MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 53/1990

Multigrid Methods

2.12. bis 8.12.1990

Die Tagung fand unter der Leitung von D. Braess (Bochum), W. Hackbusch (Kiel) und U. Trottenberg (Köln/Bonn) statt. Es nahmen 46 Personen teil, darunter Wissenschaftler aus Bulgarien, Chile, England, Israel, Niederlande, Tschechoslowakei, UdSSR und USA.

Nachdem 1984 und 1987 schon einmal Tagungen in Oberwolfach zum Thema "Mehrgitterverfahren" stattgefunden haben, war dies die dritte Gelegenheit, Wissenschaftler aus dem Umfeld der Mehrgitterverfahren an diesem einzigartigen Forschungsinstitut zusammenzubringen.

Die Vorträge behandelten Mehrgitteranwendungen auf Aufgaben der physikalischen Praxis (Navier-Stokes-Gleichungen, Probleme aus der Elektromagnetik und der Elastizitätstheorie). Viel Raum nahmen algorithmische Techniken ein: Hierarchische Finite-Element-Techniken, tau-Extrapolation, Defektkorrektur, adaptive Techniken. Es ist besonders hervorzuheben, daß wiederum neue Ansätze vorgestellt wurden: schwachbesetzte Gitter, frequenzfilternde Zerlegungen und neue ILU-Varianten). Dies macht deutlich, daß das Thema "Mehrgitterverfahren" weiterhin innovative Tendenzen zeigt. Eng verbunden mit Mehrgitterverfahren ist das Thema "Gebietszerlegungensverfahren", wie durch mehrere Beiträge belegt wird. In einen ähnlichen Rahmen fallen die ebenfalls auf der Tagung vertretenen "algebraischen Mehrstufenmethoden".

Die Beiträge waren von lebhaften Diskussionen begleitet. Eine wesentliche Rolle spielten auch die vielen fruchtbaren Gespräche am Rande. Nicht zuletzt der ebenso unauffällige wie effektive Service trug zu der angenehmen Atmosphäre bei, wofür an dieser Stelle Herrn Prof. Dr. M. Barner und seinen Mitarbeitern ein herzlicher Dank ausgesprochen sei.

Die folgenden 29 Vortragsauszüge geben einen genaueren Überblick des Tagungsverlaufes. Die Anordnung entspricht der Vortragsreihenfolge.

Vortragsauszüge

R. BANK :

The Hierarchical Basis Multigrid Method for Convection Diffusion Equations

We study the solution of the convection-diffusion equation $-\Delta u + \beta\nabla u = f$, where β is "large". The problem is discretized using piecewise linear finite elements and standard upwinding stabilization (e.g. streamline diffusion). The equations are solved by the standard hierarchical basis multigrid method. Our goal is to derive estimates for the rate of convergence which demonstrate the dependence on the mesh size h, on the number of levels, and on the strength of the upwinding term.

A. REUSKEN :

Steplength optimization and linear multigrid methods

Recently steplength parameters have been used in multigrid methods. In nonlinear multigrid such a parameter takes care of damping and for a suitable class of problems the nonlinear multigrid method then converges (more) globally. Steplength optimization is used also in linear multigrid methods. In this talk we present some theoretically understanding of the effect of steplength optimization in linear multigrid methods. It is easy to see that multigrid with steplength optimization is equal to a preconditioned steepest descent method. The results we present concern the improvement of the basic multigrid method due to steplength optimization. In particular we have some quantitative results which show that you benefit most from the steplength optimization for errors for which the multigrid performance is worst. Modifications of classical multigrid methods using steplength parameters are presented.

U. LANGER :

Multigrid Preconditioners in the Dirichlet Domain Decomposition Method

We present a new approach to the construction of preconditioners via Dirichlet Domain Decomposition (DDD) techniques. These preconditioners B involving multigrid and modified Schur complement block preconditioning operators are well suited for the solution of the finite element equations $Ku = f$ on a multiprocessor computer system with local memories and message passing principle (e.g. transputer). Estimates of the relative spectral condition number $\kappa(B^{-1}K)$ are given. The numerical experiments performed show that the conjugate gradient method preconditioned by the DDD preconditioners presented seems to be very attractive in the parallel computation. Some of the results reported here will be published in two papers by G. Haase, U. Langer and A. Meyer (1990).

A. BRANDT :

Rigorous Quantitative Analysis of Multigrid

Exact numerical convergence factors for any multigrid cycle can be predicted by local mode (Fourier) analysis. For general linear elliptic PDE systems with piecewise smooth coefficients in general domains discretized by uniform grids, it is proved that, in the limit of small meshsizes, these predicted factors are indeed obtained, provided the cycle is supplemented with a proper processing at and near the boundaries. That processing, it is proved, costs negligible extra computer work. Apart from mode analysis, a Coarse Grid Approximation (CGA) condition is introduced which is both necessary and sufficient for the multigrid algorithm to work properly. Various error norms and their relations to the orders of the intergrid transfer operators are analyzed. Global mode analysis, required to supplement the local analysis in various border cases, is developed. Partial relaxation sweeps are systematically introduced into both analysis and practice.

E. STEIN, W. RUST :

Transfer Matrices in the multigrid solution of FE approximation for Timoshenko and Reissner plates in bending

At first, linear shape functions for the displacements and the slopes of the cross–sections are investigated. Locking of thin structures is avoided by reduced integration of the shear terms in the stiffness matrix. Using interpolations of the shape functions for the prolongation matrix, the convergence factors of multigrid solvers become very bad for thin beams. A smoothing–adapted transfer matrix is gained both by using eigenmodes of the local stiffness matrix as well as by partial solution of the FE–equations for a fine grid node. Additionally, a reduced shear stiffness can be derived analytically, so that good MG–convergence holds for all limit cases.
Another FE–discretization is obtained by the Allman–type C^0– continuous hierachical linear and quadratic shape functions for the displacements. The fully integrated stiffness matrix is equal to the above reduced–integrated matrix. The prolongation following from these shape functions is the same as shear modified transfers in the first discretization concept. Reduced fine and coarse grid shear stiffnesses are necessary in order to get optimal approximation properties and MG–convergence in the whole range from thin to moderately thick beams and plates. A couple of numerical results for a Mindlin plate show excellent MG–convergence independent of the mesh size.

C. ZENGER :

The solution of elliptic differential equations on sparse grids

If rectangular grids for the solution of elliptic differential equations are used the aspect ratio of the meshsize in the coordinate directions is usually fixed. In this paper it is shown that under suitable regularity assumptions some linear combination of solutions corresponding to rectangular grids with varying aspect ratios needs much less unknowns in comparison to the conventional approach if we assume in both cases the some accuracy. For very small grids sizes the gain can be very high: If the conventional approach needs $0(n^d)$ unknowns for a certain accuracy the new approach needs only $o(n^{(1+\epsilon)})$ unknowns for every $\epsilon > 0$.

J. E. PASCIAK :

Convergence Estimates for Multigrid Algorithms without Regularity Assumptions

A new technique for proving rate of convergence estimates of multigrid algorithms for symmetric positive definite problems will be discussed in this talk. The standard multigrid theory requires a "regularity and approximation" assumption. In contrast, the new theory discussed requires only an easy verified approximation assumption. This leads to convergence results for multigrid refinement applications, problems with irregular coefficients, and problems whose coefficients have large jumps. In addition, this new theory shows why it suffices to smooth only in the regions where new nodes are being added in multigrid refinement applications.

O. WIDLUND :

Remarks on Schwarz Algorithms and Certain Multigrid Methods

Additive and multiplicative Schwarz algorithms have recently become a focus of research into iterative methods for solving large linear systems of equations, which arise when elliptic problems are discretized. Among these algorithms are different domain decomposition methods, iterative refinement methods and certain multigrid methods. In this talk, we first give a general introduction to the Schwarz algorithms and the tools available to analyse their rates of convergence. A family of nonsymmetric elliptic problems, which also can have eigenvalues in the left half plane is considered. Results recently obtained with Xiao-Chuan Cai are described, which show that many of the results previously known only for the positive definite, symmetric case can be extended to a larger family of elliptic problems.

C. WEYAND :

Multigrid method for Reissner-Mindlin plates

A multigrid algorithm for the numerical solution of the elliptic system arising from the Mindlin–Reissner formulation of the plate bending problem is presented. The convergence of a multigrid method using a standard second order finite difference discretization strongly depends on the ratio between the plate thickness t and the mesh size h, and deteriorates significantly for small t. This phenomenon is caused by the so-called locking effect. A straightforward implementation of a standard multigrid method shows that due to the locking the coarse grid correction fails. To overcome this difficulty the system of partial differential equations is reformulated by introducing two additional variables related to the transverse shear. A multigrid algorithm using a finite difference discretization on a staggered grid, a box relaxation scheme, and standard operators for inter–grid transfers is used to solve this system. Numerical tests with an implementation of this algorithm confirm that the asymptotic convergence rate of the method is bounded away from 1 independently of the parameter t. For $t \to 0$ about 0.4 is an upper bound for a W–cycle with two pre– and one postrelaxations.

U. RÜDE :

On tau-Extrapolation in Multilevel Methods

Tau-extrapolation is a technique to raise the consistency order by an implicit truncation error extrapolation within a multilevel method. It is related to Richardson-extrapolation but is more general in several respects. While Richardson-extrapolation provides the improved solution on the coarsest level, an iterative application of tau-extrapolation will deliver an improved solution on the finest level. In this case the solution on the coarser levels may even converge to different solutions. These coarse grid solutions are defined by the restriction operators. Furthermore Richardson extrapolation is based on global asymptotic error expansions for the solution and thus does not only depend on the regularity of the solution but also on a certain uniformity of the meshes. In contrast, tau-extrapolation is shown to be applicable on general finite element meshes. This analysis is based on a strictly local analysis in which the numerical quadrature for evaluating the stiffness matrix is improved by extrapolation. This is done element by element for the finite element basis functions. Thus global properties of the solution or of the mesh are not required.

P. VASSILEVSKI :

Algebraic Multilevel Iterative (AMLI-) Methods

We present a generalized V-cycle multilevel scheme for the construction of preconditioners for solving finite element elliptic equations on a sequence of meshes. The technique is based on a fundamental estimate in energy norm of the restriction operator defined by nodal interpolation. The growth of the relative condition number of the pure V-cycle preconditioner is stabilized by proper polynomial corrections which are performed at every level whose number is proportional to a given integer parameter k_0. The thus derived hybrid (generalized) V-cycle preconditioner is shown to be of optimal order for properly chosen parameters for both 2-d and 3-d problem domains. The corresponding condition number remains bounded with respect to possible jumps of the coefficients of the bilinear form as long as they are continuous within the elements from the initial triangulation. A number of generalizations are outlined including the case of nonsymmetric and indefinite bilinear elliptic forms. Some numerical illustrations are given.

M.-C. RIVARA :

GEMA-3D: A basis to develop 3D adaptive/multigrid finite element software

A 3d refinement algorithm for tetrahedral meshes is presented and discussed. An efficient data structure has been designed and used to implement a basic prototype. An empirical study of the reduction of solid angle size due repeated subdivisions has shown the algorithm is in practice a powerful and reliable tool for mesh refinement. The potentiality of the software in the context of adaptive finite-element methods and multigrid algorithms is also discussed.

T.W. FOGWELL :

Time dependent problems with discontinuous coefficients

Z. DOSTAL :

Projector preconditioning in the solution of problems with unilateral constraints

We present an algorithm for the solution of quadratic programming problems $\frac{1}{2}x^T Kx - b^T x \rightarrow \min$ for $x \in B, B = \{x : B^T x \leq c\}$ where K is positive semidefinit and B is a sparse full rank matrix. In particular, we show how the standard conjugate gradient method for solution of quadratic programming problems may be modified so that the minimum on the subspace V which is defined by current active constraints may be reduced to minimization on the intersection of V with the orthogonal complement of some auxiliary subspace U. With a special choice of U, the method becomes a variant of the domain decomposition algorithm for the solution of variational inequalities, whose particular advantage is that it uses the same Schur complement for all subspaces V. We discuss also the efficiency of the method and give a numerical example.

Yu.A. KUZNETSOV :

Algebraic Multigrid Methods with Tchebyshev Iterative Procedures

The theory of algebraic multigrid methods with inner recursively imbedded Tchebyshev iterative procedures was developed in 1987-1989. In this contribution we present a new algebraic multigrid/substructering method for a positive definite Helmholtz operator. A new preconditioner is introduced in such a way that a coarse grid modified finite-difference Helmholtz operator has twice less the condition number then in case of typical two-grid procedures. As the result we construct the Algebraic Multigrid/Substructuring preconditioner B with inner Tchebyshev iterative procedures which satisfies to the following conditions:

a.) B is spectrally equivalent to the original finest-grid Helmholtz operator;

b.) the arithmetic cost of the solution procedure $g \rightarrow B^{-1}g$ is bounded from above by cN, where c is a positiv constant independent of h and N is the number of unknowns;

c.) the coarsest-grid step size h_c is proportional to the value $\sqrt[5]{h_f^3}$, where h_f is the step size of the finest grid.

O. McBRYAN :

Rigorous Convergence Rates for the PSMG Multigrid Method

The PSMG method is a totally parallel multilevel method designed for massively parallel computers, and is most efficient if there are as many processors as fine grid points. Unlike standard multigrid, which incurs low parallelism on coarse grids, PSMG keeps all processors busy even on the coarsest grids. This is accomplished by solving multiple coarse grid problems simultaneously and in SIMD fashion. These coarse grid solutions are then combined using an interpolation procedure to produce a fine grid solution which is more accurate then any of the individual coarse grid corrections. The resulting method therefore converges faster then standard multigrid methods.
In the talk we will describe the PSMG method, prove an upper bound on convergence rates, and in fact we show that for constant coefficient problems we can exactly compute convergence rates to any prescribed accuracy. We then show convergence rates as

fast as 0.00165 for one iteration (V-cycle, 1 relaxation). Finally we present the normalized cost of the method, measured by parallel work required per digit of convergence. PSMG is shown to be four times faster then the best red-black standard multigrid (assuming as many processors as grid points).

U. RISCH :

Multigrid Methods for the Incompressible Navier-Stokes Equations

We described a multigrid algorithm for solving the stationary incompressible Navier-Stokes equations. The method is characterized by

- FE discretization using the nonconforming P1/P0 element,
- upwind discretization of the convection term,
- FAS or linear MGM,
- smoothing by blockwise Gauss-Seidel iteration,
- construction of restriction and prolongation by L^2 projection.

For large Reynolds numbers a variable control of some components of the MG algorithm is useful.

F. SCHIEWECK :

Comparison of a linear and nonlinear multigrid method for solving the incompressible Navier-Stokes equations

For solving the stationary incompressible Navier-Stokes equations we compare two methods - a nonlinear multigrid method and a linear multigrid method within a linearization process. We present some numerical experiences on the convergence and computational costs of both methods.

P. WESSELING :

Multigrid solution of the Boussinesq equations

The Boussinesq equations describe buoyancy driven flow of viscous fluids with temperature differences. A situation in which bifurcation of the solutions occur is considered. It is found that the bifurcation can be computed if the convective terms are approximated by central differences, but no bifurcation is found with upwind differences. The flow is computed with a multigrid method coupled with defect correction. Fourier smoothing analysis for a number of smoothing methods is reviewed. Robustness, efficiency, parallelizability and applicability to nonlinear systems of equations are discussed. Provided damping with a problem-independent parameter is used, a number of simple methods are robust, efficient and parallelizable.

J. LINDEN :

Multigrid Solution of 2d incompressible Navier-Stokes equations on block-structured meshes

The program package LISS has been developed for solving (systems of) PDE's in general 2d domain on parallel computer of MIMD-type. It realizes standard multigrid methods for curvilinear quadrilateral, vertex-based meshes. As an example for the use of LISS the case of the incompressible Navier-Stokes equations is considered. Here, the discretization is based on a standard finite volume technique for the viscous terms and on flux-difference splitting of Roe-type for the convective part. Results are presented for both multigrid convergence and parallel efficiency (measured on iPSC2). In addition, the behaviour of defect correction for improving the accuracy to 2nd order will be discussed for the case of Boussinesq flow in closed cavities.

M. JUNG :

Multi-level algorithms for the computation of electro-magnetic fields and thermomechanical problems

At first we consider the computation of electro-magnetic fields in electric motors. Because of the complicated interface structure in electric motors, it is in general impossible to construct an initial coarse triangulation consisting only of a few triangles and at the same time approximating all interfaces sufficiently precisely. Therefore we allow the interfaces to cross the triangles of the coarsest triangulation. We describe a hierachical mesh generator, which produces a sequence of triangular meshes, where the finest triangulation approximates the interfaces. In the multigrid algorithm we use the linear interpolation I_q^{q+1}. The restriction I_{q+1}^q is defined as usual, i.e. $I_{q+1}^q = (I_q^{q+1})^T$, and the matrices K_q on the auxiliary grids are calculated by the so-called Galerkin relation $K_q = I_{q+1}^q K_{q+1} I_q^{q+1}$, $q = l-1, l-2, \ldots, 1$. We compare various multilevel algorithms (multigrid methods, multigrid preconditioned conjugate gradient methods, algorithms based on hierachical bases) by numerical examples. In the second part of the lecture we discuss the computation of the temperature , displacement and stress fields arising from thermical and mechanical processes in the upper part of the piston of a combustion engine.

D. BRAESS :

A multigrid method for an equilibrium problem in a chaotic dynamical system

We treat a problem which arises in the analysis of a dynamical system. Particles are distributed on an $L \times L$-pattern. The particles hops from one cell to its neighbours. The hopping rates are random numbers and define a (nonsymmetric) stochastic matrix. The nonnegative eigenvector yields the equilibrium. Its computation by classical power iteration is very time consuming. Therefore, we apply the multilevel idea. 4 cells are combined to a block which is a cell of a coarse grid. The trivial injection induces a transfer such that the coarse grid matrix is stochastic again. Although the efficiency is not as good as for elliptic problems, factors of 80 and more in CPU-time are gained. The difference is understood as the effect of barriers and traps which are found in diffusion in chaotic systems.

G. WITTUM :

A new class of fast solvers for large systems of linear equations

We present a new class of fast solvers based on a special sequence of incomplete decompositions, the so-called frequency-filtering decompositions. The corresponding smoothing correction method is based on the multi-grid idea, i.e. successively filtering out certain frequencies from the error, without using coarse grids. Thus there are no basic problems with robustness as in multi-grid. The corresponding method has a asymptotic complexity of $O(n \log n)$, on grids of intermediate size, however, it is quite efficient and competes quite well with multi-grid. After presenting the algorithms we give a convergence proof and finally several examples on the performance of the new method applied to linear and non-linear equations.

J. BURMEISTER :

Incomplete LU-decomposition based on solving "Least-Squares-Problems"

A new strategy was introduced to construct an incomplete LU-decomposition of a given nonsingular matrix A. The method is based on solving overdetermined systems of equations via the normal equations. For M-matrices it was outlined that the incomplete LU-decomposition exist; in addition a stability improvement was proved. The corresponding iteration scheme converges for arbitrary initial guess. The application to 5-point-formulas are discussed. In this case the computational work simplifies to the calculation of recurrence equations. The usefullness of the iteration scheme as a smoother in a multigrid solver is answered positively by proving the smoothing property for a large class of problems.

T. DREYER :

A Single Step Variant of the Incomplete Matrix Factorization ILU

as Smoother in Nonlinear Multigrid We start from Brown's method - an iterative method for general systems of nonlinear equations. We give a new formulation in order to combine it with the idea of incomplete factorization. The result is a single step variant of ILU suited for nonlinear equations. We call it MBM (Modified Brown's Method). For the linear case one can show that MBM (and also the original method) are equivalent to a combination of ILU (or a complete factorization, resp.) providing a transformation and Gauß-Seidel's iteration, applied to the transformed system. Like other iterative methods, MBM cannot treat the Navier-Stokes equations directly. But the notions of distributive relaxation (A. Brandt, N. Dinar) and of transforming smoothers (G. Wittum) provide a framework. The equations are discretized by difference methods on staggered grids.

H. BLUM :

Defect correction for finite elements

Stable finite element schemes in blockstructured, piecewise uniform meshes are known to admit asymptotic expansions of the discretization error with respect to the mesh size parameters. This provides the theoretical basis for a posteriori increasing the accuracy of low order schemes by defect correction or extrapolation methods. It can be shown that the theory extends to several mixed and nonconforming discretizations arising from structural and fluid mechanics. Here, the standard defect evaluation frequently has to be modified by filter operations and (or) boundary corrections in order to remove local nonsmooth error contributions. The efficiency of this approach is demonstrated by results from several numerical test calculations.

W. RUST and E. STEIN :

Convergence Acceleration in Nonlinear FE-Calculation
of Structural Mechanics Problems

Two multigrid methods, Newton multigrid (NewtonMG) and nonlinear multigrid (NLMG), the latter with nonlinear smoothers of the Jacobi or Gauß-Seidel (GS) type, are implemented as solvers within the multi-purpose computer program INA-SP for nonlinear FE analysis of structural mechanics problems. Essential convergence accelerations are possible by a step-length control, a cg-step and two Quasi-Newton based methods. Simple line search procedures are discussed and shown to be effective. Numerical examples are given, consisting in plane stress and plain strain elasticity and shallow shell calculations. In the most cases the convergence factors obtained by NewtonMG and

GS-smoothed NLMG are similar and nearly constant in a wide range of the load level. Close to the critical load the Quasi-Newton based methods are longer stable than the cg-algorithm, if the step-length control is not exact. Few steps of the Pegasus line-search are sufficient to reach higher accuracy.

W. HACKBUSCH :

Convergence of the frequency decomposition method

The frequency decomposition multigrid method consists of four different coarse–grid corrections combined with a single smoothing step. The coarse–grid correction uses different coarse–grids and non–standard prolongations and restrictions. Here we consider the algorithm without any smoothing. The arising iteration can be regarded as an additive Schwarz iteration, for which sufficient conditions for convergence are known in the positive definite case. The technique is explained by which these sufficient conditions can be shown. The set of admissible indices consists of a convex linear combination of discretizations of $\partial_{xx}, \partial_{yy}$ and I. It turns out that the method converges uniformly w.r.t. the grid size and the set of indices. In particular, the method proves to be robust w.r.t anisotropic problems.

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