MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Mathematische Logik

16.12 bis 22.12.1990

Die Tagung fand unter der Leitung von W.Felscher (Tübingen), H.Schwichtenberg (München) und A.S.Troelstra (Amsterdam) statt. Im Mittelpunkt des Interesses standen Fragen aus dem breiten Gebiet der Mathematischen Logik mit Schwerpunkten in der Beweistheorie, Rekursionstheorie, Intuitionistischen u. Konstruktiven Methoden und Anwendungen in der Informatik.

VORTRAGSAUSZÜGE

M.ABRUSCI:

Non Commutative Linear Logic.

We introduce "noncommutative" linear logic, i.e. linear logic without the exchange rule. We give the sequent calculus, we prove cut elimination theorem, we give the semantics for noncommutative linear logic, and we prove the soundness and completeness theorem. Finally, we discuss open problems, perspectives, applications.

Z.ADAMOWICZ:

The End Extension Problem.

We find a Π_1 sentence τ which is a form of the consistency of $I\Delta_0$ and show a model of $I\Delta_0+\Omega_1+\tau+B\Sigma_1$ which has no proper end–extension to a model of $I\Delta_0+\Omega_1+\tau$. This is a kind of elimination of the "Exp" assumption in the WILKIE-PARIS construction of a model of $I\Delta_0+\mathrm{Exp}+B\Sigma_1$ with no proper end–extension to a model of $I\Delta_0+\mathrm{Exp}$.

K.AMBOS-SPIES:

Undecidability and 1-types in the Recursively Enumerable Degrees.

In the last decade some global results on the first order theory $Th(R;\leq)$ of the partial ordering of the r.e. degrees have been proved using the $0'''$-priority method (also called "monster method"): E.g. in 1982 HARRINGTON and SHELAH have announced that $Th(R;\leq)$ is undecidable (still unpublished) and AMBOS-SPIES and SOARE have shown that $Th(R;\leq)$ has infinitely many 1-types (published in 1989). Here we describe some joint work with R.A.SHORE, which opens a new approach to these questions. It allows us to obtain the above results by simpler means, namely by the $0'$-priority method (infinite injury method). The key idea is to code finite partial orderings into $(R;\leq)$ by naximal branches of branching degrees. Our coding schema yields, besides undecidability, not only ω many 1-types, as AMBOS-SPIES and SOARE's original approach, but in fact continuously many. Moreover, since our technique combines with the permitting method, we obtain corresponding results for the theory of any nontrivial principal ideal of $(R;\leq)$. Furthermore, it seems that our technique can be combined with a technique introduced by SLAMAN for proving that the branching degrees are dense in $(R;\leq)$ to get undecidability and continuously many 1-degrees for the theory of any nontrivial interval of $(R;\leq)$. This, however, requires a $0'''$-argument.

E.BÖRGER:

Eine logische Semantik für die Gesamtsprache PROLOG.

I report on recent developments of my work on formal specification of logic programming systems using dynamic algebras (recently introduced by Y.GUREVICH and applied by him to Modula-2 (Proc. CLS'87) and Occam (Proc. CLS'89)), see my papers in MFCS'90, CLS'89, Berkeley MSRI workshop on Logic from Computer Science '89. In particular I discuss the extension of my PROLOG algebras to PROLOG III algebras (joint paper with P.SCHMITT, CLS'90), to the Warren abstract machine (joint paper with D.ROSENZWEIG, CLS'90), to a PROLOG machine (paper in preparation with C.BEIERLE) and to PROLOG algebras (paper in preparation with E.RICCOBENE). I discuss the relation to denotational semantics (starting from a joint paper in preparation with N.NORTH).

C.CALUDE:

Recursive Baire Classification, Speedable Functions and Independence Statements.

Using recursive variants of Baire notions of nowhere dense and meager sets in some recursive topological spaces we study the 'size' of infinitely often speedable functions and sets, in a machine-independent framework. A characteriza- tion of the measure-independent speed-ups and a study of the 'size' of speedable points in both undecidable and decidable theories is also presented. Finally, some 'large' sets of independent statements connected with the infinitely often speed-up phenomenon are displayed.

1. Topologies on subsets of partial recursive functions.
2. Speedable/hard functions and sets.
3. Prefix topology.
4. Infinitely often speed-ups.
5. Independent statements.

A.CANTINI: *Property Theory Variants of Frege Structures.*
The present work is experimantal in nature: we test theories of self-referential truth, which should be helpful in discovering natural and interesting reformulations of Frege's naive abstraction principle (recent applications: logic for knowledge representation, semantics for natural languages). In particular, we concentrate upon a system BT_0 which inccludes simple schemata on the truth predicate T and which supports Frege's principle in the form: $T(\forall x(x \in \{u:A\}) \leftrightarrow TA(u/x))$ (A arbitrary, $x \in y := T(yx)$, TA is shortening for $T(A^\frown)$, $A^\frown$ being a canonical term representing A). The truth predicate T can be given a provability intertpretation by means of a suitable cut free infinitary calculus (or, alternatively, by means of a variant of supervaluation semantics). In contrast to related systems by FEFERMAN, ACZEL et al., BT_0 can define $\mathbb{N}$ and prove full number theoretic induction; indeed, BT_0 yields an inner model for PA. This feature can be generalized and leads to simple theories of truth of higher level BT_n, whose union has the same arithmetical theorems as $ID_{<\omega}(\mathcal{O})$.

J.H.GALLIER:

Strong normalization and Kripke Structures in the Coquand-Huet Theory of Constructions.

We present a fairly simple proof of strong normalization for COQUAND/HUET theory of constructions. The proof does not use infinite contexts. Instead, it uses a kind of higher-order Kripke structure.

R.O.GANDY:

Sequentiality in Higher Types.

R.PLATEK in his thesis laid down the structure of the hereditarily monotonic functions at all finite types, and taking the natural numbers + 'undefined' as type 0, gave a definition of (partial) recursive function at all types, based on the taking of fixed points. Later S.C.KLEENE in a series of papers gave a more datailed treatment, and elaborated the way in which such functionals can be 'computed'. We limit attention to continuous monotone functionals, and then 'recursive' is equivalent 'computable in the language $\mathscr{L}$CF'. It is known that some functions (e.g. 'parallel on' which are 'computable' in the sense of SCOTT and ERSCHOV are not recursive in the above sense. We are loooking for, and believe we have found a more or less extensional notion 'recursively sequential' which will be equivalent to 'recursive'; at least for simple examples this enables one to decide whether a given function is or os not recursive. It should also provide a 'fully abstract model' for $\mathscr{L}$CF. The basic idea (analogous to Kleene's oracles) is to consider e.g. the computation of $\Gamma^3(F^2)$ as proceeding by a dialogue or game between 'the examiner' Γ and 'the candidate' F. They take it in turns to 'move'; a move consists either of asking or answering a question, subject to certain restrictions. If either refuses to, or if the game never terminates, then $\Gamma(F)$ is undefined. G.PANI has shown (at all types) that Recursive ⇒ Recursively Sequential. The converse holds for type 2, and I believe will soon be proved for type 3 and then for higher types.

L.GORDEEV:

Logic of Three Variables.

While analyzing a recent publication of A.TARSKI and S.GIVANT: *A formalization of set theory without variables*, AMS CP, 41 (1987), I isolate its crucial ideas concerning logical formalism of (only) three variables. I notice an important disadvantage of the authors' Equation Calculus (EC) implementation of this formalism which is caused by the very nature of EC: the familiar noneliminabilty of the transitivity rule (Tr):

$$\frac{A = C \qquad C = B}{A = B}$$

For, when trying to deduce $A = B$ via (Tr), it is generally impossible to pick a desired C by any reasonable algirithm. That is to say, EC has no good proof search algorithm. I present a direct proof theoretical treatment of the logic of three variables in form of a suitable cut free Reduction/Term-rewriting Calculus. The resulting 'Universal Logic Calculus' Θ_U (whose language contains four binary predicates, three variables and one individual constant) is simpler than virtually all known formalizations of Predicate Calculus because the notion of provability in Θ_U employs only a small (finite) number of trivial substitutions (the provability in familiar Predicate Calculi must necessarily use/produce all possible substitutions of terms for infinitely many distinct variables). As being cut free by nature, Θ_U has a reasonable proof search algorithm. Moreover, any problem (sentence) S in Predicate Calculus can be polynomially translated into the language of Θ_U such that S is valid iff its translation T(S) is derivable in Θ_U.

P.HAJEK:

Some Applications of the Low Basis Theorem in Arithmetic.

A strong form of the Low Basis Theorem is proved in $I\Sigma_1$ (the fragment of Peano arithmetic with induction for Σ_1–formulas). An ω–interpretation of WKL_0 (the fragment of second–order arithmetic with weak Koenig lemma) in $I\Sigma_1$ as well as interpretation of $B\Sigma_2$ (collection for Σ_2–formulas) in $I\Sigma_1$ is exhibited.

E.HERRMANN:

Recursively Enumerable Sets, Lattice Properties, and General Properties.

Inside the class of recursively enumerable (r.e.) sets there can be defined many important relations. Two of them are the usual set–inclusion and the many–one reducibility. In the talk there are compared both relations (inside the class of r.e. sets). In particular the sets m–equivalent to the maximal sets are considered, also the 1–degrees inside an m–degree with a maximal set, and other results in this respect.

J.HUDELMAIER:

Cut Elimination in Intuitionistic Propositional Logic.

The usual presentation of the intuitionistic propositional logic, formalized in Gentzen's calculus LJ, poses two problems which are easily solved for the corresponding classical calculus:

1. The cut elimination procedure is nonelementary.
2. There is no bound on the length of deductions.

Problem 1 is solved by introducing a suitable cut degree and a new reduction operator which at each step reduces this cut degree. Problem 2 is solved by presenting a new calculus which is derived from LJ but has the property that the length of every deduction depends linearly on the size of its endsequent.

G.JÄGER:

Systems of Explicit Mathematics with Nonconstructive μ-Operator.

Systems of explicit mathematics were introduced by Feferman and provide axiomatic theories of operations and classes for the abstract development and proof–theoretic analysis of a variety of constructive and semi–constructive approaches to mathematics. In particular, two systems T_0 and T_1 were introduced where T_1 is obtained from T_0 by adding a single axiom for the non–constructive but predicatively acceptable quantification operator e_N over the natural numbers. Much precise proof–theoretic information was subsequently obtained about T_0 and various of its subsystems. Corresponding work on subsystems of T_1 has been slower to be achieved. This talk was concerned with weak theories of operations and numbers which may contain the unbounded μ–operator. The emphasis was put on two first order theories of this kind one with full induction and the other with induction restricted to (abstractly) decidable sets. A proof–theoretic analysis of both systems was given.

H.R.JERVELL:

Large Finite Sets.

For arbitrary well–founded homogeneous trees S we introduce the notion of an S–large finite set. We can then use recursion and induction on the homogeneous trees. This is used to transfer the usual proof of Ramsey theorem for natural numbers using double induction to a proof of a 'Ramsey theorem' for S–large finite sets generalizing the Ketonen–Soloway result.

References:

KETONEN, SOLOWAY: *Rapidly growing Ramsey functions*, Ann. of Math. (1981)
JERVELL: *Large finite sets*, Zeits. math. Log. Grundl. Math. (1985)

U.KOHLENBACH:

Proof Theoretical Analysis of Basic Proofs in Approximation Theory.

Functional interpretation combined with a pointwise variant of W.A.Howard's majorization of ineffective uniqueness proofs in analysis extracts a–priori–moduli of unicity. Applied to basic proofs in best approximation theory (Tchebycheff, Borel, dela Valee Poussin, Young, Rice, Natanson) this yields new uniform moduli of unicity, Lipschitz constants and moduli of continuity for best Tchebycheff approximation. Estimates obtained by D.BRIDGES (1980-1982) are significantly improved. We discuss how the numerical data and the extraction itself depend on the log. form in which premises (e.g. the alternation theorem) are used in different proofs. It is shown that proofs of $\Lambda V_{\leq}\Lambda$–lemmas (with "$V_{\leq}$" bounded by a compact set) are irrelevant for these data.

A.KUCERA:

Algorithmical Randomness.

1–random sets are defined equivalently either by a variant of Kolmogorov complexity or by means of "effective" measure. A–random sets (i.e. 1–random relatively to an oracle A) arise by an obvious relativization. It was an open question whether there exists a nonrecursive set A such that there is an A–random set B for which $A \leq_T B$. It was known that such A could not be itself 1–random, and also other restrictions on possible candidates were known. However, we can prove the following

Theorem. There is a nonrecursive recursively enumerable set A such that there is an A–random set B and A is recursive in B.

H.LUCKHARDT:

Complexity versus the Church-Rosser Property and Confluence.

There is a conflict between the proof complexity $\mathfrak{C}(\mathrm{T})$ of a computational theory T or even of fibres $\mathfrak{F}$ in T, the complexity $\mathfrak{C}(\triangleright)$ of Noetherian reductions $\triangleright$ in T, and the Church–Rosser (or the confluence) property of $\triangleright$. The following properties exclude each other:

(i) $\triangleright$ is Church–Rosser on T (or on $\mathfrak{F}$ in T).

(ii) $\mathfrak{C}(\triangleright)$ is weaker than $\mathfrak{C}(\mathrm{T})$ (in the sense that there is a suitable class $\mathfrak{C}$ of complexities such that $\mathfrak{C}(\triangleright)\in\mathfrak{C}$ and $\mathfrak{C}(\mathrm{T})\notin\mathfrak{C}$.

When $\triangleright$ satisfies certain additional properties the same is true for confluence. Conditions under which $\mathfrak{C}(\mathrm{T})$–decidability is equivalent to $\mathfrak{C}(\triangleright)$–complexity and the Church–Rosser (confluence) property of $\triangleright$ as well as applications to propositional logic, various λ–calculi and Presburger arithmetic are given.

J.R.MOSCHOVAKIS:

Lawlike, Lawless and Choice Sequences in Intuitionistic Analysis.

Under the assumption that a particular Δ^2_1 well ordered subclass of Baire space is countable, one can give a classical model for a substantial part of the theory of lawless sequences (cf. *Relative lawlessnes in intuitionistic analysis*, JSL 52, No1, 1987). The same assumption also leads to a natural realizability interpretation for a substantial part of the intuitionistic theory of lawlike, lawless and choice sequences, extending Kleene's FIM.

Y.N.MOSCHOVAKIS:

Sense and Denotation as Algorithm and Value.

We propose a mathematical modeling of Frege's sense (Sinn) for formal languages with reflection, which uses the recursive algorithmus: in fact, in the appropriate context, the sense of a sentence is identified with the (recursive, infinitary) algorithm which is meant (by the form of the sentence) to compute its truth–valie. The main mathematical result is that (at least) for the simple extension of Lower Predicate Calculus by reflection (recursion), sense identity is decidable but at least as hard as the graph isomorphism problem.

J.van OOSTEN:

Extension of Lifschitz' Realizability.

Lifschitz' realizability [LIFSCHITZ,1979] results from replacing in Kleene's definition the existential clause by: $e\,\underline{\mathrm{r}}\,\exists x A(x)$ iff $[e]\neq\emptyset$ & $\forall f\in[e]((f)_1\,\underline{\mathrm{r}}\,A((f)_0))$; here $[e]$ denotes the finite set $\{x\leq(e)_1\,|\,(e)_0\circ x\uparrow\}$ (I write $\circ$ for partial recursive application). The other clauses are the same as in Kleene's definition. In this talk the construction of an elementary topos is sketched which generalizes Lifschitz' realizability in the sense that for the natural number object in this topos, called Lif, exactly those sentences of arithmetic are valid as are realized in Lifschitz' sense. The construction is very analogous to that of the 'effective topos' of [HYLAND,1982]. I discuss some logical properties of Lif. It satisfies Markov's principle, Church's Thesis in the form

$(\forall f{:}\mathbb{N}\to\mathbb{N})\exists z\forall x\exists y(T(z,x,y)\wedge U(y)=f(x))$, and the Uniformity Principle. It does not satisfy AC_{00}: $\forall x\exists y A(x,y)\to(\exists f{:}\mathbb{N}\to\mathbb{N})\forall x A(x,f(x))$. As an application of the construction, a principle, due to F.RICHMAN, is discussed: $\forall^d X(\forall^d Y(X\subseteq Y\vee X\cap Y=\emptyset)\to\exists n(X\subseteq\{n\}))$, where $\forall^d$ means that the quantifier runs over the decidable subsets of $\mathbb{N}$. This principle (RP) is shown in [BLASS & SCEDROV,1986] to be not derivable in HAH, by the use of sheaf models. I show that the principle RP holds in the effective topos whereas it is refuted in Lif; showing that HAH+MP+CT is not sufficient to derive it (in fact, HAH+MP+CT+¬RP is consistent).

[BLASS & SCEDROV,1986]: *Small decidable sheaves*, JSL 51, 726–31
[LIFSCHITZ,1979]: CT_0 *is stronger than* $CT_0!$, Proc.AMS 73, 101–106
[HYLAND,1982]: *The effective topos*, The L.E.J.Brouwer Centenary Symposium, 165–216.

H.PFEIFFER:

Ordinal Notations Using Mahlo Ordinals.

Generalizing a notation system for ordinals, G.JÄGER gave in [2], a recursive notation system is established on the basis of a hierarchy $J\alpha$ of weakly inaccessible Mahlo numbers. This hierarchy is added to the hierarchy $I\alpha$ of inaccessible regular ordinals used already by JÄGER. Following W.BUCHHOLZ [1] and JÄGER [2], for both Mahlo and simply regular ordinals σ collapsing functions $\psi\sigma$ are defined such that for every Mahlo ordinal μ out of the hierarchy of the $J\alpha$, $\psi\mu\beta$ is a regular ordinal κ such that $I\kappa 0=\kappa$. For these regular κ again collapsing functions $\psi\kappa$ are defined, which have strongly critical non–regular values. To get a proper order into the collapsing procedure, a pair of ordinals is associated to each pair (σ,α), and the definition of $\psi\sigma\alpha$ is given by recursion on a suitable well–ordering of these pairs. Thus a fairly large system of ordinals can be established. It seems rather straightforward how to extend this setting further.

[1] W.BUCHHOLZ: *A new system of proof-theoretic ordinal functions*, APAL 32, 195–207 (1986)
[2] G.JÄGER: *ρ-accessible ordinals, collapsing functions, and a recursive notation system*, Arch.Math.Log.Grundl. 24, 49–62 (1984)

P.PUDLAK:

Bounded Arithmetic and Complexity of Computations.

We give a survey of witnessing theorems proved for fragments S_2^i, T_2^i of Bounded Arithmetic. We show a uniform way of proving them. The proof is based on an extension of BUSS's concept of witnessing sequents in a Gentzen proof. Different theorems are obtained by different strategies in handling the induction rules.

M.RATHJEN:

Ordinal Analysis: Recent Results.

In the late 70's JÄGER initiated proof–theoretical investigations of suitable subsystems of set theory. As a main tool he used partial cut–elimination in systems of ramidied set theory with infinitary rules. Unfortunately, the known cut elimination procedures only enable one to treat varoius sorts of Π_2–reflection. We have developed a new cut elimination technique along with a strong system of ordinal representation which is sufficient to carry out the ordinal analysis of theories KP+Π_n–reflection for arbitrary $n\in\mathbb{N}$. Here KP stands for Kripke–Platek set theory with infinity; Π_n–reflection denotes the schema $A\to\exists x((x\ \textit{is admissible})\wedge A^x)$, where A^x arises from A by restricting any unbounded quantifier by x.

G.RENARDEL de LAVALETTE:

Fragments of Intuitionistic Propositional Logic.

This talk reports on research done in collaboration with D.deJONGH and L.HENDRIKS. Intuitionistic propositional logic (IpL) is based on the connectives $\neg,\wedge,\vee$ and $\rightarrow$. Unlike in classical logic, they are not interdefinable (with the exception of $\neg A = A\rightarrow\bot$ if $\bot$ is present). *Fragments* of IpL are obtained if we take only a subset of $\{\neg,\wedge,\vee,\rightarrow\}$, possibly augmented with the defined connectives $\{\neg\neg,\leftrightarrow\}$; this yields twenty–seven nonequivalent fragments. The *diagram* of such a fragnment is the set of equivalence classes of its formulae partially ordered by the derivability relation. Given a finite number of atoms, twenty–one of these fragments yield finite diagrams, the other six (containing $\vee$ and $\rightarrow$) are infinite. The finite diagrams can be studied profitably by their *exact models*, i.e. Kripke models which exactly classify the corresponding diagram. Exact models are in general much cmaller than the diagram they characterize: the fragment $\{p,q,r,\neg,\wedge,\rightarrow\}$, e.g., has an exact model with 6423 nodes and a rather transparent structure, whereas the corresponding fragment has approximately 2.4×10^{1923} elements. In the talk, attention is also paid to the interaction of the research on fragments and the development of theorem provers, and some nice drawings of diagrams and models are presented.

M.M.RICHTER:

Remarks on the Logic of Time Intervals.

ALLON's calculus of time–intervals is treated as a first order theory. The consistency problem of the theory for quantifier free formulas is known to be NP–complete. The question arises to find interesting fragments for which the consistency problem is polynomial. We introduce the notion of a convex clause (K.NÖKEL) and show that this defines such a fragment. The proof uses the notion of 3–consistency and relations to the description of intervals in terms of endpoints.

A.SCEDROV:

Bounded Linear Logic: A Modular Approach to polynomial Time Computability.

Typing is a way of describing the interactive behavior of algorithms. Usual typing systems are mainly concerned with input–output specifications, e.g., given terms $f{:}A\Rightarrow B$ and $a{:}A$, the computation of $f(a)$ by normalization yields a result of type B. However, one can dream of more refined typings that would not only ensure ethereal termination, but would for instance yield feasible resource bounds. It seems that time complexity does not lend itself to modular manipulation. We seek something more primitive. Girard's work on linear logic shows that the nontrivial part of the dynamics of usual type systems lies in the contraction rule. In the opposite two extremes one can either: remove the contraction rule, obtaining real–time dynamics but trivial expressive power, or: freely restore contraction through the unlimited exclamation mark connective $!A$, obtaining tremendous expressive power without any realistic control on the dynamics. We propose the system of Bounded Linear Logic, BLL, lying between these two extremes. In linear logic, exclamation mark is a typing instruction which indicates that the datum is available in the memory for an unlimited number of calls. Instead, bounded linear logic contains bounded exclamation marks, new kind of typing instructions which indicate that the datum is in the memory for at most x

(direct or indirect) calls, where x is the bound. The familiar Gentzen rules, when rewritten as logical rules for bounded exclamation marks, naturally generate polynomials. This work contains two basoc results:

1. BLL normalizes in polynomial time. To any proof (that is, a typed algorithm) one can associate a polynomial which, in the interesting case, majorizes the length of the computation in the size of the input,
2. Every polynomial–time computable function can be typed in BLL. (This result makes use of a very limited part of the full system BLL.).

U.SCHMERL:

A cut Elimination Procedure for the Evaluation of Proofs as Programs.

We define a weak fragment of predicate logic that can be used as a programming language. A program in this language is a formal proof in this fragment of predicate logic of a formula $\forall x \exists y A(x,y)$, where $A(x,y)$ can be understood as a formal specification describing the relation between input x and output y. In order to evaluate proofs as programs in this system we describe a cut–elimination procedure having the following performance:

Given a proof P of a formula $\forall x \exists y A(x,y)$ and a concrete input value e, the procedure provides an output value a as follows:

1. P is specified to a proof P_e of the formula $\exists y A(e,y)$
2. Cut–elimination is applied to P_e transforming it into a normal form of the following very simple shape
$$\frac{A(e,t_e)}{\exists y A(e,y)}$$
where t_e is a term depending on e
3. The output a of P_e is the value of t_e.

In this way, the cut–elimination procedure can be used as an interpreter for proofs as programs. The interpreter has been implemented on a PC; it has been applied to many examples.

G.TAKEUTI:

From Basic Feasible Functionals to Intuitively Feasible Functionals.

COOK and URQUHART defined the basic feasible functionals by introducing

$$\mathcal{R}(y,Z,W,x) = \begin{cases} \text{if } x=0 \text{ then } y \\ \text{else } \ \mathrm{Card}(t \dot- W(x), t, W(x)), \text{ where } t := Z(x, \mathcal{R}(y,Z,\lfloor \tfrac{1}{2} x \rfloor) \end{cases}$$

to the typed Lambda Calculus. Then COOK found a feasible functional which is not in BFF (basic feasible functionals). The recursor $\mathcal{R}$ has a bound W. Therefore $\mathcal{R}$ does not produce any new bigger bound. COOK's example shows that we need recursion which produces a new bound. Therefore our program is to find some recursion without any bound which always produces a feasible functional from feasible functionals. If we find such a recursion and if we believe that it is the strongest among them, the new recursion together with the basic feasible functionals would define the feasible functionals. First we define $\mathcal{R}^0$–recursor

$$\mathcal{R}^0(0,a,g,W) = a$$
$$\mathcal{R}^0(i,a,g,W) = \mathcal{R}^0(\lfloor \tfrac{1}{2} i \rfloor, a,g,W) \cdot |W(\mathcal{R}^0(\lfloor \tfrac{1}{2} i \rfloor, a,g,W), i,a,g)|$$

and show that COOK's functional can be defined by using $\mathcal{R}^0$. Then we discuss B.KAPRON's stronger $\mathcal{R}'$-recursor

$$\mathcal{R}'(0,a,g_1,g_2) = a$$
$$\mathcal{R}'(i,a,g_1,g_2) = \mathcal{R}'(\lfloor \tfrac{1}{2}i \rfloor,a,g_1,g_2)\cdot g_1(i,|g_2(i,\mathcal{R}'(\lfloor \tfrac{1}{2}i \rfloor,a,g_1,g_2))|)$$

and then discuss the program to show that the basic feasible functionals together with $\mathcal{R}'$ is the totality of the feasible functionals.

W.VELDMAN:

Intuitionistic Ramsey Theorems.

Let S be the set of all strictly increasing sequences of natural numbers. In intuitionistic analysis one may prove, using bar induction:

(I) For all $A,B \subseteq \mathbb{N}$: If $\forall \gamma \in S \exists n \in \mathbb{N}[A(\gamma(n))]$ and $\forall \gamma \in S \exists n \in \mathbb{N}[B(\gamma(n))]$, then $\forall \gamma \in S \exists n \in \mathbb{N}[A(\gamma(n)) \wedge B(\gamma(n))]$.

(II) For all $R,S \subseteq \mathbb{N}^2$: If $\forall \gamma \in S \exists m \in \mathbb{N} \exists n \in \mathbb{N}[m<n \wedge R(\gamma(m),\gamma(n))]$ and $\forall \gamma \in S \exists m \in \mathbb{N} \exists n \in \mathbb{N}[m<n \wedge S(\gamma(m),\gamma(n))]$, then $\forall \gamma \in S \exists m \in \mathbb{N} \exists n \in \mathbb{N}[m<n \wedge R(\gamma(m),\gamma(n)) \wedge S(\gamma(m),\gamma(n))]$.

We discuss the question what further theorems of this kind are provable intuitionistically.

A.VISSER:

Local and Global Interpretability and Proofs of Falsehood.

By Goedel's Second Incompleteness Theorem theories can consistently prove their own inconsistency. In this talk I address the question what more such theories can know about their own inconsistency proofs. Specifically can theories prove that their inconsistency proofs are "small" in the sense that they lie in definable cuts? We show e.g. that for every finitely axiomatized, sequential theory U there is a definable cut I sich that U does not prove: *there is an inconsistency proof of* U *in* I. (The work reported here builds on KRAJICEK's work on proofs of Falsehood.)

S.S.WAINER:

Ordinal Complexity of Recursive Definitions.

Kleene's equation calculus was formalized as a system of deductions $\mathbf{n}:\mathbb{N} \vdash^\alpha f(\mathbf{n}):\mathbb{N}$ with "tree-ordinal" bounds α measuring uniformity of definition of f. Then $\vdash^\alpha f:\mathbb{N}^k \to \mathbb{N}$ means $\forall \mathbf{n} \in \mathbb{N}^k(\mathbf{n}:\mathbb{N} \vdash^\alpha f(\mathbf{n}):\mathbb{N})$. Computation is formalized by the Cut/Composition Rule. Complexity is measured by bounding functions B_α where $B_0(n) = n+1$, $B_{\alpha+1}(n) = B_\alpha \circ B_\alpha(n)$, $B_\lambda(n) = B_{\lambda(n)}(n)$.

Theorem. $\vdash^{2\alpha} f:\mathbb{N} \to \mathbb{N}$ iff f has complexity $\leq B_\alpha$.

Removing Cut results in a system of term-rewriting, for which the bounding functions are G_α where $G_0(n) = n$, $G_{\alpha+1}(n) = G_0(n)+1$, $G_\lambda(n) = G_{\lambda(n)}(n)$. Reduction of computation to term-rewriting is then measured by an ordinal transformation $\alpha \mapsto \varphi_{\alpha^+}(\omega)$.

Application (CICHON). Prim.Recursion $\cong$ Rec.Path.Ordering.

P.WOJTYLAK: *Generalization of Proofs.*
Suppose that a proof system $\vdash$ is given and $\vdash^k A$ means that the formula A is provable (with $\vdash$) in k steps. We consider the following schema of generalizing provable propositions:

For each formula $A(x)$ and each number k there exist terms $t_1,\ldots,t_n$ such that

(a) if $\vdash^k A(t)$ then $t = t_i[\sigma]$ for some substitution σ and some i,

(b) $\vdash^k A(t_i)$ holds for each i.

The question of generalization arises in connection with Kreisel's conjecture ([1], problem 34) and one could easily show that the conjecture is true in each arithmetic system in which the above generalization holds. One should also notice that the above does not imply decidability for k-provability as in (b) above we claim only provability of $A(t_i)$, not k-provability. It turns out that the considered form of generalization holds for (the usual formalization of) propositional logic, predicate logic, each finite theory of first order and aruthmetic with order proncipIe. The generalization is not true for PA (arithmetic with successor induction) and predicate logic with the identity schema:

$x = y \rightarrow t(x) = t(y)$, for each term t.

Some proofs of these results can be found in [2],[3],[4]. In all situations (in the positive cases) argumentation is similar. One uses known proof–theoretic results e.g.:

- cut elimination theorem (with bounds on cut–free derivations),
- Herbrand's theorem (with bounds on the number of diskuncts in the Herbrand disjunction),
- (reduction of predicate logic to) Hilbert's ε-calculus,

to reduce the problem to the propositional level, and then one deals with the problem by use of propositional methods.

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