MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 41/1990

Random Graphs and Combinatorial Structures

23.09. bis 29.09.1990

Die Tagung fand unter der Leitung von Herrn A.D. Barbour (Zürich) und Herrn B. Bollobás (Cambridge) statt.

Das Treffen führte Mathematiker aus verschiedenen Gebeiten zusammen: Kombinatoriker, Graphentheoretiker, Wahhrscheinlichkeitsttheoretiker, Statistiker und angewandte Mathematiker. Zusätzlich zur Vorstellung zahlreicher wichtiger neuer Ergebnisse ergab die Zusammensetzung des Treffens viele lebhafte und fruchtbare Diskussionen. Unter den Themen von besonderem Interesse waren Grenzverteilungen von Statistiken stochastischer Graphen, der Phasenübergang im stochastischen Graphenprozeß, die Theorie stochastischer Bäume und anderer kombinatorischer Strukturen, die Anwendung von erzeugenden Funktionen, Perkolationstheorie, die asymptotische Struktur von Graphen ohne verboten Untergraphen, Modelle für Epidemien, randomisierte Algorithmen und verschiedene Heuristiken.

Vortragsauszüge

I. ALTHÖFER:
Pathology in random game trees and other recursion trees

Several people in computer science and computer chess have investigated the problem of quality in game tree searching, when the heuristic evaluation function is not free of errors. In random models with independent leaf values and independently occuring errors a phenomenon of pathology was observed: the deeper the search in the tree, the worse the final estimate of the root value [Judea Pearl, "Heuristics: Intelligent Search Strategies for Computer Problem Solving", Addison Wesley PC, Reading, MA (1984)]. The main aim of this talk is to present a new result, proved with Imre Leader, showing that the pathology is not a feature only of game trees, but appears in any 'non-trivial' bi-valued recursion tree with independent leaf values and independently occuring errors.

E. BOLTHAUSEN:
On the construction of the three dimensional polymer measure

A polymer measure on the set of continuous functions $:[0,1] \to \mathbf{R}^d$ with a coupling constant $\lambda > 0$ is formally defined by

$$\frac{d\hat{P}_\lambda}{dP} = \exp\left(-\lambda \int_0^1 \int_0^1 \delta(p_s - p_t)\, ds dt\right) / z$$

where p is Brownian motion, P the Wiener measure, δ the delta function and z is the appropriate norming. The exponent needs a regularization and the polymer measure is defined by a convergence procedure to get rid of the regularization.

A refinement of a method of Bovier, Felder and Fröhlich (Nucl.Phys. 1984) is presented which works for $d < 4$.

G. BRIGHTWELL:
Models of random partial orders

The theory of random partial orders has not had the sweeping successes achieved by that of random graphs. Part of the reason for this is the difficulty in finding a suitable model which is easy to work with, while capturing the intuitive notion of a "typical" partial order. Here, we discuss several suggested models.

1. "Almost all orders." A result of Kleitman and Rothschild states that almost every partial order on n points has just three layers.
2. "Random graph orders." These are constructed by taking a random graph on $\{x_1, \ldots, x_n\}$, and interpreting each edge $x_i x_j$ with $i < j$ as a relation $x_i < x_j$.
3. "Random k-dimensional orders." These can be constructed either by taking the intersection of k linear orders on an n-set, or by taking n points at random in a k-dimensional unit cube. This latter method can be generalised by replacing the cube with any partially ordered measure space in which every pair of non-trivial intervals is isomorphic up to a scale-factor.

G. DUECK:
New optimization heuristics and applications

Two new general purpose optimization heuristics are presented: the Threshold Accepting algorithm and the Great Deluge algorithm. Both procedures formally resemble the well-known Simulated Annealing approach; they work, however, with different (and simpler) acceptance rules. Both new principles give definitively better results than Simulated Annealing. For large industrial applications, such as mixed integer programming and chip placement, the algorithms yielded much better results than previously known by any other program.

M.E.DYER:
Randomized greedy matching

We consider a randomized version of the greedy algorithm for finding a large matching in a graph. We assume that the next edge is always randomly chosen from those remaining. We analyse the performance of the algorithm when the input graph is fixed. We show that there are graphs for which this Randomized Greedy Algorithm (RGA) usually only obtains a matching close in size to that guaranteed by worst-case analysis (i.e., half the size of the maximum). For some classes of sparse graphs (e.g., planar graphs and forests) we show that the RGA performs significantly better than the worst-case. Our main theorem concerns forests. We prove that the ratio to maximum here is at least 0.7690..., and that this bound is tight.

P. FLAJOLET:
Singularity Analysis and Random Structures

It is known from analytic number theory and analysis that there are strong relations between the singularities of a function and the asymptotic form of its Taylor coefficients. This talk centres on these relations in the context of generating functions arising from combinatorial enumerations.

In a first part, we discuss a method based on contour integration (à la Hankel) that is an attractive alternative to Tauberian theorems and Darboux's method. It is due to P. Flajolet and A. Odlyzko (*SIAM J. on Discrete Math*, 1990), and has been called *singularity analysis*. With it, one can often translate local information on a generating function to an asymptotic form of coefficients.

The method is applicable to explicit generating functions (and its use leads to generalized 0–1 laws). It is also applicable to functions only implicitly known through functional equations (e.g., analysis of the expected heights of trees), or with an exotic singular behaviour (e.g., representation of trees by dags). It can finally be used to derive several limit laws in combinatorics in a uniform manner.

E. GODEHARDT:
Probability models for random multigraphs, with applications in cluster analysis

Let t random graphs $G_{n,p_1}, \ldots, G_{n,p_t}$ — with probabilty p_l per graph that pairs of vertices become linked together by an edge — be chosen independently of each other. In each random graph G_{n,p_l}, we expect $\mathsf{E}N_l = \binom{n}{2}p_l$ edges. Superposition of these random graphs defines a random multigraph $G_{t,n,(p_1,\ldots,p_t)}$ with $\mathsf{E}N = \mathsf{E}N_1 + \cdots + \mathsf{E}N_t$ expected edges altogether. Let the s-projection of a multigraph with t different layers be defined as the graph with the same vertex set and exactly those edges (i,j) for which the corresponding vertices i and j are linked together by at least s of the possible t edges in the original multigraph (are s-fold connected). Then the s-projection can be considered as the realization of a random graph G_{n,p^*} with p^* as the probability of two vertices being s-fold connected. With $S \subset T = \{1,2,\ldots,t\}$, we get

$$p^* = \sum_{S\subset T, |S|\geq s} \prod_{l\in S} p_l \prod_{m\in T\setminus S} (1-p_m),$$

which for $p_1 = \cdots = p_t = p$ gives the tail of a binomial distribution with parameters t and p. Clearly, for $s = 1$ and $\max p_l \to 0$, $p^* = \sum_{l=1}^t p_l$, and for $s = t$ we have $p^* = \prod_{l=1}^t p_l$. We give results for some properties of multigraphs $G_{t,n,(p_1,\ldots,p_t)}$. For example, we give conditions on how to choose $p_1,\ldots,p_t$ maximising p^*, given that $p_1 + \cdots + p_t = tp$.

W. Gutjahr:

The asymptotic shape of random binary trees

Let $\mathcal{B}_n$ denote the family of extended binary trees with n internal nodes. The *shape* of a tree $t \in \mathcal{B}_n$ is given by the sequence of the heights of its leaves. (It is easy to see that each t is determined by its shape.) Assume uniform distribution on $\mathcal{B}_n$, i.e., suppose that each $t \in \mathcal{B}_n$ is equally likely. Then the shape of t becomes a stochastic process.

Our aim is to present the following recent result, proved jointly with G. Phlug, on the convergence of this process as $n \to \infty$: the limiting process is a Bessel Bridge (the residual part of a 3-dimensional Brownian Bridge). We shall indicate some applications to Computer Science.

S. Janson:

Orthogonal decompositions and limit theorems for random graph statistics

Consider a random graph that evolves in time by adding new edges at random times, each edge being added at a time independent of the others. We describe a class of graph statistics, indexed by unlabelled graphs, which can be regarded as orthogonalised versions of subgraph counts. These constitute a basis, so every graph statistic can (in principle, and often in practice) be written as a linear combination of them.

A general functional limit theorem is proved for these statistics. This gives, using the decomposition, limit theorems for a variety of graph statistics. Results are obtained both for G_{np} (look at the evolving random graph at a fixed time) and G_{nm} (look at it at the random time it has m edges); the results for these two models may differ.

Both normal and non-normal limits may appear; the limit is normal when the leading terms in the decomposition are all indexed by connected graphs.

For example, the number of induced subgraphs of a given type in G_{np} (p fixed) is asymptotically normal in most cases, although there are some exceptional cases. On the other hand, the number in G_{nm}, for $m/\binom{n}{2}$ (almost) constant, is always asymptotically normal.

T. KRÜGER:
Random graphs and the epidemic dynamics of sexually transmitted diseases

We introduce a discrete time stochastic process as a model for the epidemic dynamics of sexually transmitted diseases which incorporates the structure of sexual contacts in a society as a random graph and reflects the inherent stochasticity of the transmission dynamics. Thus in computer simulations, a very detailed description of the configuration as well as of the dynamics is possible. The model is mainly applied to the case of AIDS. For simplified models we can find asymptotic values of the expected number of infected people and the threshold parameters.

H. LEFMANN:
Rigid codes

Let C be a subspace of a linear space over $GF(2)$. The automorphisms are given by permutations of the coordinates. If C admits only the trivial automorphism, then C is said to be rigid. Automorphisms are preserved in the dual $C^{\perp}$ of C. Let V^n be an n-dimensional linear space, $n > n_0$. Then for $d < \log_2 n$ or $d > n - \log_2 n$ no d-dimensional subspace of V^n is rigid. Moreover for $d \leq 2\log_2 n$ or $d \geq n - 2log_2 n$ the proportion of the non-rigid d-spaces of V^n is bigger than $\frac{1}{4}$. On the other hand, for given $\epsilon > 0$, if $(2+\epsilon)\log_2 n \leq d \leq n - (2+\epsilon)\log_2 n$, almost every d-subspace of V^n is rigid. This is joint work with K. Phelps and V. Rödl.

T. ŁUCZAK:
The phase transition – an intuitive approach

We describe the behaviour of a random graph process $\{G(n,M)\}_{M=0}^{N}$, $N = \binom{n}{2}$, near the point of the phase transition, i.e., for $M = n/2 + s$, where $s = o(n)$. In particular, we characterize in detail the structure of the giant component L which emerges when $M = n/2 + s, s^3/n^2 \to \infty$, and relate properties of the graph $G(n, n/2+s) \setminus L$ to those of $G(n, n/2 - s)$. Finally, we give a heuristic argument which shows that the diameter of $G(n,M)$ (i.e., the largest distance between two vertices which belong to the same component of the graph) is maximised for $M = n/2 + O(n^{2/3})$.

D.W. MATULA:
A Randomized Algorithm for the Majority Element Problem

The majority element problem is to determine whether or not any element l^* occurs more than $n/2$ times in the list (multiset) $l_1, \ldots, l_n$. Using only equality test comparisons ("does $l_i = l_j$ hold?"), it is known that the MEP can be solved by making at most $\lfloor \frac{3}{2}(n-1) \rfloor$ comparisons, and that this result is best possible. We develop a graph model of this problem and a new proof of the lower bound $\lfloor \frac{3}{2}(n-1) \rfloor$, utilizing as an 'adversary' a sequential edge 2-colouring algorithm.

Our main result is that this problem admits a randomized algorithm of improved performance. Our randomized algorithm is shown to always solve the MEP utilizing an expected number of comparisons that is at most $\frac{7}{6}n + O(1)$. Furthermore, when the most-frequent-element frequency is outside the interval $[\frac{1}{2} - \epsilon, \frac{1}{2} + \epsilon]$ for $\epsilon > 0$, the expected number of comparisons required is less than n and approaches $n/2$ as this frequency approaches either extreme of unity or zero.

C. McDiarmid:
On the expected number of hamilton cycles in a random graph

We determine exactly the expected number of hamilton cycles in the random graph obtained by starting with n isolated vertices and adding edges at random until each vertex degree is at least two. This complements recent work of Cooper and Frieze. There are similar results concerning expected numbers, for example of perfect matchings, spanning trees and directed hamilton cycles.

D. Mollison:
Random graph models for epidemics

I first explain how a wide class of epidemics can be modelled using directed graphs, concentrating on the basic case of the Reed-Frost process, which has independent links and can therefore be modelled by the simple undirected random graph, $G(n,p)$ [Barbour & Mollison, 1990].

A more complex class of models appropriate to animal and plant diseases can be defined on a 2-D lattice, with their contact structure isomorphic to a directed percolation model [Mollison & Kuulasmaa, 1985]. A very interesting strong law result, showing that the epidemic with removal with nearest neighbour contacts spreads as a circle in an appropriate norm, has recently been obtained [Cox & Durrett, 1988].

For the contact structure of human diseases, some kind of graph, intermediate between homogeneous models such as $G(n,p)$ and the extremely localised case of spatial models, seems appropriate. I describe here a generalisation of $G(n,p)$ with correlated links,

$$P(ac \mid ab, bc) = P(ac)f(n)$$

of which a preliminary study has been made using simulated annealing [Smith, 1990], in the hope that random graph experts will take up the challenge of its analysis.

L. Mutafchiev:
Component distribution probabilities in random relational structures

A local limit theorem for the distribution of the number of components in random labelled relational structures of size n (e.g., a type of random graphs on n vertices, random permutations of n elements, etc.) is proved as $n \to \infty$. The asymptotic value of the expected number of components of such structures is also derived. The case when the corresponding exponential generating functions diverge at their radii of convergence is considered.

Z. Palka:
Asymptotic properties of random overlap graphs

Let G_p denote the ordinary random graph on $[n]$. For $0 \leq p \leq 1$, W_p will denote the random digraph on $[n]$ obtained from G_p by orienting each edge $\{i,j\}$ of G_p from the vertex with the smaller number to the vertex with the larger number. The random overlap graph $G(W_p)$ of W_p is a graph on $[n]$ that has an edge $\{i,j\}$ between distinct vertices $i,j \in [n]$ if and only if there is a third distinct vertex $k \in [n]$ such that (k,i) and (k,j) are arcs of W. The random overlap graph $G(W_p)$ differs from the ordinary random graph G_p because, in $G(W_p)$ the probability of an edge between two vertices depends on the vertices; for example, the probability of the edge $\{i, i+1\}$ in $G(W_p)$ is a monotonic increasing function of $i = 1, \ldots, n-1$ starting from 0 for the edge $\{1,2\}$. In this talk we report some results about random overlap graphs such as vertex degrees, clique number, intervality and triangulation. Several open problems will also be stated.

H.-J. PRÖMEL:
The asymptotic structure of graphs without forbidden subgraphs

For a finite graph H let $Forb(H)$ denote the class of all finite graphs which do not contain H as a (weak) subgraph. Kolaitis, Prömel and Rothschild (*Trans. Amer. Math. Soc*, 303, 1987, 637–671) showed that almost all graphs in $Forb(K_{l+1})$ are l-colourable, where K_{l+1} denotes the complete graph on $l+1$ vertices.

In the first part of this talk we give a characterization of all those graphs H which have the property that almost all graphs in $Forb(H)$ are l-colourable. We show that this class of graphs corresponds exactly to the class of graphs whose extremal graph is the Turán graph $T_n(l)$. An earlier result of Simonovits (*Discrete Math.* 7, 1974, 349–376) shows that these are exactly the $(l+1)$-chromatic graphs which contain a colour-critical edge.

In the second part of the talk we investigate the structure of 'almost all' graphs in $Forb(K_{l+1})$ in more detail and use structural properties to present a linear expected time algorithm to colour all graphs which do not contain a clique of size $l+1$ as a subgraph with a minimal number of colours. This extends a result of Dyer and Frieze (*J. Algorithms*, 10, 1989, 451–489).

Both results were obtained jointly with A. Steger.

A. RUCIŃSKI:
Ramsey properties of random graphs

I shall sketch the proofs of two threshold theorems, due to T. Łuczak, B. Voigt and myself, concerning Ramsey properties of random graphs. Let $F \to (G)^1_r$ $[F \to (G)^2_r]$ mean that for every r-colouring of the vertices [edges] of F there is a monochromatic copy of G. A rational d is said to be *crucial* for property $\mathcal{A}$ if, for some constants c and C, the probability that $K(n,p)$ has $\mathcal{A}$ tends to 0 when $np^d < c$, and tends to 1 when $np^d > C$, as $n \to \infty$. We prove that $\max\{e(H)/(v(H)-1) : H \subseteq G\}$ is crucial for $K(n,p) \to (G)^1_r$ and 2 is crucial for $K(n,p) \to (K_3)^2_2$. One of the proofs gives rise to the investigation of the density of Ramsey graphs, a new topic in Ramsey Graph Theory.

K. SCHÜRGER:
Extensions of Kingman's subadditive limit theorem and an application to random graphs

This talk deals with two almost subadditive extensions of Kingman's (1968) ergodic theorem. These almost sure results, being valid under weak moment conditions, are obtained by short proofs. One of these proofs is completely elementary and does not even make use of Birkhoff's ergodic theorem which, instead, is obtained as a by-product. We also obtain that a recent almost subadditive convergence theorem of Derriennic

(1983) holds under a much weaker moment condition. An application to random graphs is given.

J. SPENCER:
The first order world of random graphs

A property is first order if it may be described by a finite sentence involving equality, adjacency, the usual Boolean connectives, and existential and universal quantification over vertices.

For a given first order property A, and a graph probability $p = p(n)$, we examine $\lim \Pr[G(n,p) \models A]$. For $p = n^{-\alpha}, \alpha \notin \mathbb{Q}$ this is either 0 or 1. An examination of extension statements plays a key rôle in the argument. When $p = c/n$, in some sense the 'double jump' is invisible as the limit is a C^∞ function $f(c)$. When $p = n^{-\frac{1}{3}}$, evaluation of the limit is undecidable. In the range $p = \Theta(\frac{\ln n}{n})$, there is a doubly infinite sequence, outside of which the 0-1 law holds. The threshold for connectivity $p = \frac{\ln n}{n}$ is a member of this sequence.

A. STEGER:
The asymptotic structure of graphs without induced forbidden subgraphs

In this talk we investigate asymptotic properties of the class $Forb_n^*(H)$ of graphs which do not contain a given graph H as an induced subgraph. For $H = C_4$ we use the Kleitman-Rothschild method to show that almost all graphs in $Forb_n^*(C_4)$ are *split graphs*. For general graphs H we introduce the notion of an extremal graph in $Forb_n^*(H)$ and define a new parameter $\tau(h)$ which generalizes the chromatic number and the clique covering number. Using these notions we are able to extend the theorems of Erdős, Stone and Simonovits and of Erdős, Frankl and Rödl concerning weak subgraphs. In the final part we characterize the asymptotic structure of almost all graphs without induced C_5. A corollary of this result is that almost all Berge graphs are perfect. The results are joint work with H.-J. Prömel.

B. VOIGT:
Large monochromatic antichains in coloured probability trees

A probability tree is a rooted tree T together with a mapping $prob: T \to [0,1]$ such that $prob(\text{root}) = 1$ and $\sum_{b \in Succ(a)} prob(b) = 1$ for all $a \in T$, where $Succ(a)$ is the set of immediate successors of a. This induces probabilities $prob(Cone(a)) = \prod_{b \in Pred(a)} prob(b)$ on the cones $Cone(a) = \{b \in T \mid a \leq b\}$, where $Pred(a) = \{b \mid b \leq a\}$ is the set of predecessors of a.

If $A \subseteq Cone(a)$, $A \neq \{a\}$, is an antichain in $Cone(a)$, then

$$val(A,a) = \sum_{b \in A} \frac{prob(b)}{prob(Cone(a))}$$

is the value of the antichain A in $Cone(a)$.

Given a colouring $\Delta: T \to [r] = \{1, \ldots, r\}$ of the vertices of T, a set S of vertices is *monochromatic* if $\Delta|_S$ is constant. Let

$$f(T_n^2, r) = \inf_{\text{prob functions}} \; \inf_{\Delta: T \to [r]} \; \max_A val(A,a),$$

where T_n^2 is the binary tree of height n, and A runs over all monochromatic antichains. We shall present the following inequalities, proved jointly with I. Althöfer:

$$1 - \frac{1}{1+\frac{n}{r}} \leq f(T_n^2, r+1) \leq 1 - \frac{1}{1+\lceil\frac{n}{r}\rceil}.$$

We conjecture the upper bound to be correct, whereas the lower bound is the exact value with respect to the class of all trees of uniform height n.

K. WEBER:
Random subgraphs of the cube

In 1987 Dyer and Frieze introduced a method to generate all random subgraphs $G_n = (V_n, E_n)$ of the n-cube graph Q_n: V_n is randomly sampled from the vertex set of Q_n so that $\Pr(x \in V_n) = p_v$ independently for each vertex $x \in Q_n$. E_n is now randomly sampled from the set of edges induced by V_n in Q_n so that $\Pr(xy \in E_n) = p_e$ independently for each induced edge xy. We investigated such graph theoretic properties and parameters of G_n as degree, independence number and related parameters, thickness, genus, component structure, raduius and diameter. In the lecture, we shall give bounds for the radius $R(G_n)$ and the diameter $D(G_n)$; the results were obtained jointly with A.A. Sapozhenko.

I. WEGENER:
Bottom-up heap-sort, a variant of heap-sort beating quick-sort on average

Bottom-up heap-sort is a variant of heap-sort. It is a general and internal sorting algorithm which is easy to implement. It can be proved that (in the worst case) the number of comparisons is bounded by $1.5n\log n$, but this upper bound may be far from optimal. Probabilistic models are presented which explain the good average case behaviour. Bottom-up heap-sort beats quick-sort if $n \geq 400$ and the median-of-three version of quick-sort if $n \geq 16000$. McDiarmid and Reed presented a non-internal version of bottom-up heap-sort which uses n extra bits. The worst case number of comparisons for this sorting algorithm is $n\log n + 1.1n$.

D. WELSH:
Voter and Antivoter Models and Colouring Algorithms

In the voter (antivoter) models on a finite graph, a clock rings at random, at each ring a random vertex chooses a neighbour at random and adopts (respectively rejects) the colour of that neighbour. Many properties of both processes can be easily settled (they are only finite state Markov chains), but some very basic questions are not settled. For example, we cannot prove that in the $m \times n$ toroidal square lattice two adjacent points have probability $< \frac{1}{2}$ of being the same colour.

A 3-colour version of the antivoter model is used to set up a heuristic randomised 3-colouring algorithm which works in linear expected time except when the 3-colourable graph is of average degree approximately 5.

J.C. WIERMAN:

Critical Probability Bounds in Percolation Models

The exact critical probability value is known for percolation models on only a few two-dimensional lattices, and bounds for unsolved lattices are very poor. A new substitution method produces a sequence of upper and lower bounds for the critical probability, improving the best previous bounds. The Kagomé lattice bond percolation critical probability satisfies $.5182 \leq p_c \leq .5335$, compared to the previous bounds $.4045 \leq p_c \leq .6180$. The square lattice site percolation critical probability is less than .6795, improving Zuev's (1987) bound of .6819. The substitution method also proves that the critical exponents of the triangular and hexagonal lattices are equal, the first such result for two-dimensional lattices. The substitution method also provides simple conditions for establishing strict inequalities for critical probabilities of sublattices in any dimension, where previous conditions were difficult to check and valid only in two dimensions.

Berichterstatter: B. Bollobás

Tagungsteilnehmer

Dr. Ingo Althöfer
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Walter Deuber
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Andrew Barbour
Institut für Angewandte Mathematik
Universität Zürich
Rämistr. 74

CH-8001 Zürich

Prof.Dr. Gunter Dueck
IBM Deutschland GmbH
Wissenschaftliches Zentrum - IWBS
Tiergartenstraße 15

6900 Heidelberg

Prof.Dr. Bela Bollobas
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Prof.Dr. Martin E. Dyer
Dept. of Mathematics and Statistics
Teeside Polytechnic

GB- Middlesborough, Cleveland TS1 3BA

Prof.Dr. Erwin Bolthausen
Institut für Angewandte Mathematik
Universität Zürich
Rämistr. 74

CH-8001 Zürich

Peter Eichelsbacher
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Dr. Graham R. Brightwell
Dept. of Mathematics
London School of Economics
Houghton Street

GB- London WC2A 2AE

Prof.Dr. Paul Erdös
Mathematical Institute of the
Hungarian Academy of Sciences
P.O. Box 127
Realtanoda u. 13-15

H-1053 Budapest

Prof.Dr. Philippe Flajolet
INRIA
Bat. 8, Domaine de Voluceau-
Rocquencourt
B. P. 105

F-78153 Le Chesnay Cedex

Prof.Dr. Svante Janson
Dept. of Mathematics
University of Uppsala
Thunbergsvägen 3

S-752 38 Uppsala

Prof.Dr. Alan M. Frieze
Dept. of Mathematics
Carnegie-Mellon University

Pittsburgh , PA 15213-3890
USA

Prof.Dr. Michal Karonski
Instytut Matematyki
UAM
ul. Matejki 48/49

60 769 Poznan
POLAND

Dr. Erhard Godehardt
Abt. Kardiovask. Chirurgie B
Universität Düsseldorf
Gebäude 12,41
Moorenstr. 5

4000 Düsseldorf 1

Dr. Tyll Krüger
Physik
BIBOs Forschungszentrum
Universität Bielefeld
Wellenberg 1

4800 Bielefeld 1

Prof.Dr. Friedrich Götze
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Imre Leader
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Dr. Walter Gutjahr
Institut für Statistik und
Informatik
Universität Wien
Universitätsstr. 5/3

A-1010 Wien

Dr. Hanno Lefmann
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Matthias Löwe
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. Ljuben Mutafchiev
Centre for Mathematics and
Mechanics
Bulgarian Academy of Sciences
P. O. Box 373

1090 Sofia
BULGARIA

Prof.Dr. Thomas Luczak
Instytut Matematyki
UAM
ul. Matejki 48/49

60-769 Poznan
POLAND

Prof.Dr. Krystoff Nowicki
Dept. of Statistics
University of Lund
Box 7008

S-220 07 Lund

Prof.Dr. David W. Matula
Dept. of Computer Science
Southern Methodist University

Dallas , TX 75275
USA

Prof.Dr. Zbigniew Palka
Instytut Matematyki
UAM
ul. Matejki 48/49

60 769 Poznan
POLAND

Prof.Dr. Colin McDiarmid
Department of Statistics
Oxford University
1, South Parks Road

GB- Oxford , OX1 3TG

Prof.Dr. Hans Jürgen Prömel
Forschungsinstitut für
Diskrete Mathematik
Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof.Dr. Denis Mollison
Department of Actuarial Mathematics
and Statistics
Heriot-Watt University
Riccarton, Currie

GB- Edinburgh EH14 4AS

Prof.Dr. Andrzej Rucinski
Instytut Matematyki
UAM
ul. Matejki 48/49

60 769 Poznan
POLAND

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Prof.Dr. Klaus Schürger
Statistische Abteilung
Inst. f. Gesellschafts- u. Wirt-
schaftswissensch. der Universität
Adenauerallee 24-42

5300 Bonn 1

Dr. Karl Weber
Bereich Mathematik
Hochschule für Seefahrt
Warnemünde-Wustrow

O-2598 Wustrow

Prof.Dr. Joel Spencer
Courant Institute of
Mathematical Sciences
New York University
251, Mercer Street

New York , N. Y. 10012
USA

Prof.Dr. Ingo Wegener
Institut für Informatik II
Universität Dortmund
Postfach 50 05 00

4600 Dortmund 50

Dr. Angelika Steger
Institut für ökonometrie und
Operations Research
Universität Bonn
Nassestr. 2

5300 Bonn 1

Prof.Dr. Dominic J. A. Welsh
Institut für ökonometrie und
Operations Research der Uni Bonn
Nassestr. 2

5300 Bonn 1

Dr. Bernd Voigt
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

4800 Bielefeld 1

Prof.Dr. John C. Wierman
Dept. of Mathematical Sciences
Johns Hopkins University
220 Maryland Hall

Baltimore , MD 21218
USA

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