

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1990

Risk Theory

16.9. bis 22.9.1990

Die Tagung fand unter der Leitung von H. Bühlmann (Zürich), P. Embrechts (Zürich), H. Gerber (Lausanne) und J. Teugels (Heverlee) statt. Auf der Grundlage von insgesamt 37 Vorträgen diskutierten die Teilnehmer aus 15 Staaten über ein breites Spektrum von Fragen aus der Versicherungsmathematik. Die angeschnittenen Themen reichten von mathematisch statistischen Problemen, wie zum Beispiel neuen Ergebnissen zu U-Statistiken, bis zur Analyse realen Datenmaterials aus der Versicherungswirtschaft und zu Simulationen. Besondere Schwerpunkte bildeten dabei

- Ruintheorie:            analytische Modelle  
                              Numerik  
                              Schätztheorie
- Kreditabilitätstheorie einschließlich ihrer Verbindung zur robusten Statistik
- Stochastische Modelle in der Lebensversicherung
- Beziehungen zwischen mathematischer Finanztheorie und Versicherungsmathematik.

## Vortragsauszüge

*E. Arjas:*

### Token causation: Can probabilities explain why an accident happened?

Several authors distinguish between two kinds of probabilistic causality: "the tendency of  $C$  to cause  $E$ " and "the degree to which  $C$  actually caused  $E$ ." The former, a generic form of causation, can be discussed by comparing two prediction probabilities: one conditional on the occurrence of  $C$ , and the other on its "counterfactual" event where  $C$  does not occur. The latter, a singular form, is often called *token causality* and corresponds to finding a causal explanation to the occurrence of an event after it has been observed to happen. The purpose of this talk was to formulate token causality by using the mathematical framework of marked point processes (MPPs) and their associated prediction processes. The formulation was illustrated by considering a (hypothetical) example concerning a child who was vaccinated against an infectious disease and who soon thereafter developed an allergy reaction and died.

*Ph. Artzner (and F. Delbaen):*

### Life insurance with stochastic interest rates

The paper is a first attempt to incorporate random interest rates into the valuation of life insurance benefits and premium payments.

In the case of fair variable premiums, the central limit for sums of identically distributed independent martingales is used to approximate the distribution of the aggregate loss process, after introduction of the stochastic discounting factor.

*S. Asmussen:*

### Computational methods in risk theory: a matrix-algorithmic approach

We survey some modern methods for giving numerical values of ruin probabilities. These methods are inspired from the work of Marcel Neuts and others on matrix-geometric stationary distributions for discrete Markov chains, but have been adapted here to deal also with continuous state space problems. From a computational point of view the basic ingredients are the evaluation of matrix-exponentials  $e^{Qt}$  and the iterative solution of matrix fixpoint problems (though this step can be avoided in the classical risk model with Poisson arrivals), and replace traditional tools like rootfinding in the complex plane and Laplace transform inversion. The basic assumption states that the claim size distribution  $B$  is of phase type (if not, one can always approximate  $B$  with such a distribution), and we look at classical compound Poisson risk processes, Sparre Andersen processes with renewal arrivals, and risk processes evolving in a Markovian or periodic environment.

*P. L. Brockett:*

### Fun with moment bounds

This talk reviewed the theory of Chebycheff Systems of functions as it relates to obtaining bounds on the expectation of functions of a random variable  $X$  when all that is known about the distribution is that it is supported by the interval  $[a,b]$  and has a prescribed finite set of moments. Explicit extremal distributions for the case of 2,3, and 4 moments given were presented and the results were applied to the finding bounds on the probability of ruin in the classical moment problem by obtaining maximally tight bounds on the adjustment coefficient. The results were then also applied to show that within a decision making context, a risk averse investor will always have times in which, when forced to choose between two prospects with equal means and ordered variance, he will prefer the prospect with higher variance, and even will occasionally prefer the prospect with higher mean, lower variance, and lower positive skewness. In general it was shown that the moment ordering for decision making in financial decision making (as in the Capital Asset Pricing Model for example) is never uniformly equivalent to expected uti-

lity ordering by rational decision makers except in the absurd polynomial utility case. Numerical illustrations of all of these applications were given.

*B. Chan:*

### Ruin probability for translated combination of exponential claims

We show that the coefficients in the combination of exponentials ruin probability are

$$C_k = \vartheta \prod_{i \neq k} (\beta_i - r_k) / r_k (1 + \vartheta) \prod_{i \neq k} (r_i - r_k)$$

by considering the residue at  $r_k$  of a rational function.

*F. Delbaen:*

### Convergence speed in the law of large numbers and the moments of ruin time in classical risk theory

We define the ruin time as  $R_0 = \inf\{t \geq 0 \mid t - S_t < 0\}$  where  $S_t$  is a compound Poisson process i.e.  $S_t = \sum_{k=1}^{N_t} X_k$ , where  $N_t$  is a Poisson process with intensity  $\lambda$  and  $X_k$  are strictly positive random variables with  $E[X_k] = \mu < \infty$ . Of course we suppose  $\lambda\mu < 1$ . In this case we can prove that  $E[R_0^\alpha \mathbb{1}_{\{R_0 < \infty\}}] < \infty$  if and only if  $E[X_k^{\alpha+1}] < \infty$  ( $\alpha > 0$ ). The result is related to a theorem of Katz, itself a generalization of a theorem of Erdős. Part of Erdős' result can be recovered from known methods in risk theory.

*N. De Pril (and J. Dhaene):*

**Error bounds for some collective approximations of stop-loss premiums**

For computational reasons actuaries often prefer to approximate the individual risk theory model by a collective model. We derive explicit lower and upper bounds for the error in the net stop-loss premiums resulting from certain collective methods, such as compound Poisson approximations and the natural approximation. In particular, the upper bound for the stop-loss error in the classical compound Poisson approximation, obtained by Gerber (1984), is improved.

*E. Eberlein:*

**Modelling questions in security valuation**

After giving a brief introduction into the theory of option pricing we describe several deficiencies of the Black-Scholes model in order to motivate approximations of continuous time models by discrete time ones. The main result concerning approximations states that for any return distribution with finite moment of order  $2 + \delta$ , a sequence of discrete time models exists which approximates the continuous time Black-Scholes model on the same probability space pathwise almost surely with a certain rate. Some consequences concerning convergence of trading strategies and of the financial gain are discussed.

*P. Embrechts:*

**Absolute ruin**

Consider the classical risk process:

$$\{N(t); t \geq 0\}$$

$$\{Y_i; i \in \mathbb{N}\}$$

Poisson ( $\lambda$ )-claim arrival process  
i. i. d.  $\exp(1/\mu)$ -claim size process

- u initial capital  
c premium income rate

The method of piecewise deterministic Markov processes will be used to solve the following two problems:

Problem 1:

Introduce the force of interest  $\delta > 0$  by replacing  $c$  in  $R_t = u + ct - \sum_{i=1}^{N(t)} Y_i$  by  $c = c(r) = c + \delta r$  ( $r < 0$ ). Denote the corresponding risk process by  $R_t^\delta$  and define

$$\tau_{\text{abs}} = \inf \{t \geq 0 : R_t^\delta \leq -c/\delta\}.$$

Explicit estimates for  $P(\tau_{\text{abs}} < \infty)$  are given.

Problem 2:

Within the class of barrier strategies, we find the optimal strategy over the lifespan  $(0, \tau_{\text{abs}})$  of the company by maximizing

$$E\left(\int_0^{\tau_{\text{abs}}} c I(R_s^\delta = W) e^{-\beta s} ds\right)$$

where  $W$  is the dividend level and  $\beta$  some discount factor.

*J. Garrido:*

**Minimum quadratic distance estimation for a parametric family of discrete distributions defined recursively**

The c.d.f. of a random sum can easily be computed iteratively when the distribution of the number of i.i.d. elements in the sum is itself defined recursively. Classical estimation procedures for such recursive parametric families often require specific distributional assumptions (e.g. Poisson, Negative Binomial). The minimum distance estimation procedure proposed here allows for the selection of the best fitting family member without further distributional assumptions. It is shown that the estimator thus obtained is consistent and that it can be made either robust or asymptotically efficient. Its

asymptotic distribution is also derived and an illustration with Automobile Insurance data is included.

*H. Gerber:*

### **Risk theory with the gamma process**

The aggregate claims process is modeled by a process with independent, stationary and nonnegative increments. Such a process is either compound Poisson or else a process with an infinite number of claims in each time interval, for example a gamma process. It is shown how classical risk theory, and in particular ruin theory, can be adapted to this model. A detailed analysis is given for the gamma process, for which tabulated values of the probability of ruin are provided.

*A. Gisler :*

### **Robust Credibility**

Outlier observations caused by big claims or by an event producing a series of claims are a special problem in rate-making and in tariff-calculation. The natural idea to cope with this problem is not to fully charge outliers to the claims load of smaller groups but rather to distribute them to a bigger collective. There are two methods traditionally used in the insurance practice. The first method is credibility theory where each claim is not fully charged but only with the fraction of the credibility weight. The second is robust statistics the basic idea being to limit the influence of single observations. Robust Credibility combines the two methods. The main idea is to robustify the individual claims experience by using a robust estimator  $T_i$  instead of the individual mean  $\bar{X}_i$  and then to use a credibility estimator based on the robust statistics  $\{T_i : i = 1, 2, \dots\}$ . Choosing an  $M$ -estimator of scale and a particular  $\phi$ -function leads to data trimming with an observation dependent trimming point.

An empirical robust credibility estimator is provided not only for the case of identical volumes, but also for the case where each risk is characterized by a specific volume.

While the method is robust large claims are not just discarded and the obtained tariff is financially balanced taken over the whole collective.

*M. Goovaerts:*

#### **Why and how to calculate ruin probabilities**

It has been shown how infinite time ruin probabilities, bounded by elementary bounds, can be applied to real insurance problems.

In addition an algorithm is given for numerical evaluation of finite time ruin probabilities based on an equation by H. Seal. In addition it is shown how ruin probabilities can be calculated numerically in case the epochs of claims form a renewal process. Some comments on the applicability have been given.

*U. Herkenrath:*

#### **On some smoothing procedures for adjusting insurance premiums to claims evolution**

The problem of adjusting insurance premiums to claims evolution by smoothing procedures is considered. If the sequence  $(X_n)_{n \geq 1}$  of random variables describes a claims evolution process in discrete time, then the premium  $W_n$  in period  $n$  is updated after observation of  $X_n$  to a premium  $W_{n+1} = u(W_n, X_n)$  by means of a smoothing function  $u$ .

We propose several schemes of smoothing functions  $u$ , which induce premium sequences  $(W_n)_{n \geq 1}$  with desired convergence properties. The procedure of exponential smoothing is contained therein. In contrast to the case of i.i.d. random variables  $(X_n)_{n \geq 1}$  a "continuous" dependence of the  $X_n$  from  $W_n$  is allowed, which should be interesting for applications.

Special examples of smoothing functions are discussed.

*C. Hipp:*

### Hedging and proportional transaction costs

In the Cox–Rubinstein binomial model, the use of a hedging strategy leads to a riskless portfolio in a market without transaction costs. If transaction costs have to be paid, then the use of the same hedging strategy is no longer self financing, the accumulated costs are random and depend on the probability structure. In Boyle and Vorst (1990), a different hedging strategy for the case with proportional transaction costs is presented which leads to a riskless portfolio and gives an exact option pricing formula. For this, a continuous time approximation similar to the logarithmic Brownian motion is not possible.

*W. Hürlimann:*

### Predictive stop-loss premiums

Consider the following model of aggregate claims in a Bayesian set-up:

$$Y = \sum_{k=1}^r \sum_{i=1}^s m_k X_{ki}$$

$X_{ki}$  counts the number of claims of amount  $m_k$  with probability of occurrence  $\Theta_1$  (e.g.  $\Theta_1 = q_x$  prob. of death in life insurance).

Let  $\Theta = (\Theta_1, \dots, \Theta_s)$  be an unknown parameter vector with structure function  $u(\Theta) = \prod_{i=1}^s \Gamma(\Theta_i; \alpha_i, \beta_i)$  (= product of Gamma densities). Let  $N_{ki}$  count the number of risk units (e.g. individual life policies) in a portfolio of risks with amount at risk  $m_k$  and prob. of occurrence  $\Theta_1$ .

Model assumption: Conditionally on  $\Theta$   $X_{ki}$  are independent Poisson with parameter  $N_{ki} \Theta_1$ .

Results:

- (1) The predictive mean, in form of a multivariate credibility formula, is derived.
- (2) Using previous computational results on linear combinations of random variables (Methods of O.R. 63, Ulm, 1989), a three-stage recursive algorithm for the exact evaluation of the predictive density is derived.

- (3) Knowledge of the predictive mean and predictive density allows the recursive evaluation of predictive stop-loss premiums.

*J. L. Jensen:*

### Saddlepoint approximation for the distribution of the total claim amount

In the talk I considered approximations for  $P(\sum_1^{N(t)} Y_i > x)$ , with  $N(t)$  either a Poisson or a Polya distribution, and with  $Y_i$  an I.I.D. sequence with density  $g(y)$ . In the Poisson case it was shown that the Esscher approximation gives a relative error tending to zero for  $x \rightarrow \infty$  if either  $g(y)$  is "Gamma-like", "Beta-like" or log concave for large values of  $y$ . In the Polya case no such restrictions on the density are needed because the Polya distribution has a finite radius of convergence of the generating functions.

Finally, I mentioned in the talk generalizations to a sum  $\sigma B(t) + \sum_{k=1}^d \sum_{i=1}^{N_k(t)} Y_{k,i}$ , where  $B(t)$  is Brownian motion,  $N_k(t) \sim \text{Poisson}(\lambda_k t)$  and  $Y_{k,i} \sim F_k$ , and generalization to certain types of processes with inflation:  $\sum_1^{N(t)} Y(T_i)$ .

*C. Klüppelberg:*

### Tail estimation

We estimate the upper tail of a distribution function by means of "asymptotic maximum likelihood estimation" based on the upper  $k$  order statistics of a sample. With this method we determine consistent estimators for a two-parametric Pareto-like tail, and also for two-parametric distribution tails in the domain of maximal attraction of the Gumbel law which have not yet been investigated. We also obtain the asymptotic distributions of our estimators and derive rates of convergence.

*E. Kremer :*

### On large claims in risk theory

Part A: Evaluating the distribution function of LCR's total claims amount

A generalization of the classical LCR treaty, the so-called GLCR, is investigated again. The problem of evaluating the distribution function of its total claims amount is treated in details. Two approaches for evaluating are proposed. The first approach consists in reformulating the total claims amount with help of the so-called spacings and in applying results on the distributions of the spacings. The second approach consists in deriving formulas for a certain conditional characteristic function and in applying methods of the Fourier analysis. Satisfying results are given for both approaches under certain specialized and generalized model assumptions.

Part B: Large claims in credibility

The problem of how to cope with large claims in credibility rating is reconsidered. New credibility techniques are introduced leading to so-called M- and L-credibility estimators. Under certain conditions the corresponding new premium predicting formulas can be shown to be robust with respect to low-probable (extraordinary) large claims amounts. The possibility of giving a robust procedure for estimating the structural parameters is pointed out. An extended study on this topic will be presented at the next ASTIN-congress in 1991 in Stockholm. The lecturer refers to that forthcoming study.

*U. Küchler:*

### On first hitting time distributions for gap-diffusions

Let  $m$  be a non-decreasing function on  $[0, \infty)$  and  $X = (X_t)_{t \geq 0}$  the strong Markov process on  $E_m = \{x \geq 0 \mid m(\cdot) \text{ increases at } x\}$  having the second order generalized differential operator  $D_m D_x$  (together with some boundary conditions) as its infinitesimal generator.  $X$  is called a gap diffusion.

The spectral theory of  $D_m D_x$ , in particular Krein's inverse spectral theorem, allows, at least principally, to express all properties of  $X$  in terms of the spectral measure of

$D_m D_x$ . Examples are the spectral representations of first hitting time distributions for gap-diffusions.

*M. Lourdes-Centeno:*

#### **An insight into the excess of loss retention limit**

In this paper we provide an algorithm to calculate the optimum retention of an excess of loss reinsurance arrangement of a risk, optimum in the sense that it maximizes the adjustment coefficient, assuming that the reinsurance premium is calculated according to the expected value principle, the loading coefficient of which is dependent on the retention limit.

*V. Mammitzsch:*

#### **An application of the discretized sequential probability ratio test to the risk process**

A discretized sequential probability ratio test (SPRT) is defined to be a test in a sequential testing problem with continuous time which allows terminal decisions only at a given discrete set of moments. It is shown that just as in the case of the (general) SPRT, a discretized distinguishes with arbitrarily small error probabilities between two simple hypotheses  $P_1$  and  $P_2$  iff  $P_1$  and  $P_2$  are orthogonal. Moreover, this result is applied to distinguish between two classical risk processes.

*A. Martin-Löf:*

#### **Ruin theory for a recurrent risk process**

The standard model in risk theory is defined by  $X_t = x + pt - S_t$ , with  $S_t =$  compound Po-process given by  $E(\exp(zS_t)) = \exp \lambda t \int_0^{\infty} (\exp(zx) - 1)F(dx) \equiv \exp tg(z)$ . It

stops at the time of ruin  $T = \min \{t, X_t < 0\}$ . When  $p > \lambda\mu \equiv g'(0)$  the ruin probability is  $< 1$  and we have the famous asymptotic formula as  $x \rightarrow \infty$ :  $P(T < \infty) \sim ce^{-RX}$ , where  $R$  is determined by  $g(R) = pR$ . An unrealistic feature of this model is that it is transient, it can only avoid ruin by escaping to  $+\infty$ . We can prevent this by letting  $p$  be a sufficiently rapidly decreasing function of  $X$ , but then in general  $P(T < \infty) = 1$ , so the ruin problem seems to be trivial. However, it is still an interesting problem to study the probability of ruin in a finite interval,  $P(T < t)$ , and try to find simple asymptotic formulas for it. We do this here in the special case considered by e.g. K. Borch when there is an upper limit  $u$  for  $X_t$  achieved by taking  $p(X) = p$  for  $X < u$ ,  $p(X) = 0$  for  $X = u$ . If  $u$  is not too small the path of the process up to  $T$  will consist of a large number of excursions between successive visits to  $u$ . The hitting points will form a renewal process, and the excursions will be independent processes. During each there will be a small probability  $\rho$  of ruin before returning to  $u$ . This means that the number of the excursion,  $K$ , when ruin occurs has a geometric distribution with  $P(K = k) = (1 - \rho)^{k-1}\rho$ ,  $k = 1, 2, \dots$ . It has  $E(K) = 1/\rho =$  very large if  $\rho$  is small. This means that we ought to have  $P(T > t) \approx e^{-\rho t/m}$ , where  $m = E(T_1 - T_0)$ . By using Wald's martingale it is possible to show that  $\rho \sim c \cdot e^{-uR}$  when  $u \rightarrow \infty$ . This means that in this recurrent case we have a constant very small rate of ruin per unit time  $\rho/m$ , and that it decreases as  $(c/m)e^{-uR}$ . The natural measure of the safety is therefore  $e^{-uR}$ , although ultimate ruin is certain.

*T. Mikosch:*

### Some limit theory for random quadratic forms

We consider quadratic forms in Gaussian or in independent random variables with constant coefficients. These forms behave asymptotically very much like sums of independent random variables. Indeed, quadratic forms obey the ordinary and the functional central limit theorem, the strong law of large numbers, the ordinary and the functional law of the iterated logarithm with nonrandom normalization. The proofs base on several technical results as estimates of tail probabilities, maximal inequalities, Berry-Esseen-type inequalities and strong approximation results for quadratic forms.

*F. Moriconi:*

**Semi-deterministic vs. stochastic immunization and the surplus stream**

When the asset and liability cashflows are known at the decision date, the semi-deterministic immunization theory provides a useful measure for interest rate risk without requiring any probabilistic assumption on the random evolution of the yield curve. By applying the risk minimizing strategy to the portfolio of a financial intermediary, it is possible to identify the surplus stream, that is the maturity structure of the surplus after the outstanding liabilities have been optimally immunized by part of the outstanding assets.

If interest-rate-dependent cashflows are considered, stochastic valuation models are needed to perform asset-liability management. Stochastic immunization theorems suggest strategies to control interest rate risk, but the definition of a surplus stream is strongly model-dependent and not so general as in the semi-deterministic framework.

*H. Müller:*

**Price equilibria and non linear risk allocations in capital markets**

The theory of risk exchange is applied to a model of financial risk. In particular, Bühlmann's notion of a price equilibrium is used in this context. For a special class of utility functions (HARA, non increasing absolute risk aversion) the equilibrium price density and the risk allocation are analyzed in detail. The characteristics of individual investors are related to their policy choice (portfolio insurance, etc.).

*R. Norberg:*

**A theory of payment streams and discounting with application to life insurance**

Streams of payments are represented as measures on the real line. A set of consistency requirements leads to valuating any payment stream by the integral of a positive discount function with respect to the payment measure. Inter-relationships between

generalized annuities and lump sum payments are obtained by the general rule of integration by parts. By placing probability distributions on the discount function and the payment measures, a wide class of actuarial and related problems is put on the same footing. The unifying power of the theory is illustrated by applications to life insurance. General formulas of moments of present values of benefits and premiums are established. Retrospective reserves are defined as conditional expected values, and are shown to satisfy a set of differential equations generalizing the Kolmogorov forward equations. It is pointed out that the Thiele differential equations for the prospective reserves generalize the Kolmogorov backward equations. The novel concept of retrospective reserve forms the basis for ideas on distribution of surplus.

*J. Paris:*

#### **Statistical methods applied to insurance problems**

The premium of the excess of loss working-cover treaties depends on the information supplied by the insurer and the estimation technique. A simulation based on a Pareto-Poisson model indicates that in any case the maximum likelihood method underestimates the premium and the maximum product of spacings overestimates the premium.

The most general compound Poisson law which is also infinitely divisible depends on a Bernstein function  $\eta(t) = \delta t + \Theta(t)$  where  $\delta \geq 0$  and  $\Theta$  is also a Bernstein function. This implies that the intensity of the corresponding process contains a constant which is met again in each premium system based on the number of claims.

*K. D. Schmidt:*

#### **Convergence of Bayes and credibility premiums**

For a risk whose annual claim amounts are conditionally i.i.d. with respect to a risk parameter, it is known that the Bayes and credibility premiums are asymptotically optimal in terms of losses. We show that the Bayes and credibility premiums actually converge almost surely and in  $L^2$  to the individual premium. We also characterize the coincidence of the Bayes and credibility premiums by a martingale property.

*E. Shiu:*

**An application of the Black–Scholes formula**

Consider a financial company which issues guaranteed investment contracts in the form of zero coupon bonds. Due to competitive pressure, the company may need to guarantee a potential customer that, if he commits himself today to buy a zero coupon bond, the interest rate that he will earn is the maximum of today's rate and the interest rate on the day when his money arrives. We show that the value of this option, expressed as an interest rate spread throughout the lifetime of the zero coupon bond, is 40 percent of the standard deviation, as seen from today, of the interest rate on the day when the customer's money is to arrive.

*J. Steinebach:*

**On the estimation of the adjustment coefficient in risk theory via intermediate order statistics**

A sequence of intermediate order statistics is proposed to estimate the adjustment coefficient in risk theory. The underlying random variables may be viewed as maximum waiting times in busy cycles of GI/G/1 queuing models under light traffic. Strong consistency is discussed as well as rates of convergence are studied. We give an example and present some simulation studies for illustrating the behaviour of the proposed estimator. The results are based on joint work with M. Csörgő (Ottawa/Canada).

*B. Sundt:*

**Allocation of excess of loss premiums**

In this talk we discussed the question of how to allocate the reinsurance premium between the subportfolios when an excess of loss reinsurance is to be shared between several subportfolios. Different allocation schemes based on the expected value principle and the standard deviation principle were suggested. The calculations are relatively simple with

unlimited free reinstatements. However, with limited and/or paid reinstatements the situation becomes rather tricky, and we therefore suggested a simulation scheme.

*G. Taylor:*

### A Bayesian interpretation of Whittaker–Henderson graduation

The primary purpose of the paper is to place Whittaker–Henderson graduation in a Bayesian context and show that this determines in a precise manner the extent to which goodness-of-fit should be traded off against smoothness in the Whittaker–Henderson loss function. This is done in Section 2.

Section 3 generalizes the set of admissible graduating functions to a normed linear space. A specific example of this generalization is Schoenberg graduation, which is strongly related to Whittaker–Henderson but leads to particular spline graduations. These are placed in a Bayesian context parallel to that of Section 2. The similarity to shrunken or Stein-type estimators is pointed out.

Section 4 considers the practical implications of these theoretical developments. Transformations of observations under graduation are examined and shown to be natural in some circumstances. The precise trade off mentioned above is enlarged upon, and the main conclusions reached here are seen to carry over to general spline graduation. The relation between Whittaker–Henderson and spline graduation is identified.

*J. Teugels:*

### A statistical problem connected with ruin analysis

The total claim amount  $Y(t) = \sum_{i=1}^{N(t)} Y_i$  has a compound distribution that almost never can be evaluated in closed form; approximations are therefore necessary. If  $N(t)$  comes from a Pascal process with parameter  $p$  and  $\{Y_i\}$  is exponentially bounded  $P\{Y(t) > x\} \sim Ce^{-|\tau|x} g(\tau)$  where  $C$  is a constant and  $\tau$  satisfies the equation  $pf(\tau) = 1$ ; here  $f$  is the Laplace transform of the claim size distribution. In practice,  $\tau$  has to be estimated. Together with S. Csörgö we derived asymptotic normality for

$\tau_n$ , the estimator obtained from  $pf_n(\tau_n) = 1$  where  $f_n$  is the empirical Laplace transform. Consequently the approximation can also be estimated and leads to normal limits. We give a number of other applications of this technique in the classical ruin context, in branching theory and in queuing.

#### *N. Veraverbeke:*

##### **Nonparametric estimation of the probability of ruin**

We consider the infinite time probability of ruin in the classical Poisson risk model and show how it can be estimated, based on a random sample of size  $n$  from the claim amount distribution. The estimator has the form of a linear combination  $\sum_{r=1}^m a_r U_{nr}$  where  $\{m_n\}$  is a sequence of natural numbers,  $m_n \leq n$ ,  $\lim_{n \rightarrow \infty} m_n = +\infty$ , and where for each  $r = 1, \dots, n$ ,  $U_{nr}$  is a U-statistic with symmetric kernel  $h_{nr}(x_1, \dots, x_r)$ . We obtain a general theorem on asymptotic normality for such linear combinations of U-statistics with varying kernel. This generalizes techniques of Frees (1986) who dealt with the special case where the kernel is of the form  $I(x_1 + \dots + x_r \leq x)$  (independent of  $n$ ) and  $a_r = 1$  for all  $r$ .

#### *H. Waters:*

##### **The recursive calculation of the moments of the profit on a sickness insurance policy**

The speaker discussed a 3-state (semi-Markov) model for sickness insurance currently being studied in the U.K. with a view to its practical implementation. The particular problem discussed was the calculation of the moments of the present value of the profit on a single policy and on a portfolio of policies, both with a fixed interest rate and with a stochastic (random walk) model for the interest rate. Integral formulae can be derived for the moments which can be solved by means of recursions.

*W. Wolf*

### Stable approximation of U-statistics

If the first absolute moment of the kernel function  $H$  exists, then the asymptotic behaviour of U-statistics  $U_n$  is determined by the asymptotic behaviour of a sum  $S_n$  of independent random variables with common distribution function  $F$  and a remainder term. For special kernel functions  $H$  it is shown that a stable  $G_\alpha$ ,  $1 < \alpha \leq 2$ , is the limit law of the normalized  $U_n$  whenever  $F$  belongs to the domain of attraction of  $G_\alpha$ .

Moreover, if the rate of convergence of the normalized  $S_n$  to  $G_\alpha$  is known then, under a mild condition of  $H$  depending on the tail behaviour of  $F$ , the normalized  $U_n$  tend to  $G_\alpha$  with the same rate.

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