

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Wahrscheinlichkeitsmaße auf Gruppen

4.11. bis 10.11.1990

Die Tagung fand unter der Leitung der Herren H. Heyer (Tübingen) und L. Schmetterer (Wien) statt.

Von den 52 Teilnehmern aus 9 europäischen Ländern, Australien, Japan, Kanada, Tunesien, der UdSSR und den USA wurden insgesamt 40 Vorträge gehalten. Dabei kamen vielfältige wahrscheinlichkeitstheoretische Fragestellungen auf unterschiedlichen Strukturen, namentlich auf Gruppen, Hypergruppen, Halbgruppen und Bäumen, zur Sprache. So wurden etwa (zentrale) Grenzwertsätze, Einbettungssätze, Transienz Kriterien, Invarianzprinzipien, De Finetti-Sätze sowie potentialtheoretische Sätze vorgestellt. Von besonderem Interesse dürften neue Methoden zur Konstruktion von Hypergruppen gewesen sein.

Am Anfang des täglichen Programms stand jeweils ein einstündiger Hauptvortrag. Bei den Hauptvorträgen handelte es sich um Übersichtsvorträge zu den Themen: Halbgruppen in der Wahrscheinlichkeitstheorie; das Verhalten im Unendlichen und harmonische Funktionen bei Irrfahrten auf Bäumen; theoretische Aspekte der Formenanalyse; von Quantengruppen abgeleitete positive Faltungsstrukturen; Symmetriegruppen von Markov-Prozessen. Neben den übrigen zwanzigminütigen Vorträgen gab es Abendsitzungen zu den Themen: Transienz von Halbgruppen von Wahrscheinlichkeitsmaßen auf kommutativen Hypergruppen und Typenkonvergenz von Wahrscheinlichkeitsmaßen auf Lie-Gruppen.

Die Tagung endete mit einem ausführlichen Vortrag über die Einbettung unendlich teilbarer Wahrscheinlichkeitsmaße auf algebraischen Gruppen. Das in diesem Vortrag angesprochene Problem, das vor kurzem in großer Allgemeinheit gelöst worden ist, hat eine mit Oberwolfach eng verknüpfte Vergangenheit. Insbesondere war es bereits Gegenstand ausführlicher Diskussionen in der ersten der bisher 10 Tagungen zum Thema vom Jahre 1970.

## Abstracts

**N. AOKI:**

### Probability measures and $C^1$ -perturbations of dynamical systems

Let  $M$  be a closed manifold and  $\text{Diff}^1(M)$  the set of all diffeomorphisms of  $M$  endowed with the  $C^1$ -topology. The following is an important theorem in dynamical systems.

*Theorem* (Kupka, Smale). The set of all diffeomorphisms  $f \in \text{Diff}^1(M)$  satisfying

- (i) all periodic points of  $f$  are hyperbolic,
- (ii) the stable manifold  $W^s(x) := \{z : \lim_{n \rightarrow \infty} d(f^n(x), f^n(z)) = 0\}$  and the unstable manifold  $W^u(y) := \{z : \lim_{n \rightarrow \infty} d(f^{-n}(y), f^{-n}(z)) = 0\}$  meet transversally for all pairs  $(x, y)$  of periodic points  $x, y$

is a residual set of  $\text{Diff}^1(M)$ .

Now define the following

$$\begin{aligned} J^1(M) &= C^1\text{-interior of } \{f \in \text{Diff}^1(M) \mid f \text{ satisfies (i)}\} \\ KS^1(M) &= C^1\text{-interior of } \{f \in \text{Diff}^1(M) \mid f \text{ satisfies (i) and (ii)}\}. \end{aligned}$$

Then by the  $C^1$ -perturbation based on probability measures we have that each diffeomorphism belonging to  $J^1(M)$  satisfies Axiom A and has no cycle, and that each diffeomorphism belonging to  $KS^1(M)$  satisfies Axiom A and strong transversality. This result is a positive answer to a question raised by Mañé and Palis.

**J.P. ARNAUD**

### Stationary processes indexed by a homogeneous tree

Let  $T$  be the set of vertices of a homogeneous tree and  $X = (X_t)_{t \in T}$  a second order real valued process such that  $E(X_s, X_t)$  depends only on the distance between the vertices  $s$  and  $t$ . We construct a measure space  $(K, m)$  and an isometry of the closed subspace  $\mathcal{H}(X)$  spanned by  $X$  onto  $L^2(m)$ . With such an isometry every question which concerns to the linear structure of  $X$  (like prediction, filtering) will be translated in a question about the concrete space  $L^2(m)$  of functions on  $K$ .

**M.S. BINGHAM**

### An approximate martingale characterization of Brownian motion on locally compact groups

Let  $G$  be a locally compact second countable Abelian group and let  $\hat{G}$  be its dual

group. Denote by  $\langle x, y \rangle$  the value of the character  $y \in \hat{G}$  at the point  $x \in G$  and let  $g$  denote a local inner product on  $G \times \hat{G}$ . Let  $(\Omega, \mathcal{I}, (\mathcal{I}_t)_{t \in [0,1]}, P)$  be a stochastic basis which satisfies the "usual conditions" of stochastic analysis. Consider a  $G$ -valued stochastic process  $X = \{X(t), t \in [0, 1]\}$  on  $(\Omega, \mathcal{I})$  adapted to  $(\mathcal{I}_t)$ , with continuous sample paths and satisfying  $X(0) = e$ , the identity of  $G$ . Call  $X$  an  $(\mathcal{I}_t)$ -Wiener process corresponding to  $(\phi_{s,t})$  if  $E\{X(t) - X(s), y | \mathcal{I}_s\} = \exp[-\frac{1}{2}\phi_{s,t}(y)]$  for all  $y \in \hat{G}$ ,  $0 \leq s \leq t \leq 1$  where  $\{\phi_{s,t} : 0 \leq s \leq t \leq 1\}$  is a collection of continuous nonnegative forms on  $\hat{G}$  such that  $\phi_{s,t}(y) + \phi_{t,u}(y) = \phi_{s,u}(y)$  for all  $y \in \hat{G}$ ,  $0 \leq s \leq t \leq u \leq 1$  and  $\phi_{0,t}(y) \rightarrow \phi_{0,0}(y)$  as  $t \rightarrow 0$ . For each positive integer  $n$  choose points  $0 = t_{n,0} < t_{n,1} < \dots < t_{n,k_n} = 1$  such that  $\max_j(t_{n,j} - t_{n,j-1}) \rightarrow 0$  as  $n \rightarrow \infty$ . Denote  $\Delta_{n,j}X := X(t_{n,j}) - X(t_{n,j-1})$ ,  $\Delta_{n,j}\phi := \phi_{t_{n,j-1}, t_{n,j}}$ ,  $\mathcal{I}_{n,j} := \mathcal{I}_{t_{n,j}}$ . Then  $X$  is an  $(\mathcal{I}_t)$ -Wiener process corresponding to  $(\phi_{s,t})$  if and only if

- (1)  $\sum_{j=1}^{k_n} |E[g(\Delta_{n,j}X, y) | \mathcal{I}_{n,j-1}]| \xrightarrow{P} 0$  as  $n \rightarrow \infty$
- (2)  $\sum_{j=1}^{k_n} |E[g(\Delta_{n,j}X, y)^2 | \mathcal{I}_{n,j-1}] - \Delta_{n,j}\phi(y)| \xrightarrow{P} 0$  as  $n \rightarrow \infty$ .

## J.S. BONDAR:

### Statistical applications of probabilities on groups

An important problem in statistical decision theory is the construction of optimal (smallest risk) estimators which are invariant under the action of a group  $G$  of transformations. If  $G$  is the real line, acting linearly on  $R^n$ , an effective construction has long been known. We shall discuss the generalization of this construction to arbitrary topological groups  $G$ . A key to this generalization is finding certain probability measures on  $G$ ; this in turn can often be done using a result of Effros that (loosely speaking), a transformation group either acts very nicely, or very badly.

## Ph. BOUGEROL:

### Random walks on the symplectic group and the Kalman-Bucy filter

We consider the linear system

$$\begin{aligned} X_n &= A_n X_{n-1} + F_n \varepsilon_n \\ Y_n &= C_n X_n + \eta_n \end{aligned}$$

where  $(\varepsilon_n, \eta_n)$  is a white noise and  $(A_n, F_n, C_n)$  is a sequence, known at time  $n$ , of parameters which is stationary ergodic independent of the noise. Let  $\hat{X}_n = E(X_n | Y_1, \dots, Y_n)$  and  $P_n = E((X_n - \hat{X}_n)(X_n - \hat{X}_n)^* | Y_1, \dots, Y_n)$ . Then the recursive Kalman equation says that

$$P_n = M_n \cdots M_1 \cdot P_0$$

where  $M_n$  are the Hamiltonian elements of  $Sp(d, \mathbb{R})$ , acting on the space  $\mathcal{P}_0$  of symmetric positive definite matrices by  $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot Z = (AZ + B)(CZ + D)^{-1}$ . We show that this set of Hamiltonians acts by contraction on  $\mathcal{P}_0$ , if  $\mathcal{P}_0$  is equipped with the usual Riemannian metric. We deduce from this that under a weak controllability and observability condition:

- (i) There is a stationary process  $\bar{P}_n \in \mathcal{P}_0$  such that  $\|P_n - \bar{P}_n\| \rightarrow 0$  a.s. for all  $P_0$ .
- (ii) The filter is a.s. exponentially stable.

The underlying ideas come from the theory of products of random matrices.

## I. DRYDEN:

### Theoretical aspects of shape analysis

Let  $G$  be a Lie group which acts smoothly on a differentiable manifold  $M$ . Then  $G$  acts smoothly on  $M^N$ , the space of  $N$ -tuples of  $M$ . The general shape space  $M^N/G$  is the set of orbits for this action. We investigate when  $M = \mathbb{R}^k$  and  $G$  is the Euclidean similarity group of mappings,

$$\mathbb{R}^k \rightarrow \mathbb{R}^k : x \rightarrow c\Gamma x + v \text{ where } c \in \mathbb{R}^{++}, \Gamma \in SO(K), v \in \mathbb{R}^k.$$

In this case the geometry of the orbit space (the shape space  $\Sigma_K^N$ ) has been studied by D.G. Kendall. It is of fundamental interest to study probability distributions of shape. If the  $N$  points in  $\mathbb{R}^k$  have independent isotropic Gaussian distributions with different means and common variance, then the resulting probability distribution of shape in  $\Sigma_K^N$  is known for  $K = 2$ .

For  $K \geq 2$  the QR decomposition can be used to obtain suitable shape coordinates. Techniques from classical multivariate analysis, such as Bartlett's decomposition and integration over  $O(K)$ , are adapted to obtain shape distributions. Further cases of interest are when  $G$  is the isometry group  $SO(K) \times \mathbb{R}^k$  ( $c = 1$ ) and the acceptance of reflections ( $O(K)$  instead of  $SO(K)$ ). The densities of the resulting distributions involve hypergeometric functions of matrix argument and zonal polynomials. Recent developments in shape distribution theory are reported, in addition to established results.

## P. EISELE:

### A generalization of the Kawada-Ito theorem

The classical Kawada-Ito theorem states that the sequence of the  $n$ -th convolution powers of a probability measure  $\mu$  on a compact group converges (weakly, and necessarily to an idempotent measure) iff the support of  $\mu$  is not contained in a coset

$Hx = xH \neq H$  of a compact subgroup  $H$  of  $G$ . In order to generalize this theorem we considered a sequence  $\varrho_n := \lambda T(\lambda) \cdots T^{n-1}(\lambda)$  where  $\lambda$  is a probability measure on an arbitrary (locally compact or polish) group  $G$  and  $T$  an automorphism of  $G$ . We got the following result:

Assume the family  $\{T^{-n} : n \in \mathbb{N}\}$  to be equicontinuous. Then  $\varrho_n$  converges (necessarily to an idempotent  $T$ -invariant measure) if and only if the following two conditions are fulfilled:

- (1) The support of  $\lambda$  is not contained in a coset  $Hx = xT(H) \neq H$  of a compact subgroup of  $G$ ;
- (2) The support of  $\lambda$  is contained in a compact  $T$ -invariant subgroup of  $G$ .

Other assumptions on  $T$  were discussed and it was shown by the example  $G = \mathbb{R}$ ,  $T(x) = \alpha x$  with  $\alpha \in (0, 1)$  that the above result cannot be valid for arbitrary automorphisms.

**L. ELIE:**

**Examples of statistical models related to random walks on groups**

The starting point of this talk is to show that heteroscedastic models such as ARCH models which first were introduced by Engle and which are used for financial time series can be constructed by using random walks on the affine group of  $\mathbb{R}^k$ .

Then we give a general frame for models with heteroscedastic errors which is the following: we consider models of the form

$$X_n = f(X_{n-1}) + g(X_{n-1})\varepsilon_n$$

where  $X_n \in \mathbb{R}^k$ ,  $\varepsilon_n$  a sequence of i.i.d. random variables in  $\mathbb{R}$  such that  $E(\varepsilon_n) = 0$ ,  $E(\varepsilon_n^2) = 1$  and  $\varepsilon_n$  independent of  $\{X_p, p \leq n\}$ . We study the existence of a strictly stationary solution, the moments and the ergodicity of the solution. In general these models can't be embedded in an "affine" frame, but we show that if we impose Lipschitz conditions on the functions  $f$  and  $g$ , we can find sufficient conditions for the existence of a strictly stationary ergodic solution in terms of the Lyapunov exponent. We show by some examples simple conditions, which are not difficult to use in practice, in order to obtain solutions with moments of order 1 or 2. One example is the model

$$X_n = a_1 X_{n-1} + \cdots + a_p X_{n-p} + \sqrt{\beta_0 + \beta_1 X_{n-1}^2 + \cdots + \beta_q X_{n-q}^2} \varepsilon_n$$

which generalizes autoregressive models with ARCH errors.

G.M. FEL'DMAN:

**Characterization problems of mathematical statistics on Abelian groups**

Let  $X$  be a locally compact Abelian group. The main result presented in this talk is: Let  $X_i$  be independent random variables with values in the group  $X$ , having distributions  $\mu_i$  with nonvanishing characteristic functions, and let  $\{a_i\}$ ,  $\{b_i\}$  be sets of integers admissible for  $X$ . Independence of the forms  $L_1 = a_1\xi_1 + \dots + a_s\xi_s$ , and  $L_2 = b_1\xi_1 + \dots + b_s\xi_s$ , implies that all  $\mu_i$  are Gaussian distributions if and only if either  $X$  is a group without torsion or all elements in  $X$  have the order  $p$ , where  $p$  is a prime number.

L. GALLARDO:

**A central limit theorem on a two-dimensional hypergroup**

For  $\alpha > 0$ , disk polynomials  $R_{m,n}^\alpha(z, \bar{z})$  are defined (in polar coordinates) in terms of the normalized Jacobi polynomials by

$$R_{m,n}^\alpha(\varrho, \theta) = R_{m \wedge n}^{\alpha, |m-n|}(2\varrho^2 - 1)\varrho^{|m-n|}e^{i(m-n)\theta}$$

The linearization formula for these polynomials has coefficients which are positive. This allows us to define a convolution on  $\mathbb{N}^2$ , say  $*_\alpha$ . Then  $(\mathbb{N}^2, *_\alpha)$  is a commutative hypergroup. For the random walks associated with this structure we obtain the following theorem: Let  $S_k = (X_k, Y_k)$  the position at time  $k$  of a random walk of law  $\mu$  ( $\mu$  adapted). Suppose  $E(X_1 - Y_1) = 0$  and  $a = E((X_1 - Y_1)^2)$  finite and also  $b = \sum_{m,n} \mu(m, n) \left(\frac{2}{\alpha+1}mn + m + n\right) < \infty$ . Then the sequence of random vectors  $(k^{-\frac{1}{2}}X_k, k^{frac{1}{2}}Y_k)$  converges in distribution to the probability measure

$$\frac{2^{\alpha+1}}{\sqrt{2\pi a} b^{\alpha+1} \Gamma(\alpha+1)} (xy)^\alpha (x+y) e^{-\frac{2}{a}xy} e^{-\frac{1}{2a}(x-y)^2} dx dy \quad (x \geq 0, y \geq 0).$$

For the simple random walk (i.e.  $a = b = 1$ ), the sequence of the random walks  $\frac{X_k}{\sqrt{k}}$  converges in distribution to the following probability measure on  $\mathbb{R}$ :

$$\pi^{-\frac{1}{2}} 2^{\alpha+\frac{1}{2}} x^\alpha e^{-\frac{x^2}{4}} D_{-\alpha}(x) dx \quad (x \geq 0),$$

where  $D_{-\alpha}$  is the cylinder parabolic function of index  $-\alpha$ .

O. GEBUHRER:

Transience of semigroups of probability measures on commutative hypergroups

M. Itô proved in 1983 the following theorem: Let  $\{\mu_t\}$  be a semi group of probability measures on a locally compact Abelian group; then  $\kappa = \int_0^\infty \mu_t dt$  is a Radon measure (that is the semi group  $\{\mu_t\}$  is transient) if (and only if)  $Re(\frac{1}{\psi})$  belongs to  $L^1_{loc}(\hat{G}, d\gamma)$  where  $\psi$  is the exponent of  $\{\mu_t\}$ . This proof answers a question of C. Berg of a purely analytical proof for that criterion; however having in mind the same question for commutative locally compact hypergroups, it is necessary to look for a new analytical approach, avoiding the structure theorems of locally compact Abelian groups. We expose here how to make a step in that direction: Under the supplementary hypothesis  $\liminf_{x \rightarrow \infty} Re(\psi) > 0$ , we show that in fact the "if" part of the theorem is still true for power type growth hypergroups.

J. GLOVER:

Symmetry groups of Markov processes

Algebraic structures have enjoyed a special rôle in Markov processes from the very beginning of the subject. Probabilists focussed much of their attention initially on independent increment processes in the group  $\mathbb{R}^d$ . As Markov processes matured, probabilists began to study them in more general settings, and there is often no algebraic structure in evidence on the state space these days. We will explore methods of introducing algebraic structures on the state space which are naturally associated with the process. In several instances, after an invertible probabilistic transformation, they will become independent increment processes in the new group structure.

C. HASSENFORDER:

Characterization of some particular Gaussian processes indexed by a homogeneous tree

Let  $T$  be the set of vertices of a homogeneous tree and  $(X_t)_{t \in T}$  a second order real or complex valued process such that the expected value  $E(X_s X_t)$  depends only on the distance between the vertices  $s$  and  $t$ . Arnaud and Letac have constructed the Karhunen representation for such a process (i.e. a measure space  $(K, \mathcal{H}, m)$  and an isometry of the closed subspace of  $L^2(\Omega, \mathcal{A}, P)$  spanned by  $(X_t)_{t \in T}$  onto  $L^2(m)$ ). Hence, we have now a marvellous tool to study processes like  $(X_t)_{t \in T}$ . Here we consider two simple problems.

(1) Which processes are such that there exist  $p$  in  $[-1, 1]$ ,  $\kappa > 0$  with  $E(X_s X_t) = \kappa p^{d(s,t)}$ ? For Gaussian time series, this is the standard Ornstein-Uhlenbeck discrete

process, which can be characterized by a Markov property. We shall extend this Markov property to the tree in a suitable sense.

(2) Which stationary processes  $X$  on a tree have a real spectral measure  $\mu_X$  concentrated on one point? These processes can be characterized by the following property: If  $T_1$  is a finite subtree such that the valencies of the vertices are either  $q + 1$  or 1 (for the set  $T_2$  of end points) then the knowledge of  $(X_t)_{t \in T_2}$  gives the deterministic knowledge of  $(X_t)_{t \in T_1}$ .

### V.A. KAIMANOVICH:

#### Boundaries of random walks on solvable groups

Let  $G$  be a discrete group and  $\mu$  a probability measure on  $G$ . Then consider the Poisson boundary: the spectrum of the algebra of bounded  $\mu$ -harmonic functions on  $G$ . One can ask about a description of the Poisson boundary in terms of intrinsic properties of the group  $G$ . Here we consider two examples:

1)  $G = \text{Aff}(\mathbb{Z}[\frac{1}{2}])$  - the affine group of the dyadic rational line. Let  $\alpha = E \log a$ , where a group element  $g$  is written as  $g = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$ . It is well known that if  $\alpha < 0$  then for a.e. trajectory  $y_n = \begin{pmatrix} s_n & \varphi_n \\ 0 & 1 \end{pmatrix}$  of the random walk there exists a limit  $\varphi_\infty = \lim \varphi_n \in \mathbb{R}$ . This is also true for the group  $\text{Aff}(\mathbb{R})$ , but here one can get nontrivial behaviour at infinity also in the case  $\alpha > 0$ . In this case  $\varphi_\infty = \lim \varphi_n \in \mathbb{Q}_2$  the field of  $z$ -adic numbers. Using the entropy technique one can prove that this is in fact the whole Poisson boundary, i.e. it coincides with  $\mathbb{R}$  for  $\alpha < 0$ , with  $\mathbb{Q}_2$  for  $\alpha > 0$ , and is trivial for  $\alpha = 0$ . An analogous description is also obtained for the case where

2)  $G$  is a polycyclic group. On the other hand, the problem of proving maximality of a certain natural boundary for a class of non-polycyclic solvable groups is still open.

### E. KANIUTH:

#### The Pompeiu problem for groups

Let  $G$  be a locally compact group. A relatively compact Borel set  $E$  in  $G$  of positive Haar measure is called Pompeiu set for  $L^1(G)$  (equivalently, for the measure algebra  $M(G)$ ) if for  $f \in L^1(G)$  the condition  $\int_x E y f(t) dt = 0$  for all  $x, y \in G$ , implies  $f = 0$ .  $G$  is said to be a Pompeiu group if each relatively compact Borel subset of  $G$  with positive measure is a Pompeiu set. The appropriate tool to deal with such problems is group representation theory. In fact,  $E$  is a Pompeiu set if  $\pi(\chi_{E^{-1}}) \neq 0$  for all  $\chi$  in some dense subset of the dual  $\hat{G}$  of  $G$ . It turns out that if  $G$  is first countable, then there exist Pompeiu sets in abundance. The problem of when  $G$  is Pompeiu is much more intricate, a necessary condition being that  $G$  contains no nontrivial compact normal subgroup. However, it can be solved for various classes of l.c. groups as

follows:

- (i) A nilpotent l.c. group is Pompeiu if and only if it contains no nontrivial compact element;
- (ii) Every simply connected solvable Lie group is a Pompeiu group;
- (iii) A discrete group  $G$  is Pompeiu iff its finite conjugacy class subgroup is torsionfree.
- (iv) A semi-direct product  $G = K \rtimes N$ , where  $K$  is a compact Lie group and  $N$  is Abelian, is Pompeiu if and only if  $N$  contains no compact elements  $\neq e$  and for every  $\lambda$  in some dense subset of  $\hat{N}$ , the stability subgroup of  $\lambda$  in  $K$  is trivial.

(Joint work with A. Carey and W. Moran, U. of Adelaide.)

## T. KOORNWINDER:

### Positive convolution structures obtained from quantum groups

A quantum group is a virtual object which can be studied from the Hopf algebra of "polynomial" functions on it. If the Hopf algebra is moreover a Hopf  $*$ -algebra generated by the matrix elements of a unitary corepresentation and if there is a suitable  $C^*$ -algebra closure then we are in the situation of Woronowicz's compact matrix pseudogroups and analogues of harmonic analysis on compact groups, like the Haar functional and the Schur orthogonality relations, are available. We also define the quantum analogue of a Gelfand pair  $(G, K)$ , ( $G, K$  compact). Then the subalgebra of "normal" elements of the Hopf  $*$ -algebra provides a very natural example of a discrete, possibly noncommutative hypergroup. In particular, we can thus prove the positivity of linearization coefficients for certain little  $q$ -Jacobi polynomials and certain Askey-Wilson polynomials.

## R. LASSER:

### On polynomial hypergroups

There is a close relationship between certain orthogonal polynomial sequences and polynomial hypergroups. Two theorems are given describing structural aspects of polynomial hypergroups.

*Theorem 1:* Given an orthogonal polynomial sequence  $(\tilde{P}_n(x))_{n \in \mathbb{N}_0}$  defined by  $x \tilde{P}_n(x) = e_n \tilde{P}_{n+1}(x) + d_n \tilde{P}_{n-1}(x)$ ,  $\tilde{P}_0(x) = 1$ ,  $\tilde{P}_1(x) = x/e_0$ , the minimal parameter sequence  $(c_n)_{n=0}^\infty$  of  $(d_n e_{n-1})_{n=1}^\infty$  defines a polynomial hypergroup if  $d_n \leq d_{n+1}$ ,  $d_n + e_n \leq d_{n+1} c_{n+1}$  and  $d_n \leq e_n$ , provided  $(d_n c_{n-1})_{n=1}^\infty$  is a chain sequence.

The proof heavily depends on a recent result of Szwarz.

*Theorem 2:* Given an orthogonal polynomial sequence  $(P_n(x))_{n \in \mathbb{N}_0}$  with  $P_n(1) = 1$ , which defines a polynomial hypergroup on  $\mathbb{N}_0$  and for which  $c_n(1 - c_{n-1}) \rightarrow \frac{1}{4}$  as  $n \rightarrow \infty$ , then property (T) holds, (i.e.  $P_n(x) = \sum_{k=0}^n a_{n,k} T_k(x)$ ,  $T_k(x) = \cos(k \arccos x)$  and  $a_{n,k} \geq 0$ ).

As an application of theorem 1 a lot of new polynomial hypergroups are given, as associated ultraspherical polynomials, Pollaczek polynomials and random walk polynomials.

**M. LEITNER:**

**On the prediction of weakly stationary processes indexed by a commutative hypergroup**

A family  $(X_a)_{a \in K}$  of square integrable scalar random variables on a probability space  $(\Omega, \Sigma, P)$  indexed by a commutative hypergroup  $(K, *, -, e)$  is called  $K$ -weakly stationary, if

- (i) the means are constant, i.e.  $EX_a = c$  for all  $a \in K$ ;
- (ii) the covariance function given by  $\rho(a, b) = E((X_a - c)(X_b - c))$  is bounded, continuous and satisfies

$$\rho(a, b) = \int_K \rho(t, e) da * b(t).$$

*Results:*

- (i) There is a generalized Wold decomposition for  $K$ -weakly stationary processes.

For the special case  $K = \mathbb{N}_0$ ,  $K$  induced by an orthogonal polynomial sequence  $(P_n)_{n \in \mathbb{N}_0}$  with respect to an orthogonalization measure  $\pi$ , we have

- (ii) Modified moving-average processes can be defined.
- (iii) If  $\pi$  is continuous, every purely nondeterministic process with spectral density  $f \in L^1(\pi)$ , is a moving-average process.
- (iv) If a polynomial weakly stationary process has a spectral density  $f \in L^1(\pi)$  with  $1/f \in L^1(\pi)$ , then it is purely nondeterministic.

**C. MARKETT:**

**Hypergroups of Sturm-Liouville type and polynomial hypergroups:  
An approach via partial differential and difference equation techniques**

Sturm-Liouville hypergroups are convolution algebras of Borel measures on the positive half-line or on a compact interval, which arise from a generalized translation

operator acting multiplicatively on the eigenfunctions of a given Sturm-Liouville problem, i.e.,  $T_y : \varphi_y(x) \rightarrow \varphi_y(x)\varphi_y(y)$ . An appropriate way to find a closed representation of this operator is to solve a corresponding hyperbolic initial boundary value problem. For this purpose, an approach is presented which is based on Riemann's integration method and extensions of it. Some typical examples associated with classical singular Sturm-Liouville equations of both compact and noncompact type are discussed. We also present a discrete analogue of Riemann's method which enables one, in principle, to derive the linearization formula for a given sequence of orthogonal polynomials and thus to find new examples of polynomial hypergroups.

**M. McCrudden:**

**Embedding infinitely divisible probabilities on almost algebraic groups**

*Theorem.* Let  $G$  be an almost algebraic (real) group, suppose  $\mu$  is an infinitely divisible probability measure on  $G$ . Then there is a continuous homomorphism  $t \mapsto \mu_t$  of  $\mathbb{R}_+$  into  $\mathcal{P}(G)$ , the space of probability measures on  $G$ , such that  $\mu = \mu_1$ . *Outline of proof:* (i) We can go to a subgroup  $G^*$  of  $G$ , which is almost algebraic, such that  $\mu$  is infinitely divisible on  $G^*$  and such that  $Z(\mu, G^*) = \{x \in G : xy = yx, \text{ for all } y \in \text{sup}(\mu)\}$  is a compact extension of a simply connected nilpotent almost algebraic group  $L$ . (ii) We now use the factor compactness theorem, the "root set sequence" construction, and a property of  $L$  called "affine root rigidity" to produce a root  $v$  of  $\mu$ , and a closed subgroup  $H$  of  $G$  such that  $v$  is root compact and infinitely divisible on  $H$ . We then embed  $v$  by standard arguments, thus embedding  $\mu$ .

**P. Milnes:**

**Haar measure for compact right topological groups**

These groups arise in topological dynamics as the enveloping semigroups of distal flows. An example is  $G = \mathbb{T} \times E(\mathbb{T})$  with multiplication  $(w', h')(w, h) = (w'wh' \circ h(e^i), h' \circ h)$  (where  $\mathbb{T}$  is the circle group and  $E = E(\mathbb{T})$  is the set of all endomorphisms of  $\mathbb{T}$ ). Typical in this situation is that right translations  $t \rightarrow ts$ ,  $G \rightarrow G$  are all continuous and that  $\Lambda = \{a \in G \mid t \rightarrow st \text{ is continuous}\}$  ( $= \mathbb{T} \times \mathbb{Z}$  here) is dense in  $G$ . Noting that  $L_1 = \mathbb{T} \times \{1\} \triangleleft G$  and  $G/L_1 \cong E$ , we can define a measure  $\mu$  on  $G$  by  $\mu(f) = \int_{G/L_1} (\int_{L_1} f(st) dt) d\bar{s}$  ( $f \in C(G)$ ),  $dt$  and  $d\bar{s}$  indicating integration with respect to Haar measures on  $L_1$ , and  $E$ , respectively. Calculation shows that  $\mu$  is right invariant and unique as such, hence also left invariant. Our mild strengthening of Namioka's structure theorem for compact right topological groups  $G$  permits us to use the ideas above and his construction of a left invariant measure on  $G$  (which need not be unique) and demonstrate the existence of a (unique) invariant measure on  $G$ , accordingly called Haar measure.

## A. MUKHERJEA:

### Semigroups, random walks, and attractors

In this talk, we discuss attractors  $\mathcal{A}(x)$  defined by

$$\mathcal{A}(x) = \{y \in E \mid \Pr(W_n x \in N(y) \text{ i.o.}) > 0 \text{ for every open set } N(y) \ni y\}$$

where  $W_n = X_n X_{n-1} \cdots X_1$ , and  $(X_i)_{i=1}^\infty$  is an i.i.d. sequence of random variables in a given family of functions from a complete metric space  $E$  into itself. In particular, we consider several examples where  $E = (\mathbb{R}^d)^+$  or  $\mathbb{R}^d$ . The following result was used in the discussion: If  $E = (\mathbb{R}^d)^+$ , then for  $x = (x_1, x_2, \dots, x_d)$  with each  $x_i > 0$ ,

$$\mathcal{A}(x) = \{Bx \mid B \text{ is a recurrent state of the random walk } (W_n)\}.$$

## N. OBATA:

### Isometric Operators between $L^1$ -algebras of hypergroups

In this talk an analogue of Wendel's theorem for hypergroups and its application to characterization of locally compact groups are discussed. For a hypergroup  $K$  (always assumed to have a left invariant measure) we denote by  $\tilde{K}$  the set of all characters  $\chi$  of  $K$ , namely, (i)  $\chi : K \rightarrow \mathbb{T}$  continuous; (ii)  $\chi(x * y) = \chi(x)\chi(y)$ . For two hypergroups  $K_1$  and  $K_2$  a map  $\tau : K_1 \rightarrow K_2$  is called (hypergroup) isomorphism if (i)  $\tau$  is a homeomorphism; (ii)  $\tau_*(\delta_x * \delta_y) = \delta_{\tau(x)} * \delta_{\tau(y)}$ . If  $\omega_1$  and  $\omega_2$  are left invariant measures of  $K_1$  and  $K_2$ , respectively,  $\tau_*\omega_1 = c\omega_2$  for some  $c > 0$ .

*Theorem.* Let  $K_1$  and  $K_2$  be hypergroups and,  $\mathcal{A}_1$  and  $\mathcal{A}_2$  be subalgebras of  $M(K_1)$  and  $M(K_2)$ , respectively, such that  $L^1(K_i) \subset \mathcal{A}_i \subset M(K_i)$ ,  $i = 1, 2$ . Assume  $T : \mathcal{A}_1 \rightarrow \mathcal{A}_2$  to be an isometric algebra-isomorphism. Then, there exist an isomorphism  $\tau : K_1 \rightarrow K_2$  and a character  $\chi \in \tilde{K}_2$  such that  $T\mu = \chi \cdot \tau_*\mu$ ,  $\mu \in M(K_1)$ .

If we take  $\mathcal{A}_i = L^1(K_i)$ , we get immediately an analogue of Wendel's theorem for hypergroups. As a simple application we obtain the

*Proposition.* For  $x \in K$  we define  $\phi_x \in B(L^1(K))$  by  $\phi_x f = \delta_x * f * \delta_x^\vee$ . If  $\phi_x$  is an isometric operator for all  $x \in K$ , then  $K$  is a locally compact group.

## G. PAP:

### Rate of convergence in the central limit theorem on stratified groups

Let  $H$  be the Heisenberg group ( $\mathbb{R}^3$  with the product  $(x_1, x_2, x_3) \circ (y_1, y_2, y_3) = (x_1 + y_1, x_2 + y_2, x_3 + y_3 + \frac{1}{2}(x_1 y_2 - x_2 y_1))$ ). Let  $\delta_t : H \rightarrow H$  be defined by  $\delta_t(x) = (tx_1, tx_2, t^2 x_3)$  for  $t > 0$ ,  $x = (x_1, x_2, x_3) \in H$ . Let  $\rho(x) = (x_1^4 + x_2^4 + x_3^2)^{\frac{1}{4}}$  for  $x \in H$  and  $B(a, r) := \{x \in H : \rho(a^{-1}x) < r\}$  for  $a \in H$ ,  $r > 0$ . We obtain the following result for the speed of convergence in CLT on  $H$ .

*Theorem.* There is a constant  $c > 0$  such that one has for all  $\mu \in \mathcal{P}(H)$  with

$$(M1) \int_H x_i \mu(dx) = 0, \quad i = 1, 2, 3, \quad (M2) \int_H x_i x_j \mu(dx) = \delta_{ij}, \quad i, j = 1, 2,$$

$$(M3) \int_H x_i^6 \mu(dx) < \infty, \quad i = 2, 3,$$

and all  $n \geq 4$ ,  $a \in H$ ,  $\tau > 0$  the inequality

$$|\delta_{1/\sqrt{n}} \mu^n(B(a, \tau)) - \nu(B(a, \tau))| \leq c K(a, \tau) (M_3(\mu) + M_6(\mu)) n^{-1/2},$$

where  $K(a, \tau) = (1 + \min\{\rho(a), \tau\})(1 + \frac{1+\rho(a)}{\tau})$  and  $\nu \in \mathcal{P}(H)$  is the "standard" Gaussian measure on  $H$  and  $M_k(\mu) = \int_H (\rho(x))^k \mu(dx)$ ,  $k \in \mathbb{N}$ .

The generalization of this theorem is obtained for the class of *stratified* groups defined as follows:

Let  $G$  be a simply connected, nilpotent Lie group.  $G$  is called *graded*, if its Lie algebra  $\mathcal{G}$  has the subvector space decomposition  $\mathcal{G} = \bigoplus_{j=1}^s V_j$  such that  $[V_i, V_j] \subset V_{i+j}$  for  $i + j \leq s$ , and  $[V_i, V_j] = \{0\}$  for  $i + j > s$ .  $G$  is called *stratified*, if in addition  $V_1$  generates the whole  $\mathcal{G}$  as an algebra.

The reasons of the nonuniformity of the above estimate are discussed.

## M. PICARDELLO:

### Martin boundaries of products of Markov chains

Let  $P_i$  be stochastic transition operators on state spaces  $X_i$ , ( $i = 1, 2$ ), transient and irreducible. For  $0 < \alpha < 1$ , let  $R_\alpha = \alpha(P_1 \otimes Id) + (1 - \alpha)(Id \otimes P_2)$  on  $Z = X_1 \times X_2$ . For every  $t \geq \rho(R_\alpha)$  (the spectral radius of  $R_\alpha$ , equal to  $\alpha\rho(P_1) + (1 - \alpha)\rho(P_2)$ ) there exist positive eigenfunctions of  $R_\alpha$  with eigenvalue  $t$ . Denote by  $\mathcal{E}(R_\alpha, t)$  the minimal positive  $t$ -eigenfunctions. Moreover, let  $I_t$  be the segment  $I_t = \{(r_1, r_2) \in \mathbb{R}^2 : r_i \geq \rho(P_i), \alpha r_1 + (1 - \alpha)r_2 = t\}$ . We show that

$$\mathcal{E}(R_\alpha, t) = \bigcup_{I_t} \mathcal{E}(P_1, r_1) \otimes \mathcal{E}(P_2, r_2)$$

and determine the topology of the minimal part of the Martin boundary. This complements (and completes) results of Molchanov for the operator  $P_1 \otimes P_2$ .

## P. RESSEL:

### Semigroups in probability theory

A survey is given on the applications of harmonic analysis on (Abelian) semigroups to problems in probability theory. The main tool is the generalized Fourier-Laplace transform, generalizing the classical Fourier transform, Laplace transform, generating function, distribution function, and also infinite dimensional versions thereof. A very general De Finetti/Schoenberg-type theorem is presented from which many

representations of symmetric probability distributions are derived; these include new results in mixtures of non-homogeneous product measures. Finally a classical inequality of Hoeffding is extended to general semigroups, showing for example a rather peculiar different behaviour of the actual variance and the mean empirical variance of partial sums of independent variables.

## H. RINDLER:

### Weak containment properties and tall groups

If  $G$  is a group acting on a measure space  $(X, \mu)$ ,  $\mathcal{H}$  is the orthogonal complement of the  $G$ -invariant functions in  $L^2(X, \mu)$ ,  $\pi$  the regular representation of  $G$  on  $\mathcal{H}$ . If  $1 < \pi$  (i.e. if  $\pi$  weakly contains the trivial representation) then  $G$  behaves similar to simple or commuting transformations. If  $1 \not< \pi$ ,  $G$  has very strong ergodic properties.

Chou stated in Trans. AMS, 317, 229-253 (1990) that, if  $G_k$  are pairwise nonisomorphic simple groups, then  $G = \prod_{k=1}^{\infty} G_k$  is a tall group (i.e. there exist only finitely many inequivalent irreducible representations of degree  $n$  for all  $n \in \mathbb{N}$ ) and  $1 < \pi$  ( $\pi$  acts on  $L_0^2(X) = \{f \in L^2, \int f dx = 0\}$ ,  $dx$  denotes the Haar-measure). Lemma 3.3 and Prop. 3.4 in that paper are wrong. A counter example is given by  $G = \prod_{p \in \mathbb{P}} \text{PSL}(n, \mathbb{F}_p)$ ,  $n \geq 3$ ,  $\mathbb{P}$  set of primes,  $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$ . The statement of Chou, remains valid for finite simple groups  $G_k$  and suitable positive integers  $n_k$  ( $G = \prod_{k=1}^{\infty} G_k^{n_k}$ ). As a consequence there exist compact groups that do not have the dual Bohr approximation property. (Some problems with partial solutions are mentioned).

## I.Z. RUZSA:

### Infinite convolution of distributions on discrete commutative semigroups

We consider the convergence of an infinite convolution  $\mu_1 \mu_2 \mu_3 \dots$  of probability measures on a discrete commutative semigroup  $H$ . For a  $h \in H$  we say that this is stubbornly convergent at  $h$ , if for every sequence  $\lambda_1, \lambda_2, \lambda_3 \dots$  of measures such that  $\lambda_j = \mu_j$ , for all  $j < j_0$  the sequence  $(\lambda_1 \lambda_2 \dots \lambda_n)(\{h\})$  is convergent. A necessary and sufficient condition for stubborn convergence at  $h$  is given. A simple corollary is the following: if  $\liminf \mu_n(I) > 0$  for the set  $I = \{x \in H : x^2 = x\}$  of idempotents, then  $\nu_n = \mu_1 \mu_2 \dots \mu_n$  converges to some  $\nu$  vaguely. An application to the logarithmic density of certain sets of integers is found also.

R. SCHOTT:

### Constrained random walks

The basis of this talk is a joint work with G. Louchard (Université de Bruxelles). We consider random walks (i.e. diffusions processes) inside bounded domains of  $\mathbb{R}^n$ . The boundary is partly reflecting and partly absorbing. Such kind of domains permit to modelize the behaviour of some distributed algorithms (ex: the two stacks problem, the banker's algorithm etc.). We use techniques familiar for people working on random walks / diffusions on groups in order to give the explicit form of the hitting time and place of the absorbing boundary (the probability distributions are expressed in terms of the probability measure and of the size of the absorbing boundary). We show also how to obtain  $P_{g,g}^{(n)}$ , the probability that starting from the point  $g$  the random walk comes back to the same point in  $n \in \mathbb{N}$  steps. The distribution of the last leaving time (from  $g$ ) before absorption is also given. Finally we mention some open problems in this area and discuss the possibility of using conformal mapping techniques.

M. SCHÜRMAN:

### Unitary evolutions with $q$ -independent increments

Let  $q$  be a complex number,  $|q| = 1$ . The unitary processes with independent, stationary increments of [M. Sch, Probab. Th. Rd. Fields 84 (1990)] are generalized to a  $q$ -version ( $q = 1$  yields the old processes): the increments of disjoint intervals are no longer assumed to commute but to  $q$ -commute, i.e. commutators are replaced by  $q$ -commutators. Again the processes can be realized as solutions of quantum stochastic integral equations in the sense of [R.L. Hudson, K.R. Parthasarathy, Commun. Math. Phys. 93 (1984)]. The presence of a  $q$  result in a second quantization of multiplication by powers of  $\bar{q}$  factor in the corresponding integral equation. The result can be generalized to quantum stochastic processes on  $q$ -\*-bialgebras [M. Sch., Preprint, Heidelberg 1990].

A.L. SCHWARTZ:

### Polynomial hypergroups and generalizations

There are possibly two hypergroups associated with orthogonal polynomials  $\{P_n\}_{n=0}^\infty$ . Formally; for sequences  $a$ ,  $b$ , and  $c$  the implication  $a * b = c \Leftrightarrow (\sum_k a_k P_k)(\sum_k b_k P_k) = \sum c_n P_n$  may define a discrete polynomial hypergroup, and if  $\alpha$  is the measure of orthogonality normalized to mass 1, the implication  $f * g = h \Leftrightarrow (\int f P_n d\alpha)(\int g P_n d\alpha) = \int h P_n d\alpha$  may define a continuous polynomial hypergroup.

There are many examples of the discrete ones, and new ones are continually being

discovered. If  $M(N_0)$  is a Banach algebra with respect to  $*$ , necessary and sufficient conditions are given that the convolution arises as above from orthogonal polynomials. One consequence is that there can be no polynomial hypergroups when the measure has unbounded support.

Of the continuous variety, it is shown that the only possibilities are those associated with the Jacobi polynomials.

But, this class can be generalized to Jacobi type hypergroups which include all hypergroups on a compact interval known to the speaker, the recent results on Fourier-Bessel series of Markett and the eigenfunctions of the perturbed ultraspherical equations due to the speaker, Markett and Connett.

**P.M. SOARDI:**

### Random walks on the edge graph of a tiling of the plane

Suppose that  $\mathcal{T} = \{T\}$  is a tiling of  $\mathbb{R}^2$  such that  $\mathcal{T}$  is locally finite, every tile has at most  $N$  edges, the diameters of the tiles are uniformly bounded and the following holds: There is a  $K > 0$  such that  $R_T \leq r_T$  for all  $T \in \mathcal{T}$ , where  $R_T$  and  $r_T$  are the circumradius and the inradius of the tile  $T$ . Call such a tiling quasi-normal. The simple random walk on the edge graph of  $\mathcal{T}$  is defined by assigning for each vertex equal probability of moving in every direction. Then:

*Theorem:* The simple random walk in the edge graph of a quasi-normal tiling is recurrent.

The proof is obtained by using the properties of the extremal length of a set of paths and properties of the Dirichlet spaces.

**K. TRIMÈCHE:**

### Permutation operators and central limit theorem associated with partial differential operators

We consider the partial differential operators:

$$\Delta_1 = \frac{\partial}{\partial x}, \quad \Delta_2 = \frac{\partial^2}{\partial r^2} + \frac{2\alpha + 1}{r} \frac{\partial}{\partial r} - \frac{\partial^2}{\partial x^2}$$

with  $(r, x) \in ]0, +\infty[ \times \mathbb{R}$  and  $\alpha \in \mathbb{R}, \alpha \geq 0$ .

In this work we determine permutation operators of  $\Delta_1, \Delta_2$  into  $\Delta_1, \frac{\partial^2}{\partial r^2}$ . Next we study a harmonic analysis and we prove a central limit theorem associated with the operators  $\Delta_1, \Delta_2$ .

**T.C. VREM:**

**The  $L^p$ -conjecture for hypergroups**

$L^p$ -conjecture: Let  $K$  be a l.c. hypergroup. If  $L^p * L^p \subset L^p$  for some  $1 < p < \infty$  then  $K$  is compact.

In 1988 S. Saeki proved the  $L^p$ -conjecture for an arbitrary locally compact group. The result also holds for certain classes of l.c. hypergroups and various values of  $p$ .

1. If  $K$  is discrete then the  $L^p$ -conjecture holds for  $p > 2$  and if  $K$  is also Abelian then the  $L^p$ -conjecture holds for all  $1 < p < \infty$ .
2. If  $K$  is Abelian then the  $L^p$ -conjecture holds for all  $p \geq 2$ .
3. If  $K$  is a  $Z$ -hypergroup with compact maximal subgroup then  $K$  satisfies the  $L^p$ -conjecture for all  $p \geq 2$ .
4. If  $K$  is a hypergroup join where  $K = H \vee I$ ,  $H$  compact and  $I$  discrete then  $K$  satisfies the  $L^p$ -conjecture for all  $1 < p < \infty$  if  $I$  is Abelian and  $K$  satisfies the  $L^p$ -conjecture for  $p > 2$  if  $I$  is not Abelian.

**M.E. WALTER:**

**Some remarks on finite groups**

If  $G$  and  $H$  are locally compact groups and

$$P_1(G) = \{p : G \rightarrow \mathbb{C} \mid p \text{ is continuous, positive definite, } p(e) \leq 1\},$$

$P_1(H)$  similarly defined, we have the following

*Theorem.*  $P_1(G) \cong P_1(H) \Leftrightarrow H \cong G$  where the first isomorphism is one of ordered convex semigroups (under pointwise operations  $+$  and  $\cdot$ ).

Two groups with nonisomorphic enveloping  $C^*$ -algebras are said to have different *orientations*. This concept is useful in partially classifying finite groups.

**W. WOESS:**

**Behavior at infinity and harmonic functions for random walks on graphs**

Let  $(X, E)$  be a locally finite, connected, infinite graph, and  $\{Z_n\}$  a random walk with state space  $X$  and transition matrix  $P$ , in some sense adapted to the graph structure (minimal requirement: all states communicate).

The problems addressed here are the following:

- A) Type problem: Is  $\{Z_n\}$  recurrent or transient.
- B) Suppose that  $\{Z_n\}$  is transient. How does  $Z_n$  tend to infinity? I.e., consider a "natural" compactification  $\bar{X}$  of  $X$ . Is it true that  $Z_n \rightarrow Z_\infty$  a.s., where  $Z_\infty$  is a random variable on the boundary  $\partial X = \bar{X} \setminus X$ ?

- C) In the situation of B), is the Dirichlet problem solvable with respect to the given compactification? I.e., does every continuous function on  $\partial X$  extend continuously to a function on  $\bar{X}$  which is harmonic on  $X$  with respect to  $P$ ?
- D) How good is our compactification? (Observe that "good" means "big", such that B) still admits positive answer.)
- D.1) Does  $\partial X$  give the Poisson boundary, which serves for determining all bounded  $P$ -harmonic functions?
- D.2) Does  $\partial X$  even coincide with the Martin boundary, which serves for the description of all positive  $P$ -harmonic functions?

We discuss these questions and their answers in several selected cases, such as integer lattices, trees and hyperbolic graphs.

## K. YLINEN:

### Some group-related polymeasures with applications

Let  $X_1, \dots, X_n$  be locally compact Hausdorff spaces. A bounded  $n$ -linear form  $P : C_0(X_1) \times \dots \times C_0(X_n) \rightarrow \mathbb{C}$  is called a polymeasure. A noncommutative polymeasure  $P : A_1 \times \dots \times A_n \rightarrow \mathbb{C}$  involves general  $C^*$ -algebras  $A_1, \dots, A_n$ . It is well known that  $M(A_1 \otimes \dots \otimes A_n) \subset CB(A_1, \dots, A_n) \subset PM(A_1, \dots, A_n)$  where  $PM(A_1, \dots, A_n)$  is the space of noncommutative polymeasures,  $CB(A_1, \dots, A_n)$  is the space of completely bounded multilinear forms in the sense of Christensen and Sinclair, and  $M(A_1 \otimes \dots \otimes A_n)$  is the space of linear forms continuous with respect to the maximal  $C^*$ -norm and identified with the corresponding  $n$ -linear forms on  $A_1 \times \dots \times A_n$ . The talk is concerned with construction of polymeasures essentially using techniques of harmonic analysis, showing that the above inclusions are in nontrivial situations proper (except in the case governed by the Grothendieck inequality). Applications to random fields on locally compact groups are indicated.

## A. ZEMPLÉNI:

### Counterexamples concerning Hun semigroups

Ruzsa and Székely introduced the notion of Hun semigroups as a generalization of Kendall's Delphic semigroups. They proved the analog of Hincin's decomposition theorem in arbitrary Hun semigroups and the infinite divisibility of antiirreducibles for normable ones.

We investigated  $D(\mathbb{R}^2, V)$ , the maximum semigroup of independent,  $\mathbb{R}^2$ -valued random variables. By the help of Balkema-Resnick's characterization of max-infinitely divisible distributions and by a sufficient condition for  $F \in D(\mathbb{R}^2, V)$  being antiirreducible we showed that in this structure not all antiirreducible elements are

infinitely divisible.  $D(\mathbb{R}^2, V)$  is normable but not Hun. This shows the relevance of this property in algebraic probability theory.

The stability of Hun semigroups was introduced by Ruzsa and Székely as well. They showed that this notion ensures a lot of "nice" properties of the semigroups. An other direction of the use of stability was my theorem concerning the Hun property of  $D(S)$  where  $S$  is a stable Hun semigroup. I gave an example which showed that without the stability this statement is not true.

**Hm. ZEUNER:**

### Invariance principles for one-dimensional hypergroups

Let  $(K, \star)$  be either a polynomial hypergroup (on  $\mathbb{N}$ ) or a Sturm-Liouville hypergroup on  $\mathbb{R}_+$  with  $A$  being the density of the Haar measure. Consider an i.i.d. sequence of  $K$ -valued random variables  $X_n$  and the corresponding random walk  $(S_n)_n$  defined by the randomized sums  $S_n := \Lambda \sum_{j=1}^n X_j$ . We suppose that the generalized variance  $\sigma^2$  of  $X_j$  is finite and define  $\mu := E_\star(X_1)$  as the generalized expectation. Then we have the following results:

*Theorem.* If  $\alpha := \lim x \frac{A'(x)}{A(x)}$  exists then the càdlàg processes  $(\frac{S_{[nt]}}{\sqrt{n}} : t \geq 0)$  converge in probability towards the Bessel process  $(Y_{\sigma^2 t} : t \geq 0)$  with parameter  $\alpha + 1$  (in the Skorohod space  $D[0, \infty[)$ ).

A similar result holds for polynomial hypergroups with  $n[\epsilon_n \star \epsilon_1(n+1) - \epsilon_n \star \epsilon_1(n-1)]$  converging under mild assumptions.

*Theorem.* Suppose that  $\frac{A'(x)}{A(x)} = 2\varrho + o(\frac{1}{\sqrt{x}})$  as  $x \rightarrow \infty$  with  $\varrho > 0$ . Then the processes  $(\frac{S_{[nt]} - [nt]\mu}{\sqrt{n}} : t \geq 0)$  converge in probability towards the Brownian motion  $(B_{\sigma^2 t} : t \geq 0)$ .

**V.M. ZOLOTAREV:**

### Limit theorems for linear models as stability theorems

At the present time there are some generalizations of the classical theory of limit theorems for  $\mathbb{R}^n$ ,  $H$  and some Abelian groups and semigroups. They are based on the condition of independence of "summands" only. In all of these theories criteria of convergence of distributions of "sums" in double array schemes have a universal form, so there is a reason to think that this form of convergence criteria will be a universal one for all Abelian groups in which characteristic functions or their analogues can be introduced. The criteria of convergence have an equivalent form in the theory of stability for the corresponding problem of characterization of distributions. Perhaps this phenomenon is also a universal one at least for some classes of Abelian groups.

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