MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 51/1990

Stochastische Approximation und Optimierungsprobleme in der Statistik

25.11 bis 1.12.1990

Die Tagung fand unter der Leitung von G. Pflug (Wien) und H. Walk (Stuttgart) statt, gleichzeitig mit der Tagung 'Lineare Modelle und multivariate statistische Verfahren'. Mit den Organisatoren der Paralleltagung (O. Krafft, E. Sonnemann, H. Drygas) wurde deswegen vereinbart, daß gemeinsame Plenarvorträge angeboten werden, um die Teilnehmer der einen Tagung mit neueren Resultaten aus dem anderen Gebiet vertraut zu machen. Plenarvorträge hielten G. Styan, R. J-B Wets, V. Fabian, B.K. Sinha und H. Kushner. Insgesamt wurden in der Tagung 'Stochastische Approximation und Optimierungsprobleme in der Statistik' 24 Vorträge abgehalten, davon 3 Plenarvorträge. Der Schwerpunkt lag einerseits auf 'Averaging'-Methoden für stochastische Approximation und andererseits auf Anwendungen rekursiver Verfahren in der statistischen Schätztheorie. Auch dem Sequentialaspekt rekursiver Verfahren waren einige Vorträge gewidmet.

E. Berger

Some remarks concerning the finite sample behaviour of the Robbins-Monro procedure

The talk deals with the possiblity of deriving probability estimates concerning the finite sample behaviour of the Robbins-Monro procedure under "classical" conditions. The starting point of the considerations is a decomposition of the domain of the regression function into three sets:

1. an "outer" region, where only a lower bound for the absolute value of the regression function is available,
2. an "intermediate" region, where the regression function is quasi-linear,
3. a small "inner" region (about the unknown point θ, where the regression function takes the value 0), where the regression function is virtually linear.

The behaviour of the process, when it approaches θ is separately analyzed in these three regions. The essential tools are exponential estimates for real- and vector-valued martingales (e.g., analogues of the Bennett-Hoeffding inequality). These considerations can be utilized to establish estimates for deriving upper bounds of probabilities of the form $P\{X_n \geq \varepsilon\}$ and for estimating the rate of convergence in the central limit theorem.

C. Bouton

Gaussian approximation for stochastic algorithms with Markovian dynamic

We study the problem of gaussian approximation for stochastic algorithm of the type:

$$\vartheta_{n+1} = \vartheta_n - \gamma_{n+1} f(\vartheta_n, y_{n+1})$$

with markovian dynamic, that means, such that:

$$L(Y_{n+1} | \sigma(\vartheta_i, y_i); \quad i \leq n) = \pi_{\vartheta_n}(y_n, \cdot)$$

where $\{\pi_\vartheta, \vartheta\}$ is a family of Markov chains depending on $\vartheta \in \mathbb{R}^d$. We define processes $U_t = \frac{\vartheta_n - \vartheta^*}{\sqrt{\gamma_n}}$, if $t_n \leq t < t_{n+1}$ where $t_n = \sum_{i=1}^n \gamma_i$, and prove that in the case of convergence of the algorithm to ϑ^* the translated processes $U_t^N = U_{t+t_N}$ converge in law to a gaussian diffusion. In the case of a constant step γ, we define a "mean differential equation" $\frac{d\overline{\vartheta}_t}{dt} = -h(\overline{\vartheta}_t)$, using a hypothesis of existence of an invariant probability for Π_ϑ, and we study the processes $U_t^\gamma = \frac{\vartheta_t - \bar{\vartheta}_t}{\sqrt{\gamma}}$ where $\vartheta_t = \vartheta_n$ if $n\gamma \leq t < (n+1)\gamma$. We prove the convergence in law of U_t^γ to a gaussian diffusion. (References: Métivier, Priouret-Benveniste, Kushner.)

J. Dippon

Asymptotic Confidence Regions in Banach Spaces

In the theory of stochastic approximation in Banach spaces quite often the Gaussian random variable

$$G = \int_{(0,1]} u^{A-I} dW(u)$$

occurs as weak limit of a sequence (Z_n). We propose two methods to estimate the distribution of $||G||$ in order to obtain asymptotic confidence regions for the sequence (Z_n). The first method requires estimation of the covariance structure of the Brownian motion W, the second generates a sequence of converging empirical distribution functions of approximations of G.

V. Dupač

Stochastic approximation with delayed observations

The Robbins-Monro and the Kiefer-Wolfowitz stochastic approximation procedures are considered, where at each time-point $n = 1, 2, \ldots$ one experiment is to be performed, but at the same time, the results of experiments may become known only after a random time delay. Two possible ways are investigated, how to handle with delays: In the first one, experiments are allocated into K parallel series that are either closed or open and at any time instant an experiment is made in one of the open series, if there is such one. The global approximation to the sought point θ is obtained as an average of approximations in all K series. The efficiency of the procedure with respect to the procedure without delays is found by means of the apparatus of the queuing theory. The other possibility is to use averages of observations that have become known during the time-interval $[n, n+1)$ to correct the n^{th} approximation to the $n+1^{st}$ one. Efficiencies of both approaches are compared.

V. Fabian and J. Dippon

A stochastic approximation method estimating a point of a global minimum of a function

Stochastic approximation methods for locating a point of minimum of a function have been studied since a paper by Kiefer and Wolfowitz in 1952. An open problem since then was how to obtain the convergence if, in addition to the point ϑ of minimum, there are possible other points with vanishing derivative. A solution is offered by constructing a method that is a combination of an L_∞ regression estimated and a stochastic approximation method. The new method has properties of the stochastic approximation method (and some more) even without the assumption that the derivative vanishes only at ϑ.

L. Györfi

Good news and bad news in distribution estimation

Let $T(\mu, \gamma)$ be the variational distance of the probability measures μ and γ. Given $X_1, X_2, \ldots, X_n$ i.i.d. random variables with absolutely continuous distribution μ, there is a distribution estimate μ_n the (integral of the histogram) for which

$$T(\mu, \mu_n) \to 0 \qquad a.s.$$

If μ can be arbitrary then for all estimate μ_n there is a μ such that

$$\inf_n T(\mu, \mu_n) \geq \frac{1}{4} \qquad a.s.$$

If the nonatomic part of μ is dominated by a known σ-finite measure then it is possible to construct an estimate μ_n for which

$$T(\mu, \mu_n) \to 0 \qquad a.s.$$

U. Herkenrath

Recursive smoothing procedures for adjusting forecasts to means of random observations

We consider the situation that for a sequence of random observations (X_n) sequentially forecasts (W_n) have to be made. W_n should be a good estimate for the mean of X_n, but moreover the evolution of the sequence (W_n) should be smooth in some sense. As example we regard the case, where the (X_n) represent claim variables and (W_n) the corresponding net insurance premiums. We propose different types of smoothing procedures u for the construction of the (W_n) by means of the recursive formula $W_{n+1} = u(W_n, X_n)$. Convergence properties of the sequence (W_n) are studied. The procedure of exponential smoothing is covered by our schemes. In addition to the usual case of i.i.d. random observations (X_n) a dependence of the distribution of X_n from W_n is admitted. Special examples of smoothing functions are discussed.

A. Irle

Minimax results for games of stopping

Two-person zero-sum games of the following form are considered. Player I chooses a stopping time and player II a distrubution of an underlying stochastic process. The pay-off is given by the expected value of a stopped stochastic process where stopping occurs according to the strategy of player I and expectation is taken with respect to the strategy of player II. Different concepts of randomization for player I are discussed and their equivalence is shown. Furthermore, minimax results for such games are derived.

A. King

Asymptotics for non-smooth M-estimation and stochastic programming

In stochastic programs and certain M-estimation problems, the physical requirements of the model and/or the specification of the loss function are such as to introduce significant technical complications. One may need to treat cases where solutions lie on boundaries or at points of non-differentiability. I discuss two asymptotical arguments one leading to asymptotic distributions for solutions, the other to asymptotic confidence estimates; both derive from the asymptotic normality of the sample means of random continuous functions. The first, developed jointly with R.T. Rockafellar, generalizes the classical pattern that proceeds from the "normal equations" to the asymptotics via a generalized implicit function theorem for nonsmooth generalized equations. The results apply to $\mathbf{C}^1$ (but not necessarily $\mathbf{C}^2$) loss functions subjected to constraints, and yield explicit representations of the (conical-normal) asymptotic distributions. The second argument proves confidence intervals from information gained in the course of an algorithm based on cutting lower-envelope approximations (e.g. cutting planes); as the number of samples increases the asymptotic normality of the objective approximations provides the asymptotics for the solutions through the geometry of the lower approximating envelope approximation.

F. Konecny

Optimal filtering and parameter estimation of Poisson cluster processes

We consider a class of Neyman-Scott processes which can be represented as doubly stochastic Poisson processes with a shot-noise process as intensity. By a measure transformation and the application of the moment generating function we split the problem of recursive computation of the optimal filter in the integration of a simple p.d.e. of first order and updating at the times of observed events. In some cases the filter can be written down explicitly. In the case of unknown intensity parameters we obtain also a representation of the likelihood functional for the case of partial observations.

H.J. Kushner

On the Polyak-Juditsky stochastic approximation algorithm

Consider the stochastic approximation
$(*)X_{n+1} = X_n + a_n f(X_n, \xi_n)$, $\quad \overline{X}_n = \frac{1}{n}\sum_1^n X_i$, $\quad a_n = \frac{A}{n^\gamma}$, $\quad \gamma \in (0,1)$.
P - J proved that the rate of convergence of $\overline{X}_n$ is equal to the "optimal" rate for (*) for $\gamma = 1$. The proofs were by direct construction in special cases, and represent a breakthrough in Stochastic Approximation. More general results are possible. Define $U_n = X_n/\sqrt{a_n}$, and $U^n(\cdot)$ by $U^n(0) = U_n$,

$$U^n(t) = U_i \qquad \text{on} \quad \left[\sum_{j=0}^{i-1} a_{j+n}, \sum_{j=0}^{i} a_{j+n}\right]$$

Under quite broad conditions $U^n(\cdot)$ converges weakly to the stationary solution to $dU = -HU dt + R_0^{1/2} dW$, where $-H$ is a stable matrix. Then $\frac{1}{\sqrt{ta_n}} \sum_n^{n+t/a_n} X_i \to N(0, H^{-1}R_0H'^{-1} + O(e^{-\lambda t}))$ for some $\lambda > 0$.

H.J. Kushner

The Monte-Carlo optimization of nonlinear stochastic systems of high dimension

Let $X(\cdot)$ satisfy $(*)dX = (b(X(\cdot)), t, \alpha)dt + \sigma(X(\cdot), t)dW$, and given the cost function $E(C(X(\cdot), \alpha)) = V(\alpha)$. Three methods for getting unbiased estimators of $V_\alpha(\alpha_0)$ are discussed: Finite differences; mean square derivative; derivative of likelihood ratio. Each has its own advantages, with the latter method being preferable for many nonlinear and high dimensional cases. Computable approximations to (*) are discussed (discrete time, Markov chain), and unbiased estimators of the derivatives of the cost functions for these approximations derivated. Those approximations converge to unbiased estimators of $V_\alpha(\alpha_0)$.

V. Mammitzsch

Optimal kernels and orthogonal series expansion

Kernel type estimators (and statistics) occur in various branches of statistics. We present a list of problems in

regression analysis (Gasser & Müller 1979)
density estimation (Reiss 1981, Falk 1983)
stochastic approximation (Walk 1985)
hypothesis testing (Nadaraya 1985)
average derivative estimation (Härdle, Hart, Marron, Tsybakov 1989)
hazard rate estimation (Wang 1990+ε)

and indicate how these problems may be solved by means of orthogonal series expansion.

K. Marti

Semi-stochastic approximation by response surface methods

Stochastic approximation methods, e.g. stochastic gradient procedures, can be accelerated considerably by using improved step directions (deterministic descent directions or improved gradient estimations) at certain iteration points. More exact gradient estimations are obtained here by the Response Surface Method (RSM). Using estimates of the objective function F at certain (design-)points in a certain neighborhood of the last iteration point, first estimates $\hat{F}$ of first and second order of F are computed by fitting first and/or second order polynomials to F. The accuracy of this estimator is studied and the convergence behavior of the resulting semi-stochastic approximation procedure is considered.

A. Pechtl

Arithmetic means in stochastic approximation

The talk deals with an invariance principle concerning arithmetic means won by the observations U_n of a recursion formula of the type

$$U_{n+1} = (I - \frac{A}{n^\alpha})U_n + \frac{1}{n^\alpha}V_n.$$

Furthermore an application of the result to the Robbins-Monro process is presented.
The invariance principle is introduced in the following form:

$U_{n+1} = (I - \frac{1}{n^\alpha}A)U_n + \frac{1}{n^\alpha}V_n$; $n \in \mathbb{N}$, $0 < \alpha < 1$; $\mathbb{H}$ real separable Hilbert space; $A : \mathbb{H} \to \mathbb{H}$ bounded linear operator; U_1, $V_n\,(n \in \mathbb{N})$ $\mathbb{H}$-valued random variables with V_n MDS with respect to an isotonic family of sub-σ-fields $(\mathcal{F}_n)_{n\in\mathbb{N}}$ and satisfying the conditions

(1) ~~$\frac{1}{n}\sum_{i=1}^{n} E(\|V_i\|^2|\mathcal{F}_{i-1}) \xrightarrow{P}$ trace S (S should be an S-operator)~~

(2) $\forall\,\delta \in \mathbb{R}^+; \forall x \in \mathbb{H} : \frac{1}{n}\sum_{i=1}^{n} E(\langle V_i, x\rangle^2\chi[|\langle V_i, x\rangle| \geq \delta\sqrt{n}]|\mathcal{F}_{i-1}) \xrightarrow{P} 0$

(3) $\forall\, t \in [0,1]; \forall x \in \mathbb{H} : \frac{1}{n}\sum_{i=1}^{[nt]} E(\langle V_i, x\rangle^2|\mathcal{F}_{i-1}) \xrightarrow{P} t\langle Sx, x\rangle$; $n \to \infty$.

Then $(Z_n)_{n\in\mathbb{N}}$ in $C_{\mathbb{H}}[0,1]$ defined by $Z_n(t) := \int_0^t \sqrt{n}U_{[ns]+1}ds$ converges in distribution to $A^{-1}W$, where W is the Wiener-Process in $\mathbb{H}$ with covariance operator S.

G. Ch. Pflug

The "simulated annealing" aspect of stochastic approximation

The "simulated annealing" aspect of S.A. consists in considering the Markovian process

$$(*)\qquad X_{n+1}^a = X_n^a - af(X_n^a) - a^\varepsilon W_n, \qquad 1/2 \leq \varepsilon \leq 1,$$

studying its ergodic behavior (the coefficient of ergodicity), its invariant laws and their week limits.
Two methods of identifying the invariant laws are presented. One consists in defining an order structure of measures and to find bounds for the invariant law in terms of known distributions. The other takes the limits as $a \to 0$ as approximants. It is shown that the invariant law of $(*)$ is close to the distribution ν with density

$$\text{const.}\ \exp\left(-\frac{F(x)}{a^{2\varepsilon-1}}\right).$$

In the case $\varepsilon < 1$, $F'(x) = f(x)/\sigma^2$, where $\sigma^2 = \mathrm{Var}(W_n)$, whereas, if $\varepsilon = 1$, $F'(x) = f(x)/(\sigma^2 + f^2(x))$.
An example is given, where the procedure $(*)$ converges to different points, if we choose $\varepsilon = 1$ or if we choose $\varepsilon < 1$.

B. Polyak

Acceleration of stochastic approximation by averaging

The idea of averaging along a trajectory of a stochastic approximation procedure is not new. However, all attempts to improve the rate of convergence via this approach were unsuccessful. The proposed method is based on the idea which seems to be unnatural – the process under averaging should be a slow one. For this method almost sure convergence is proved, and mean square rate of convergence is obtained. Asymptotic normality results are also considered. Such extended stochastic approximation procedure has the highest possible rate of convergence; it is simple and reliable. Applications to identification problems are considered.

A. G. Ramm

Random fields estimation theory

Let $D \subset R^2$ be a bounded region with a smooth boundary Γ. Let $u(x) = s(x) + n(x)$, where $s(x)$ and $n(x)$ are random fields, $s(x)$ is a useful signal, $n(x)$ is noise. Suppose that $\overline{n(x)} = \overline{s(x)} = 0$, $\overline{u^*(x)u(y)} = R(x,y)$, $f(x,y) = \overline{u^*(x)s(y)}$.
Given $R(x,y)$ and $f(x,y)$, and the observations of $u(x)$ in D one wishes to find a linear estimate

$$Lu := \int_D h(x,y)u(y)dy$$

which is an optimal estimate of $s(x)$ (or some operator $(As)(x)$) in the sense

$$\overline{|Lu - s(x)|^2} = \min . \tag{1}$$

A class of the kernels $R(x,y)$ is introduced for which (1) is solved analytically. The contents of the lecture is presented in the monograph A. G. Ramm, Random fields estimation theory, Longman/Wiley, N.Y., 1990.

J. Renz

Rate of convergence in central limit theorems and of asymptotic confidence intervals in stochastic approximation

In the context of the Fabian scheme which is given by the recursion formula

$$X_{n+1} - \theta = (1 - \frac{1}{n}A_n)(X_n - \theta) + n^{-\frac{1+\beta}{2}}B_nV_n + n^{-(1+\frac{\beta}{2})}T_n \ (n \in \mathbb{N}) \text{ with real r.v.}$$

the rate of convergence to the asymptotic level $1-\alpha$ for $d \to 0$ of asymptotic fixed-width confidence intervals $[X_{N(d)} - d,\ X_{N(d)} + d]$ for the unknown quantity θ in connection with a Sielken type stopping time $N(d) = \inf\{n \in \mathbb{N} | \hat{\sigma}_n \le n^\beta d^2/u^2_{\alpha/2}\}$ and an a.s. estimator $\hat{\sigma}^2_n$ of the variance $\hat{\sigma}^2$ in the central limit theorem is discussed. The main result of this discussion concerns the rate of convergence in the central limit theorem for the randomly stopped Fabian scheme in a martingale setting and e.g. states the best obtainable rate in the case of arithmetic means under a moment condition of order $3 + \varepsilon$. One example of the classical and one of the modified Robbins Monro method are investigated.

Further the estimator $\hat{\sigma}_n^2 = \frac{1}{\ln n} \sum_{i=1}^{n} i^{\beta-1}(X_{i+1} - \frac{1}{\gamma_n} \sum_{j=1}^{n} j^{\beta-1} X_{j+1})^2$ with $\gamma_n = \sum_{j=1}^{n} j^{\beta-1}$ is introduced.

M. Schäl

Estimation and control: maximum-likelihood estimation and the Howard algorithm

The paper is concerned with a Markov decision model with finite state space and compact action space where the law of motion depends on an unknown parameter ϑ. It is assumed that under each ϑ the Markov decision model is communicating. The Certainty Equivalence Principle (CEP) proposes: At time n behave as if the estimated parameter $\hat{\vartheta}_n$ were the true parameter. As estimation procedure, the MLE is studied here. Then the CEP is used for the Howard algorithm: At each time n do the evaluation and the policy-improvement step within the model belonging to the estimated parameter. It is shown that the twofold problem to optimize and to collect information about ϑ can be solved in an annealing-like way. As time n goes on, choose the action (control) with increasing probabilities according to the CEP and with decreasing probabilities according to a kind of uniform distribution on the action space.

N. Schmitz

Games against a prophet

Prophet theory is concerned with comparisons of the functionals

$$M(X_1, X_2, \ldots) := E(\sup_{i \in \mathbb{N}} X_i); \quad M(X_1, \ldots, X_n) := E(\max_{1 \leq i \leq n} X_i)$$

and

$$V(X_1, X_2, \ldots) := \sup\{EX_\tau : \tau \in T\}; \quad V(X_1, \ldots, X_n) := \sup\{EX_\tau : \tau \in T^n\}$$

where T and T^n are the sets of stopping rules with infinite and finite horizon resp. The difference and the ratio comparison allow a game-theoretical interpretation as Bayes-risks for games of a prophet against a statistician; the famous prophet inequalities of Krengel/Sucherton and Hill/Kertz for the independent case yield minimax-values for these games. But it turns out that the games are not strictly determined; so mixed strategies are considered. For the ratio and the difference case saddle-point results are proved for the independent case as well as for the general case yielding minimax strategies for the prophet and for the statistician; the value is n for the ratio case and $((n-1)/n)^n$ for the difference case.

R. Schwabe

On the rate of convergence in the Robbins-Monro process

We consider the one-dimensional Robbins-Monro stochastic approximation process $X_n = X_{n-1} - a_n(f(X_{n-1}) - U_n)$ for estimating the root of an unknown function f. Our aim is to clarify the fact that this recursive scheme can essentially be decomposed in a weighted average of the error terms U_n and a slightly perturbed difference equation. Both parts can then be treated by the appropriate methods: deterministic stability theory for difference equations and convergence results for weighted averages. With these methods the rate of convergence in the Robbins-Monro process can be obtained for any sequence of steplengths satisfying the natural conditions $a_n \to 0$, $\sum_{n=1}^{\infty} a_n = \infty$. In particular this can explain the structural change in the asymptotic behaviour in dependence on the steplengths. The results can readily be extended to higher dimensions and adaptive procedures.

H. Walk

Almost sure convergence of stochastic approximation processes under ergodicity assumptions

Two generalizations of the Widrow algorithm – of Robbins-Monro type – for recursive estimation of a solution of a linear equation in a Banach space under ergodicity assumptions are investigated. The first concerns the treatment of a nonlinear equation in view of global convergence by the averaging method using an argument of Sanchez-Palencia (1975; Sanders and Verhulst 1985). The second concerns a recursion for learning processes with partial foregetting in linear models with forecast feedback (Kottmann, Mohr 1988–90), for which a loglog invariance principle is formulated via a stochastic functional differential equation.

R. J-B Wets

Estimation and optimization

Constrained estimation, in particular constrained estimation of the parameters of a regression model is used as an example to illustrate the development of a general theory for the analysis of consistency and asymptotics for problems of estimation and stochastic optimization that involve nondifferentiable loss functions and constraints on the choice of the estimator.

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