MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematical Economics

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This was the fifth meeting on Mathematical Economics in Oberwolfach. The conference concentrated on the following topics:

1. Decision Theory
2. Experimental Economics
3. Financial Economics
4. Incomplete Markets

In preparing the conference we invited the leading experts in each of these fields who in our view had contributed most to the recent achievements. Since most of them accepted, the conference was a great success.

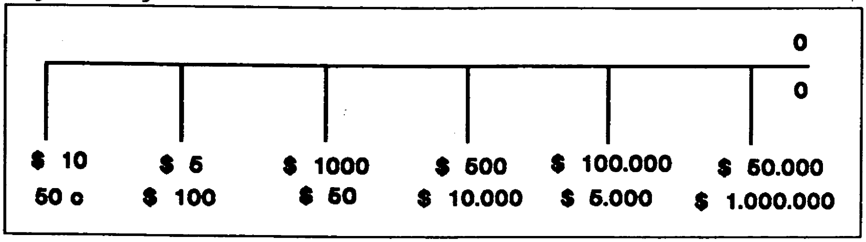
This report follows the organisation of the program which devoted one full day to each of the above topics. Each field was introduced by a carefully prepared survey on the state of the art. We thank all participants for their inspiring contributions and discussions. Special thanks are due to Robert Aumann, Charles Plott, Vernon Smith, Stan Pliska, David Heath and Wayne Shafer for their excellent survey presentations.

Once again the warm hospitality provided by the Mathematisches Forschungsinstitut combined with the excellent facilities created a stimulating atmosphere which was appreciated by all participants.

MONDAY, 10-29-1990: DECISION THEORY

R. Aumann: Irrationality In Game Theory

In the "centipede" games of R. Rosenthal, all equilibria prescribe that the first player "leaves" the game at the first opportunity, thus foregoing opportunities for large payoffs to both players. It is pointed out that it is possible for the first player to stay in, and in fact for both players to stay in for a considerable sums of money, while still being perfectly rational and even maintaining fairly high levels of mutual rationality. At the same time, the "expected irrationality" in systems of this kind can be tiny. For example, in Megiddo's game



Player 1 can "stay in" although the "expected irrationality" may be as small as 3 promil of one cent.

Edi Karni: Subjective Expected Utility Theory With State-Dependent Preferences

This work extends Savage's subjective expected utility theory to include state-dependent preferences, (i.e. the possibility that the decision-maker's preferences over consequences are not independent of the prevailing state of nature). The dependence of the decision maker's preferences over consequences on the states of nature is represented by state-specific mappings of the set of consequences, X, onto itself. Thus, for each state of nature, p, we have ρ_1: X $\rightarrow$ X onto. Savage's postulates are reformulated within this framework and a generalization of Savage's theorem is obtained. In particular, it is shown that there exist subjective expected utility representations of the preference relation on acts, (i.e. functions on the set of states into the set of consequences) with a unique, nonatomic, finitely additive probability measure on the algebra of all subsets of states, and a state-dependent utility function on the set of consequences. The utility functions are bounded and unique up to positive linear transformations.

David Schmeidler (joint work with Sergiu Hart and Salvatore Modica) :
A Neo^2 Bayesian Foundation Of The Maxmin Value For Two-Person Zero-Sum Games

Using a decision theoretic approach we obtain a joint derivation of utitility and value in the theory of two-person zero-sum games: Acts map states to consequences that are lotteries over prices. Preferences between acts are complete, transitive, continuous, monotonic and satisfy the axiom of certainty-independence. The set of states is a product of two finite sets (m rows and n columns), and a new axiom describing the attitude toward the substitution of risk for uncertainty is satisfied by the preferences. The new axiom can also be described as representing the decision maker's attitude toward restrictions on strategic flexibility. The axioms imply that the consequences can be represented by a von Neumann-Morgenstern utility so that each act is represented by a m x n matrix, i.e., a two-person zero-sum game. Moreover, the preferences between acts are represented by the (maxmin) values of the corresponding games. An alternative statement of the result shows that preferences between conditional acts are represented by the values, for all finite two-person zero-sum games. (This is a joint work with Sergiu Hart and Salvatore Modica.)

Clemens Puppe: Distorted Probabilities And Choice Under Risk

Rank-Dependent Utility theory is a class of models of transitive preferences over probability distributions where the representation of preferences is based upon a (generalized) utility function defined on the outcome/probability - plane. Several functional forms of this (generalized) utility function have been proposed: Expected utility theory corresponds to <u>linearity</u> in the probability; anticipated utility involves a (uniform) transformation of the probability; here the assumption of <u>homogeneity</u> in the probability is examined.
It turns out that this assumption (together with stochastic dominance) captures an <u>optimistic</u> point of view towards gambling while being compatible with <u>risk aversion</u>. Furthermore, this assumption is consistent with well-known empirical patterns like the behaviour exhibited in the Allais-Paradox and the common ration effect.

<u>TUESDAY, 10-30-1990: EXPERIMENTAL ECONOMICS</u>

Charles R. Plott: Experiments With Market Stability And With General Equilibrium Economies

Questions have been posed concerning the nature of the stability of equilibria in a single market with multiple equilibria. Additional and fundamental questions exist about the ability of the general competetive equilibrium model to capture the behavior of economies. The report here consisted of two separate parts, each of which addressed these two classes of question.
A set of experiments was designed to test the (walrasian) theory of price adjustment ($dP/dt = F(D(p)-S(p))$ against the (marshallian) theory ($dQ/dt = Q(P_D(Q)-P_S(Q))$. If $S'(p)<0$ the two theories give opposite predictions about the nature of the stability of equilibria. The experiments show that:

(1) the concept of stability itself is appropriate for markets,

(2) the appropriate stability concept is dependent upon factors other than market organizations and the slopes of demand and supply, in particular the Marshallian concept is appropriate when the downward slope of supply is due to an externality. The Walrasian concept is appropriate if the negative slope is due to "backward binding" of individual supplies.

The second set of results leads to the following conclusions:

(1) It is possible to create and observe the operation (in a laboratory) of an economy with the essential features of the general equilibrium model. The technology exists. Such economies function.
(2) The price ratios, and the allocation approach the static competitive equilibrium of the natural model of the economy.
(3) Nominal prices are effected directly by the quantity of money in the system.
(4) Real variables were not affected by monetary variables.
(5) Adjustments were smooth and without cycles or other phenomena that might suggest an inability of the decentralized markets to coordinate the system. These economies used the double auction process. Related experiments suggest that the Tatonnement process will work.

Vernon L. Smith: Stock Market Bubbles In The Laboratory

Inexperienced subjects invariably exhibit price bubbles relative to fundamental value in an environment in which all investors receive the same dividend from a known (iid) dividend distribution at the end of each of 15 trading periods. When traders are experienced the bubble is reduced; when the same group returns for a third market, prices fluctuate around the period-by-period decline in dividend value. Changes in mean prices are explained largely by the equation

$$\bar{P}_t - \bar{P}_{t-1} = \alpha + \beta(\beta_{t-1} - \theta_{t-1}), \quad t = 2, \ldots, 15$$

where $\alpha = -E(d)$, the one-period expected dividend value, $\beta > 0$, and lagged net bids (number of bids minus number of offers) is a surrogate measure of excess demand arising from homegrown capital gain (losses) expectations. Thus phenomenon can be interpreted as a form of temporary myopia from which agents learn that capital gains expectations are only temporarily sustainable, ultimately inducing common expectations. These results are not significantly modified by the introduction of short selling, margin buying, transaction fees, equal endowments, and dividend uncertainty. Bubbles are, however, significantly dampened by informed insiders (who are given excess bids information and are knowledgeable about these experimental markets), and by the introductions of a futures contract on period 8 (midhorizon) which helps to give trades common expectations at midhorizon. It appears that bubbles arise because of agent uncertainty about others' behaviour. A futures market helps to resolve this uncertainty for a future period.

Finally Changing price limit rules (constraints) make bubbles worse; probably because the floor on how much price can decline gives subjects the perception that downside risk is reduced so that bubbles carry for then before the crash occurs.

Reinhard Selten: Duopoly Strategies Programmed By Experienced Players

In field studies and game playing experiments one can observe plays but not strategies. However, one can ask experienced players to program strategies. This "strategy method" was applied to a 20-period supergame of a Cournot-duopoly with linear cost and demand. The subjects were participants in a student seminar. They first gained experience in three game playing round. Then they programmed strategies which were matched in a computer tournament. In a second and a third round of strategy programming strategies would be revised in the light of earlier tournament results.

The evaluation of the final strategies reveals a typical approach to the strategic problem. First an "ideal point" - a pair of outputs for both players - is chosen as a cooperative goal. The choice is based on some fairness criterion like equal profits or equal additional profits above Cournot profits. The strategy programmers then face the problem how to achieve cooperation at his own ideal point.

Typically different output determination rules were used in three phases of the supergame, an initial phase of up to 4 periods, a main phase and an end phase of up to 4 periods. A fixed sequence of outputs decreasing in the direction of one's own ideal output is specified for the initial phase. In the main phase a "measure for measure policy" is used. A policy of this kind which responds to movements of the opponent toward or away from one's own ideal cooperation is abolished in the end phase for which non-cooperative outputs are specified, Cournot-outputs in most cases.

The structure of a typical strategy can be described by 13 characteristics. A measure of "typicality" based on these characteristics can be applied to the strategies. A strategy tends to be the more successful in the final tournament, the more typical it is (1% significance).

The structure of a typical strategy suggests a new duopoly theory fundamentally different from existing ones. No expectations on the opponent's outputs are formed and no attempt is made to optimize against his conjectured behavior. Instead of this a measure for measure policy is used in order to induce the opponent to cooperate at one's own ideal point.

John Ledyard: Experiments In General Equilibrium And Mechanism Design

In a general equilibrium environment with preferences suggested by Gale, there is a unique competitive equilibrium, which is locally unstable under the Walrasian Tatonnement. In our experiment prices and trades were accomplished with a double oral auction (DOA). We found that prices and quantities

converged to the competitive equilibrium. The conclusion is that Walrasian dynamics do not describe the forces driving prices in the DOA.
In mechanism design, I presented a problem arising from work with NASA to design ways to price and allocate resources on space shuttles. The environment has a unique Pareto-optimum but many non-convexities, similar to those in a Koopmans-Beckman Transformation problem. Standard mechanisms such as administrative procedures (first come - first served) on markets (DOA) do not perform well in this environment - yielding at best around only 10% efficiencies. The reason for bad markets performance is that there is no competitive equilibrium. Two new specially designed mechanisms, generalizing English and second price auctions to multidimensional knapsack problems, are shown to be about 90% efficient in this environment. This illustrates the ability to design practical mechanisms to solve difficult allocation problems using experimental methods to test bad alternate designs, in much the same way an aeronautical engineer uses a wind tunnel.

<u>WEDNESDAY, 10-31-1990: TOPICS IN MATHEMATICAL ECONOMICS</u>

Victor Polterovich (joint work with Gennady Henkin): Some Evolutionary Equations Of Innovation Imitation Processes

We study the evolution of efficiency distribution in an industry with many firms involved in the processes of creation and imitation of more profitable production methods. Let $F_n(t)$ be the share of firms which have an efficiency level n or less at time t, and the variable n is an integer. The following infinite system of nonlinear difference - differential equations is a natural model of the evolution

$$(1) \qquad dF_n/dt = \rho(F_n)(F_{n-1}-F_n), \quad n=0,1,\ldots$$

where $\rho(F_n)$ is the speed of the process on the level n. If the imitation process prevails then ρ decreases, and any solution of equation (1) with natural initial conditions converges to some "standard" distributions function. The behaviour is more complex for non-monotonic ρ. We discuss some hypotheses for this case including the many waves phenomenon, as well as some related models.

Martin Beckmann: Income And Expenditure In A Spatial Economy

We study the generation of income through the rational diffusion of expenditures. Income y(x) at location x gives rise to an expenditure $c \cdot y(x) \cdot e^{-m|r-x|}$ at location r. Here "interaction decreases exponentially with distance", a favorite assumption in Regional Science. We consider a closed one-dimensional region $-R \le x \le R$. Under what conditions is there an equilibrium of incomes and expenditures? What is its spatial structure? Mathematically we solve the Fredholm integral equation

$$e^{-x}\int_{-R}^{x} e^{r}y(r)\,dr + e^{x}\int_{x}^{R} e^{-r}y(r)\,dr = \lambda y(r) \qquad \text{with} \qquad y(-R) = y(R) = 1$$

Time and a second spatial dimension are also considered. Income decrease away from the center according to $y(x)=a \cos wx$ where $w = (\theta+(\pi/2))/R$ and θ is determined by $\theta+R+g\theta=-\pi/2$.

Michael Maschler: Playing Non-Cooperative Games With Changing Utilities

These are games in extensive form in which agents of a player may have different utility functions. One application is that during playing the game utilities change. Another application is when a "player" is actually a group of individuals - each having a different utility function. We assume that the game has perfect recall.
The purpose is to generalize the concept of Nash equilibrium for this situation. Playing Nash in the agent-form is not good enough, because Nash protects only against a deviation by one player, whereas here several agents of the same player may cooperate to their advantage. We introduce the concept of "credible deviation" of agents of the same player and define "a credible equilibrium" to be an n-tuple of strategies such that no credible deviation exist.
This set is not empty, because it contains all agent-perfect equilibria (i.e. trembling-hand perfect equilibria in the sense of Selten).
If all utilities of the agents of a player are the same, and this is true for each player, then s is credible if it is a Nash equilibrium. Other results were also presented as well as a few examples. It was pointed out that it is highly desirable to generalize these concepts to games with imperfect recall.

Birgit Grodal: Externalities And Walras Equilibria With Coordination

In a Walras Equilibrium with coordination each consumer takes as given the Walrasian market with a price system p, and thereby the individual budget-constraint; but when choosing the consumption plan on the market, he can coordinate his decision with other consumers in accordance with the given coordination functions. The traditional Walras equilibria are the special case, where each consumer only coordinates with himself i.e. takes the consumption plan of all the other consumers as given.
The existence of a Walras equilibrium with coordination in a pure exchange economy with externalities and exogeneously given coordination functions is ensured under standard assumptions on the economy.
We especially consider a pure exchange economy with externalities in which consumers have preferences on the states of the economy, which can be represented by a utility function of the form

$$\tilde{U}_i:\prod_{i=1}^{m} X_i \to \mathbb{R}, \quad \tilde{U}_i\ (x_1,\ldots,x_m) = U_i(v_i(x_1),\ldots,v_m(x_m))$$

Also, the coordination functions in the economy are special in that the consumers coordinate in subgroups (which are not

necessarily disjoint).
In this economy we prove that a Walras equilibrium is a Walras equilibrium with coordination, for any coordination structure. If consumers are benevolent then for any coordination structure the two sets of equilibria coincide.
We also, in the general case, discuss whether coordination will improve the state of the economy compared to Walras equilibria. It is easy to construct examples where the Walras equilibria with coordination are Pareto inferior to the Walras equilibria without coordination. The intuition behind this is parallel to the examples given by O. Hart (75), where introduction of more market possibilities give rise to Pareto inferior equilibria due to the effect on equilibrium prices.

Karl Vind: Probability, Uncertainty, And Statistics

Let (X,Q,P) be a σ-field on an arbitrary set X with a correspondence $P: Q \rightarrow 2^Q$, assume independence

$$\left.\begin{matrix} A \in P(B) \\ C \in P(D) \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} A_1 \cup C_2 \in P(B_1 \cup D_2) \text{ or} \\ A_2 \cup C_1 \in P(B_2 \cup D_1) \end{matrix}\right.$$

Then there exist two measures on Q such that

$$A \in P(B) \Leftrightarrow \lambda(A) - \lambda(B) > \mu(A \,\Delta\, B). \quad (\lambda \geq \mu \geq 0)$$

λ is the probability and μ an uncertainty measure. Similar theorems for function spaces give a foundation for statistics, which have a probabililty and an uncertainty measure on a parameter space given sub σ-fields of (Ω, Q). The Bayesian approach comes out as the special case where the uncertainty is 0.

Alan Kirman (with Vincent Brousseau) :
The Dynamics Of Learning In Mispecified Environments

We consider a simple example of a duopoly, in which the two firmes are unaware of each others existence. (This is no difference from a n person game in which each of the n ignores the existence of one other player.)
At time t the demand for each player i is generated by

$$d_i(\tau) = \alpha - \beta p_i(\tau) + \gamma p_j(\tau) \quad i \neq j$$

They <u>believe</u> that it is generated by

$$d_i(\tau) = a_i - b_i p_i(\tau) + \varepsilon_\tau$$

where ε_i is an error term.
They use ordinary least square learning to estimate the parameters a_i and b_i.
One of us has shown that for a larger class of pairs (p_1, p_2) there are initial conditions which will cause the learning process to converge to those prices which are not"equilibria" of the true model. Here we show that
1) The apparent convergence of the learning process in general

is due to the increasing lengthening of memory in ordinary, least square estimation. Each observation modifies the estimate less and less.

2) Equilibria are generically unstable.
3) If memory is truncated cycles can occur.

H.G. Tillmann : The Need For Impatience (Myopia) For Existence Of Pareto-Optima And Equilibria

Results of Araujo (Econometrica 53, 1985) are generalized from $E=\mathbb{R}^1$ to infinite dimensional spaces. Let E be a reflexive Banach Lattice or loc. convex solid Riesz space.
Let $E=l^{\infty}(E)$, $E'=l_b^1(E')$, $\tau_{MA}=\tau_{MA}$, $\langle E,E'\rangle$ the Mackey technology, $\tau_{\infty}\supset\tau\supset\sigma(E,E')$ a topology on E. With τ,E we associate the class $\xi_\tau=\xi_\tau(E)$ of exchange economies $\varepsilon=\{E_+, (\succsim_i, w_i)\}$ with a finite number of agents i with total, transitive, convex preferences that are τ-continuous.

<u>Theorem 1:</u> There is a general existence theorem for individual rational Pareto optimal states $\Leftrightarrow\tau_{MA}\equiv\tau_{SM}\supset\tau$

<u>Cor. 1:</u> $\tau_{MA}\supset\tau\Leftrightarrow\tau$ is strongly myopic (in the sense of Brown-Lewis, Econometrica 49, 1981)

For the subclass $\xi_\tau^0\subset\xi_\tau$ characterized by an additional non-satiation property A3 and conditions (A4) on the endowment w_i we have:

<u>Theorem 2:</u> Let E be a reflexive DF-Riesz space, E' separable. A general existence theorem for equilibria in all economies in ξ°_τ exists $\Leftrightarrow\tau_{MA}\supset\tau$.

<u>THURSDAY, 11-1-1990: Financial Economics</u>

Stanley R. Pliska: Survey Of Stochastic Process Models In Finance

David Heath: The Term Structure of Interest Rates and Valuation of Contingent Claims

This survey talk concerned several descriptions of the current term structure of interest rates (prices of default-risk-free pure discount bonds, their yield, and forward rates) and presented the framework developed by Robert Jarrow, Andreas Morton and the presentator for modelling the stochastic motion of this term structure. Within this framework the no-arbitrage condition is simple to check, as is that for market completeness. Under the equivalent martingale measure the motion is of the form

$$d_t f(t,T) = \sum_{i=1}^{k} \sigma_i(t,T)\, dW_t^{(i)} + \alpha(t,T)\, dt$$

where f(t,T) is the forward rate observed at time t for an infinitesimally short loan at time T and
Three models were presented in this framework: The model of Cox, Ingersoll and Ross; a continuous time limit of the model

$$\alpha(t,T)=\sum_{i=1}^{k}\sigma_i(t,T)\int_t^T\sigma_i(t,v)\,dv$$

of Ho and Lee; and a model derived from historical U.S. bond data. For all of these models the price at time 0 of a contingent claim paying the random amount C at time T is given by

$E[\, C\cdot \exp\{ -\int_0^T f(u,u)\,du \}\,]$, the expectation being computed

under the martingale measure.

Hans Föllmer: Why Should Brownian Motion Appear On A Financial Market?

Bachelier (1900) introduced Brownian motion (B_t) as a model for continuous price fluctuations on a speculative market, based on a loose equilibrium argument which concluded that (B_t) should be a martingale, and anticipating Lévy's theorem that a continuous martingale with homogeneous increments must coincide with Brownian motion. A justification of "geometric Brownian motion", the standard reference model for the analysis of options, can be given in a similar manner. A more rigorous derivation in terms of "rational expectations equilibria" involves quite delicate assumptions and does not explain the fundamental role of this model. Instead, we derive it via an invariance principle applied to a sequence of equilibrium prices resulting from individual random demands. This look into the anatomy of geometric Brownian motion opens new possibilities for including stochastic interaction in the formation of demands. Some preliminary examples for such interactions (joint work with A. Kirman) lead to deviations from geometric Brownian motion in the form of spontaneous "bubbles and crashes".

Sigrid Müller (jointly with Martin Klein): ECU – Bond Pricing

We consider arbitrage pricing of bonds in the European Currency Unit (ECU) in periods during which ECU basket adjustments are being implemented. The composition of the ECU may be

changed for two reasons:
(i) new currencies are added
(ii) there are large deviations of the actual currency weights from given tangent values.

First, it is shown that in the case of ECU basket adjustments the usual arbitrage pricing relationship between the ECU bond and the ECU member currency bonds breaks down. Second, a new arbitrage pricing relationship is derived and it is shown that it can be obtained from the arbitrage pricing relationship in the absence of basket adjustments properties of the pricing formula and factors. Finally, a "peso problem" for ECU-bonds in periods of impending ECU basket adjustments is identified.

Klaus Sandmann and Dieter Sondermann: Arbitrage And Interest Rate Derivatives

Stochastic models for the price processes of pure discounted bonds are used by arbitrage pricing theory to price options on bonds. These processes are designed to capture special features of zero coupon bonds; but these models do not satisfy the no-arbitrage requirement with respect to the term structure of interest. Therefore it is necessary to model the whole term structure of interest in one step.
We presented a binomial model for the term structure of interest which is based on the spot rate process. The local expectation hypothesis links the spot rate to the bond price. Therefore the excess return per unit risk of every bond is equal to zero.We can generalize this to be equal to a risk premium function.
Under the equivalent martingale measure the local expectation hypothesis is fulfilled. Therefore the martingale measure P depends on the risk premium. The no-arbitrage condition thus guarantees the existence of a transformation from the original model with risk premium to a model without risk premium.

In a discrete time setting we prove that for every transition probability and every volatility specification of the spot rate process there exist a unique path independent binomial spot rate process with non-negative realisations, such that the observed bond prices at t_0 are explained if these bond prices are strictly decreasing with respect to their maturity. Both, the transition probability and the volatility function must be predictable with respect to the filtration induced by {r}.
Obviously a limit result has to depend on the transition probability and the volatility function. For constant transition probability and a reasonable volatility function the limit spot rate process is given by a geometric Brownian motion. The derived term structure model describes a complete market structure. Within this structure interest rate derivatives like bond options or caps and floors can be priced under arbitrage. As suggested by the limit result, simulations have shown that in the limit the arbitrage price and the trading strategies are independent of the transition probability (- constant case -). Furthermore the model can be generalized to include more factors by changing the assumption of a binomial process for the spot rate.

Philipp Artzner and Freddy Delbaen: Bank Insurance

We consider an arbitrage-free market containing
- a money market account
- a default-free zero coupon bond maturing at date T, with a (discounted) price process M
- a (for example variable rate) loan with a possibility of default on interest and principal payments, at some stopping time σ.

The lender's security is:

$X=$ (fixed rate of insurance payments
+ variable interest rate payments
+ payments of principal at date $T)^{\sigma-1}$
(process stopped at $\sigma-$),

and $\Pi(X)$ is the discount market price of X.

It is asked whether the "pure insurance contract" of 1 at date σ is "on the market", i.e. whether the compensation of the process $1_{[[\sigma,T]]}$ is defined unambiguously, i.e. is independent of the choice of an element P in the (non-empty) set $\mathbb{P}$ of probablity measures equivalent to the original one, which make M and $W = X + \Pi(X)$ martingales. This is true under the following assumptions:

- The stopping time σ is totally inaccessible.
- The martingale M generates all continuous $\mathbb{P}$ martingales by stochastic integration.
- The only possible discontinuities of W are at σ.

F. Delbaen: Completeness And Uniqueness Of Martingale Measures

If V is the vector space of marketable assets then we look for measures P under which all elements of V become martingales. Under the usual conditions of a given filtration $(\Omega, (\mathcal{F}_t)_{t\in[0,T]}, P_0)$ the following is known:
The market is complete if and only if the set of martingale measures $M(V)$ only contains one point. The theorem is true in either of the two cases
(a) all elements of V are continuous
(b) V is constructed using a finite number of basic assets.
In case (a) an elementary proof using stopping times can be given. In the case (b) when V is spanned by an infinite number of basic assets Heath and Artzner gave a counterexample. If we look for completeness under absolute continuity conditions the role of extreme points is well known. It can be proved in case (a) and (b) that extreme points yielding complete markets are given by the condition $P(A)=0$ and for all $Q\in M(V)$, $Q\neq P$ $Q(A)>0$. In case (a) P is strongly exposed in $M(V)$.

Martin Schweizer: Risk-Neutral Option Pricing In An Incomplete Model

We consider the following continuous trading model for the hedging of contingent claims:
There is a risky asset described by a special semimartingale $X = X_0 + M + A$,
a riskless asset $Y > 0$ with continuous paths of finity variation and
a contingent claim given by a random variable $H\in\mathcal{F}_T$.
Since we do not assume the spanning property for X, this model will in general be incomplete. For a criterion of maximizing local variances of cumulative hedging costs, one can show that the existence of an optimal hedging strategy is equivalent to the existence of a representation
(' always denotes a discounted quantity).

$$H' = H_0 + \int_0^T \zeta_s^H dX_s + L_T^H$$

Denoting by W the wealth of an optimal hedger, we can decompose every valuation process $V' = E_Q[H' | \mathcal{F}]$ for H as

$$V'_t = W'_t + Cov(L_T^H, R'_T | \mathcal{G}_t),$$

where R′ is a measure for the risk attitude implicit in the use of Q for pricing. This suggests to view W as the intrinsic value process of H. There is also a second motivation for this interpretation: we show that every hedgeable claim H corresponds to a special semimartingale W with $W_T = H$ and satisfying the local CAPM

$$\frac{dB_t}{W_{t-}} - \frac{dY_t}{Y_t} = \frac{d\langle W, X\rangle_t}{d\langle X\rangle_t} \cdot \frac{X_{t-}}{W_{t-}} \cdot \left(\frac{dA_t}{X_{t-}} - \frac{dY_t}{Y_t}\right)$$

(where $W = W_0 + N + B$). To compute the optimal wealth process, one can use the minimal martingale measure $\underline{P} \approx P$ for X′ which is characterized by the property that $R' \equiv 0$.

FRIDAY, 11-2-1990: Incomplete Markets

Wayne Shafer: A Survey Of Incomplete Marktes

This talk briefly surveys results obtained in the last few years for general equilibrium models with incomplete financial markets. We look at a standard two period Radner model in which the number of assets traded may be less than the number of states of nature. The basic results are as follows:

(1) Existence: Regardless of the number or type of assets, if the returns matrix has rank independent of spot prices, then equilibria exist. If the number of assets is greater than or equal to the number of states, the under certain regularity conditions equilibria exist generically in either endowments or assets. If the number of assets is less than the number of states, then equilibria exist generically in both endowments and assets simultaneously.

(2) Determinacy: With asset returns smooth homogenous functions of spot prices, equilibria will generically be locally unique. If nominal assets are present, there will typically be with J<S an uncountable number of equilibria. In particular with all nominal assets present the degree of indeterminacy is S-1 (the number of states minus one).

(3) Efficiency: An omnipotent central planner can generically choose for each agent portfolios and portfolio costs so as to Pareto improve on an incomplete market equilibrium allocation.

Martine Quinzii (joint work with Michael Magill): Monetary Equilibrium With Incomplete Markets

We study the effects of money in an equilibrium model with nominal assets. The idea is to show the conditions under which changes in monetary policy affect the equilibrium allocation.

The economy has a finite number of goods (L) in each of two periods (t=0,1) with S possible states at date 1. Each consumer (i=1,...I) has characteristics (u^i, w^i) consisting of a utility function $u^i:\mathbb{R}^{L(S+1)} \to \mathbb{R}$ and an endowment $w^i \in \mathbb{R}^{L(S+1)}_{+}$. Let $(u,w)=(u^1,\ldots, u^I, w^1,\ldots, w^I)$. There are J nominal assets traded at date 0 (for prices $q=(q_1,\ldots, q_J)$) which deliver a matrix of nominal returns summarized in an SxJ matrix N.
Money serves as a medium of exchange and can also be used as a store of value between date 0 and date 1. A procedure is described by which a Central Exchange injects amounts of money $M=(M_0, M_1,\ldots, M_S)$ with flows through the economy permitting the exchange of goods. The resulting economy is written as $\xi_{u,N}(w,M)$.
We define a concept of <u>monetary equilibrium</u> in which agents choose consumption bundles (x^i), portfolios (z^i) and precautionary balances (z^i_0), the prices of goods on spot markets $(p=p_0, p_1,\ldots, p_S)$ and the asset prices (q) being such that these markets clear. The equilibrium is summarized by (x, z, z_0, p, q, v) where $v=(v_0,\ldots, v_S)$ denotes the velocity of circulation of money in each state. Under the assumption that the first asset (with price $q_1=i/(1+r)$, r denoting the rate of interest) is the riskless bond we prove the following. Let $\Omega \times M = \mathbb{R}^{L(S+1)I}_{++} \times \mathbb{R}^{S+1}_{++}$.

<u>Theorem 1:</u> For all $(w,M) \in \Omega\times M$ the economy $\xi_{u,N}(w,M)$ has an equilibrium. There is a generic set $\Delta \subset \Omega \times M$ such that each economy $\xi_{u,N}(w,M)$ has a finite number of equilibria each of which are locally smooth functions of (w,M) and are one of two types
(a) $r>0,\ z^i_0=0,\ v_s=1,\ s=1,\ldots, S$ or
(b) $r=0,\ v_0<1,\ v_s>1,\ s=1,\ldots, S$.

<u>Theorem 2:</u> Let the map $dM \to dx:\mathbb{R}^{S+1} \to \mathbb{R}^{L(S+1)I}$ denote the change in the equilibrium allocation induced by the local change in money dM. There is a generic set $\Delta' \subset \Omega \times M$ such that for any positive interest rate equilibrium of the economy $\xi_{u,N}(w,M)$
(1) If rank N=S then dx=o for all $dM \in \mathbb{R}^{S+1}$
(2) If rank N<S, N is in general position, J<I then there exists an S-1 dimensional subspace $F \subset \mathbb{R}^{S+1}$ such that the map $dM \to dx: F \to \mathbb{R}^{L(S+1)I}$ is injective where $F=\{ dM \in \mathbb{R}^{S+1} \mid dM_0 = 0, \sum_{s=1}^{S} dM_s = 0\}$.

<u>Theorem 3:</u> There is a generic set $\Delta^* \subset \Omega \times M$ such that for any zero interest rate equilibrium of the economy $\xi_{u,N}(w,M)$ wifh $J\leq I$ there exists an S-dimensional subspace $F^* \subset \mathbb{R}^{S+1}$ such that the map $dM \to dx: F^* \to \mathbb{R}^{L(S+1)I}$ is injective, where $F^* = \{ dM \in \mathbb{R}^{S+1} \mid dM_0 = 0\}$.

H. Polemarchakis (joint work with P. Siconolfi):
Competitive Equilibria Without Free Disposal With An Application To An Incomplete Asset Market

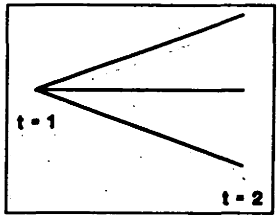
Consider an exchange economy in which individual consumption sets are compact, convex subsets of a finite dimensional commodity space, $\mathbb{R}^A$. In particular, the economy does not satisfy the assumptions of free disposal and nonsatiation that are made in the standard argument for the existence of competitive equilibria.
Suppose individual utility functions are continuous and

strictly quasi-concave and let s^h be the unique satiation point of individual h, h=1,... H. Suppose further that $\sum_{h=1...H} s^h \neq 0$; it is not feasible for every individual to attain his satiation point.
The price domain is $\mathbb{R}^A$ and the aggregate (excess) demand function is $y:\mathbb{R}^A \to \mathbb{R}^A$. Competitive equilibrium prices are such that $y(q^*)=0$.
Consider the subset of the price domain

$$Q = \{ q: qs^h \geq 0, \quad h=1,...H\} \subset \mathbb{R}^A$$

<u>Proposition 1:</u>
No competitive equilibrium prices exist in $\mathbb{R}^A$ if $Q=\{0\}$.

<u>Proposition 2:</u>
Competitive equilibrium prices exist and lie in Q if
(i) Q has a nonempty interior and
(ii) if $q \in BdQ/\{0\}$, $y(q) \notin N(q)$, where $H(q)=\{h: qs^h=0\}$ and $N(q)=\{y: y=\sum_{h\in H(q)} \lambda^h s^h, \quad \lambda^h \leq 0\}$.



An example of an economy that fits this framework is an economy that extends over two periods under uncertainty.
With one consumption good at each state at t=2 and no consumptions at t=1, equilibrium reduces to an equilibrium in the period 1 asset market. If the matrix of asset payoffs, R, is such that there exists <u>no</u> portfolio y with positive payoffs ($R_y>0$), the consumption set of each individual in the period 1 asset market is indeed compact, since the consumption of the good in period 2 must be nonnegative.

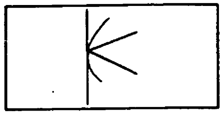
Jan Werner (joint work with Don Brown):
Arbitrage And Existence Of Equilibrium In Infinite Asset Markets

This paper develops a framework for general equilibrium analysis of markets with infinitely many assets. We consider an exchange economy with an infinite dimensional choice space in which individual consumption sets are not bounded below. The fact that consumption sets are not bounded reflects the possibility of short-sales of assets. We develop a concept of arbitrage-free prices and prove that an economy in which the set of arbitrage-free prices is non-empty has an equilibrium. The best known model of markets with infinitely many assets is the Arbitrage Pricing Theory. Our paper can be seen as a first step in the general equilibrium analysis of the APT.

Aloisio Araujo (joint work with P.K. Monteiro):
General Equilibrium With Infinitely Many Goods And Financial Theory

Let $\mathcal{E}=\{(w_i,\succeq_i) \ i=1,...,I\}$ be an economy with $I < \infty$ consumers, complete markets, and infinitely many goods (in $L^p_+(M)$, $1 \leq p \leq \infty$).

We start by presenting the approach of properness of preferences first suggested by A. Mas-Colell to obtain equilibrium, i.e. the existence of a backward cone:



<u>Theorem 1:</u> (JET 89) Suppose for each $1 \leq i \leq I$ and x there exists v and a U neighborhood of zero such that if $r>0$ is small, $x' = x + rv - rz \geq 0$ and $z \in U$ then $x' \succsim_i x$. Then there exists $(x^*,p^*) \in (L_+^P)^I \times (L_+^P)_+'$ equilibrium for ε. However, if the preferences are given by separable utilities:

$$M(x) = \int e^{-\rho t} u(x(t))\, dM(t) \qquad \rho > 0$$

as usually assumed in finance, INADA condition holds $u'(0) = \infty$, we have:

<u>THEOREM 2:</u> (JME, forthcoming) Generically on ω, equilibrium does not exist.

A second approach that is valid also when the commodity space is the space of measures, makes no properness assumptions and hence it covers cases with INADA conditions. However the price equilibria it guarantees are not in the dual, they are only in

$$L_1(w) = \left\{ p\ ,\ \int p(t)\,\omega(t)\, dM(t) < \infty \right\} \supsetneq L^q(M)\,, \quad \textit{where} \quad \omega = \sum_{i=1}^{I} \omega_i\ .$$

Hence we cannot guarantee L^2 prices if p=2. Also, we can price only the Edgeworth boxes $\{0,\omega\}$. In view of theorem 2 that is the best we can do. Moreover, prices continue to have a concrete interpretation.
In the case in which the commodity space is the space of measures our theorem gives a non-continuous price function that can be applied to non-competitive economies.

<u>THEOREM 3:</u>
(a) If $L_+^P(M)$ is the commodity space, the equilibrium always exists in $L_+'(w)$ if we restrict the commodity space to $[0,\omega]$.
(b) If $M_+(R)$ is the commodity space then equilibrium always exsists in $L_1(M) = \{p, \int p\, d\omega < \infty\}$, if one restricts the commodity space to $[0,\omega]$.

Thorsten Hens: Sunspot Equilibria In Finite Horizon Models With Incomplete Markets

The paper analyses, whether "sunspots" i.e. extrinsic uncertainty ("animal spirits", "market psychology",...), play a significant role in rational expectation models. The model chosen is a Radner general equilibrium model with real assets and incomplete markets. For this model it is known that sunspots don't matter if markets are complete (Cass-Shell Ineffectivity Theorem) and that they do matter if markets are incomplete and the underlying non-sunspot economy has multiple equilibria. In the second case sunspot equilibria are simply obtained by randomization across several equilibria of the underlying non-sunspot economy. The paper shows that this randomization argument is not necessary for sunspot equilibria to exist. It turns out that incomplete insurance against sunspot uncertainty can itself be the reason why sunspots matter. In addition we show that there exists no continuous transition (bifurcation) from an equilibrium in which only fundamentals matter to an equilibrium in which sunspots matter .Thus a transition to a sunspot equilibrium requires a "big change" in agents's expectations.

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