MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 42/1990

Diophantische Approximationen
30.09 bis 6.10. 1990

Die Tagung, die unter der Leitung von P. Bundschuh (Köln) und R. Tijdeman (Leiden) stattfand führte 48 auf dem Gebiet der Diophantischen Approximationen arbeitende Wissenschaftler zusammen. Bei ihren Vorträgen standen arithmetische Fragestellugen (Irrationalität, Transzendenz, algebraische Unabhängigkeit) sowie Diophantische Gleichungen im Vordergrund des Interesses. Einen weiteren Schwerpunkt bildeten Probleme aus der Gleichverteilungs theorie.

Vortragsauszüge:

P. G. BECKER:

Exponential Diophantine equations and the irrationality of certain real numbers.

K. Mahler proved the irrationality of the decimal fraction $0.(1)(g)(g^2)\ldots$ where $g \geq 2$ is a fixed integer and (g^n) denotes the number g^n written in decimal form. This was later generalized by various authors. Their proofs showed a strong connection between the irrationality of these numbers and the question whether certain Diophantine equations have only a finite number of solutions. We apply Schlickewei's recent result on the number of solutions of the S - unit equation to show that the number of solutions of a certain type of exponential Diophantine equations can be bounded by an effectively computable constant. This gives a further generalization of Mahler's irrationality result.

V. BERNIK:

Diophantine approximation on manifolds.

Theorem 1: Let $f_i(x) \in C^{n+1}[a,b]$ for $i = 1,\ldots,n$ and put

$$\Delta(x) = \begin{vmatrix} f_1'(x) & \cdots & f_n'(x) \\ & \cdots & \\ f_1^{(n)}(x) & \cdots & f_n^{(n)}(x) \end{vmatrix}$$

Suppose there is a $\delta > 0$ such that for every $x \in [a,b]$, we have

$$|\Delta(x)| \geq \delta.$$

Then for every $\varepsilon > 0$, every $N > c(\delta, \varepsilon)$ and every interval $I \subseteq [a, b]$ of length $> N^{-\varepsilon}$, there exist points $x \in I$ such that for every $\mathbf{a} \in Z^{n+1}$ with $H(\mathbf{a}) \geq N$ and every $w > \frac{1}{4}n^2 + n + 4(n+3)\varepsilon$, the inequality

$$(*) \qquad |a_n f_n(x) + \ldots + a_1 f_1(x) + a_0| > H(a)^{-w} \text{ is true.}$$

Theorem 2: Let $f_i(x) \in C^{n+1}[a, b]$ for $i = 1, \ldots, n$ and suppose that $\Delta(x) \neq 0$ for almost all $x \in [a, b]$. We denote by $B_n(w)$ the set of real numbers x for which inequality $(*)$ has infinitely many solutions in integral vectors $\mathbf{a}$. Then $\mu(B_n(w)) = 0$ for $w > \frac{1}{4}n^2 + \frac{3}{2}n + \frac{5}{4}$. For $n = 3$, we have $\mu(B_3(w)) = 0$ for $w > 4$.

These results generalize certain known results of Schmidt (1964), Pjartly (1969), Sprindzuk (1970), Viola(1979).

D. BERTRAND:

Algebraic groups, differential equations and algebraic independence.

Applying an analogue of the cohomological methods of Kummer theory, we give criteria for the differential Galois group of reducible differential systems to be as big as possible. As a typical corollary of the method we prove:

Theorem. Let $A_1 \in M_{p,p}(\mathbf{C}(z)), A_2 \in M_{q,q}(\mathbf{C}(z)); B \in M_{p,q}(\mathbf{C}(z))$ be matrices.

Assume

(i) the differential system $P' = A_2P - PA_1$ is irreducible over $\mathbf{C}(z)$;

(ii) the differential system $P' = A_2P - PA_1 + B$ has no solution defined over $\mathbf{C}(z)$;

(iii) $Y_0 = {}^t(Y_1, Y_2)$ is a vector solution of the system

$$Y' = \begin{pmatrix} A_1 & B \\ O & A_2 \end{pmatrix}$$

such that the components of Y_2 are algebraically independent over $\mathbf{C}(z)$. Then the components of Y_0 are algebraically independent over $\mathbf{C}(z)$.

B.BRINDZA:

On the "algebra" of Thue's equation.

(joint work with A. van der Poorten and M.Waldschmidt.)

Let $F \in \mathbb{Z}[x,y]$ be a binary form of degree $n \geq 3$ and m a positive integer. We denote by s the number of d distinct prime factors of m. Furthermore, let $(x_1, y_1), \ldots, (x_k, y_k)$ be a sequence of distinct solutions to the equation $|F(x,y)| = m$ in rational integers x, y (the solutions $(x,y)$and $(-x,-y)$ are considered as the same.) We assume that these solutions are "primitive" i.e $\gcd(x_i, y_i) = 1$. Then we have

Theorem: If F is irreducible, $k > 2(s+2)n^2$ and $c_1(n)M^4m^{2/(n-1)} \leq y_1 \leq \ldots \leq y_k$, where M is the Mahler height of F and $c_1(n)$ is an effective, positive constant, then $y_k > \exp(c_2(n){y_1}^{n-1})$.

However, this lower bound for y_k is greater than the universal upper bound for all solutions, hence $k \leq 2(s+2)n^2$.

W.W.L. CHEN:

Irregularities of distributions.

We discuss a link between irregularities of distribution relative to half-planes and relative to convex polygons. We also discuss a rather surprising phenomenon where certain L^1 - estimates of the discrepancy are extremely small compared to the corresponding L^2 - estimates.

The results represent joint work with J. Beck (Rutgers University).

T.W. CUSICK:

Inhomogeneous Diophantine Approximation.

(joint work with P. Szűsz and A.M. Rockett.)

We define the inhomogeneous Diophantine approximation constant $m(\alpha, \beta)$ for the pair α, β, where α is irrational and $\beta \neq m\alpha + n$ for all $m, n \in \mathbb{Z}$, by

$$m(\alpha,\beta) = \liminf |q| \cdot ||q\alpha - \beta||$$

and we define

$$m(\alpha) = \sup m(\alpha, \beta).$$

It follows from a theorem of Minkowski that $m(\alpha) \leq \frac{1}{4}$ for all α and it is known that $m(\alpha) < \frac{1}{4}$ if the continued fraction for α has bounded partial quotients. We give a new method for calculating the constants $m(\alpha, \beta)$ which is particularly effective when α is a quadratic irrational. The new method improves earlier ones by Davenport and Barnes and Swinnerton-Dyer because it involves no intelligent ideas in its algorithmic implementation and so can be programmed for a computer in a straightforward way. As one application, a simpler proof of Davenport's 1947 determination of the complete sequence of inhomogeneous minima

$$\delta_0 = m(\alpha) = (4\sqrt{5})^{-1}, \delta_1 = (5\sqrt{5})^{-1}, \ldots, \text{ for } \alpha = \frac{1}{2}(1+\sqrt{5}) \text{ is given.}$$

G. DIAZ:

Approximation measure for π: a new result after Fel'dman, Waldschmidt.

The result: There exists an absolute constant $c > 0$ such that for all integers $n \geq 1$ and all algebraic numbers ξ we have:

$$\log|\pi - \xi| \geq -c\frac{D_N^2}{N}(h(\xi) + \log D_N)(\log D_N)$$

where $D_N := [\mathbf{Q}(i, \xi, \exp(2\pi i/N)) : \mathbf{Q}]$ and $h(\xi)$ is the absolute logarithmic height of ξ (with $N = 1$ we have the "usual" approximation measure of π: Fel'dman 1948,1960). M. Waldschmidt (1978) had $(1 + \log N)^{-1}$ in place of N^{-1}.

Corollary: When $\xi \in \mathbf{Q}(i, \exp(2\pi/N))$ we have:

$$\log|\pi - \xi| \geq -4cD_N(\log D_N)^2(1 + \log L(\xi)/d(\xi))$$

$$(L(\xi) := \text{length of } \xi \; ; \; d(\xi) := [\mathbf{Q}(\xi) : \mathbf{Q}])$$

N.I. Fel'dman (Journal of Number Theory, 1977) had D_N^2 in place of $D_N(\log D_N)^2$. We use the Gel'fond-Schneider method with a construction of the auxiliary function which is not the standard one (the modification is of the same type as the one we introduced in a study of "large transcendence degree").

G.R. EVEREST:

On the traces of algebraic units and the Leopoldt conjecture

Suppose $\mathbf{K}/\mathbf{Q}$ is totally real, $[\mathbf{K}:\mathbf{Q}] = r+1, U_K = O_K^*$, the group of algebraic units. Then $U_K \cong \{\pm 1\} \times \mathbf{Z}^r$. Write $T : K \to \mathbf{Q}$ for the trace map $T(k) = \sum_\sigma \sigma(k)$, where σ runs through the Q - isomorphisms: $K \to \mathbf{R}$. Given $\alpha \epsilon O_K$, write $I_\alpha = \alpha U_K$, and let q be a large, positive real number.

Theorem 1: $\#\{x \epsilon I_\alpha\colon |T(x)| < q\} = A(\log q)^r + B(\log q)^{r-1} + \circ$ (i.e. an error).

Given $p > 0$, prime there are infinitely many $\alpha \epsilon O_K$ with $\liminf_{x \epsilon I_\alpha} |T(x)|_p = 0$

where $|.|_p$ denotes the p-adic absolute value. Fix $\alpha \epsilon O_K$ with this property.

Theorem 2: $\# \{x \in I_\alpha : |T(x)| \cdot |T(x)|_p < q\} = A_p(\log q)^r + B_p(q) + \circ$, where

$$A_p > 0 \text{ and } (\log q)^{r-1} \log\log q \ll B_p(q) \ll (\log q)^{r-1} \log\log q.$$

The next theorem describes A and A_p. Let $|R_K|$ denote the regulator of K and $|R_p|$ the p-adic regulator.

Theorem 3: $A = \theta_1 |R_K|^{-1}$, with $\theta_1 = \theta_1(r) \epsilon \mathbf{Q}; A_p = \theta_2 |R_K|^{-1}$, with $\theta_2 = \theta_2(r,p) \epsilon \mathbf{Q}$ and $|\theta_2|_p = |R_p|_p^{-1}$ if $e_1 = e_r, |\theta_2|_p = |R_p|_p^{-1} p^{e_r} - ^{e_1} - ^1$ if $e_1 < e_r$, where

$$\begin{pmatrix} p^{e_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p^{e_r} \end{pmatrix}$$

denotes the Smith Normal Form of R_p.

An indication was given of the role played by the p-adic Subspace Theorem, and also of the role played by the growth of the height counting function. Let $H(x) = \max_\sigma |\sigma(x)|$ denote the naive height. Then

Proposition. $\#\{x : H(x) < q\} =$

$$\frac{2(r+1)^r (\log q)^r}{r!|R_K|} + \frac{2(r+1)^{r-1}}{(r-1!|R_K|} H_K\{1 + \frac{r+1}{r-1}\}(\log q)^{r-1} + \circ,$$

where $H_K = \int_{[-1/2,1/2]^r} \sigma(\mathbf{x}) d\mathbf{x}, \sigma(\mathbf{x}) = \max_\sigma \log|\sigma(u)|, u = (\mathbf{x}), \mathbf{x}$ denotes the vector of exponents of $u \epsilon U_K$. This is a generalisation of a formula with 2 terms proved by Lang (ineffective) and Győry and Pethő (effective and explicit).

J.H. EVERTSE:

Pairs of binary forms with given resultant.

We consider the equation (1) $0 < |R(F,G)| \leq M$ in binary forms $F, G \epsilon \mathbf{Z}[X,Y]$, where $R(F,G)$ denotes the resultant of F and G. Two pairs of binary forms $(F,G),(F',G')$ are called equivalent, if there is a matrix $\binom{ab}{cd} \in SL(2,\mathbf{Z})$ such that $F'(X,Y) = F(aX+bY, cX+dY)$ and $G'(X,Y) = G(aX+bY, cY+dY)$. Then one has:

Theorem 1: For every number field L and every $M \geq 1$, there are only finitely many equivalence classes of pairs of binary forms $F, G \in \mathbf{Z}[X,Y]$ with $0 < |R(F,G)| \leq M, FG$ has splitting field L, FG is not divisible by the square of a linear form in $L[X,Y]$, and $\deg F \geq 3$ and $\deg G \geq 3$.

Theorem 1 is ineffective. Theorem 1 is a generalisation of a result of Győry's from 1988 about eq. (1) in monic binary forms F, G, i.e with $F(1,0) = G(1,0) = 1$.

We derived Theorem 1 from the following inequality. As in Theorem 1, F, G are two binary forms in $\mathbf{Z}[X,Y]$ such that FG has splitting field L, FG is not divisible by the square of a linear form in $L[X,Y]$, and $\deg f = r \geq 3$ and $\deg G = s \geq 3$.

Theorem 2: For all $\epsilon > 0, |R(F,G)| \geq C\{|D(F)|^{\frac{s}{r-1}}|D(G)|^{\frac{r}{s-1}}\}^{\frac{1}{3s}-\epsilon}$, where $D(F), D(G)$ are the discriminants of F and G, and C is an ineffective number depending on r, s, L, ϵ only.

N.I. FEL'DMAN:

On the number π

Let $\alpha_1, ..., \alpha_p \in \mathbf{A}, \alpha_1, ..., \alpha_p \neq 0, \log\alpha_1, ..., \log\alpha_p$ be any determination of the logarithms of the $\alpha's$. Assume that the numbers $\log\alpha_1, ..., \log\alpha_p$ are linearly independent.

Then ([1],p. 128) for all $\beta_1, ..., \beta_p \in \mathbf{A}$,

$$\sum_{k=1}^{p} |\log\alpha_k - \beta_k| \geq -\exp(-cn^{1+1/p}(n\log n + \log L + 1)(\log n + 1)^{-1},$$

where

$$L = \exp\left(\frac{n}{n_1}\log L_1 + ... + \frac{n}{n_p}\log L_p\right) \quad n = \deg\mathbf{K}, \mathbf{K} = \mathbf{Q}(\alpha_1, ..., \alpha_p; \beta_1, ..., \beta_p),$$

$$n_k = \deg \beta_k,\ L_k = L(\beta_k), c = c(\alpha_1, ...\alpha_p, \log \alpha_1, ..., \log \alpha_p) > 0, c \text{ effective.}$$

Theorem. Assume that $\log \alpha_2, ..., \log \alpha_p$ and πi are linearly independent. Then there exists an effective constant $c_0 = c_0(\alpha_2, ..., \alpha_p; \log \alpha_2, ..., \log \alpha_p) > 0$, such that for all $\beta_1, ..., \beta_p \in \mathbf{A}$, $(n, n_k, L, L_k$ as above)

$$|\pi i - \beta_1| + \sum_{k=2}^{p} |\log \alpha_k - \beta_k| \geq \exp(-c_0 n(n \log n + \log L + 1)(\log n + 1)^{1/p}).$$

A. GALOCHKIN:

On certain arithmetical properties of the values of hypergeometric functions

We deal with the functions

$$\phi(z) = 1 + \sum_{\nu=1}^{\infty} z^{\nu} \prod_{x=1}^{\nu} \frac{a(x)}{b(x)}, \text{where}$$

$$a(x) = (x + \mu_1)...(x + \mu_m) \in \mathbf{K}[x],$$

$$b(x) = (x + \lambda_1)...(x + \lambda_m) \in \mathbf{K}[x], m > n,$$

and $\mathbf{K}$ is an algebraic number field. The following problems were discussed.

1. To give necessary and sufficient conditions for $\phi(z^{m-n})$ to belong to the class of Siegel's E - functions.

2. To give estimates for linear forms in the values of $\phi(z), \phi'(z), ..., \phi^{(m-1)}(z)$ with the conditions that $a(x) \equiv 1$ and $\mathbf{K}$ is an imaginary quadratic field $(\mathbf{K} = \mathbf{I}, \mathbf{I} = \mathbf{Q}(\sqrt{-d}), d \in \mathbf{N})$.

3. To give results in more general cases: (i) $[\mathbf{K} : \mathbf{I}] > 1$, (ii) $a(x) \not\equiv 0$.

K. GYŐRY:

Decomposable forms with given discriminant

(joint work with J.H. Evertse)

The decomposable forms F, F' in $\mathbf{Z}[x_1, ..., x_m]$ are called equivalent if $F'(x) = F(UX)$ for some $U \in SL_m(\mathbf{Z})$. For $m > 2$, equivalent binary forms have the same discriminant. It is known that there are only finitely many equivalence classes of binary forms in $\mathbf{Z}[x_1, x_2]$ with a given degree $n(\geq 2)$ and a given non - zero discriminant. This was proved in the case $n = 2,3$ by Lagrange and Hermite, respectively, in an effective form, and in the general case by Birch and Merriman in an ineffective way. We gave an effective and quantitative version of this result of Birch and Merriman. Further, we extended the concept of discriminant to decomposable forms in $m \geq 2$ variables and obtained the following generalization.

Theorem. There are only finitely many equivalence classes of primitive, square-free decomposable forms in $\mathbf{Z}[x_1, ..., x_m]$ with a given non - zero (generalized) discriminant and a full set of representatives of these classes can be effectively determined.

We proved our theorems in quantitative and more general forms, over a ring of S - integers of an arbitrary algebraic number field. Our results have several applications, among others to algebraic numbers with given discriminant, $\mathbf{Z}$ - modules with given discriminant and decomposable form equations.

M. HINDRY:

Heights and abelian varieties

Motivated by research on the Mordell-Weil group of an abelian variety A over a number field K, we study the following conjectures:

Conjecture A: $\#(A(K)_{\mathrm{torsion}}) \leq C(K, \dim A)$;

Conjecture B: $\hat{h}_{\min}(A/K) \gg \hat{h}(A/K)$, where $\hat{h}$ is the canonical Néron-Tate height, $\hat{h}_{\min}(A/K) = \min \hat{h}(P)$ where the minimum is taken over all non-torsion points P of A for which $\mathbf{Z}P$ has Zariski-closure A, and where $\hat{h}(A/K)$ is some measure of the complexity of A such as the Faltings height.

Mazur proved that $C(\mathbf{Q}, 1) = 16$ and Khamienny that $C(\mathbf{Q}(\sqrt{d}), 1) \leq C_0$. Further, Silverman and Hindry proved, that for elliptic curves, $\#(E(K)_{\mathrm{torsion}} \leq$ explicit function of $[K : \mathbf{Q}]$ and $\sigma_{E/K}$, and $\hat{h}_{\min}(E/K) \leq f([K : Q], \sigma_{E/K})\hat{h}(E/K)$, where $\sigma_{E/K}$ is Szpiro's ratio. Finally, S. David proved that $\hat{h}_{\min}(A/K) \geq \rho_v(A)^{-4g^{-2}}\hat{h}(A/K)$, where v is an archimedean place on K, $\mathrm{A}(\mathrm{K}_v) \simeq \mathbf{C}^g/(\mathbf{Z}^g + \mathcal{T}_v\mathbf{Z}^g)$, and $\rho_v(A) = \hat{h}(A/K)/||\mathcal{T}_v||$. We discuss another approach based on a local decomposition of h extending and making more precise Néron's theory of quasi-functions.

N. HIRATA-KOHNO

Linear forms on Algebraic Groups

This work is a quantitative version of a Theorem of Wüstholz's on transcendence on algebraic groups, and it improves the previous results.

Notations. Let G' be a connected,commutative algebraic group / $\bar{\mathbf{Q}}$ of dimension d, $G := \mathbf{G_a} \times G'$. We fix a basis of the tangent space $T_G(\mathbf{C})$ defined at the origin of G and we fix also an embedding of G into a projective space. Further, let $u \in T_G(\mathbf{C})$ be a point satisfying $\exp(u) \epsilon G(\bar{\mathbf{Q}})$, and W a hyperplane/ $\bar{\mathbf{Q}} \subset T_G(\mathbf{C})$. We denote by $\log H$ an upper bound for the values of Weil's logarithmic height of the coefficients of the equation defining W.

Theorem: When $u \notin W$, we have $\log \operatorname{dist}(u, W) > -c \log H(\log\log H)^{d+1}$ where c is an effectively computable positive number, depending on d, the basis of T_G $(\mathbf{C})$, the embedding of $G, h(\exp_G(u)), |u|$ and the degree of the ground field.

This improves the previous results of Philippon and Waldschmidt: $\log \operatorname{dist}(u, W) > -c(\log H)^{d+1}$. There are some refinements of this theorem in case that u is a period, G' contains some groups, or is of CM - type.

R.J. KOOMAN:

Decomposition of linear recurrences

Let $\mathbf{K} = \mathbf{R}$ or $\mathbf{C}$ and $\{p_0(n)\}, \ldots, \{p_{k-1}(n)\}$ sequences in K with $p_0(n) \neq 0$ for all $n \in \mathbf{N}$. The linear recurrence

$$(1) \qquad u_{n+k} + p_{k-1}(n)u_{n+k-1} + \ldots + p_0(n)u_n = 0$$

has a k - dimensional vector space of solutions, consisting of sequences $\{u_n\}_{n \epsilon \mathbf{N}}$ in K. It is assumed that $\lim_{n\to\infty} p_i(n) = p_i$ exists for all i, then (1) has a characteristic polynomial $X^k + \sum_{i=0}^{k} p_i X^i = \prod_{i=1}^{k}(X - \alpha_i)$. The classical theorem of Poincaré and Perron states that (1) has a basis of solutions $\{\{u_n^{(i)}\} : i = 1, \ldots, k\}$ such that $\lim_{n\to\infty} u_{n+1}^{(i)}/u_n^{(i)} = \alpha_i$, provided that $|\alpha_1| < |\alpha_2| < \ldots < |\alpha_k|$. We proved some results about (1) in the case that $|\alpha_1|, \ldots, |\alpha_k|$ are not all distinct.

Theorem 1: If (1) has a characteristic polynomial with roots $|\alpha_l| = \ldots = |\alpha_1| \neq |\alpha_j|$ for $j \geq l$, then the space of solutions of (1) has a 1 -dimensional linear subspace of sequences $\{u_n\}$ in K satisfying a linear recurrence of order l $u_{n+l} + q_{n-1}(n)u_{n+l-1} + \ldots + q_0(n)u_n = 0$ which has a characteristic polynomial

$\prod_{j=1}^{l}(X-\alpha_j)$ and in which the $q_j(n)'s$ do not converge more slowly than the $p_i(n)'s$.

Theorem 2: Assume that (1) has characteristic polynomial $\prod_{i=1}^{m}(X-\alpha_i)^{e_i}$, where $\alpha_1,\dots,\alpha_m$ are distinct, and $e_i \in \mathbb{N}$. Put $L := \max_{i=1,\dots,m} e_i$. Further, assume that $\sum_{n=1}^{\infty} n^{L-1}|p_j(n)-p_j| < \infty$ for $j = 0,..,k-1$.

Then (1) has a basis of solutions $\{u_n^{(i,j)}\}(i = 1,\dots,m, j = 1,\dots,e_i)$ such that $\lim_{n\to\infty} u_n^{(i,j)}/\alpha_i^n n^{j-1} = 1$.

Both results are derived from a "decomposition theorem" on the matrix recurrence (2) $M_n\mathbf{x}_n = \mathbf{x}_{n+1}$ in sequences $\{\mathbf{x}_n\}$ of vectors in K^k, where $M_n \in GL(k,K)$.

M. LAURENT:

Interpolation determinants in transcendence theory

In transcendence theory, or in Diophantine approximation theory, the proofs usually begin by the construction of an auxiliary function, via the resolution of some linear system of equations. We show a new method that avoids to produce a solution of this linear system. We consider the subdeterminants, extracted from the linear system: these are usually "interpolation determinants" of some set of entire functions evaluated (with their derivatives) at some set of points. It is easily seen that such determinants are "small " under suitable conditions. With this method, we give a new proof of the standard theorem of Gel'fond-Schneider on the transcendency of a^b.

J. LOXTON:

Partition Statistics

Turán proved that if n_1 and n_2 are large and close together, then almost all pairs of partitions of n_1 and n_2 contain many common parts. This result can be extended as follows. Suppose Λ is an increasing sequence of positive integers which is regular and well-distributed and suppose that

$$n \le n_1 \le \dots \le n_k \le n(1+o(1)).$$

Then almost all k - tuples $Q_1,\dots,Q_k$ of partitions of $n_1,\dots,n_k$ into distinct parts from 1 have at least $\{2^k.k^\alpha(1-2^{1-\alpha})\zeta(\alpha)^{-1}-\epsilon\}(\max \text{length} Q_i)$ common parts. Moreover, if $K(n_1,\dots,n_k,\lambda)$ denotes the number of k - types of ordinary partitions $p_1,\dots,p_k$ of $n_1,..,n_k$, respectively, with at most $(\sqrt{6}/2\pi k)\sqrt{n}\log n + \lambda\sqrt{n}$ common parts, and $n \leq n_1 \leq ... \leq n_k \leq n + o(n/(\log n)^{1+\epsilon})$ and $\lambda = o(\log\log n)$, then $K(n_1, ..., n_k, \ \lambda)/\{p(n_1)..p(n_k)\} \rightarrow \exp(-(dk)^{-1}e^{-dk\lambda})$ as $n \rightarrow \infty$. Here $p(n)$ denotes the number of partitions of n and $d = \pi/\sqrt{6}$. Such distribution functions are believed to exist for other partition problems.

This is joint work with Hang Jai Yeung.

YU. V. NESTERENKO

On types of algebraic independence for almost all numbers

I discussed the next

Problem: Is it true that for almost all points $\omega \epsilon \mathbf{R}$ with the exclusion of a set of Lebesgue measure 0 there exists a constant $c = c(\omega) > 0$ such that for every polynomial $P \epsilon \mathbf{Z}[x_1, .., x_m]$, with $P \not\equiv 0$, the inequality $\ln|P(\omega)| > -ct(P)^{m+1}$ holds, where $t(P) = \deg P + \ln\{\max|\text{coeff}.P|\}$? In other words, this means, that almost all points $\omega \epsilon \mathbf{R}^m$ have a type of algebraic independence equal to $m + 1$.

This problem is decided in the case $m = 1$. It is easy to prove that for all points ω this type is not less than $m + 1$ and in the general case it is proved that for almost all points $\omega \epsilon \mathbf{R}^m$ this type is not greater than $m + 2$. The p-adic analogue of this problem is proved for $m \leq 2$.

H. NIEDERREITER:

Quantitative inversion of orthogonal transforms and the distribution of values of Kloosterman sums

Let F_q be the finite field of order q, $F_q^* = F_q \backslash \{0\}$, and χ a fixed nontrivial additive character of F_q. Define the Kloosterman sums

$$K(q, a) = \sum_{b \in F_q^*} \chi(b + ab^{-1}) \text{ for } a \in F_q^*.$$

From the classical bound of Weil it follows that the normalized values $\omega(q, a) = K(q, a)/2q^{\frac{1}{2}})$ of the Kloosterman sums lie in $[-1, 1]$. Katz (1988) has shown that if $q \rightarrow \infty$, then the $q - 1$ numbers $\omega(q, a), a \in F_q^*$, have an asympotic distribution given by the measure $\frac{2}{\pi}(1 - t^2)^{1/2}dt$ on $[-1, 1]$. We establish the following quantitative result which describes the distribution of the $\omega(q, a)$ in a more precise manner.

Theorem 1. The discrepancy D_{q-1} of the $q-1$ numbers $\omega(q,a), a \in F_q^*$ with respect to the measure above satisfies $D_{q-1} < 10q^{-1/4}$. The proof is based on results of Katz (1988) on Kloosterman sums and on the following general principle for the quantitative inversion of orthogonal transforms.

Theorem 2. Let $P_0, P_1, \ldots$ be an orthonormal system of polynomials for the continuous weight function $\omega \geq 0$ on $[u,v]$ with $\int_u^v \omega(x)dx = 1$. Then the discrepancy D_N of any numbers $x_1, \ldots, x_N \in [u,v]$ relative to the measure $\omega(t)dt$ on $[u,v]$ satisfies

$$D_N \leq \frac{C_1 B(v-u)}{k} + C_2 \sum_{r=1}^{2k-1} b_r \left|\frac{1}{N}\sum_{n=1}^{N} P_r(x_n)\right|$$

for all $K \in \mathbf{N}$, where $C_1, C_2 > 0$ are effective absolute constants, B is an upper bound for ω on $[u,v]$, and b_r is an upper bound for $|\int_u^x P_r(t)\omega(t)dt|$ on $[u,v]$.

K. NISHIOKA:

Arithmetic properties of the values of Mahler functions

By using Nesterenko's method, we obtain a zero estimate for Mahler functions and an effective algebraic independence measure for the values of Mahler functions at algebraic points. The measure gives examples of fields of transcendence degree m with transcendence type $\leq m+2+\epsilon$. Another application of the zero estimate is the study of algebraic independence of the values of Mahler functions at transcendental numbers.

A. D. POLLINGTON:

The k - dimensional Duffin and Schaeffer conjecture

In 1941 Duffin and Schaeffer considered the case $k=1$ of the assertion that when $\alpha : N \to [0,\infty]$ the statement:

(A_k) For almost all $\theta \in (\mathbf{R}/\mathbf{Z})^k$ there are infinitely many natural numbers q for which there exist integers $a_1, \ldots, a_k$ such that $(a_1, \ldots, a_k, q) = 1$ and

$$\max_{1 \leq i \leq k} ||\theta_i - \frac{a_i}{q}|| < \alpha(q).$$

holds only if and only if

$$(S_k) \qquad \sum_{q=1}^{\infty} (\alpha(q)\phi(q))^k \text{ diverges.}$$

Sprindžuk raised this question for $k > 1$ in his book on Metric Diophantine Approximation. Together with R. C. Vaughan we have shown that the conjecture holds in the case $k > 1$. Some further cases of the $k = 1$ conjecture are also obtained.

A.J. van der POORTEN:

Folded Continued Fractions

The uncountably many power series $F_a(X) = X\sum_{h=0}^{\infty}(-1)^{a_h}X^{-2^h}$, with $a = 0.a_1\ a_2\ a_3...$ a binary decimal , all have continued fractions $[1, (-1)^{f_1}X, (-1)^{f_2}X, (-1)^{f_3}X, ...]$ where

$$(F) \qquad f_1 f_2 f_3 \cdot\ \cdot\ \cdot$$

is a paperfolding sequence: that is (F) is the sequence of creases, say with $V = 1$ and $\Lambda = 0$ of a sheet of paper folded infinitely many times in half and then unfolded (technically, a cluster point of sheets folded finitely many times). In particular it follows that the subsequence $f_1\ f_3\ f_5...$ alternates in value. Specialising to $X = 2$ we obtain the continued fraction expansions

$$[1, (-1)^{f_1}2, (-1)^{f_2}2, ...]$$

for the binary decimals (*) $2\sum(-1)^{a^h}2^{-2^h}$ (recalling the rule $1-2^{-b} = 0.11...1(b\ 1's)$ allows one to write these as proper binary decimals). To rewrite the continued fractions as regular continued fractions - with positive integer partial quotients - one uses the folding property: whereby the expansion is of the shape , say

$$[1, 2, 1, -2, b, 2, c, -2, d, 2, ...]$$

with

$$a, b, c, ... = \pm 2 - \quad \text{and the rule} \quad [x, -y, z] = [x-1, 1, y-2, 1, z-1],$$

obtaining

$$[1, 2, a-1, 1, b-1, 2, c-1, 2, d-1, ...]$$

A further application of the cited rule proves:

Theorem: The uncountably many numbers (*) have continued fractions requiring the partial quotients 1 or 2 only.

[Note: by a Theorem of Mendès France and AvdP, based on a theorem of Loxton and AvdP, all the numbers $F_a(2)$ are transcendental]

This was joint work with Jeff Shallit (Waterloo)

G. RHIN:

On irrationality measures of π^2

We denote by $\mu = \mu(\pi^2)$ an irrationality measure of π^2 i.e for all $\epsilon > 0$, there is an effective constant $c(\epsilon) > 0$ such that for all $(p,q) \epsilon \mathbf{Z}^2$ with $q \neq 0$ we have $|\pi^2 - p/q| > c(\epsilon) q^{-\mu-\epsilon}$.

I give a brief schetch of the two methods that give now good irrationality measures of π^2. The two methods use modified Beukers integrals $\int_0^1 \int_0^1 \frac{H(x,y)}{(1-xy)^{n+1}} dx\, dy$.

The first is due to Masayoshi Hata and gives $\mu(\pi^2) \leq 7.5252$. It uses a property, already proved by Rukhadze, of some Legendre - type polynomials which have coefficients with a fairly large greatest common divisor.

The second method by G. Rhin and C. Viola uses for $H(x,y)$ a product of suitable polynomials in x and y. We use an optimization method involving semi - infinite linear programming to obtain the best choice of exponents. We use computer algebra to get a complete proof which gives $\mu(\pi^2) \leq 7.66976$. Is it possible to improve these results in combining the two methods in order to prove Chudnovsky's claim $\mu(\pi^2) \leq 7.325$?

A. SCHINZEL:

A decomposition of integer vectors

Let us adopt the following notation. κ_m is the volume of the m - dimensional unit ball,

$$g_0(m) = \sup_K \inf_P \frac{vol P}{vol K}, \quad g_1(m) = \sup_K \inf_P \frac{vol P}{vol \mathcal{E}(K)} \cdot \frac{\kappa_m}{2^m},$$

where the suprema are taken over all m - dimensional convex bodies K symmetric with respect to the origin $\mathbf{0}$, the infima are taken over all m- dimensional parallelepipeds P symmetric with respect to $\mathbf{0}$ contained in K, and $\mathcal{E}(K)$ is the ellipsoid of maximal volume contained in K. For linearly independent vectors $\mathbf{n}_1, \ldots, \mathbf{n}_m \in \mathbf{Z}^k$, $H(\mathbf{n}_1, \ldots, \mathbf{n}_m)$ is the maximum of the asbolute values of the minors of order m of the $m \times k$ matrix with rows $\mathbf{n}_1, \ldots, \mathbf{n}_m$, respectively and $D(\mathbf{n}_1, \ldots, \mathbf{n}_m)$ is the greatest common divisor of the minors. Further, $h(n)$ is the maximum of the absolute values of the coordinates of the vector n. For all triples of positive integers k, l, m with $k \geq l \geq m, k > m$, we define

$$C_0(k,l,m) = \sup \inf \left(\frac{D\,(\mathbf{n}_1, \ldots, \mathbf{n}_m)}{H\,(\mathbf{n}_1, \ldots, \mathbf{n}_m)} \right)^{(k-l)/(k-m)} \prod_{j=1}^{l} h(\mathbf{p}_j),$$

where the supremum is taken over all sets of m linearly independent vectors $\mathbf{n}_1, \ldots \mathbf{n}_m \in \mathbf{Z}^k$, and the infimum is taken over all sets of L linearly independent vectors $\mathbf{p}_1, \ldots, \mathbf{p}_l \in \mathbf{Z}^k$ satisfying $\mathbf{n}_i = \sum_{j=1}^{l} u_{ij} \mathbf{p}_j$ for certain $u_{ij} \in \mathbf{Q} (1 \leq i \leq m, 1 \leq j \leq l)$. The following estimate holds:

Theorem:

$$C_0(k,l,m) \leq \min\{(l-m+1)^{l/2} g_1(m) \gamma_l^{l/2}, \frac{l!}{m!} g_0(m), \binom{l}{m}^{\frac{1}{2}} l^{(l-m)/2} g_1(l) \gamma_l^{l/2}\},$$

where γ_l is the Hermite constant.

H.P. SCHLICKEWEI:

Multiplicities of linear recurrences

Let K be a number field of degree d. Let S be a subset of the set of places $M(K)$ of cardinality s containing the set of archimedean places $M_\infty(K)$. Using my p-adic generalization of the quantitative version of Schmidt's subspace theorem, I prove:

Theorem 1: Suppose $a_1, \ldots, a_{n+1} \in K^*$. The number of projective solutions of the equation

$$a_1 \; x_1 + \ldots + a_{n+1} \; x_{n+1} = 0$$

such that no proper subsum $a_i \; x_i + \ldots + a_{i_k} x_{i_k}$ vanishes and such that the x_i are all S - units, is bounded by

$$(4sd!)^{2^{36nd! s^6}}.$$

This bound is in particular independent of $a_1, \ldots, a_{n+1}$. As another application of the quantitative subspace theorem we prove : let $u_{m+n} = \nu_{n-1} u u_{m+n-1} + \ldots + \nu_o u_m$ be a linear recurrence sequence. Write $Z^n - \nu_{n-1} Z^{n-1} - \ldots - \nu_1 Z - \nu_0 = \prod_{i=1}^{r} (Z - \alpha_i)$. Assume that K contains $\nu_{n-1}, \ldots, \nu_0, \alpha_1, \ldots, \alpha_r, \ldots, u_0, \ldots, u_{n-1}$ and denote by ω the number of prime ideals occuring in the factorization of the ideals (α_i) in K.

Theorem 2: Suppose that the sequence (u_m) is non- degenerate. Then the number of solutions of the equation $u_m = 0$ is bounded by

$$(4(\omega + d)d!)^{2^{40n!d!}(\omega+d)^6}$$

The method of proof of Theorem 2 extends to multivariable polynomial - exponential equations. Using a completely different approach via Straßmann's theorem it was shown in joint work with A. van der Poorten that the number of solutions of $u_m = 0$ is bounded by $(4(\omega + d))^{2(d+1)}(n-1)$. However, it seems that this purely p-adic method can not be generalized to the multivariable case.

W.M. SCHMIDT:

On Vojta's refinement of the Subspace Theorem

Let S be a finite set of properly normalized absolute values of $\mathbf{Q}$ containing the archimedean absolute value. For each $p \in S$, let $L_{p1}, ..., L_{p,t(p)}$ be nonzero linear forms in n variables with coefficients in an algebraic number field K.
For $\mathbf{x} \in \mathbf{Q}^n \setminus \{0\}$, set

$$\lambda_\pi(\mathbf{x}) = |L_\pi(\mathbf{x})|_p / |\mathbf{x}|_p,$$

where $|\mathbf{x}|_p$ is the maximum norm and where $|.|_p$ has been suitably extended to K.
Put

$$\Pi(\mathbf{x}) = \prod_{p \in S} \prod_{i=1}^{t(p)} \lambda_{pi}(\mathbf{x}).$$

Both $\Pi(\mathbf{x})$ and the height $H(\mathbf{x})$ are defined on projective space $\mathbf{P}^{n-1}(\mathbf{Q})$. The Subspace Topology of $\mathbf{P}^{n-1}\mathbf{Q}$ has as its closed sets any finite union of subspaces. The Subspace Theorem says that when $L_{p1}, ..., L_{pt(p)}$ are linearly independent for each $P \in S$, then for every $\epsilon > 0$, the set of solutions of
(x) $\Pi(\mathbf{x}) < H(\mathbf{x})^{-n-\epsilon}$ is not dense in $\mathbf{P}^{n-1}(\mathbf{Q})$.

In the general case, let r be the supremum of the numbers ρ such that the solutions of $\Pi(\mathbf{x}) < H(\mathbf{x})^{-\rho}$ are dense in $\mathbf{P}^{n-1}(\mathbf{Q})$.

Theorem: r is a rational number with numerator and denominator bounded in terms of n and the numbers $t(p) (p \in S)$.

As an easy consequence, we obtain Vojta's refinement of the Subspace Theorem that the solutions of (x) lie in finitely many proper subspaces *independent* of ϵ, except for finitely many solutions which may depend on ϵ.

A.B. SHIDLOVSKY:

On the transcendence and algebraic independence of the values of E - function at algebraic points.

The report is about the latest result of the author on the algebraic independence of the values of certain E - function at algebraic points, if these E - functions belong to a set of E - functions which satisfies a system of linear differential equations with coefficients from $\mathbf{C}(z)$ and which is algebraically dependent over $\mathbf{C}(z)$.

I. SHIOKAWA:

A class of normal numbers

Let $r \geq 2$ be a fixed integer and let $\theta = 0.a_1a_2... = a_1r^{-1} + a_2r^{-2} + ...$ be a real number expanded in the base r. For any integer $l \geq 1$ we put

$$R_n(\theta) = R_{n,l}(\theta) = \sup_{b_1,...,b_l \in\{0,1...,r-1\}^l} \left|\frac{N(\theta, b_1...b_l; n)}{n} - r^{-l}\right|,$$

where $n(\theta; b_1...b_l; n)$ denotes the number of occurrences of the block $b_1...b_l$ in the first n digits $a_1...a_n$. Then θ is said to be normal to base r, if $R_n(\theta) = o(1)$ for any given l. Let R be the set of all nonconstant functions $g(x)$ of the form

$$g(x) = \alpha X^{\beta} + \alpha_1 X^{\beta_1} + ... + \alpha_d X^{\beta_d}, \alpha_i \in R, \beta > \beta_1 > ... \geq \beta_d \geq 0,$$

with $g(x) > 0$ for $x > 0$. For each $g(x) \in R$, we define the number $\theta_r(g)$ by the infinite r - adic fraction

$$\theta_r(g) = 0.[g(1)][g(2)]...,$$

where each $[g(n)] = a_{n1}r^{k(n)-1}+...+a_{nk(n)}$ is replaced by the string $a_{n1}a_{n2}...a_{nk(n)}$ of r-adic digits $0, 1, ..., r-1$.

Theorem: For every $g(x) \in \mathbf{R}$ and every integer $l \geq 1$, we have $R_n(\theta_r(g)) = O(1/\log n)$.

Corollary: The number $\theta_r = [\alpha][\alpha 2^{\beta}][\alpha 3^{\beta}]...$ is normal to base r for every $\alpha > 0$ and $\beta > 0$.

Remark: The error term $O(1/\log n)$ cannot be replaced by $o(1/\log n)$.

This is joint work with Y. -N. Nakai: A class of normal numbers I, II, III. (Japanese Journal of Mathematics (1990), London MS. LN 154)

T.N. SHOREY

Some exponential diophantine equations

This is a joint work with N. Saradha. Let a, b be positive integers. We consider the equation

(1) $$a(x+1)...(x+k) = b(y+1)...(y+k+l)$$

in integers $x \geq 0, y \geq 0, k \geq 2, l \geq 0$ with $k + 1 \geq 3$. Erdős (1975) conjectured that eq, (1) has only finitely many solutions in integers x, y, k, l as above with

$x \geq y + l + k$. For several given values for $a,, b, k, l$ the complete resolution of (1) in integers $x \geq 0, y \geq 0$ is known. For instance Mordell proved, that $x = 1, y = 0$ and $x = 13, y = 4$ are the only solutions of (1) with $a = b = 1, k = 2, l = 1$. Avanesov (1967), Tzanalis and de Weger (1989), Boyd and Kisilevsky (1972), Cohn (1971) and Ponnudurai (1975) solved eq. (1) in integers x, y for several other tuples a, b, k, l. In 1970, McLeod and Barrodale proved that $x = 7, y = 0, k = 3$ is the only solution of (1) with $a = b = 1, l = k$ and $k \leq 5$. Further, for a given k they showed that there are only finitely many x, y satisfying (1) $a = b = 1$ and $l = k$. We proved

Theorem 1: Eq. (1) with $a = b = 1$ and $l = k$ has only one solution in integers $x \geq 0, y \geq 0, k \geq 2$ and it is given by $x = 7, y = 0, k = 3$.

Relative to Erdős' conjecture, we proved the following:

Theorem 2: Assume (1) with $x \geq y + l + k$. Then:

(i) $x \gg_{a,b} k^3(\log k)^{-4}$; (ii) $(\log x)^2 \gg_{a,b,\epsilon} k$ if $y < (1-\epsilon)x$; (iii) $x - y \gg_{a,b} x^{2/3}$; (iv) $\max(x, y, k, l)$ is bounded by an effectively computable number depending only on a, b and the greatest prime factor of xy.

In the proof of (iv) the theory of linear forms in logarithms is used.

J.H. SILVERMAN:

A canonical height for $K3$ surfaces.

Let $S \subset \mathbf{P}^2 \times \mathbf{P}^2$ be the smooth $K3$ surface described by equations $a_{ij}\ X_i\ Y_j = \sum b_{ijkl}\ X_i\ X_j\ Y_k\ Y_l = 0$. The projections $\pi_1, \pi_2 : \mathbf{S} \to \mathbf{P}^2$ have degree 2, so induce involutions $\sigma_1, \sigma_2 \in$ Aut (S). The subgroup $A =:< \sigma_1, \sigma_2 >\subset$ Aut S is $A \cong \mathbf{Z_2} * \mathbf{Z_2}$. If S is defined over a number field K, we define two canonical heights $h^+, h^- : S(\bar{k}) \to [0, \infty]$ satisfying :

(1). $\hat{h}^+ + \hat{h}^- = (\alpha - 1)(\hat{h} \circ \pi_1 + \hat{h} \circ \pi_2) + O(1)$

(2). $\hat{h}^\pm \circ \sigma_1 = \alpha^{\mp 1}\hat{h}^\pm, \hat{h}^\pm \circ \sigma_2 = \alpha^{\mp 1}\hat{h}^\mp$. (Here $\alpha = 2 + \sqrt{3}$).

Theorem: $\hat{h}^+(P) = 0 \Leftrightarrow \hat{h}^-(P) = 0 \Leftrightarrow \{\varphi P : \varphi \epsilon A\}$ is finite.

Corollary: Let $S_{\text{tors}} = \{P \in S(\bar{K}) : \{\varphi P : \varphi \epsilon A\}$ is finite$\}$.
Then $\{P \epsilon S_{fin} : [K(P) : K] \leq d\}$ is a finite set. More precisely, S_{fin} is a set of bounded height.

We also suggest a $K3$- analogue of Lehmer's conjecture.

C.L. STEWART:

On Thue equations with many primitive solutions

We discussed the following results. First, let f be a polynomial with integer coefficients, degree $r \geq 2$, content 1, and non-zero discriminant D. Let p be a prime and k a positive integer. Then the number of solutions modulo p^k of $f(x) \equiv 0(p^k)$ is at most $2p^{[l/2]} + r - 2$, where $l = \text{ord}_p(D)$. Secondly, put $F(x,y) = xy(x+y)$. We proved, in connection with a conjecture made at the last conference, that for infinitely many integers h, the Thue equation $F(x,y) = h$ has 18 solutions (x,y) with x and y coprime.

R. TICHY:

Some remarks to Erdős-Turán's inequality

Using a new approximation kernel due to J. Vaaler, the constant in the k - dimensional Erdős -Turán-Koksma inequality can be improved:

$$D_N(x_n) \leq C_{k,M}\left(\frac{1}{M+1} + \sum_{0<||h||\leq M} \frac{1}{R(h)} \left|\sum_{n=1}^{N} e^{2\pi i<h,x_n>}\right|\right),$$

where D_N is the discrepancy, $M \in \mathbb{N}, R(h) = \prod_{j=1}^{k} \max(1,|h_j|)$, and $C_{k,M} \leq M((1+\frac{1}{M})^k - 1)$. This improvement is independently due to Cochrane and Grabner and Tichy. Furthermore, an analogue for the spherical cap discrepency is established. Applications to the metric theory of uniform distribution and to random-number tests are given.

R. TIJDEMAN

On the number of solutions of unit equations

Erdős, Stewart and I have shown that certain upper bounds of Evertse and Silverman for the number of solutions of S - unit equations in two variables are not far from being the best possible.

In the lecture it will be indicated how it can be shown that upper bounds of

Schlickewei and others for the number of solutions of S - unit equations in any number of variables cannot be improved too much.

T. TÖPFER:

Simultaneous approximation and algebraic independence in Mahler's method.

Let $f : U_1(0) \to \mathbf{C}$ be a holomorphic function which is transcendental over $\mathbf{C}(z)$ and satisfies a functional equation $f(z) = a(z)f(z^p) + b(z)$, where $a(z)$ and $b(z)$ are polynomials with algebraic coefficients and $p \geq 2$ is an integer. Under several conditions the inequality

$$\log\max(|\alpha - \xi_1|, |f(\alpha) - \xi_2|) \geq$$
$$\geq -cd_2\log(d_1H_1)(d_1 + d_2\log H_1)\bullet$$
$$\bullet[(d_1 + d_2\log H_2)\log(d_1 + d_2\log H_2) + \frac{d_1\log H_2}{d_1 + d_2\log H_1}]$$

with a constant $C \in \mathbf{R}$ holds for $\alpha \in U_1(0), \xi_1, \xi_2 \in \bar{\mathbf{Q}}$ with $\deg\xi_i \leq d_i, H(\xi_i) \leq H_i\ (\geq 2)$ for $i = 1, 2$.

From this simultaneous approximation measure, we can deduce the algebraic independence of α and $f(\alpha)$ for some α which can be well approximated, e.g. the Liouville number

$$\alpha = \sum_{k=1}^{\infty} \gamma_k g^{-4^{4^k}}$$

with $g \in \mathbf{N}, g \geq 2, \gamma_k \in \{0, ..., g-1\}$ and $\gamma_k \neq 0$ for infinitely many $k \in \mathbf{N}$.

R. TUBBS:

Some transcendencies associated with some U - numbers

A classical line of study in transcendental number theory has involved investigations into the arithmetic of numbers of the form α^β, where $\alpha \in \bar{\mathbf{Q}}\backslash\{0,1\}$ and $\beta \in \bar{\mathbf{Q}}\backslash\mathbf{Q}$. Such a number is trancendental, and for β of degree $d > 1$ at least $[\frac{1}{2}(d+1)]$ of the numbers $\alpha^\beta, \alpha^{\beta^2}, ..., \alpha^{\beta^{d-1}}$ are known to be algebraically independent. We present here work on the arithmetic of these values for a large class of nonalgebraic α.

$\alpha \in \mathbf{C}$ is said to be $(d_0, f(T))$ - approximable if there exists $d_0 \in \mathbf{N}$ and a real valued function $f(T)$ such that there exists a sequence of algebraic numbers (a_T), deg $a_T \leq d_0$ and $ht(a_T) \leq e^T$ such that $|\alpha - a_T| < \exp(-f(T))$. For example, then the following holds:

Theorem (D. Caveny, 1989) Suppose β has degree $d \geq 3$. Then there exists a function $f(T)$ such that if α is $(d_0, f(T))$ - approximable then at least $[\frac{1}{2}(d+3)]$ of $\alpha, \alpha^\beta, ..., \alpha^{\beta^{d-1}}$ are algebraically independent.

G. TURNWALD:

On the nonvanishing of homogeneous product sums

M. Ward conjectured that the sum $H_n(a,b,c) \neq 0$ of all terms $a^i b^i c^i$ with $i+j+k = n$ never vanishes for nonzero integers a, b, c if $n > 1$. We supplement a recent investigation by T.M. Apostol by showing that $H_n(a,b,c) \neq 0$ for odd integers a, b, c; the proof is based on a general result concerning the behaviour of linear recurring sequences considered modulo prime powers. Furthermore, we point out a fatal error in Ward's purported proof of the conjecture in the case where $n+2$ is a prime.

J. D. VAALER

Effective measure of irrationality for algebraic numbers

(joint work with E. Bombieri and A. J. van der Poorten)

Let $\mathbf{k}$ be a number field, ν a fixed place for $\mathbf{K}$ and $\mathbf{K}$ a finite extension of $\mathbf{k}$ which has an embedding in the completion $\mathbf{k}_\nu$ of $\mathbf{K}$ at ν. We may identify $\mathbf{K}$ with its image in $\mathbf{k}_\nu$ - then $\alpha \in \mathbf{k}_\nu$ generates $\mathbf{K}$ over $\mathbf{k}$. We may ask how well $\beta \in \mathbf{k}$ may approximate α. In this setting, the Liouville lower bound is

$$(2h(\alpha)h(\beta))^{-r} \leq |\alpha - \beta|_\nu$$

where $[\mathbf{K} : \mathbf{k}] = r \geq 2$. We say that $\mu = \mu(\mathbf{K}, \mathbf{k}, \nu)$ is an effective exponent of irrationality for the triple $(\mathbf{K}, \mathbf{k}, \nu)$ if *for every generator* $\alpha \in \mathbf{k}_\nu$ of $\mathbf{K}$ over $\mathbf{k}$ we can effectively determine a constant $c(\alpha) > 0$ so that

$$c(\alpha)h(\beta)^{-\mu} \leq |\alpha - \beta|_\nu$$

holds for all $\beta \in \mathbf{k}$. This is nontrivial if $2 \leq \mu < r$. Our objective is the construction of such triples with $r = 3$ and μ quite close to 2. Toward this end we have:

Theorem : Suppose $\alpha \in \mathbf{k}_\nu$ generates the cubic extension $\mathbf{K} = \mathbf{K}(\alpha)$ and the minimal polynomial of α over $\mathbf{k}$ is

$$x^3 + px + q \in \mathbf{k}[x], \quad p \neq 0.$$

If $160 \leq \log h(p^3/q^2)$ and $(2/3)\log h(p^3/q^2) \leq \log^+ |(p^3/q^2)|_\nu$ then

$$\mu = \frac{2\log h(p^3/q^2)}{\log^+ |p^3/q^2|_\nu}\left(1 + \frac{10.75}{(\log h(p^3/q^2))^{1/3}}\right)$$

is an effective measure of irrationality for $(\mathbf{K}, \mathbf{k}, \nu)$.

K. VÄÄNÄNEN

On irrationality measures of certain numbers

Let $F(z) = {}_2F_1(1, b; c; z)$ denote the hypergeometric series with rational $b = G/L$ and $c = J/M$ satisfying $b, c, c - b \neq 0, -1, \ldots$ By p we denote a prime number satisyfing $p|LM$. In a joint work with Kuukasjärvi and Matala-aho we prove the following theorem on the approximation of the p-adic values of F by rational numbers: Let $\theta = r/s \neq 0$ and $\gamma = R/S$ be rational numbers and denote $\max\{|R|, S\} = H$. Suppose that $|\theta|_p < p^u$, where u is a positive number such that $p^{2^u} > C = 2^5 e^2 \mu_L L(\mu_M M)^2, \mu_n = \prod_{p|n} p^{1/(p-1)}$. For all $H \geq H_0$ we then have

$$|F(\theta) - \gamma| > \Omega H^{-\omega},$$

where $\omega = 2u\log p(2u\log p - \log C)^{-1}, \Omega = (3^\omega C^2)^{-1}$, and $H_0 = H_0(b, c, p, u)$ is an explicitly given positive constant. This result is a generalization of an earlier result of Bundschuh concerning the case $c = 1$.

M. WALDSCHMIDT:

Lower bounds for liner forms in logarithms

So far, Baker's method was the only one which enabled one to produce lower bounds for linear combinations of logarithms of algebraic numbers, at least when there are more than two terms and effectivity is required. We develop an extension to several variables of the method introduced by Schneider in his solution of Hilbert's seventh problem which we combine with some ideas of Noriko Hirata in her work on commutative algebraic groups, and this enables us to propose new methods which also yield explicit estimates of good quality. An important tool is, as usual, Philippon's zero estimate on algebraic groups. We also quote a result by D. Roy which gives a lower bound for the rank of matrices whose entries are linear combinations with algebraic coefficients of logarithms of algebraic numbers. Here again, the proof involves a generalization of Schneider's method to several variables.

R. WALLISER:

Über die Bestimmung aller imaginärquadratischen Zahlkörper der Klassenzahl 2

Es wird über Rechnungen berichtet, die Herr Ch.Wagner durchgeführt hat. Baker und Stark haben Ihre untere Abschätzungen für Linearformen in Logarithmen algebraischer Zahlen angewandt um eine Schranke für imaginärquadratische Zahlkörper der Klassenzahl 2 zu finden.Nimmt man die neueren unteren Abschätzungen von Mignotte und Waldschmidt, so kann man diese Schranken sehr verbessern. Im Falle $-d < 0$, d gerade, wird hier $d < 3 \cdot 10^{18}$ gezeigt Stark $d < 10^{440}$; im Falle ungerader d wird $d < 10^{144}$ früher $d < 10^{1030}$ gezeigt. Die Formel von Goldfeld-Oesterlé

$$h(-d) > \frac{1}{55} \prod_{p|d}^{*} \quad (1 - [2\sqrt{p}]/(p+1)). \log d.$$

liefert im Falle $h(-d) = 2$ dieselbe Größenordnung. Für gerades d kann man mit Hilfe von Kettenbruchentwicklungen schnell alle traglichen Diskriminanten finden. Im Falle d ungerade muß man auf die Methode von Stark (Math. of Comp. 29 (1975)) zurückgreifen. Auch hier können wegen der allgemeinen Schranke $d < 10^{144}$ die Rechnungen stark abgekürzt werden.

J. YU

Applications of zero estimates in finite characteristics

Using zero estimates we can obtain the following analogue for Drinfeld A- modules of the well-known theorem of Wüstholz:

Theorem 1: Let $G = (G_a^n, \varphi)$ be a Drinfeld A- module/ $\bar{k}$. Let $u \in$ Lie $G(\bar{k}_\infty)$ such that $\exp_G(u) \in G(\bar{k})$. Then the smallest $d\varphi(A)$ - invariant vector subspace of Lie G defined over $\bar{k}$ that contains u is the tangent space at the origin of a Drinfeld A - submodule of G.

From the structure of higher Carlitz modules, and by the above results one obtains:

Theorem 2: $\dim_{\bar{k}}\{1, \tilde{\pi}, \ldots, \tilde{\pi}^n, \varsigma_c(1) \ldots, \varsigma_c(n)\} = n + 1 + \#\{i : 1 \leq i \leq n, i \not\equiv 0 (\mathrm{mod}\ q-1)\}$. Here $\varsigma_c(n) = \sum_{\substack{a \in \mathbf{F}_q[t] \\ \text{monic}}} (1/a^n) \in \mathbf{F}_q((t))$.

G. Anderson has obtained the following for higher Carlitz $C^{\otimes n}$:

Theorem 3: Let $u = (\ldots, l) \in$ Lie $C^{\otimes n}$ such that for $\exp_n u \in C^{\otimes n}(\bar{k})$. Then for every integer $n' > 0$ one can always find $u' = (\ldots, l\tilde{\pi}^{n'}) \in$ Lie $C^{\otimes(n+n')}$ such that

$\exp_{n+n'} u' \in C^{\otimes(n+n')}(\bar{k})$.

Combining this with the analogue of Wüstholz's theorem, we get:

Theorem 4: If $n \not\equiv 0 (\bmod\ q-1)$, then $\varsigma_c(n)$ and $\tilde{\pi}$ are algebraically independent over $\bar{k}$.

In the above, $k = \mathbf{F}_q(\lambda), A = \mathbf{F}_q[t]$.

Probleme:

A. GALOCHKIN:

1. A problem of E.U. NIKISHIN . Let $J_0(z)$ be Bessel's function, and let α_1, α_2 be two numbers with $J_0(\alpha_1) = J_0(\alpha_2) = 0, \alpha_1 \neq 0, \alpha_2 \neq 0, \alpha_1 \neq \pm\alpha_2$.
Prove that $\alpha_2/\alpha_1 \notin \mathbf{Q}$

2. Let λ be an algebraic number of degree $k \geq 2; \lambda^{(1)}, ..., \lambda^{(k)}$ the conjugates of $\lambda, \mathbf{K} = \mathbf{Q}(\lambda), \mathbf{K}^{(j)} = \mathbf{Q}(\lambda^{(j)})$ for $j = 1, ..., k$. Prove that for all $\omega \in \mathbf{K}, \omega \neq 0$, there is a j with $1 \leq j \leq k$ such that

$$\varphi_\lambda^{(j)}(\omega^{(j)}) \notin \mathbf{K}^{(j)} = \mathbf{Q}(\lambda^{(j)})$$

where

$$\varphi_\lambda(z) = \sum_{n=0}^{\infty} \frac{z^n}{(\lambda+1)...(\lambda+n)}.$$

This is a simpler problem than to prove that $\varphi_\lambda(\omega) \notin \bar{\mathbf{Q}}$. For $k = 2$ and $k = 3$ some partial cases were proved.

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