaces and their applications

Due to the lack of Lebesgue measure on infinite dimensional spaces it is impossible to describe measures on such spaces by means of densities. However we can imbed some infinite dimensional spaces into hyperdimensional ones in a natural way. For function spaces we can use hyperfinite discretisation, for example $L^2(0,1) \ni u \to (U_k) \in {}^*\mathbf{R}^N, U_k = N\int_{k/N}^{(k+1)/N} {}^*u(x)dx, N$ – infinite, or we can use an orthonormal basis extending it to a hyperfinite one. Now a measure can be fully described by the Loeb extension of the internal measure on ${}^*\mathbf{R}^N$ given by $\psi(A) = \int_A F(U)dU$ for suitable F.

One possible application is the investigation of statistical solutions of PDEs, that is, families of measures $\mu_t(A) = \text{prob}(u(t) \in A)$, where $u(t)$ is a solution of a PDE with random coefficients. Nonstandard densities of statistical solutions satisfy certain differential equation. This approach gives some tools to construct invariant measures and to tackle the problem of existence of intrinsic turbulence, that is, a statistical solution which is the Dirac measure for $t = 0$ and non–Dirac for some $t > 0$.

N. Cutland:

Applications of NSA to Malliavin calculus and related topics

In the past 12 years a comprehensive calculus has been developed for functionals on Wiener space, based around the intuitive idea of "differentiation with respect" to $db_t = \mathfrak{k}_t$, i. e. $\frac{d\varphi}{dx_t}$ where φ is a Brownian (Wiener) functional.

The nonstandard framework, with $\Delta B_t = \mathfrak{k}_t$ for discrete time t, and $\mathfrak{k}_t \in {}^*\mathbf{R}$ allows this idea to be made precise, and then the fundamental ideas and results of Malliavin calculus, the gradient operator on Wiener space, the Skorohod integral, etc. all reduce to undergraduate calculus. We shall outline these ideas.

A. Delcroix:

Study of critical points of slow–fast differential equations

A critical point of the slow–fast differential equation
(1) $dy/dx = \varepsilon^{-1} f(x,y), \varepsilon > 0$ infinitesimal,
is a point S on the slow curve C of equation $f(x,y) = 0$ such that $f'_y(x,y) = 0$.
Assuming that C is the union of a finite number of branches and that the taylor set $T(f) = \{(m,n) \in \mathbf{R}^2 : \partial^{m+n} f/(\partial^m \times \partial^n y(S) \neq 0\}$ of f is non empty, we can build one (or more) magnifying glass:
(2) $(x,y) \to (X = \varepsilon^{-\alpha} x, Y = \varepsilon^{-r\alpha} y)$
such that the image of the slow–fast equation under (2) is a near standard equation having rivers. These rivers describe the behaviour of the slow trajectories of the slow–fast growing equation (1) in the neighbourhood of the critical point.

F. Diener:

Existence of canards and "summation" of asymptotic series

In this talk, I show an existence theorem of canard, related with the delayed bifurcation phenomenon. I consider a differential equation $\varepsilon\frac{du}{dt} = f(t,u)$ near (t_0, u_0) for which the slow curve is attracting for $t < t_0$, repelling for $t > t_0$ and such that the fast dynamic spirals around the slow curve ${}^0f(t,u) = 0$. The theorem, first proved by Neishtadt (1988), says that if f is analytic in t and u near (t_0, u_0), the equation has a canard at that point. To prove it, we first show that an exponentially small perturbation of an equation having a canard still has a canard. Then we obtain, by a Borel–Laplace summation of the formal series approximating the canard, a solution, not of the equation itself but of an ε–exponentially small perturbation of it, and then, by the first remark, the existence of a canard for the initial equation. The summation procedure uses the crucial fact that the asymptotic expansion of the canard is of Gevrey–1 type (Sibuya 1989 and Canales–Durand, 1990).

M. Diener:

Canards: new approaches and results

There exist at this time at least two situations involving canards elementary and important enough to give one more talk on canards.

First consider the slow–fast growing differential equation $\varepsilon\frac{du}{dt} = u(t^2-u^2)+\varepsilon d(t,u,d \in \mathbf{R}, \epsilon > 0, \epsilon \simeq 0)$; for $d = \pm 1$ or 0, $u = \pm t$ or 0 are respectively evident slow solutions, none being a canard. Now, by using the usual continuity argument applied to the solution $\overline{u}_d$ such that, say $\overline{u}(-1) = -1$, one shows that there exists $\overline{d} \in [-1, +1]$ s.th. $\overline{u}_{\overline{d}}$ is a canard (following $u = t$ for $t << 0$ and $u = 0$ for $t >> o$); $\overline{u}$ is not S^1 (i. e. ${}^0\overline{u} \in \mathfrak{C}^1$). Using the "untangling magnifying glass" $t = \varepsilon^{\frac{1}{3}}T$, $\quad u = \varepsilon^{\frac{1}{3}}U$, this shows the existence of one solution $\overline{U}(T)$ for $dU/dT = U(T^2 - U^2) + {}^0\overline{d}$ which is a "river" at both $T = -\infty$ and $T = +\infty$: but this solution has different "incoding" $\sum_{n\geq -1} a_n T^{-n}$ and $\sum_{n \geq +1} b_n T^{-n}$ by divergent series at $T = \pm\infty$.

The second situation is related with the question of delayed bifurcation, the role of canards in this problem pointed out by Lobry, and the existence in the analytic case shown by Neishtadt (see the talk of F. Diener).

M. Dresden:

Chaos, predictability, undecidability and non standard methodology

Non standard analysis is rapidly developing in a general methodology, which can advantageously be applied to many problems in Theoretical Physics. Two recent developments strongly suggest the use of non standard ideas. One is the investigation of chaotic phenomena, the other the surprising recognition that a number of physical systems (both continuous and discrete) exhibit the features of computational irreducibility and undecidable propositions. In connection with chaotic phenomena a study has been initiated on the proper definition of non standard stability and the ensuing bifurcation structure, which differs in interesting ways from the standard patterns.

It is as yet unclear whether systems which show undecidable features or computational irreducibility in a standard treatment, will continue to do so in a non standard treatment. Since nonstandard analysis is a conservative extension of the classical logic, one might expect that. However, an examination of specific systems shows that whereas the behaviour of a quantity Q as $t \to \infty$ might be computationally irreducible, meaningful statements, can still be made about Q at star finite ${}^*t$.

R. Fittler:

Asymptotical Nonstandard Quantumelectrodynamics

Classical Quantumelectrodynamics is modified using different–size–of–infinity cutoff of space, momentum, particle number and of order of permutation resp., as well as infinitesimal photon restmass and energy modifications. The resulting theory only "infinitesimally approximates" classical Quantumelectrodynamics, but it is manifestly consistent. The asymptotic behaviour of the temporal development is investigated for a large class of Feynman diagrams whose "skeleton" is "stable". They yield asymptotic limits which agree with the classical ones, as long as no time derivatives of fields are involved. In the latter case agreement is reached only after appropriate charge renormalizations are performed.

C. W. Henson:

(Non)–determinacy of hyperfinite games

In 1980 Nigel Cutland posed the question whether all Borel hyperfinite games are hyperdetermined. (In such a game two players alternately choose 0 or 1 and thereby generate an internal sequence $\vec{y} = (y_1, y_2, \dots, y_H)$ in $\{0,1\}^H = P$. The game is specified by giving a set $A \subseteq P$; player I is said to win the play $\vec{y}$ if $\vec{y} \in A$ and otherwise II wins. The game is Borel if A is in the σ–algebra generated by the algebra of internal subsets of P. The game is hyperdetermined if there is an internal winning strategy for one of the players.) Cutland showed that such a game is hyper–determined if A is either $\sum_1^0$ or Π_1^0, but he left even the $\sum_2^0$ and Π_2^0 cases open (Zeit. für Math. Logik u. Grund. der Math. 1985). In this talk we presented two examples of Δ_2^0 games which are *not* hyperdetermined, thus giving a negative answer to Cutland's question. In fact, in the second example neither player has any winning strategy at all, neither external nor internal.

J. Hirshfeld:

The inverse function theorem

We present the inverse function theorem and the implicit function theorem in the general framework of functions between metric spaces. We also prove that a major part of the theorems holds even without any assumptions on the continuity or uniformity of the derivatives.

Ch. Impens:

Nonstandard upgrading of Weierstrass Approximation

Polynomial approximation of real continuous functions, of real analytic functions and of complex analytic functions is characterized in nonstandard terms, and thereby considerably upgraded: a continuous function on a G_δ–set is the standard part of a hyperreal polynomial (and conversely), a real analytic function on an open set is the "absolute" standard part of a hyperreal polynomial (and conversely), a complex analytic function on an open set with connected complement in $\mathbf{C} \cup \{\infty\}$ is the standard part of a hypercomplex polynomial (and conversely).

H. J. Keisler and J. H. Schmerl:

Making the hyperreal line both saturated and complete

In a nonstandard universe, the κ–saturation property states that any family of fewer than κ internal sets with the finite intersection property has a nonempty intersection. An ordered field F is said to have the λ–Bolzano Weierstrass property iff F has confinality λ and every bounded λ–sequence in F has a convergent λ–subsequence. We show that if $\kappa < \lambda$ are uncountable regular cardinals and $\beta^\alpha < \lambda$ whenever $\alpha < \kappa$ and $\beta < \lambda$, then there is a κ–saturated nonstandard universe in which the hyperreal numbers have the λ–Bolzano–Weierstrass property. The result also applies to certain fragments of set theory and second order arithmetic.

P. E. Kopp:

A nonstandard approach to option pricing

The celebrated Black–Scholes formula on the rational price of a European call option has been greatly extended using martingale representation theory. A discrete analogue was given in the form of a binomial option pricing formula by Cox, Ross and Rubinstein. It is known that this model is related to that of Black and Scholes by a weak convergence argument. We construct a hyperfinite version of this model and show that the Black–Scholes construction appears as the standard part of our model. This enables us to find explicit versions of the trading strategy generating the call option. The methods developed here have wide applicability for the pricing of contingent claims.

P. A. Loeb:

A reduction technique for limit theorems in analysis and probability theory and the martingale convergence theorem

The gist of many theorems in analysis and probability theory is as follows: Fix an appropriate class M of measures and a reference measure $\sigma \in M$; $\forall \mu \in M$, $d\mu/d\sigma$ is equal σ–almost everywhere to a limit along a directed set $\mathfrak{I}$ of ratios $R_i(\mu, \sigma), i \in \mathfrak{I}$. In this joint work with Jürgen Bliedtner, we show that the desired result is established once it is shown that $\forall$ measurable E and $\forall \nu \in M$ with $\nu(E) = 0, \limsup_i R_i(\nu, \sigma) \leq 1$ σ–a. e. on E. Applications include measure differentiation theorems, boundary limit theorems, and the martingale convergence theorem. For the latter, we use our technique to give a simple proof of the following theorem of Andersen and Jessen: Let $\mathfrak{A}_i$ be a countable, increasing family of σ–algebras and fix a probability measure P on $\mathfrak{A} = \sigma(\bigcup \mathfrak{A}_i)$. Let μ be a finite signed measure on $\mathfrak{A}$, and $\forall i$, let $\mu_i = \mu|\mathfrak{A}_i$. Then $d\mu/dP = \lim_i d\mu_i/dP$ P–a. e. Not all martingales are generated by a measure. If, however, one goes to a nonstandard model and uses the Andersen–Jessen theorem there, one obtains the usual convergence theorem for L^1–bounded martingales on $\{\mathfrak{A}_i : i \in \mathfrak{I}\}$.

G. Lumer:

Diffusions with discontinuous boundary behaviour, macroscopic convergence and approximation of (generalized) solutions

We consider problems of the type $\partial u/\partial t = A(x, D)u, u(0, \cdot) = f(\cdot)$, $u(t, \cdot)|\partial\Omega$ for $t > 0$, where A is a quite general elliptic operator of order 2 with real coefficients, and $A(x, D)1 \leq 0$, posed in sup-norm, i. e. in $X = C(\overline{\Omega})$. Such problems are then written as $du/dt = A_{\overline{\Omega}}u, u(0) = f \in C(\overline{\Omega})$, where because of the above conditions we must have $D(A_{\overline{\Omega}}) \subset C_0(\Omega)$; $A_{\overline{\Omega}}$ cannot generate a semigroup, but will generate a locally lipschitz integrated semigroup $S(t)$, and $\mathfrak{A}(t) = (1/t)S(t)$ (for $t > 0$) supplies a generalized average solution even for initial $f \in C(\overline{\Omega}) \setminus C_0(\Omega)$. We get, using Hunt theory, the asymptotic formula $-\frac{1}{t}A_{\overline{\Omega}}^{-1}$ as $t \to +\infty$, for $\mathfrak{A}(t)$. Moreover we can study in much more detail the solutions $(\mathfrak{A}(t)f)(x) = u(t, x)$ by an approximation result proved via nonstandard analysis, which is a Trotter theorem for locally lipschitz integrated semigroups on variable spaces $X_k \to X$ (for $\{A_k\}$ stable, $A_k \to A$ in the appropriate sense, we get $S_k(t) \to S(t)$), in particular for $A_k \in B(X)$ one gets that $S(t) = \lim_k \frac{1}{t} \int_0^t e^{sA_k} ds$. In the proof, the notion properties of "macroscopic convergence" on variable spaces, as well as Crandall–Liggett theory, and (sometimes a bit delicate) estimates for expressions of the type $(e^{tA_{k0}} - (1 - \frac{t}{N}A_k)^{-N})A_k^{-1}Q_k f(N$ infinitely large, $0 \leq t \leq t_0$ standard, f standard,

A_{k0} being "the part of A_k in $\overline{D(A_k)}$"), and related estimates, obtained via Crandall–Liggett theory and otherwise, are used, and later combined with the use of macroscopic (strong, uniform on t–compacta) convergence (and of course transfer).

W. A. J. Luxemburg:

Representations via internal functions

Using $*$–finite sets and their internal objects we shall review various representations of mathematical objects by internal ones. In doing so one presents an internal characterization or internal description of standard objects.

M. Machover:

The place of nonstandard analysis in mathematics and in mathematics teaching

This autum, Nonstandard Analysis will be thirty years old. Its acceptance into mainstream mathematics has not been as rapid as most of us had hoped and as this elegant and powerful method surely deserved. Nevertheless, in the last few years it has clearly gone quite a long way towards gaining that long–overdue acceptance. No longer the hobby–horse of a few enthusiasts, it is increasingly being appropriated by researchers in various fields of mathematics, whose interest in Nonstandard Analysis is mainly instrumental: they value it is a tool in their main fields of research.

I feel that this welcome development has reached a point where it may be desirable to take an over–view of Nonstandard Analysis and assess its place within the edifice of mathematics as a whole. Beyond the obvious interest that such an assessment may have from the viewpoint of the philosophy and methodology of mathematics, there is also a very important pragmatic aspect: that of teaching. The spread of Nonstandard Analysis among the new generation of mathematicians, and the benefit that they will be able to draw from it, will depend to a considerable extent on the way Nonstandard Analysis is taught and on the place of such teaching within the curriculum of mathematics departments in institutes of higher education.

A. Makhlouf:

"Rigidity of complex associative algebras"

The environment where the infinitesimals exist is useful for the study of deformations. Intuitively an object is rigid if every deformation leaves it unchanged. Here, we change the classical notion of deformation, which is an infinite formal series, by the nonstandard notion of perturbation: a law μ is a perturbation of a law μ_0 if $\mu(X,Y) \simeq \mu_0(X,Y) \forall^{St} X, {}^{St} Y \in \mathbf{C}^n$. Using the GOZE decomposition of a perturbation, we resolve completely the perturbation equation, then the deformation equation. We said that a law μ_0 is rigid if every perturbation is isomorphic. The closure of rigid law give an irreducible component of the variety. Our approach of the rigidity of the assocative law is based on the perturbation of idempotents. To each idempotent X, $X \neq 1_A$ of the algebra A, we associate the decomposition $A = X \cdot A \cdot X \oplus X \cdot A \cdot (1_A - X) \oplus (1_A - X)A \cdot X \oplus (1_A - X) \cdot A \cdot (1_A - X)$.

It shows that the right and left translations by X are diagonalizable and their eigenvalues are 0 or 1. We prove that a perturbation of an algebra with an idempotent independent of the unit has an idempotent with the same decomposition. As illustration we construct several families of associative rigid laws and we give the six–dimensional classification.

H. Osswald:

A functional approach to stochastic analysis

After we have introduced an appropriate notion of vector valued Loeb measures, we show that the stochastic integral with respect to L_p–bounded martingales, $1 \leq p < \infty$, on an adapted Loeb space is the Lewis–integral with regard to a Loeb vector measure.

The main tools in the proofs are internal representation (called $\mathfrak{S}$–liftings) of linear continuous functions on the Banach spaces of L_p–bounded martingales. Because of Doob's inequality, the proofs in the cases $1 < p < \infty$ are not difficult. We give a short sketch of proof in the case $p = 1$ and point out, where the difficulties arise.

Moreover, we give examples, where $\mathfrak{S}$–liftings always exist ($L^P(\Delta)$, where $1 \leq p < \infty$ and Δ is a Loeb–probability space) and where $\mathfrak{S}$–liftings don't exist in general ($L^\infty(\Delta)$; nonstandard hulls of Banach spaces).

Y. Peraire:

Analyse relative

Dans diffèrentes versions de l'Aanalyse nonstandard, les object sont classés en deux classes: celle des objets internes et celle des objets externes. La première classe est elle même divisée en deux sous-classes: objets standard — objets nonstandard.

La pratique nous a enseigné qu'il etait souhaitable de considérer plutot que tout objet interne a un degré de standardicité. Ceci a pour buts:

1. de rendre universelles les caractérisations externes des propriétès internes.
1. de donner des outils supplémentaires à l'exploration des objets externes.

Nous montrerons que ces objectivs peuvent etre réalisés dans le cadre d'une theorie des ensembles internes "a la Nelson" dans laquelle le prédicat unaire "st" est remplacé par un prédicat binaire.

V. Pestoff:

A nonstandard glance upon infinite-dimensional Lie theory

We consider an extension of some notions from the finite dimensional nonstandard Lie theory to infinite dimensions. The construction of the nonstandard hull of a Lie group is discussed. As a major application, a new enlargibility theorem for Banach–Lie algebras is presented. In particular, it results in a new super version of Lie's Third Theorem concerning finite dimensional super Lie algebras over algebras of super-coefficients satisfying a duality type condition.

M. Reeken:

External constructions in internal set theory

Internal set theory (IST) possesses a syntactic transformation into ZFC. In this sense both theories have the same content but they express it in different languages, that of IST being the more expressive one. Even if one accepts the language of IST as the appropriate one there remains the problem to translate a given internal context into a standard context: in order to communicate with "standard" mathematicians and because the standard context may have a greater appeal and more structure. The reduction algorithm of Nelson does not work in this sense because it typically destroys the mathematical context a given theorem of IST belongs to. It also results in formulas of dramatically increased complexity making them incomprehensible. We propose the following translation procedure which rests on the following theorem: Given an internal set S which possesses a standardization ${}^{s}S$ and given an external equivalence relation $\simeq$ then there exists a standard set T in the universe of IST the standard elements of which are in (external) bijection with the classes of $\simeq$, provided that $\simeq$ satisfies a certain admissibility condition. If S has an algebraic structure which is "compatible" with $\simeq$ then the bijection mentioned before carries it to the standard set T.

D. Ross:

Nonstandard methods for random sets

Let X be a partially ordered Polish space; a theorem of Strassen gives conditions on a pair P_1, P_2 of probability measures on X necessary and sufficient for the existence of an X^2 valued random vector having P_1 and P_2 as marginals, concentratred on the order.

I use a Loeb measure construction to extend this result to arbitrary spaces, provided P_1 and P_2 are Radon and the order satisfies a simple condition. The hyperfinite approximation reduces the proof to a "marriage lemma" argument.

The intended applications are to the stochastic ordering of random closed sets, random measures, point processes, and related stochastic objects. Time permitting I'll discuss some of these applications.

Ch. L.Thompson:

Extending the scope of elementary non-standard analysis

The author's experience of teaching non–standard analysis to third year undergraduates is described. The students seem to prefer an axiomatic approach to an ultrapower construction, and so a simple set of axioms including first order transfer is adopted. However, some of the more interesting applications of non–standard techniques e. g. the Peano existence theorem for differential equations, are usually not presented in such an elementary framework because they involve internal sets and permanence principles. It is suggested that many of these interesting applications may nevertheless be presented in elementary non–standard analysis by making use of "semi–standard" objects in place of internal ones. In this way a richer and more useful body of results becomes accessible by non–standard methods to those who are either unable or unwilling to undertake the study of a superstructure construction.

N. Vakil:

Representation of nonstandard hulls in IST for certain uniform spaces

Let X be a standard infinite set and D a standard family of pseudometrics on X endowing X with a Hausdorff unifrom structure. Two points x, y in X are called conditionally infinitely close if $d(x,p) \simeq d(y,p)$ for every standard p in X and every standard d in D. In general, this is a weaker condition than the well–known relation of being infinitely close. They are, however, equivalent in case either x or y is pre–near–standard. Let's call a uniform space admissible if the equivalence of these two concepts also holds for all finite points. Thus, for example, every HM–Space is an admissible space. Restricting ourselves to admissible unifrom spaces, we present, in this paper, a method of constructing nonstandard hulls which can be carried out within the framewok of internal set theory.

F. Wattenberg:

Economics, simulated annealing and nonstandard analysis

This work is motivated by the striking analogies between the bahaviour of algorithms based on simulated annealing and the behaviour of agents searching for an optimum in an economy. Using nonstandard methods we develop a simple and intuitive framework for studying the effects of deterministic (or centralized) forces, aléatoire (or decentralized) forces, and the structure of the economy on the efficiency of the economy at equilibrium and the speed with which an economy adjusts to change.

H. von Weizsäcker:

Discrete versus continuous time reversal of Markov processes

This talk gives an example both of an intuitive and of a formal application of infinitesimal resp. Nonstandard ideas. The first part shows how two papers of Komogorov can be related to each other much more directly than Kolmogorov does himself. The question is the characterization of those Markovian dynamics for which the steady state is reversible. The PDE method in one paper can be substituted by a nonstandard version of the discrete argument in the other.

The second part of the talk describes the following result of D. Zimmermann: Let $(X, \mathfrak{B})$ be a measurable space and let $\mathfrak{T}$ be a set of transition kernels on $\mathfrak{B}$ then the decomposition $\mu = \int_{\varepsilon} \nu dp$ of an invariant probability measure μ into ergodic components ν is unique (provided it exists). The proof uses heavily Loeb theory.

W. Willinger:

A nonstandard approach to the study of modern financial markets

Stochastic modeling of modern financial markets provides an excellent environment for studying both martingale theory and stochastic integration theory. As an example, the economically meaningful condition of "no arbitrage" in a market model (also known as the "no free lunch"–assumption), is related to the probabilistically desirable property that the securities price process becomes a martingale under a certain change of measure. Similarly, "completeness" of the financial market (i. e., every option can be uniquely priced), is equivalent to the martingale representation property of the securities price process.

Whereas the theory of finite markets (discrete time and finite probability space) is well understood, a similar insight into continuous–time extensions is missing. We propose using methods from nonstandard probability theory to study continuous market models via appropriately 'built–in' discrete models. We illustrate some of the appealing features of this nonstandard approach in the context of the well–known Black–Scholes option pricing model.

M. Wolff:

An application of spectral calculus to the problem of saturation in approximation theory

Let $\mathfrak{L} = (L_\alpha)$ be net of bounded linear operators on the Banach space E converging strongly to the identity on E. For a given complex–valued function f of a fixed type we consider the net $f(\mathfrak{L}) = (f(L_\alpha))$. Among other things we shall show that under reasonable conditions the saturation space of $\mathfrak{L}$ with respect to a given net $\phi = (\varphi_\alpha)$ of positive real numbers converging to zero is equal to that one of $f(\mathfrak{L})$. More generally we consider nets $(f_\alpha(L_\alpha))$ where (f_α) is a net of complex–valued functions. Our results generalize theorems of Butzer as well as such ones of Nishishirao.

B. Zivaljevic :

Hyperfinite transversal theory

The Hall's type theorem has been proved in the case when the ordinary, standard cardinality function has been substituted by the Loeb measure of an internal, uniformely distributed, hyperfinite measure space $(\Omega, L(\mathfrak{A}), L(\mu))$. Different types of results are obtained for graphs Γ being of different "complexity" with respect to the Borel and projective hierarchies of sets. It turns out that the full analogue of Hall's theorem is true when the graph Γ is

i) Π_1^0;

ii) $\sum_1^1$ graph all of whose Y–sections are of cardinality $\leq n$ where n is a standard integer;

iii) Borel graph all of whose Y–sections are internal sets;

iv) Borel over the family of all squares $A \times B$ and all of the Y–sections being Π_1^0;

v) $\Gamma = U_{k=1}^n (M_k \times N_k)$ where M_k, and N_k are Loeb measurable.

The approximation form of Hall's theorem holds true in one of the below cases.

i) Γ is $\sum_1^0$;

ii) Γ is $L(\mu \times \mu)$ measurable and $L(\mu)(\Gamma(x)) \geq L(\mu)(\Gamma^{-1}(y))$ for almost all $x \in X$ and $y \in Y$;

iii) Γ is $\sum_1^1$ and all of its Y–sections are at most countable.

Berichterstatter: W. Rinkewitz

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Tagungsteilnehmer

Prof.Dr. Sergio Albeverio
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

4630 Bochum 1

Prof.Dr. Leif Arkeryd
Dept. of Mathematics
Chalmers University of Technology
and University of Göteborg

S-412 96 Göteborg

Prof.Dr. Imme P. van den Berg
Interfaculteit Econometrie
Rijksuniversiteit Groningen
Postbus 800

NL-9700 AV Groningen

Prof.Dr. Marek Capinski
Instytut Matematyki
Uniwersytet Jagiellonski
ul. Reymonta 4

30-059 Krakow
POLAND

Prof. Nigel J. Cutland
Dept. of Pure Mathematics
University of Hull
Cottingham Road

GB- Hull , HU6 7RX

Dr. Antoine Delcroix
Institut de Mathematiques
Universite Louis Pasteur
7, rue Rene Descartes

F-67084 Strasbourg Cedex

Prof.Dr. Francine Diener
17 bis rue de la Barre

F-95880 Enghien les Bains

Dr. Karl-Heinz Diener
Mathematisches Institut
Universität Köln
Weyertal 86-90

5000 Köln 41

Prof.Dr. Marc Diener
17, bis rue de la Barre

F-95880 Enghien les Bains

Prof.Dr. Max Dresden
727A Loma Verde Ave.

Palo Alto , CA. 94303
USA

Prof.Dr. Robert Fittler
Institut für Mathematik II
Freie Universität Berlin
Arnimallee 3

1000 Berlin 33

Dr. Christoph Kessler
Maria-Theresiastr. 3

7800 Freiburg

Prof.Dr. C. Ward Henson
Department of Mathematics
University of Illinois
Altgeld Hall
1409, West Green Street

Urbana , IL 61801
USA

Prof.Dr. Peter Ekkehard Kopp
Dept. of Pure Mathematics
University of Hull
Cottingham Road

GB- Hull , HU6 7RX

Prof.Dr. Joram Hirshfeld
Dept. of Mathematics
Tel Aviv University
Ramat Aviv
P.O. Box 39040

Tel Aviv , 69978
ISRAEL

Prof.Dr. Detlef Laugwitz
Fachbereich Mathematik
TH Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Dr. Christian Impens
Seminarie voor Wiskundige Analyse
Rijksuniversiteit Gent
Galglaan 2

B-9000 Gent

Prof.Dr. Peter A. Loeb
Department of Mathematics
University of Illinois
Altgeld Hall
1409, West Green Street

Urbana , IL 61801
USA

Prof.Dr. H. Jerome Keisler
Department of Mathematics
University of Wisconsin-Madison
Van Vleck Hall

Madison WI, 53706
USA

Prof.Dr. Gunter Lumer
Mathematiques
Faculte des Sciences
Universite de L'Etat a Mons
Av. Maistriau 15

B-7000 Mons

Prof.Dr. Wim A. J. Luxemburg
Dept. of Mathematics
California Institute of Technology

Pasadena , CA 91125
USA

Prof.Dr. Vladimir Pestoff
Laboratory of Functional Analysis
Institute of Mathematics
Novosibirsk State University

63 0090 Novosibirsk
USSR

Prof.Dr. Moshe Machover
Philosophy of Science
King's College
Manresa Road

GB- London SW3 6LX

Prof.Dr. Michael Reeken
Fachbereich 7: Mathematik
Universität/Gesamthochschule
Wuppertal, Gaußstr. 20
Postfach 10 01 27

5600 Wuppertal 1

Prof.Dr. Abdenacer Makhlouf
21, rue de la Tour du Diable

F-68100 Mulhouse

Prof.Dr. Michael M. Richter
Fachbereich Informatik
Universität Kaiserslautern
Postfach 3049

6750 Kaiserslautern

Prof.Dr. Horst Osswald
Petersbrunn 111

8130 Starnberg

Werner Rinkewitz
Mathematisches Institut
Universität München
Theresienstr. 39

8000 München 2

Prof.Dr. Yves Peraire
Mathematiques
Universite de Metz
Faculte des Sciences
Ile du Saulcy

F-57045 Metz Cedex 1

Prof.Dr. Lothar Rogge
Fachbereich Mathematik
Universität-GH Duisburg
Postfach 10 16 29
Lotharstr. 65

4100 Duisburg 1

Prof.Dr. David Ross
Department of Statistics
Oregon State University

Corvallis, OR 97331
USA

Prof.Dr. Frank Wattenberg
Dept. of Mathematics
University of Massachusetts

Amherst , MA 01003
USA

Adrian Simpson
c/o Prof. N. Cutland
Dept. of Pure Mathematics
The University of Hull
Cottingham Road

GB- Hull HU6 7RX

Prof.Dr. Heinrich von Weizsäcker
Fachbereich Mathematik
Universität Kaiserslautern
Erwin-Schrödinger-Straße
Postfach 3049

6750 Kaiserslautern

Dr. Maria Stary
Fachbereich 7: Mathematik
Universität/Gesamthochschule
Wuppertal, Gaußstr. 20
Postfach 10 01 27

5600 Wuppertal 1

Prof.Dr. Walter Willinger
Bell Communications Research
Room 2P-372
445 South Street
Box 1910

Morristown , NJ 07960-1910
USA

Dr. Christopher L. Thompson
Faculty of Mathematical Studies
University of Southampton

GB- Southampton , S09 5NH

Prof.Dr. Manfred Wolff
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof.Dr. Nader Vakil
Dept. of Mathematics
Western Illinois University

MaComb , IL 61455
USA

Prof.Dr. Bosko Zivaljevic
Z. Josila 5

YU-71000 Sarajevo