MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 39/1990

SURGERY UND L-THEORIE

9.9 bis 15.9.1990

Die Tagung fand unter der Leitung von Herrn Bak (Bielefeld), Herrn tom Dieck (Göttingen) und Herrn Ranicki (Edinburgh) statt, mit 31 Teilnehmern und 21 Vorträgen. Im Mittelpunkt des Interesses standen die Anwendungen von Surgery und L-Theorie auf die Klassifikation von topologischen Mannigfaltigkeiten. Die neuartige Methoden der kontrollierten Topologie waren besonders stark vertreten.

The conference was organized by A. Bak (Bielefeld), T. tom Dieck (Göttingen) and A. Ranicki (Edinburgh), with 31 participants and 21 talks. The topics concerned the applications of surgery and L-theory to the classification of topological manifolds. The new methods of controlled topology were particularly strongly represented.

Tagungsteilnehmer / Participants

1. P. Akhmetiev, Troitsk
2. D. R. Anderson, Syracuse
3. A. Bak, Bielefeld
4. F. Bermbach, Mainz
5. F. Clauwens, Nijmegen
6. F. Connolly, Notre Dame
7. J. Davis, Bloomington
8. T. tom Dieck, Göttingen
9. S. Ferry, Binghamton
10. I. Hambleton, Hamilton
11. J.-C. Hausmann, Genève
12. C. B. Hughes, Nashville
13. C. Kearton, Durham
14. A. F. Kharsiladze, Troitsk
15. E. Laitinen, Helsinki
16. A. Libgober, Chicago
17. M. Morimoto, Okayama
18. H. Munkholm, Odense
19. Ngoc Diep Do, Hanoi
20. K. Pawałowski, Poznań
21. A. Pazhitnov, Moscow
22. S. Prassidis, Hamilton
23. A. Ranicki, Edinburgh
24. M. Raußen, Aalborg
25. M. Rothenberg, Chicago
26. J. Smith, Philadelphia
27. C. Stark, Gainesville
28. P. Teichner, Mainz
29. P. Vogel, Nantes
30. M. Weiss, Aarhus
31. B. Williams, Notre Dame

Vortragsauszüge / Summaries of talks

Tame immersions

P.Akhmetiev

The notions of tame immersions and tame bordism of immersions of 2-dimensional manifolds in $\mathbb{R}^3$ are defined. These notions are very similar to the usual ones of the bordism of immersions. However, we construct an invariant ы of tame bordisms, which is not an invariant of ordinary bordism.

An elementary approach to controlled Whitehead theory

D.R.Anderson

Let $p: E \to B$ be a map. Quinn has constructed a homology theory $H_*^{lf}(B, \mathbb{S}(p))$ in which the obstructions to solving the controlled end problem and the controlled s-cobordism problem lie. He has also described an Atiyah-Hirzebruch type of spectral sequence for calculating $H_*^{lf}(B, \mathbb{S}(p))$. Since Quinn's construction of this theory is quite complicated, it is desirable to have a more elementary approach to it. The talk described one such approach, based on bounded controlled algebraic K-theory.

Unanticipated general vanishing theorems in G-surgery and applications to transformation groups

A.Bak

(joint with M.Morimoto)

A *3-dimensional homology G-surgery theorem is established* and new, unexpected *general vanishing theorems for n-dimensional ($n = 3$ or ≥ 5) G-surgery obstruction groups are proved*. These results are applied to show that *there is a smooth 1-fixed point action of the alternating group A_5 on the standard sphere S^9*. The vanishing results are as follows. If π is a finite abelian group and if σ generates a group $\langle\sigma\rangle$ of order 2 then the semi-direct product $\pi \ltimes \langle\sigma\rangle$ where σ acts on π by inverting each of its elements is denoted by (π, σ). If G is a finite group let $G(2) = \{g \in G \mid g^2 = 1, g \neq 1\}$.

Theorem The G-surgery obstruction groups $W_n^{s \text{ or } h}(\mathbb{Z}[G], \Gamma(G(M)), \omega)$ are trivial in any one of the following situations

i) $n = 4k+3$, $G(M) = G(2)$, the 2-hyperelementary subgroups of G are abelian, ω trivial.

ii) $n = 4k+3$, $G = (\pi, \sigma)$, $G(M) \supseteq \pi\sigma$ and if $\pi_2 \neq 1$ then $G(M) \cap \pi_2 \neq \emptyset$, ω trivial.

iii) $n = 4k+2$, $G, G(M), \omega$ as in ii).

iv) $n = 4k+1$, $G = (\pi, \sigma)$, $G(M) \supseteq \pi\sigma$, $\omega(\sigma) = -1$, $\omega(\pi) = 1$.

v) $n = 4k$, $G, G(M)$ and ω as in iv) with the additional restriction $|\pi/\pi^2| \leq 2$.

Diffeomorphism classification of simply-connected 7-manifolds

F.BERMBACH

Motivated by the problem of classifying simply-connected 7-dimensional compact homogenous spaces, the surgery obstruction in Kreck's l-monoids for transforming a normal bordism $(W^8, \bar{\nu})$ of normal 2-smoothings $\bar{\nu}_i : M_i^7 \to X$ into an s-cobordism is analysed.

First an introduction to Kreck's manifold approach to surgery theory is given (normal k-smoothings, normal k-type), followed by the definition of the corresponding surgery obstructions (l-monoids). The analysis of the l-obstructions leads to the following

THEOREM Suppose given two simply-connected compact manifolds M_i^7 - both or neither Spin - with $\pi_2(M_i) \cong \mathbb{Z}^r$ and $H^4(M_i)$ finite. (The normal 2-type of such manifolds is $H(w_2) \oplus p : X = (\mathbb{CP}^\infty)^r \times BSpin \to BO$). Then M_1 is diffeomorphic to M_2 if and only if there exist

(1) an isomorphism $\alpha : H^4(M_1) \to H^4(M_2)$ preserving the linking form,

(2) normal 2-smoothings $\bar{\nu}_i : M_i \to X$ such that

$$\alpha \bar{\nu}_1^* = \bar{\nu}_2^* : H^4(X) \to H^4(M_1) \to H^4(M_2) ,$$

(3) a normal bordism $\bar{\nu} : W \to X$ (with $\bar{\nu}$ a 4-equivalence) satisfying the following conditions:

   (a) $\mathrm{sign(W)} = 0$,

   (b) $\langle \bar{\nu}^* x \cup p^{*-1} \bar{\nu}^* y, [W, \partial W] \rangle = 0$ for all $x, y \in H^4(X)$ with p^* the rational isomorphism $H^4(W, \partial W) \to H^4(W)$

   (c) the partial quadratic refinement of the linking form induced by the bordism vanishes on

$$\Delta = \{(z, \alpha z) \mid z \in H^4(M_1)\} \subseteq H^4(\partial W) .$$

Remark: Condition (3) can be dropped if $|H^4(M_i)|$ is odd or $H^4(M_i)$ is generated by products.

Higher order Arf invariants

F.CLAUWENS

With my student Paul Wolters. This material is in his thesis.

Given a ring with antistructure (R, α, u), a new one (S, β, v) is defined by taking

$$S = R[t]/t^n, \qquad \beta\left(\sum r_i t^i\right) = \sum \alpha(r_i) \left(\frac{-t}{1+t}\right)^i, \qquad v = u(1+t)$$

Then the map $p: L_0^h(S) \to L_0^h(R)$ given by $t \mapsto 0$ is an isomorphism. Composing p^{-1} with the discriminant map yields a map $\omega_1: L_0^s(R) \to H^0(C_2, K_1(S))$. This map can be viewed as the combination of all ordinary Arf invariants.

If $x \in ker(\omega_1)$ then $p^{-1}(x)$ is the image of some $y \in L_0^s(S)$; take its image in $H^1(C_2, K_2(S))$ under the Hasse-Witt invariant. We thus get a map

$$\omega_2: ker(\omega_1) \to coker(d: H^1(C_2, K_1(S) \to H^1(C_2, K_2(S,(t)))$$

The indeterminacy $im(d)$ of this "secondary Arf invariant" is not too bad: if R is commutative it is given by $x \mapsto \{x, -u\}$. If moreover R is equipped with a partial λ-structure e.g. for $n = 3$ with a map $\theta: R \to R$ such that $\theta(a+b) = \theta(a)+\theta(b)+ab$ and $\theta(ab) = \theta(a)b^2 + a^2\theta(b) - 2\theta(a)\theta(b)$ then we can express the target group in terms of the module of differentials Ω_R. (Little is won by taking $n > 3$). This invariant suffices to detect all of $L(\mathsf{F}_2[x,y])$. It is however unpractical for noncommutative rings.

These invariants are best tested on the subgroup $Arf(R) \subset L_0(R)$ of all differences $(M,\lambda,\mu)-(M,\lambda,\tilde{\mu})$ between quadratic forms with the same underlying symmetric form.

We also defined an invariant

$$\Upsilon: Arf(R) \to coker(1+\phi: HQ_1(R/2R) \to HQ_1(R/2R))$$

where HQ_* denotes quaternionic homology, and ϕ a certain natural operation on it. It coincides with the combination of ω_1 an ω_2 for commutative rings. In case that R is a group ring $\mathsf{F}_2\pi$ then $HQ_1(R)$ and hence $coker(1+\phi)$ can be easily computed: they split as direct sums indexed by classes of elements of π under the relation generated by conjugation, inversion, and squaring. The part corresponding to the class of an element g is typically the quotient of the group $\{h \in \pi\,;\, hgh^{-1} = g \text{ or } g^{-1}\}$ by the subgroup generated by all squares and by all elements h for which $h^{2^n} = g$ for some n.

Theorem This invariant is injective on $Arf(\mathsf{F}_2\pi)$ if π has an (infinite) cyclic subgroup of finite index.

Corollary There are groups of this type (in the smallest example the index is 24) for which the map into the relative KQ_0 group in theorem 12.2 of Bak's book does not vanish.

Nil groups as finitely generated $\mathbb{Z}N$-modules

F. Connolly

The Nil-K theory groups $N^iK_j(R)$ are never finitely generated groups unless they are zero, by a result of F.T.Farrell. But we prove the following:

Let $N = \{1,2,3,4,\ldots\}$, a monoid under multiplication. Let $N^i = N \times N \times \ldots \times N$.

Theorem 1 $N^iK_j(\ )$ is actually a functor from rings to $\mathbb{Z}[N^i]$-modules. If $R = \mathbb{Z}[G]$, G a finite group we then have

a) $N^iK_j(R) = 0$ for all $i \geq 1, j \leq -1$

b) $N^iK_0(R) = \mathbb{Z}[N^i] \otimes_{\mathbb{Z}[N]} NK_0(R)$. The map $N \longrightarrow N^i$ is the diagonal.

c) $NK_0(R)$ is a finitely generated $\mathbb{Z}[N]$-module.

Analogous statements, with K_1 replacing K_0, hold when R is a finite ring.

We then consider $UNil^x_{2k}(R;R,R)$ if $x = h$ or s, and R is any ring with involution. We supply a missing calculation in the following:

THEOREM 2 $UNil^x_{2k}(R;R,R)$ is naturally a $\mathbf{Z}[N]$-module. Moreover, $UNil_2(\mathbf{Z};\mathbf{Z},\mathbf{Z})$ is isomorphic to $\mathbf{F}_2[N]$.

(Previously, it had been known to have exponent ≤ 4 and to contain $\bigoplus_1^\infty \mathbf{Z}/2\mathbf{Z}$.)

Theorem 1 is joint work with M.daSilva. Theorem 2 is joint with T.Koźniewski.

Rational geometric topology

J.F.DAVIS

Surgery theory reduces the classification of manifolds of dimension ≥ 5 to homotopy theory and the study of $\mathbf{Z}[\pi_1]$-modules. Unfortunately, the way is fraught with difficulty due to their complexities. However, the stable homotopy groups of spheres are rationally trivial, and all $\mathbf{Q}[G]$-modules are projective for a finite group G. One might hope for corresponding simplification in rational geometric topology.

THEOREM 1 A CW complex X with $\pi_1(X) = G$ finite has the rational homotopy type of a closed manifold of dimension $n \geq 5$ iff

i) for all $g \in G$-$\{1\}$, $L(g) = \sum_i(-)^i \mathrm{trace}(g_* : H_i(\tilde{X}) \longrightarrow H_i(\tilde{X})) = 0$

ii) X is a $\mathbf{Q}$-Poincaré complex of dimension n with orientation character w

iii) $\sigma^*(X) \in \Omega_n(G,w)$

iv) If $n \equiv 0(\mathrm{mod}\ 4))$ and $w = 1$, there exist classes $p_i \in H^{4i}(X;\mathbf{Q})$ so that $L(p_1,\ldots p_{n/4})[X] = \mathrm{sign}(X)$ and the Pontrjagin numbers of X are the Pontrjagin numbers of a closed manifold.

These conditions depend only on $H^*(\tilde{X};\mathbf{Q})$. Here $\sigma^* : \Omega_n(G,w) \longrightarrow L_n(\mathbf{Q}[G],w)$ is the Mishchenko-Ranicki symmetric signature. The next theorem follows from the computation of σ^*.

THEOREM 2 If $n \equiv 2(\mathrm{mod}\,4)$ or $n \equiv 0(\mathrm{mod}\,4)$ and $w \neq 1$ or $n \equiv 0(\mathrm{mod}\,4)$, $w = 1$ and $\mathrm{sign}(M) = 0$, then for any free G-action on an oriented manifold $\tilde{M}$ with $M = \tilde{M}/G$, the rational symmetric intersection form on $H^{n/2}(\tilde{M};\mathbf{Q})$ is hyperbolic.
If $n \equiv 3(\mathrm{mod}\,4)$, $\sigma^*\Omega_n(G,w) = 0$.
If $n \equiv 1(\mathrm{mod}\,4)$, $\sigma^*\Omega_n(G,w) = 0$ is generated by σ^*(lens spaces).

Much of this is joint work with J.Milgram.

Controlled topology and differential geometry

S.FERRY

In connection with the study of finiteness theorems in differential geometry, Gromov has introduced spaces $LGC(n,\rho)$ of manifolds. We prove that every precompact subset of $LGC(n,\rho)$ contains at most finitely many simple homotopy types. This involves the study of controlled simple homotopy theory over limits of spaces in $LGC(n,\rho)$. The finiteness theorem above is true in spite of the fact that these limits need not be manifolds - or even finite-dimensional spaces. We also discuss the problem of proving that such classes contain finitely many homeomorphism types.

Actions on $S^n \times \mathbf{R}^k$ with Compact Quotient

I.HAMBLETON

(joint with E.Pedersen)

What are the possible finite sub-groups of a group Γ which acts freely and properly discontinuously on some product $S^n \times \mathbf{R}^k$ with compact quotient? If a finite group G acts freely and simplicially on a complex homotopy equivalent to a sphere S^n, then G has periodic Tate cohomology. Swan proved in 1960 that this condition was also sufficient to construct such a simplicial action. A result of Connolly and Prassidis (1989) is that any countable group with finite virtual cohomological dimension and such that every finite subgroup has periodic cohomology, acts freely and properly discontinuously on some $S^n \times \mathbf{R}^k$, but their construction does not produce actions with compact quotient.

For free topological actions on S^n itself the first additional restriction is:

THEOREM (Milnor, 1957) A finite dihedral group does not act freely and topologically on S^n.

Milnor's argument used the compactness of S^n as well as the manifold structure. In contrast,

EXAMPLE Any finite dihedral group with periodic cohomology acts freely and smoothly on $S^3 \times \mathbf{R}^3$.

Another attempt to study the problem from this direction is to study actions on a sphere S^{n+k} with a standardly embedded invariant sub-sphere S^{k-1}. This situation is related to our question if we assume that the action is free away from the S^{k-1}, since $S^{n+k} - S^{k-1} \approx S^n \times \mathbf{R}^k$.

THEOREM (Anderson-Pedersen, Hambleton-Madsen, 1982) A finite dihedral group can not act semifreely and topologically on S^{n+k} with fixed standard sub-sphere S^{k-1}.

Our first result is a generalization of this. Let $\mathbf{R}_-$ denote the non-trivial 1-dimensional representation of a finite dihedral group.

THEOREM 1 Let V be a linear representation of the dihedral group $G = D_{2p}$ of order $2p$, where p is an odd prime. Then there is a topological action of G on a sphere, free off a standard sub-sphere where it is given by the unit sphere $S(V)$, if and only if V has at least two $\mathbf{R}_-$ summands.

This is used to prove the non- existence part of the following:

THEOREM 2 Let $\alpha : D_{2p} \to GL_k(\mathbf{Z})$ be a homomorphism and form the semidirect product $\Gamma = D_{2p} \ltimes \mathbf{Z}^k$. The group Γ acts freely and properly discontinuously on $S^n \times \mathbf{R}^m$ with compact quotient for some n, m if and only if $n \equiv 3 \pmod 4$, $m = k$ and α considered as a real representation has at least two $\mathbf{R}_-$ summands.

Our non–existence results concern topological actions, but the actions constructed in Theorem 2 are actually smooth. The lowest dimensional example is a co–compact action on $S^3 \times \mathbf{R}^2$. The proof of Theorem 1 uses the L–theory of additive categories due to Ranicki (1990), and further develops the "bounded surgery theory" of Ferry and Pedersen (1990).

Quasi-linear actions on spheres

J.-C.HAUSMANN

Let $\alpha : G \to O_{n+1}$ be an orthogonal representation of a compact Lie group G. The induced action on S^n is called a "linear action". If Σ^n is a codimension 1 submanifold of $\mathbf{R}^{n+1}$, diffeomorphic to S^n, with $G\Sigma = \Sigma$, this also induces an action of G on S^n. Such an action is called a quasi-linear (QL) action (associated to α).

PROBLEMS 1. Which actions of G on S^n are QL?
2. Is a QL-action DIFF (or TOP)-conjugate to its associated linear action?

We can give precise answers for G finite, $G = S^1$ or S^3 and <u>free</u> actions. The answer depends on the h-cobordism class of $G\backslash S^n$. For example:

1. For G cyclic, there exist in general QL-actions which are not TOP-conjugate to linear actions.
2. $G = S^1$ or S^3, $n - \dim G > 4$: any QL-action is DIFF-conjugate to its associated linear action.
3. $G = S^3$, standard action on $\mathbf{R}^8$: any QL-action is DIFF-conjugate to the standard action iff the smooth Poincaré conjecture is true in dimension 4.

QL-actions occur naturally around isolated extrema of equivariant maps $f : V \to \mathbf{R}$, where V is a G-manifold.

Neighborhoods in stratified spaces

B.HUGHES

The main result is a structure theorem for neighborhoods of strata in the manifold homotopically stratified spaces introduced by Frank Quinn.

TEARDROP STRUCTURE THEOREM If X is a manifold homotopically stratified space with no strata of dimension less than 5, then each stratum has a teardrop neighborhood in X.

If Y is a stratum of X, then such a neighborhood is given by first constructing a stratified manifold approximate fibration over $Y \times \mathbf{R}$ and then adjoining Y to the total space of this map with a natural topology.

Moreover, a very global result establishes a homotopy equivalence between simplicial sets of stratified manifold approximate fibrations over $Y \times \mathbf{R}$ and neighborhood germs of Y. Then a classification theorem for these maps (generalizing an unstratified result of myself, L.Taylor and B.Williams) yields a classification of neighborhood germs. The obstruction to the existence of mapping cylinder neighborhoods becomes transparent.

Other applications include a multiparameter isotopy extension theorem (generalizing the single isotopy result of Quinn), the local contractibility of the homeomorphism group of a compact stratified space (generalizing Siebenmann's result in the locally conelike case), the existence of tangent bundles (generalizing Anderson-Hsiang's result in the locally conelike case), and a topological version of Thom's First Isotopy Theorem.

The main result was first discovered in the case of two strata in the course of joint work of myself, L.Taylor, S.Weinberger and B.Williams.

A spectral sequence in surgery

A.F.KHARSILADZE

(joint with I.Hambleton)

For all integers n, m the spaces $X_{n,m}$ are constructed so that $X_{0,0} = \mathbb{L}_0(G)$, $X_{1,1} = \mathbb{L}_{-1}(G^-)$, and so on, with $\mathbb{L}_i(G)$ the Quinn-Ranicki spectra. Maps $X_{n,m} \longrightarrow X_{n-1,m}$ and $X_{n,m} \longrightarrow X_{n,m+1}$ are defined so that the square

$$\begin{array}{ccccc} & & X_{n-1,m} & & \\ & \nearrow & & \searrow & \\ X_{n,m} & & & & X_{n-1,m-1} \\ & \searrow & & \nearrow & \\ & & X_{n,m+1} & & \end{array}$$

is a homotopy pullback. Now the homotopy spectral sequence of infinite filtration

$$X_{\infty,0} \to \ldots \to X_{n,0} \to X_{n-1,0} \to \ldots \to X_{0,0} \to X_{-1,0} \to \ldots \to X_{-\infty,0}$$

is defined with $E_1^{p,q} = \pi_{q-p}(X_{p,0}, X_{p+1,0}) = LN_q(\pi \longrightarrow G^-)$ and $E_2^{p,q} = H^p_\phi(\mathbb{Z}/2;$-$LN_q(\pi \longrightarrow G^-))$; the last being the Tate cohomology group of $\mathbb{Z}/2$ with Browder-Livesay group $LN_q(\pi \longrightarrow G^-)$ as coefficients with respect to the involution ϕ acting on LN_q. If (M, f) is a quadratic form over $(\mathbb{Z}[\pi], \alpha, u)$ then ϕ takes it to (M', f') with $m'r = m(trt^{-1})$, and $f'(m,n) = w(t)t^{-1}f(m,n)t$.

Complex structures on manifolds homeomorphic to Grassmanians

A.LIBGOBER

Kodaira and Hirzebruch gave the following characterization of $\mathbb{C}P^n$: if X is a Kähler manifold homeomorphic to $\mathbb{C}P^n$ then X is analytically equivalent to $\mathbb{C}P^n$. (The case $c_1 = -(n+1)\sigma$, σ a positive generator of $H^2(X;\mathbb{Z})$ does not occur as follows from Yau's work). Brieskorn gave a similar characterization of quadrics (of dimension > 2). For Grassmanians $Gr(n, n+m)$ of n-spaces in $\mathbb{C}^{n+m}$ the following is true:

THEOREM Let X be a compact Kähler manifold such that there exists $\phi : X \to Gr(n, n+m)$ which is a homeomorphism and Q is a rank m holomorphic bundle generated by global sections, the Chern classes of which are ϕ^* images of Chern classes of the universal quotient bundle of $Gr(n, n+m)$ then X is analytically equivalent to $Gr(n, n+m)$.

There is an extension of this theorem to all hermitian symmetric spaces (irreducible) of classical type.

Applications of equivariant surgery

M.MORIMOTO

We have recently proved that G-surgery obstruction groups frequently vanish when the set of special involutions $G(X)$ is very large. This has encouraged us to seek constructions of various G-actions, using G-surgery where $G(X)$ is very large. The following theorems were obtained using this philosophy.

THEOREM I S^n has smooth one fixed point actions of A_5 if $n \geq 6$ and $n \neq 8$.

THEOREM II For an arbitrary fake lens space X^{2m+1} ($m \geq 2$), there exists a fake lens space Y with involution σ such that

i) Y is simple homotopy equivalent to X, and

ii) Y^σ is diffeomorphic to $\begin{cases} S^m & (|\pi_1(X)| \text{ odd}) \\ \mathbf{RP}^m \coprod \mathbf{RP}^m & (|\pi_1(X)| \text{ even}) \end{cases}$.

THEOREM III The number of the A_5-homeomorphism classes of singular sets Σ_s of homology 3-spheres Σ having one fixed point A_5-actions, is 4.

[Theorems II and III were obtained in joint work with A.Bak.]

Open cones in controlled topology

H.J.MUNKHOLM

Chapman's and Quinn's solutions to the controlled end problem and the controlled h-cobordism problem led to controlled K-theoretic groups which are not well understood from an algebraic viewpoint. Pedersen, and later on Anderson and Munkholm, developed a boundedly controlled analogue of Chapman's and Quinn's work when the algebra is much nicer, especially where the controlling space is an open cone. In fact, in that case the boundedly controlled K-theory is a homology theory. Pedersen (and coworkers) have applied this theory succesfully to important problems, but basically the question of the actual occurrence of open cones (in the controlled end problem, say) has not been asked. In this lecture I show that - under slightly stronger ab initio conditions than those imposed by Chapman, or Quinn - any controlled end problem $p : M \to B$ over a compact metric space can be "enhanced" to a problem $\bar{p} : M \to O(B)$ controlled over the open cone of B, and such that the K-theory of $(M, \bar{p})$ "at ∞" is equal to that of a suitably defined open cone $O(q) : O(X) \to O(B)$ for some $q : X \to B$.

I also give explicit geometric, and algebraic, descriptions of the term "at ∞" above.

The talk is strongly related to the talk by Anderson at this conference.

Morse theory of closed 1-forms and related algebraic constructions

A.V.PAZHITNOV

Let M be a manifold, ω be a closed 1-form. ω is said to be a Morse form if locally $\omega = dh$, h a Morse function. In the early 80's Novikov proposed the theory which enables one to obtain the analogues of the Morse inequalities for this situation. The analogue of the Morse complex here is the chain complex over a special completion Λ_ξ^- of the group ring Λ of $\pi_1(M)$ (or some suitable quotient G of $\pi_1(M)$), where $\xi = [\omega]$ is the deRham cohomology class. This completion has the good algebraic properties for at least two cases: 1) G is free abelian and ξ determines a monomorphism $G \to \mathbf{R}$, (J.-C. Sikorav) 2) G is free and $\xi = [\omega]$ is any cohomology class (the author). In the case 1) Λ_ξ is a principal ideal domain and in the case 2) Λ_ξ is a semifir, i.e. any f.g. right ideal is free.

Using the property 1) one obtains the Morse-type inequalities. They are sharp for the following cases: $\pi_1(M^n) = \mathbf{Z}$, $n \geq 6$ (M.Farber); $\pi_1(M^n) = \mathbf{Z}^m$, $n \geq 6$, ξ is in general position (the author).

The generalization of these results for dimension 5 were also discussed.

Equivariant pseudo-isotopy

M.ROTHENBERG

Let G be a finite group, V a finite dimensional representation, and $Top_G(V)$ the group of G-homeomorphisms. Ib Madsen and myself have been engaged in a long study of the homotopy properties of $Top_G(V)$. The key to deriving stability properties is an analysis of the homotopy fiber $P_G(V)$ of $Top_G(V) \to Top_G(V+\mathbf{R})$, which is also the group of G-pseudo-isotopies. Here are some recent results of ours.

THEOREM 1 Suppose V has codimension k gaps between its fixed point sets, p is prime to $|G|$ and let $m = \dim V$. Then for $0 < i \leq m$,

$$\pi_i(P_G(V))_{(p)} \cong \bigoplus_H \tilde{K}_{i-m+1}(\mathbf{Z}[NH/H])_{(q(H,P))} ,$$

where H varies over the conjugacy classes of isotropy subgroups for V, q is the maximal p-ideal of the p-localized Burnside ring asociated to H, and where $\tilde{K}_j = Wh, \tilde{K}_0, K_{-1}$ for $j = 1, 0, -1$. The same holds for $i = 0$ when G has odd order.

If one does not localize away from the group order of G these homotopy groups are far more complicated; they are not just equivariant algebraic K-groups.

Given representations V, W of G we construct a G-map $\sigma : P(V) \to \mathrm{Map}(\dot{W}, P(V+W))$ where $\dot{W}$ is the one point compactification, and $P(V)$ is the group of all pseudo-isotopies. We then have the following stability theorem:

THEOREM 2 Suppose V has codimension k gaps between the fixed sets and that V and $V+W$ have the same set of isotropy subgroups. Then σ is $(\dim V + k - 3)$-connected on Γ-fixed point sets for all $\Gamma \subseteq G$.

Let us define $R_G(V) = P_G(V)/Ppl_G(V)$, where Ppl denotes piecewise linear pseudo-isotopies. $Ppl_G(V)$ is connected up to the range of dimensions in theorem 1, and thus $R_G(V)$ also satisfies Theorem 1. Define an equivariant spectrum

$$PR(V) = \mathrm{colim}_W \ \mathrm{Map}(\dot{W}, \mathbf{R}(V + W + \mathbf{R}))$$

The associated $RO(G)$-graded homology and cohomology groups are denoted by $PR_\alpha(X)$ and $PR^\alpha(X)$, with α in $RO(G)$. Theorem 2 tells us that

THEOREM 3 For $i < 0$ $PR_i(X) = K^{PL}_{G,i}(X)$, where the K-groups are the controlled equivariant pl K-groups of Steinberger-West.

Theorem 1 gives us a calculation of the groups of a point localized away from the order of the group.

Algebraic Homotopy

J.R.SMITH

This talk discusses applications of work in homotopy theory to the problem of computing surgery obstructions of fiber bundles.

The homotopy-theoretic work originally studied the problem of determining the cohomology ring structure of the total space of a fibration. The question of computing the chain-complex was already solved some time ago by Szczarba, Brown, Gugenheim and others (from various points of view, and with varying degrees of elegance and generality).

The speaker developed a functor $\mathcal{C}(*)$ that incorporates the chain complex of a space with a "higher diagonal" structure.

DEFINITION If X is a simplicial complex $\mathcal{C}(X)$ consists of the chain complex of X together with a sequence of S_n equivariant diagonal maps $\mathrm{R}(S_n) \otimes C(X) \to C(X)^n$. Here, $C(X)$ is the simplicial chain-complex of X, S_n is the symmetric group, and $\mathrm{R}(S_n)$ is the bar resolution of $\mathbb{Z}$ over $\mathbb{Z}\, S_n$.

These higher diagonal structures were shown to determine the (geometric) coproduct structure (in fact, the ***higher*** coproduct structure) of the cobar construction and of the total space of fibrations. (In fact, it can be shown to determine the ***homotopy type*** of a pointed simply-connected simplicial complex).

This is applied to compute the symmetric surgery obstruction of a surgery problem that is the total space of a fiber bundle over another surgery problem.

Let M^m and N^m be compact manifolds. Let G be a topological group acting on M and N and let $f : M \to N$ be a degree-1 normal map that is G-equivariant. Now let B^k be an additional compact manifold and suppose that $B \times_\xi M$ and $B \times_\xi N$ are fiber bundles, where $\xi : B \to B_G$ is the classifying map of a principal bundle inducing both of the above. Then $1 \times f : B \times_\xi M \to B \times_\xi N$ is also a degree-1 normal map. Any framing of this map restricts to a framing for $f : M \to N$, so we get (symmetric) surgery obstructions $\sigma^*(1 \times f)$ and $\sigma^*(f)$. We give an explicit formula for computing $\sigma^*(1 \times f)$ from $\sigma^*(f)$ and "higher" symmetric invariants $\mathfrak{D}_i \in H^*(S_i, \mathbb{Z}^t \otimes_{\mathbb{Z}\pi} C(\tilde{B})^i)$.

The formula for $\sigma^*(1\times f)$ (as a chain-map from $W = \mathrm{R}(S_2)$ to $(C(B)\otimes_\xi C(f))^t\otimes_{\mathbf{Z}\pi}$ $(C(B)\otimes_\xi C(f))$ is the following:

$$\begin{aligned}\sum_{\alpha_1\geq 1,\alpha_2\geq 1} &(1\otimes a)\otimes(1\otimes a)\circ(1\otimes\mu^{\alpha_1-1}\otimes 1)\\ &\otimes(1\otimes\mu^{\alpha_2-1}\otimes 1)\circ(1\otimes\xi^{\alpha_1}\otimes 1)\otimes(1\otimes\xi^{\alpha_2}\otimes 1)\\ &\circ V(\alpha_1,\alpha_2)\circ(\mathfrak{D}_{\alpha_1+\alpha_2+2}\otimes 1)\circ(y_{\alpha_1,\alpha_2}\otimes\sigma^*(f))\circ\Delta_R\\ &+\text{untwisted product}\\ &:\mathrm{R}(S_2)\to(C(B)\otimes_\xi C(f))^t\otimes_{\mathbf{Z}\pi}(C(B)\otimes_\xi C(f))\end{aligned}$$

Here:

1. $\Delta_R:\mathrm{R}(S_2)\to\mathrm{R}(S_2)\otimes\mathrm{R}(S_2)$ is the coproduct;
2. $C(f)$ is the chain-complex of the algebraic mapping cone and $a: C(G)\otimes C(f)\to C(f)$ is the chain-evel description of the action of G;
3. $\mu: C(G)\otimes C(G)\to C(G)$ is induced by the product.
4. $y_{\alpha_1,\alpha_2}:\mathrm{R}(S_2)\to\mathrm{R}(S_{\alpha_1+\alpha_2+2})$ is a degree-$\alpha_1+\alpha_2$ map (*never a chain-map*).
5. $\xi: C(B)\to C(G)$ is (by abuse of notation) the twisting cochain induced by the map $\xi: B\to B_G$ classifying the fiber bundles in question.
6. $V(\alpha_1,\alpha_2): C(B)^{\alpha_1+1}\otimes C(B)^{\alpha_2+1}\otimes C(f)\otimes C(f)\to C(B)\otimes C(B)^{\alpha_1}\otimes C(f)\otimes C(B)\otimes C(B)^{\alpha_2}\otimes C(f)$ is the isomorphism that shuffles the factors.

   The y-functions are defined inductively via the formula:

$$\begin{aligned}y_{\alpha_1,\alpha_2}=&(-1)^{\alpha_1+\alpha_2}\varphi\Bigg\{y_{\alpha_1,\alpha_2}\circ\partial\\ &+\sum_{t=1}^{\alpha_1+\alpha_2}\mathfrak{T}_{\underbrace{1,\dots,2,\dots,1}_{2\text{ in }t^{\text{th}}\text{ position}}}\circ\begin{cases}y_{\alpha_1-1,\alpha_2}, & \text{if } 1<\alpha_1<t+2\\ y_{\alpha_1,\alpha_2-1}, & \text{if } \alpha_1\geq t+2, \alpha_2>1\\ 0, & \text{otherwise}\end{cases}\\ &+(-1)^{\alpha_1}(3,\alpha_1+2)^{-1}\mathfrak{T}_{2,1,\dots,1}\circ\begin{cases}y_{\alpha_1,\alpha_2-1}, & \text{if } \alpha_1\geq 2,\alpha_2>1\\ 0, & \text{otherwise}\end{cases}\\ &+\sum_{\substack{\alpha_1'+\alpha_1''=\alpha_1\\ \alpha_2'+\alpha_2''=\alpha_2}}S([\alpha_1'+2,\alpha_2'+2],[\alpha_1''+2,\alpha_2''+2])\circ y_{\alpha_1',\alpha_2'}\otimes z_{\alpha_1'',\alpha_2''}\circ\Delta_R\Bigg\}\end{aligned}$$

where:

1. $\varphi:\mathrm{R}(S_{\alpha_1+\alpha_2})\to\mathrm{R}(S_{\alpha_1+\alpha_2})$ is the canonical contracting homotopy.
2. $y_{0,0}=\mathrm{id}:\mathrm{R}(S_2)\to\mathrm{R}(S_2)$;
3. $y_{0,i}=y_{i,0}=0$, if $i>0$;
4. $S([\alpha_1'+2,\alpha_2'+2],[\alpha_1''+2,\alpha_2''+2])$ is the permutation that shuffles these two sequences together (with its associated sign);
5. The map $\mathrm{T}_{\alpha_1,\dots,\alpha_k}: S_k\to S_{|\alpha|}$ (where $|\alpha|=\sum_{i=1}^k\alpha_i$), is defined to send the permutation $\sigma\in S_k$ to the permutation $\begin{pmatrix}L_1 & \dots & L_k\\ L_{\sigma(1)} & \dots & L_{\sigma_k}\end{pmatrix}$, where L_i is the

sequence of integers $\{1+\sum_{j=1}^{i-1}\alpha_i,\dots,\sum_{j=1}^{i}\alpha_i\}$. For example $T_{2,1}(1,2)$ is the permutation $\begin{pmatrix}\{1,2\} & \{3\}\\ \{3\} & \{1,2\}\end{pmatrix}=\begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix}$.

6. The maps $\mathfrak{T}_{\alpha_1,\dots,\alpha_k}: \mathrm{R}(S_k)\to \mathrm{R}(S_{|\alpha|})$ are defined to send $a_1\cdot[a_2|\dots|a_t]$ to

$$T_{\alpha_1,\dots,\alpha_k}(a_1)\cdot \mathfrak{T}_{a_1^{-1}\{\alpha_1,\dots,\alpha_k\}}(1\cdot[a_2|\dots|a_t])$$

and $1\cdot[a_1|\dots|a_t]$ to

$$[T_{\alpha_1,\dots,\alpha_k}(a_1)|\mathfrak{T}_{a_1^{-1}\{\alpha_1,\dots,\alpha_k\}}(1\cdot[a_2|\dots|a_t])$$

7. the $z_{\alpha_1,\alpha_2}: \mathrm{R}(S_2)\to \mathrm{R}(S_{\alpha_1+\alpha_2})$ are maps of degree $\alpha_1+\alpha_2-1$ defined inductively by the formula

$$\begin{aligned} z_{\alpha_1,\alpha_2} =&(-1)^{\alpha_1+\alpha_2}\varphi\Bigg\{ z_{\alpha_1,\alpha_2}\circ\partial \\ &+\sum_{t=1}^{\alpha_1+\alpha_2}(-1)^t\mathfrak{T}_{\underbrace{1,\dots,2,\dots,1}_{2\text{ in }t^{\text{th}}\text{ position}}}\circ\begin{cases} z_{\alpha_1-1,\alpha_2}, & \text{if } \alpha_1>1\\ z_{\alpha_1,\alpha_2-1}, & \text{if } \alpha_1=1,\text{and } \alpha_2>1\\ 0, & \text{otherwise}\end{cases} \\ &+\sum_{\substack{\alpha_1'+\alpha_1''=\alpha_1\\ \alpha_2'+\alpha_2''=\alpha_2}} S([\alpha_1',\alpha_2'],[\alpha_1'',\alpha_2''])\circ * \circ z_{\alpha_1',\alpha_2'}\otimes z_{\alpha_1'',\alpha_2''}\circ\Delta_R\Bigg\}\end{aligned}$$

where $*: \mathrm{R}(S_{\alpha_1+\alpha_2})\otimes \mathrm{R}(S_{\alpha_1+\alpha_2})\to \mathrm{R}(S_{\alpha_1+\alpha_2})$ is the twisted shuffle product, and $z_{0,0}=0, z_{0,1}([\,])=z_{1,0}([\,])=[\,]$.

The speaker has a Pascal program for computing these functions.

Groups acting on $S^n\times \mathbf{R}^k$: Variations on a theme of Hambleton–Pedersen

C.W.Stark

Ian Hambleton and Erik Pedersen have shown that $\mathbf{Z}^k\ltimes D_{2p}$ can act freely, properly discontinuously, and cocompactly on $S^n\times\mathbf{R}^k$ (see their paper, "Bounded surgery and dihedral group actions on spheres"). This talk generalizes their construction to show that dihedral subgroups are actually quite common in groups acting on a product of a sphere and a Euclidean space, with the properties listed above. The argument, which follows theirs, smooths a Poincaré complex constructed as a singular bundle over a 2–orbifold with an even number of elliptic points of order 2; the generic fiber is a homotopy lens space and the singular fiber is a Swan complex for the dihedral group. A variation replaces the 2–orbifold with an aspherical manifold of higher dimension that carries an effective torus action.

A vanishing theorem for Nil-groups

P.VOGEL

Let A be a ring. Denote by $\mathcal{L}_A$ the smallest class of modules containing projective A-modules, which is stable under direct limit and such that

for every short exact sequence of A-modules, if two are in $\mathcal{L}_A$ the third is also in $\mathcal{L}_A$.

DEFINITION A is called *weakly regular* if every A-module belongs to $\mathcal{L}_A$.

EXAMPLE If every finitely presented A-module has finite homological dimension A is weakly regular.

THEOREM Let A be a weakly regular ring. Let S_1 and S_2 be two right and left-projective A-bimodules.
Then the Waldhausen spectra $\widetilde{\mathcal{NIL}}(A, S_1)$ and $\widetilde{\mathcal{NIL}}(A, S_2)$ are contractible.

COROLLARY Let $\mathcal{C}$ be the smallest class of groups containing free groups, which is stable under direct limit (with injections) and amalgamated free products (also with injections).
Then for every $G \in \mathcal{C}$ the spectrum $\mathcal{WH}(G)$ is contractible.

REMARK The class $\mathcal{C}$ is stable under subgroups and extensions.

Automorphisms of manifolds

M.WEISS

(joint with B.Williams)

Given a compact manifold M^n we construct a commutative diagram

$$\begin{array}{ccc} \Omega(G(M)/TOP(M)) & \longrightarrow & \Omega(G^{\infty}(\tau_M)/TOP(\tau_M)) \\ \Big\downarrow d & & \Big\downarrow \\ \Omega LA_{\%}(M,\nu,-n) & \xrightarrow{\ \partial\ } & \Omega LA^{\%}(M,\nu,-n)^{\%} \end{array}$$

such that

i) if $n > 4$ the vertical maps are equivalences through the concordance stable range,

ii) ∂ is the connecting map associated to the assembly for the functor LA, and

iii) the functor LA is an algebraic concoction of surgery L-theory and Waldhausen's A-theory.

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