MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 44/1990

Geometrie

14.10. bis 20.10.1990

Die Tagung fand unter der Leitung von V. Bangert (Freiburg) und U. Pinkall (Berlin) statt. In 25 Vorträgen berichteten Teilnehmer über ihre neuesten Forschungsresultate. In den nachstehenden Vortragsauszügen spiegeln sich die Vielfalt des Tagungsthemas "Geometrie" und dessen enge Beziehungen zu andern Teilen der Mathematik. Schwerpunkte des Vortragsprogramms lagen in der Geometrie der symmetrischen Räume, der Riemannschen Geometrie, der Variationsrechnung und der Theorie der Untermannigfaltigkeiten. Die Zeit zwischen den Vorträgen und die Abende boten lebhaft genutzte Möglichkeiten zu mathematischen Gesprächen und Diskussionen.

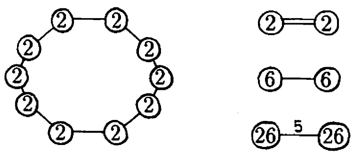
Vortragsauszüge

U. Abresch (joint work with V. Schroeder):
End structures of manifolds of negative curvature

Consider a complete Riemannian manifold (M^n, g) with $\mathrm{vol}_{M^n} < \infty$. By a result of P. Eberlein, the hypothesis $-1 \leq K_{M^n} \leq -\epsilon^2 < 0$ implies that the ends of M^n are diffeomorphic to $W^{n-1} \times \mathbf{R}_+$ where W^{n-1} is an infranilmanifold. Typical examples are the maps in $\mathbf{C}H^1/\Gamma$, where W^{2n-1} is the quotient of a Heisenberg group. The weaker hypothesis $-1 \leq K_{M^n} \leq 0$ holds for many more space, e.g. the symmetric spaces of rank ≥ 2. There are also further possibilities for the ends. A simple, but nevertheless typical example is $M^4 = \Sigma_{2,1} \times \Sigma_{2,1}$, where $\Sigma_{2,1}$ denotes a Riemann surface of genus 2 with 1 puncture and $K \equiv -1$; in this example there is just one end, which is modelled on the graph manifold $W^3 := (\Sigma_2 \setminus D^2) \times S^1 \cup S^1 \times (\Sigma^2 \setminus D^2)$. The third curvature hypothesis to be mentioned here is $-1 \leq K_{M^n} < 0$. It is strong enough for a rich structure theory (due to Gromov) based on the Margulis Lemma and the idea of using the injectivity radius as a Morse function.

The new result presented in this lecture is that the hypothesis $-1 \leq K_{M^n} < 0$ also allows for lots of 4-manifolds (M^4, g) of finite volume, whose ends are modelled on graph manifolds W^3 rather than infranilmanifolds. In particular, $W^3 := (\Sigma_2 \setminus D^2) \times S^1 \cup S^1 \times (\Sigma^2 \setminus D^2)$ describes the end of such an (M^4, g). Note that this manifold cannot be diffeomorphic to $\Sigma_{2,1} \times \Sigma_{2,1}$ because of rigidity theorems. There is a long rich list of similar new examples. Let us mention just a few of them.

Theorem 1: The 4 graphs listed below and all the subgraphs obtained by deleting some vertices and all the adjacent edges describe graph manifolds W^3 s.th. each occurs as the end of a complete manifold (M^4, g) with finite volume and $-1 \leq K_{M^4} < 0$.



In fact, the basic existence result is much more general:

Theorem 2: Given $n \geq 4$ and $2 \leq k \leq \frac{n}{2}$, there exist countably many compact hyperbolic manifolds H^n/Γ' together with a finite family of compact, totally geodesic, codimension 2 submanifolds $\overline{V}_i^{n-2} \subset H^n$, $1 \leq i \leq N$, which intersect pairwise orthogonally in a set of codimension 4 and such that $k = \max\{\#$ sheets through one point $p_0\}$. On the open set $M^n := (H^n/\Gamma') \setminus \bigcup_{i=1}^{N} \overline{V}_i^{n-2}$ the hyperbolic metric g_0 can be deformed into a complete metric g with finite volume and $-b^2 \leq K_g < 0$.

The latter theorem is obtained upon combining basic facts about arithmetic groups with detailed curvature calculations for a specific metric deformation, which is in fact a stretching into the directions $\mathrm{grad\,dist}(., \overline{V}_i^{n-2})$. The concrete examples listed in

Theorem 1 are related to the hyperbolic 120-cell space: the tesselation of H^4 induced by the Coxeter group Γ with diagramm $\bullet\overset{5}{—}\bullet—\bullet—\bullet\overset{5}{—}\bullet$ defines a tiling of H^4 into solid 120-cells, each of which consists of 14400 fundamental orthoschemes for the action of Γ. It has been shown by Davis in 1984 that such a solid 120-cell in H^4 is itself the fundamental domain for the action of a torsionfree, normal subgroup $\Gamma' \triangleleft \Gamma$, thereby getting a quotient manifold H^4/Γ' similar to the hyperbolic dodecrahedral space in dimension 3. For the detailed calculations in the group Γ, which are required in the proof of Theorem 1, it is easiest to think of Γ as an index 2 subgroup in $PO(E,g)$ where $E = \mathbb{Z}[\frac{1}{2}(1+\sqrt{5}]^5$ and g is the Coxeter matrix associated to $\bullet\overset{5}{—}\bullet—\bullet—\bullet\overset{5}{—}\bullet$; then Γ' can be defined as the intersection of Γ with $\ker\big(PO(E,g) \to PO(E_{F_5},\overline{g}) \to PO(E_{F_5}/\operatorname{rad}(\overline{g}),\overline{\overline{g}})\big)$. Using these techniques, one finds 2 totally geodesic surfaces of genus 2 in H^4, which intersect orthogonally in precisely 2 points and allow for an automorphism of order 5 each. This explains the case ②═② of Theorem 1. The other cases are obtained using appropriate coverings.

J. Berndt:
Nearly symmetric manifolds

Let R denote the curvature tensor of a Riemannian manifold M. Associated with each geodesic γ in M is the self-adjoint Jacobi-operator $R_\gamma := R(.,\dot{\gamma})\dot{\gamma}$. We are concerned with the problem: "What Riemannian manifolds are characterized by the property that for any geodesic the eigenspaces of the associated Jacobi operator are invariant with respect to parallel translation, or in other words, that the Jacobi fields can be calculated by solving appropriate scalar Jacobi equations?" In these manifolds the geometry of tubes around "curvature-adapted" submanifolds develops easily from the geometry of the latter one. Examples of such manifolds are locally symmetric spaces and 2-dimensional manifolds. In the talk we outline a local classification for such 3-dimensional manifolds which is valid almost everywhere. It turns out that in 3 dimensions the problem is related to the question whether one can obtain solutions of the Helmholtz equation (a special kind of the Schrödinger equation) by separation of variables.

U. Brehm:
Some geometric constructions in the complex projective plane

There is a canonical bijection between bisectors and geodesics. The intersection of any two bisectors in general position is an embedded smooth torus which is contained in exactly three bisectors.

Each general geodesic triangle Δ is contained in a unique such torus $T(\Delta)$. $T(\Delta)$ is the set of points having the same distance from the three points of intersection of pairs of altitudes of Δ. $T(\Delta) = T(\Delta^p)$, where Δ^p denotes the polar triangle. $T(\Delta)$ contains exactly six geodesics. These induce a tiling of the torus $T(\Delta)$ by two triangles, nine quadrangles and one right-angled hexagon.

$T(\Delta) = \{p \in \mathbf{C}P^2 \mid$ there are geodesics connecting p and the three vertices of Δ, having coplanar tangent vectors at $p\}$.

We give also another characterization of $T(\Delta)$. We classify the classes of general geodesic triangles which are equivalent under collineations. If Δ_1 and Δ_2 are equivalent then there exists a collineation mapping $T(\Delta_1)$ onto $T(\Delta_2)$.

Similar results hold also in the complex hyperbolic plane.

Y. Burago:

A. D. Alexandrov's length spaces with curvature bounded from below

In the talk there were presented the results of two papers, the first by M. Gromov, G. Perelman and myself and the second by Perelman only. In those papers the theory of finite dimensional spaces of curvature bounded from below (FSCBB) is constructed.

In particular it is proved that for every $\epsilon > 0$ such a space M^n ($n = \dim M$) contains an open dense set X_ϵ which is a topological manifold and that every point $p \in X_\epsilon$ has a nbh which is ϵ-almost isometric to a ball in $\mathbf{R}^n$. At every point $p \in M^n$ the tangent cone exists and is a cone under $(n-1)$-dimensional space with curvature ≥ 1. Toponogov comparison theorem holds for FSCBB.

The strongest result which belongs to Perelman, is the following: 1) Any point of FSCBB M^n has a nbh isometric to the tangent cone of M^n at this point. 2) M may be stratified into topological manifolds. 3) For any FSCBB M there exists a constant $\nu = \nu(M) > 0$ such that if FSCBB M' has the same dimension and the same lower bound of curvature as M, and if Hausdorff-Gromov distance $d(M, M') < \nu$, then M' is homeomorphic to M. (In 1) - 3) it is assumed, that boundary of M^n is empty).

W. Degen:

Can there be an infinite number of conics on an algebraic surface of order $n \geq 4$ without singularities?

In 1865, E. Kummer gave a classification of algebraic surfaces F_4 with at least one one-parametric family of conics on it; supplements to this catalogue were made by C. M. Jessop, 1916, and W. F. Meyer 1921-1934. But it seemed not to be clear whether if this classification, which contained only F_4 with singular points, was complete, nor if there would exist a one-parametric family of conics on a non-singular F_4. Using more advanced techniques of algebraic geometry, a student of mine, S. Oehms, gave the answers to both questions: 1) Up to some special cases (which he added) the list is complete. 2) There are only a finite number of conics on a regular F_4 in $P^3(\mathbf{C})$.

Combining methods of algebraic geometry with those of differential geometry (moving frames) the last result can be generalized to algebraic surfaces F_n of any order $n \geq 4$.

B. V. Dekster:

Width and constant width of a manifold

A set C in a Riemannian manifold M^n is called definitely convex if (i) C is compact and each two points of C can be connected in C by a rectifiable curve; (ii) any shortest path in C between two points of C is a geodesic; (iii) any geodesic in C is the unique shortest path in C between its ends; (iv) each geodesic segment in C contains no pair of conjugate points along this segment.

Let K be a definitely convex set in M^n homeomorphic to a ball. It is called a body of constant width $W > 0$ if for any $p \in \partial K$ and any exterior unit normal n of ∂K at p there exists the geodesic pq of length W having direction $-n$ such that $K \supset pq$ but K contains no longer geodesic $pq' \supset pq$.

We prove that $\operatorname{diam} K = W$ and inradius plus circumradius equals W as it is in $\mathbf{R}^n$. We introduce also a width of K as function of its normal and prove that K is of constant width if and only if this newly introduced width is constant. Some other properties of constant width are extended to Riemannian case.

P. Dombrowski:

Some mechanical problems in spaces of constant curvature

R. Thom formulated in one of his lectures (in ~1958) his conviction, that the quality of "laws of nature" should not depend on the precise value of certain "constants of nature", but should be stable under small perturbations of these constants. – In this talk this question was discussed for certain mechanical problems which were treated in the 3-dim. 1-connected, complete spaces M^3_κ of constant curvature $\kappa \in \mathbf{R}$ according to results of J. Zitterbarth (Köln) in his doctoral thesis [which partially rely on resp. extend results of H. Liebmann (1902) on the planetary motion]. Emphasis was put on the dependence of the results on the curvature constant κ, the "limit" $\kappa \to 0$ should imply the Newtonian case in $M^3_0 = E^3$. – A gravitational potential for M^3_κ had been already proposed by J. Bolyai (about 1848-1851). For the corresponding planetary motion main qualitative results are e.g.: Every maximally defined orbit $c : J \to M^3_\kappa$ of a "planet" attracted by the "sun" is plane, i.e. is contained in a totally geodesic M^2_κ of M^3_κ through the sun. Total energy E and angular momentum L (i.e. its scalar value) are constant, Kepler's second law holds in an extended version (first found for M^3_{-1} by Liebmann 1905). Orbits with angular momentum zero ($L = 0$) are radial, i.e. $c(J)$ is contained in a geodesic line through the sun. Orbits with $L \neq 0$ are bounded iff they are periodic. All orbits with $L \neq 0$ (even the unbounded) are connected components of "conics" in the sense, that $c(J)$, lying in M^2_κ (see above), is the set of points in M^2_κ

which have the same distance from the sun F (= focus) and a complete oriented curve C in M^2_κ of constant (signed) curvature, where F lies in the "positive half-plane" of C in M^2_κ. The durance $T(a,\kappa)$ of a "planetary year" for a periodic orbit of great half-axis $a \in \mathbf{R}_+$ depends (for fixed $a \in \mathbf{R}_+$) monotonicly decreasing on κ. The amount of change of $T(a,\kappa)$ was estimated for our earth. – Finally a result on spherically symmetric gyroscopes moving without external forces in M^3_κ was mentioned, which gives a mechanical interpretation of Levi-Civita's parallelism in M^3_κ.

W. Henke:

Vollständige Untermannigfaltigkeiten in einem geodätischen Tubus und ein verallgemeinertes Maximumprinzip

Bericht über die Dissertation von Ulrich Gauß (Köln). Sei $\varphi : M \to N$ eine isometrische Immersion einer vollständigen Riemannschen Mannigfaltigkeit M in eine Riemannsche Mannigfaltigkeit N, deren Bild $\varphi(M)$ enthalten ist in einem geodätischen Tubus $\tau(P,\lambda)$ vom Radius λ um eine reguläre Untermannigfaltigkeit P von N. ($\tau(P,\lambda)$ ist diffeomorphes Bild der abgeschlossenen λ-Umgebung des Nullschnittes des Normalenbündels unter der normalen Exponentialabbildung.) Dann lassen sich in sehr allgemeinen Situationen Mindestwerte jeweils für das Supremum der mittleren Krümmung H^φ und der Schnittkrümmung K_M angeben. In Standardraum-Situationen gilt in den Abschätzungen Gleichheit. Spezialfälle dieser Resultate sind bekannt und stammen von Jorge, Koutroufiotis, Hasanis, Kitagawa u.a. Wesentlich für den Beweis ist eine Verallgemeinerung des Satzes von Omori (für nicht notwendig nach oben beschränkte Funktionen auf Riemannschen Mannigfaltigkeiten, deren Schnittkrümmung nach unten nicht beschränkt zu sein braucht).

H.-C. Im Hof:

The generalized pentagramma mirificum

The pentagramma mirificum of spherical trigonometry and its generalization to higher dimensional spherical geometry is presented. Then its proper translation into hyperbolic geometry will be discussed. The study of Napier cycles in $\mathbf{R}^{n,1}$ will show that the hyperbolic counterpart of the pentagramma mirificum is more complicated. We will exhibit three pentagramma figures in H^2.

Under a suitable crystallographic condition Napier cycles in $\mathbf{R}^{n,1}$ generate hyperbolic Coxeter groups.

M. Katz:

The filling radius, the Kuratowski embedding, and diameter-extremal sets in CP^2

A new Riemannian invariant called the filling radius was introduced by M. Gromov in 1983 to prove the isosystolic inequality conjectured by M. Berger. The filling radius is defined relative to the Kuratowski embedding of V in $L^\infty(V)$. The computation of the filling radius makes use of Serre spectral sequences and the Schubert calculus, and requires the knowledge of the homotopy type of tubular neighborhoods of $V \subset L^\infty(V)$. This homotopy type is determined by a Morse theoretic argument. The "Morse function" in question is the diameter functional on the set of subsets of V. A study of the extrema of this functional in the case of the projective plane makes use of P. A. Shirokov's theorem of cosines in complex projective trigonometry.

R. Kellerhals:

On polylogarithms and volumes of hyperbolic polytopes

It is a very difficult problem to calculate volumes of hyperbolic polytopes of higher dimensions. Restricting the problem to the case of 3-dimensional orthoschemes, Lobachevsky derived an explicit formula in terms of the Lobachevsky function $\Pi(\alpha) = -\int_0^\alpha \log|2\sin t|dt$, $\alpha \in \mathbf{R}$, which depends on the dihedral angles. Since every polyhedron can be represented by orthoschemes (after cutting and pasting), the 3-dimensional volume problem is solved in terms of Euler's Dilogarithm $Li_2(z) = \sum_{m=1}^\infty \frac{z^m}{m^2}$ related to $\Pi(\alpha)$ by $Li_2(e^{2i\alpha}) = 2\Pi(\alpha)$.

Considering the volume problem for higher dimensional polytopes, one remarks a fundamental difference between even and odd dimensions. We present Schläfli's Reduction principle to show how volumes of even-dimensional orthoschemes can be expressed in terms of the volumes of certain lower (odd) dimensional ones. Finally, we talk about the philosophy treating the volume problem for asymptotic orthoschemes in $\overline{H^{2n+1}}$, $n \geq 2$, and relate the ideas to number- and K-theoretical aspects of the (expected) volume functions $Li_n(z) = \sum_{m=1}^\infty \frac{z^m}{m^n}$, the polylogarithms.

P. Kohlmann:

A generalization of the Liebmann/Süss theorem and a splitting theorem for convex sets

In view of the Liebmann/Süss theorem we ask, whether an ovaloid in $\mathbf{R}^{n+1}$ with one mean curvature H_l close to 1 for one $1 \leq l \leq n$ is also close to a unit ball with respect to the Hausdorff metric. We give an affirmative answer to this question in the context of the curvature measures developed by Federer.

For $b > a > 0$ let $\tilde{M}^{(n,l)}_{(a,b)} := \{C \subset \mathbf{R}^{n+1} \mid \partial C, \mathrm{int}\, C \neq \emptyset, C$ closed, convex, $a\Phi_n(C,.) \leq \Phi_{n-l}(C,.) \leq b\Phi_n(C,.)\}$, $\tilde{M}^{(n,l)}_\delta := \tilde{M}^{(n,l)}_{(1-\delta,1+\delta)}$ for $1 > \delta > 0$ and $M^{(n,l)}_\delta := \tilde{M}^{(n,l)}_\delta \cap \{C$ compact$\}$ and Φ_r the curvature measures normalized so that for Borel sets B $\Phi_r(C,B) = \int_{B\cap\partial C} H_{n-r} d\partial C$. Then the following theorems hold.

Theorem 1: For $\epsilon > 0$ ex. $\delta > 0$ so that all $C \in M^{(n,l)}_\delta$ contain a ball of radius $1-\epsilon$ and are contained in a ball of radius $1+\epsilon$. For $0 < \delta < (n^{n/(n-1)} - n - 1)/(n^{n/(n-1)} + n + 1) =: \xi_n$ exists an upper bound for the diameters of the elements of $M^{(n,l)}_\delta$.

The case $l = n$ and with additional restrictions also $l = 1$ have been treated by Koutroufiotis, Moore, Diskant and Schneider.

Theorem 2 (splitting theorem): There exists $\delta > 0$ so that all $C \in \tilde{M}^{(n,l)}_\delta$ are congruent to $\mathbf{R}^i \times C'$ with $0 \leq i \leq n-l$ and C' is a compact convex body. For $l = n-1$ the δ can be chosen as any positive number smaller ξ_n.

Theorem 3: Let $C \in \tilde{M}^{(n,l)}_{(a,b)}$ for some $b > a > 0$. Then C is either a compact convex body or a multiple of the closure of the projection of C on some suitable hyperplane lies in $\tilde{M}^{(n-1,l)}_{(a,b)}$ and the dimension of the recession cone of C is not greater $n-l$.

The proofs of the theorems make use of generalized symmetrization processes and generalized Minkowski integral formulas for compact convex bodies.

W. Kühnel (joint work with T. Banchoff):
Equilibrium triangulations of CP^2

Starting with the 7-vertex triangulation of the ordinary solid torus we construct a 10-vertex triangulation of CP^2 which fits the equilibrium decomposition into three 4-balls around $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ in the simplest possible way. By suitable positions of the vertices the full automorphism group of order 42 is realized by a discrete group of isometries in the Fubini-Study metric. A slight subdivision leads to an elementary proof for the theorem of Kuiper-Massey saying that CP^2 modulo conjugation is PL homeomorphic to the standard 4-sphere. The branch locus in this case is a fairly simple 7-vertex triangulation of $\mathbf{R}P^2$. In the quotient with 13 vertices there is an opposite 6-vertex $\mathbf{R}P^2$, and these two are PL analogues of the two opposite Veronese surfaces in S^4.

N. H. Kuiper:
On the gradient flow near a critical point

Vector fields in the plane can have integral curves $\kappa(t)$ spiraling to a singular point $0 \in \mathbf{R}^2$, so that $\kappa(t)$ has no tangent at $\kappa(\infty) = 0$, and the celestial trace $\omega(t) = \kappa(t)/\|\kappa(t)\|$ has as limit set $\Omega \subset S^1$, the unitsphere, which is all of S^1 and not one point. Now let $f : \mathbf{R}^n \to \mathbf{R}$ be an analytic "height" function with development

$$f = \sum_{l \geq k} \overline{H_l}(\kappa), \quad \kappa = (\kappa_1, \ldots, \kappa_n), \quad H_l = \overline{H_l} | S^{n-1},$$

$\overline{H_l}$ homogeneous of degree l. Let $\kappa(t)$ be a curve of steepest descent to $\kappa(\infty) = 0 \in \mathbf{R}^n$, solution of $-\dot{\kappa} = \operatorname{grad} f$ $(-d\kappa_i/dt = \partial f / \partial \kappa_i)$. R. Thom conjectures that then $\kappa(t)$ has a tangent at 0, that is the trace has as limitset Ω one point in S^{n-1}.

It was shown that trace $\omega(t)$ must converge to one component Σ of the algebraic critical set $\{\omega \in S^{n-1} : \operatorname{grad} H_k(\omega) = 0\} \subset S^{n-1}$. Then it can be shown that Ω is one point if $n = 2$ (known), and for $n = 3$ if $H_k(\Omega) = c > 0$. For $n = 3$ and $H_k(\Omega) = 0$ the trace $\omega(t)$ can show essential trace speeds in S^2; near to Σ, along Σ. They are however locally guided by (decrease of) a potential function on Σ. This should prevent Ω from being different from a point, but I have no complete proof yet.

U. Lang:
Quasiminimalflächen im hyperbolischen Raum

Sei M eine 2-dim., vollständige, einfach zusammenhängende Riemannsche Mannigfaltigkeit. Eine Kurve $c : \mathbf{R} \to M$ heisst quasigeodätisch mit Konstante Q, wenn für je zwei Punkte p, q von c die Länge des Kurvenstückes zwischen p und q nicht grösser als das Q-fache der Länge des kürzesten Weges von p nach q in M ist. Hat M Krümmung $K \leq -a^2 < 0$, so zeigte Morse (1924), dass ein quasigeodätisches Segment mit Endpunkten auf einer Geodätischen stets in einer Tubenumgebung dieser Geodätischen mit beschränktem Abstand liegt, wobei die Abstandsschranke nur von a und Q abhängt.

Analog lassen sich Quasiminimalflächen (genauer: homotop quasiminimierende Flächen) in 3-dim. Riem. Mannigfaltigkeiten (M, g) definieren. Ist z.B. g' eine zu g Lipschitz-äquivalente Metrik auf M, so ist eine minimale Untermannigfaltigkeit von (M, g) bezüglich g' noch quasiminimal. Wir beweisen eine zum Resultat von Morse analoge Aussage für Quasiminimalflächen im 3-dim. hyperbolischen Raum und beantworten damit eine aktuelle Frage von M. Gromov.

E. Leuzinger:
The generalized Laws of Sines and Cosines for symmetric spaces

For a Riemannian symmetric space S of non-compact type we determine the space of invariants for the relative position of two points and that of an ordered pair of Weyl chambers, respectively. Points in these spaces are called intervals resp. angles. We use them to attach to a congruence class of marked, regular geodesic triangles six geometric quantities, three sides and three angles. This six quantities cannot be arbitrary, they must satisfy a "closing condition". We obtain this fundamental relation by following the path traced by a Weyl chamber, when it is moved along a geodesic triangle.

It turns out that a congruence class is essentially determined by two sides and two angles. By trigonometry of the space S we mean a (minimal) set of functional relations by means of which the third side and the two remaining angles can be computed.

We obtain the generalized laws of cosines by first embedding S into its group of isometries and then by applying invariant theory to the fundamental relation.

The laws of sines in spaces of constant curvature (and also those of rank 1) can be deduced from an integral of the geodesic flow. This is generalized to the construction of integrals for the Weyl chamber flow. From there we in turn can derive the generalized laws of sines.

B. Opozda:

Locally symmetric affine connections on hypersurfaces in $\mathbf{R}^{n+1}$

Let $f : M \to \mathbf{R}^{n+1}$ be a non-degenerate hypersurface endowed with the Blaschke structure. Assume that the Blaschke connection (i.e. defined by the affine Gauß formula) is locally symmetric. If $n > 2$, then it is known (Verheyen-Verstraelen 1985) that f is an improper affine sphere or $f(M)$ lies on a quadric. The proof given by V-V is mainly algebraic. If $n = 2$ and M is compact, then $f(M)$ must be an ellipsoid. It can be proved by using, for instance, the Riemann-Roch theorem. For the remaining case of arbitrary surfaces the methods mentioned above do not work. By using conormals we can prove:

Theorem: Let $f : M \to \mathbf{R}^3$ be a non-degenerate (connected, oriented) surface affinely locally symmetric. If the affine shape operator has rank 2, then f is affinely equivalent to a constantly curved semi-Riemannian surface with respect to some standard scalar product on $\mathbf{R}^3$.

The problem of describing hypersurfaces in $\mathbf{R}^{n+1}$ with locally symmetric affine connections can be formulated and solved in case of affine connections induced by an arbitrary (not necessarily the affine normal) transversal vector field.

B. Palmer:

Stability in Geometric Variational Problems

We discuss the stability of a Willmore surface $M \subset S^3$. For such a surface it is known that the image of it's conformal Gauß map is a weak minimal surface $\tilde{M} \subset S^4_1$ where S^4_1 denotes the unit sphere in 5-dimensional Minkowski space. For a nowhere umbilic surface the second variation of the Willmore functional may be computed from the second variation of area for $\tilde{M}$ with the variation fields constrained to satisfy an appropriate P.D.E. involving the Jacobi operator of $\tilde{M}$. This is then used to derive sufficient condition for the instability of certain Willmore tori.

F. Pedit (joint work with D. Ferus, U. Pinkall):
Pluri-harmonic tori in symmetric spaces

We address the problem to find all pluri-harmonic maps $f : \mathbf{C}^r/\Gamma \to N$ of a complex torus $\mathbf{C}^r/\Gamma$ into a symmetric space $N = G/K$. This approach subsumes previous work (for $r = 1$ and special N) on tori of constant mean curvature, tori of constant negative Gauß curvature, minimal tori and harmonic tori in Lie groups. We show that the pluri-harmonic maps of $\mathbf{C}^r/\Gamma$ are in 1-1 correspondence with special flat connections $A : T\mathbf{C}^r \to \tilde{g}$, A Γ-periodic, in the symmetric loop algebra

$$\tilde{g} = \{X : S^1 \to g | X = \sum_{k \in \mathbf{Z}} X_k \lambda^k,\ X_{\text{even}} \in k,\ X_{\text{odd}} \in p\},$$

where $g = k \oplus p$ is the symmetric decomposition of $N = G/K$. The connection A is of the form $A = \lambda^{-1}\overline{A}_1 + A_0 + \lambda A_1$. We then show that flat connections of above type are gotten as solutions to a complete integrable finite dimensional ODE-system on $\tilde{g}$ in lax-pair form. Furthermore, under suitable non-degeneracy conditions on A_1, every such flat connection is obtained from this ODE-system. This gives a classification of all pluri-harmonic maps $f : \mathbf{C}^r/\Gamma \to N$ of a complex torus into a symmetric space with a certain non-degeneracy condition on ∂f.

H.-B. Rademacher:
Generalized Killingspinors and conformal Killingfields

A non trivial spinor Ψ on a complete Riemannian spin manifold is called a generalized Killingspinor with imaginary Killing function if

$$(*) \qquad \nabla_X \Psi = ibX \cdot \Psi$$

for a real valued function b and for all vectorfields X. Baum classified all manifolds admitting a solution of (*) with $b =$ const (imaginary Killingspinor), then Lichnerowicz studied the above equation. Extending his results we classify all manifolds admitting a solution of (*). In the proof we use that the vectorfield V defined by $\langle V, X \rangle := i\langle \Psi, X \cdot \Psi \rangle$ for all X is a non-isometric conformal closed Killing field. The classification of the manifolds with such Killing field follows from results by Brinkmann, Tashiro, Bourguignon, Kerbrat and Kühnel.

K.-D. Semmler:
Fenchel-Nielsen parameters and $z * w = t$

While Fenchel-Nielsen parameters (gluing together pairs of pants build up from hyperbolic hexagons) give a geometrically satisfying picture of the deformation or Teichmüller space (T-space) of a Riemann surface, they do not reflect any of the complex structure of this space. So Teichmüller and others introduced "natural" complex parameters (from the conformal point of view) on this space which, unfortunately, can not be inerpreted in hyperbolic geometry terms. The latest advances in complex parameters of T-space is the $z * w = t$ construction first mentioned (to my knowledge) by Earle and Marden around 80. In the Helsinki conference of 87 Earle and Marden and especially Wolpert gave lectures about this construction and by this interested many people. The most comprehensive paper has been given by Kra in 89.

Take a neighborhood V of a closed geodesic of a Riemann surface S. For simplicity assume that this geodesic divides S into two open sets U and W. Let $z = z(p)$ and $w = w(p)$ be two conformal charts mapping V to a neighborhood of the unit circle, such that $p = q$ iff $z(p) * w(q) = 1$. Now a one complex parameter family of conformal structures can be defined on the sets $S_t = (U \cup V) \uplus (W \cup V)/ \approx$ where $p \approx q$ iff $z(p)$ and $w(q)$ are defined and $z(p) * w(q) = t$. Obviously $S_1 = S$ serves as an origin for this transformation. The mapping $\log t \mapsto S_t$ is a complex parameter of T-space defined on a neighborhood of $1 \in \mathbf{C}$. Kra and Maskit have a similar construction out of thrice puntured spheres which yield global parameters with an origin on the boundary of T-space.

When we uniformize S on the upper halfplane such that our geodesic is covered by the imaginary axis we see that moving t on the unit circle corresponds to the Fenchel-Nielsen twist. For real t the deformation is not easily desribed. In the simple case of the marked cylinder we see that it corresponds to a length deformation of the girth geodesic. In general we have to use the Riemann mapping theorem to reuniformize. Thus we enter the Kleinian Group setting and Kra has interpreted these parameters as building Kleinian groups via Maskits combination theorems. But there the hyperbolic geometry stays mysterious.

I. Taimanov:

Periodic extremals of many-valued functionals

We consider (possibly many-valued) functionals which in local coordinates x^j_α have the form

$$l^{(\alpha)}(\gamma) = \int_\gamma \left(\sqrt{g_{ij}(x)\dot{x}^i_\alpha \dot{x}^j_\alpha} + A^{(\alpha)}_i(x)\dot{x}^i_\alpha\right) dt.$$

Here the expression $F_{ij} = \partial_i A^{(\alpha)}_j - \partial_j A^{(\alpha)}_i$ is uniquelly defined on the whole M^2 as a closed but possibly non-exact 2-form; $\gamma \subset U_\alpha \subset M^2$ (x^j_α are the local coordinates on U_α and $A^{(\alpha)}_i dx^i_\alpha$ is a 1-form defined on U_α).

Using the spaces of films, we prove: if F is exact and l is not-everywhere-positive, or if F is non-exact and analogue of not-everywhere-positivity condition holds, then exists a closed non-selfintersecting extremal of l with index $= 0$.

In the case, then M^2 is a 2-sphere and F is exact, this theorem was proved by Novikov and author by means of different method.

E. Teufel:

A generalization of the isoperimetric inequality in the hyperbolic plane

T. F. Banchoff, W. F. Pohl, J. L. Weiner generalized the classical isoperimetric inequality to non-simple closed curves in the euclidean or the spherical plane. We give the isoperimetric inequality for non-simple closed curves in the hyperbolic plane H:

$$L^2 \geq 4\pi \int_H w_f^2(p)\, dH_p + \int_{H\times H} m_f(p,q)\, dH_p \wedge dH_q;$$

equality holds only for geodesic circles transversed in the same direction a number of times. $w_f(p)$ denotes the winding number of the curve f w.r.t. p; and $m_f(p,q)$ is a certain homology invariant of the curve f in $H \setminus \{p,q\}$. Furthermore we discuss in this spirit the isoperimetric inequality for non-simple closed curves on arbitrarily curved surfaces.

L. Vanhecke:

A theorem of Archimedes and two-point homogeneous spaces

As is well known, Archimedes showed that the ratio of the volumes (or the ratios of the areas) of a sphere and a circumscribing cylinder is constant in E^3. Of course, the same is true in E^n.

On a Riemannian manifold one may consider geodesic spheres, balls, disks and circumscribing tubes and determine the associated ratios of the volumes.

We prove that the property given above characterizes locally Euclidean space and we derive similar theorems to charaterize the other two-point homogeneous spaces.

Finally, we derive a general formula for the volume of a (circumscribing) tube in a two-point homogeneous space and use it to generalize a result of Weyl concerning the volume of tubes. In fact we prove that the volume of a tube of radius r about a curve in a two-point homogeneous space only depends on the radius r and the length of the curve.

K. Voss:

Umbilics on Willmore surfaces

An immersion $x : M^2 \to \mathbf{R}^3$ defines a Willmore surface, if $\delta \int (H^2 - K) dA = 0$ where $H = \frac{1}{2}(k_1 + k_2)$, $K = k_1 k_2$; $k_1, k_2 =$ principal curvatures, $dA =$ area element, or – equivalently – if $\Delta H + 2H(H^2 - K) = 0$. Using isothermic coordinates $z = u + iv$ for x and the family Y of central spheres of x (conformal Gauß map), a differential $\Omega = \langle Y_{zz}, Y_{zz} \rangle dz^4$ can be defined, which – in case of Willmore surfaces – is holomorphic.

For a "general" family of spheres the zeros of Ω are umbilics of one of the enveloped surfaces $x, \hat{x} : M^2 \to \mathbf{R}^3$. But this is not true for the family of central spheres. To clear the situation, the following properties of Willmore surfaces are indicated: 1) There exist surfaces with umbilical lines, e.g. Willmore cylinders or certain surfaces of revolution. According to a theorem of Thomsen (1923) there exist conformal transformations of the space, sending the mentioned surfaces (but only there non umbilical parts) to hyperbolic minimal surfaces. 2) In general, zeros of Ω are not umbilics of x, 3) An umbilic of x is a zero of Ω iff $\Phi_z H_z = 0$, where $\Phi_z dz^2$ is the Hopf differential of x. 4) The index of an isolated umbilic on a Willmore surface does not take the value $j = 1$, but all other values $j = \frac{1}{2}, 0, -\frac{1}{2}, -1, \ldots$

Berichterstatter: U. Lang

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