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Affine Differentialgeometrie

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Affine differential geometry is once again a field of active interest. The first Oberwolfach conference on this subject in November 1986 had great influence on the direction of research, as reflected by the lectures at this conference and by the fact that new research groups in affine differential geometry have emerged at several different places.

Summarizing the lectures of Dillen, Kozlowski, Li, Wang, Vrancken and Magid we can state our first observation that the class of affine spheres is larger than expected. This is demonstrated by various new methods of construction (use of homogeneous polynomials by Dillen, use of complex function theory by Kozlowski, use of PDE's by Wang-Simon).

The subclass of locally strongly convex affine spheres of constant sectional curvature metric is now classified (Vrancken-Li-Simon); these investigations were started by a lecture of Li at the 1986-conference. The corresponding problem for indefinite metrics is still open for higher dimensions (except the results by Magid-Ryan for affine 3-spheres with nonzero Pick invariant).

A surprising progress has also been made in the classification of affine surfaces with constant curvature metric and constant affine mean curvature (independently by Dillen-Vrancken and Martinez-Milan). Li has finally given a complete solution of the Calabi conjecture about complete hyperbolic affine spheres so that the locally strongly convex affine complete affine spheres are now finally classified.

Other complete or partial classifications of surfaces/hypersurfaces concern:

a) the homogeneous affine surfaces (Nomizu-Sasaki),
b) the locally symmetric affine surfaces (Opozda),
c) pseudo-symmetry conditions (Deszcz),

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d) quasi-umbilical affine hypersurfaces (Vrancken, Deszcz),
e) curves and hypersurfaces of finite type (Verstraelen).

Some of these lectures also demonstrate the strong progress made since 1986 in the application of recent methods from Riemannian geometry to affine geometry, including investigations in spectral theory and their application to relative geometry (Bokan-Gilkey).

Pinkall's lecture presented a natural symplectic structure coming from centroaffine curve theory which motivates KdV equation in soliton theory. Kurose's contribution emphasized relations of affine geometry to statistics.

There has been more systematic progress in local affine theory (see above) than in global theory (Bokan, Pinkall, Verstraelen, Voss, Simon, Li, Heil, Teufel, Walter), and progress in higher codimension is still singular (Walter). The lecture by Leichtweiß showed that different approaches to the theory of affine area (without differentiability) of convex bodies are in fact equivalent (including recent results of Leichtweiß, Lutwak, Schütt-Werner); this question was open in 1986.

The progress in affine differential geometry influences also projective geometry; besides other recent publications this was shown in the lectures of Nomizu-Sasaki and Podestà. Also problems in Euclidean hypersurface theory can be solved by affine methods (Heil).

At the end of this report we shall indicate the current status of the conjectures and open problems that appeared in the Proceedings of the conference in 1986. We are planning to publish the new Proceedings in which most of the contributions are expected to be published together with a list of recent papers that have appeared or will appear. Some of these have been presented at the conference.

Abstracts

N. Bokan:

Asymptotics of Laplacians defined by Symmetric Connections

This is a lecture about the joint paper of the lecturer and P. B. Gilkey. We study the heat equation asymptotics of the generalized Laplacian $\mathcal{P}$ defined on the tangent bundle by a symmetric connection.

In the first section, we calculate the first three terms in the asymptotic expansion of the heat equation for $\mathcal{P}$, using results from the paper of P. Gilkey: "The spectral geometry of a Riemannian manifold", J. Diff. Geo. 10 (1975), 601 - 608.

In the second section, we study the symmetric connections arising in affine differential geometry.

R. Deszcs:

Pseudosymmetry curvature conditions on hypersurfaces in affine spaces

Let (∇, h, ω, S) be the Blaschke structure of a non-degenerate hypersurface M in the real affine space $\mathbb{A}^{n+1}$, $n \geq 3$, with the affine normal ξ. We define on M a generalized curvature tensor R^* by

$$R^*(X,Y)Z = R(X,Y)SZ, \quad X,Y,Z \in \mathfrak{X}(M),$$

where R is the curvature tensor of ∇. From the Gauss equation of M in $\mathbb{A}^{n+1}$ it follows that

$$R^*(X,Y)Z = S(Y,Z)SX - S(X,Z)SY.$$

In this talk we present some results on hypersurfaces of $\mathbb{A}^{n+1}$ satisfying certain curvature conditions of pseudosymmetric type imposed on the tensors R, R^* and $\mathrm{Ricc}(R^*)$. Moreover, we give a curvature characterization of affine quasi-umbilical hypersurfaces of dimensions ≥ 4. The hypersurface M is said to be affine quasi-umbilical if at every point x of M the tensor $\overline{S}$ is of the form: $\overline{S} = \alpha h + \beta a \otimes a$, $\alpha, \beta \in \mathbb{R}$, $a \in T_x^*(M)$. Namely, we state that M, $\dim M \geq 4$, is affine quasi-umbilical if and only if the Weyl curvature tensor $W(R^*)$ of the tensor R^* vanishes on M.

F. Dillen:

Calabi-type composition of affine spheres and homogeneneous hypersurfaces

In 1969 Calabi introduced his formula for composing hyperbolic affine spheres. We introduce here three other types of formulas to compose affine spheres.
Whereas Calabi's method creates again an affine sphere, our method gives hypersurfaces whose shape operators are simple (they have at most two eigenvalues, 0 and a nonzero constant).
As an application we find a procedure to obtain proper affine spheres out of arbitrary improper affine spheres. These formulas also create a lot of examples of homogeneous hypersurfaces (this means that the hypersurface is the orbit of a point under a subgroup of the equiaffine transformations).
In particular, we can show that this procedures produces all locally strongly convex 3-dimensional homogeneous hypersurfaces and all locally strongly convex homogeneous hypersurfaces whose shape operators have rank one.

E. Heil:

A problem of euclidean surface theory solved by means of affine geometry

A point of a surface in euclidean 3-space is called a vertex if there are no cubic terms in the Euler representation. Marcanti claimed that an ovaloid has at least 6 vertices (counting multiplicities). But the following argument will show that there are ovaloids without vertices. At a vertex the affine normal coincides with the euclidean normal and Pick's invariant vanishes. Given an ovaloid there always is an affine mapping which destroys the coincidence of the two normals at all zeros of Pick's invariant. Therefore this affine image has no vertices.

M. Kozlowski:

Constructions of an Improper Affine Sphere

By the Weierstrass method one can construct a Euclidean minimal surface using meromorphic functions. Using ideas of Heinz and Jörgens I describe a way how to construct an improper affine sphere from a given holomorphic function. In this way one also gets one-parameter families of improper affine spheres.
Hence the class of improper affine spheres is almost "as large as" the class of holomorphic functions.

T. Kurose:

Dual Connections in Affine Geometry

Let ∇ be a torsion-free affine connection on a pseudo-Riemannian manifold (M, g) of dimension $n \geq 3$. A triple (M, g, ∇) is called a *statistical manifold* if ∇g is symmetric. We say a statistical manifold (M, g, ∇) is *1-conformally flat* if there exist a flat affine connection ∇^* of M and a function φ on M satisfying $g(\nabla^*_X Y,Z) = g(\nabla_X Y,Z) - d\varphi(Z)\, g(X,Y)$ for arbritrary vector fields X, Y and Z.
Statistical manifolds play an important role in the geometric research of mathematical statistics, and the notion of 1-conformally flat arose from the study of asymptotic inference.
Our main results are as follows:

THEOREM 1: There exists a (1,3)-tensor field W_1 and a (0,2)-tensor field W_2, which are canonically determined for each statistical manifold, such that a statistical manifold is locally 1-conformally flat if and only if both W_1 and W_2 vanish identically.

THEOREM 2: Let (M, g, ∇) be a simply connected and connected statistical manifold. Then there exists an equi-affine immersion of M into (n+1)-dimensional affine space with the second fundamental form g and the induced connection ∇, if and only if (M, g, ∇) is 1-conformally flat.

K. Leichtweiß

On the history of the affine surface area for convex bodies

There have been made many attempts to define a good notion for the affine surface area of a compact convex body in the n-dimensional affine space by Blaschke, Busemann, Petty, Firey, Lutwak, Schütt-Werner and myself. It is the aim of the lecture to give a survey about these efforts which turn out to be converging to some point of interest in affine differential geometry.

A.-M. Li:

Complete Affine Hyperspheres

Calabi made the following conjecture about complete affine hyperspheres: Every locally strongly convex affine complete hyperbolic affine hypersphere is asymptotic to the boundary of a convex cone with vertex at the center.
S. Y. Cheng and S. T. Yau answered this conjecture for locally strongly convex Euclidean complete affine hyperbolic hyperspheres. We prove the following theorem:

THEOREM: Every locally strongly convex affine complete hyperbolic affine hypersphere is Euclidean complete.

Combining this theorem with the result of S. Y. Cheng and S. T. Yau, we answer the Calabi conjecture.

M. Magid:

1. Surfaces which are timelike and affine minimal

2. Affine hyperspheres with constant affine sectional curvature

1. I found the equations of surfaces which are locally timelike and affine minimal. They are translation surfaces $f(u,v) = \alpha(u) + \beta(v)$ with $\alpha'(u) = (1, \cos\sigma(u), \sin\sigma(u))$ and $\dot{\beta}(v) = (1, \cos\psi(v), \sin\psi(v))$, where σ' and ψ satisfy the o. d. e.

$$2y^4 + 2yy'' - \frac{7}{2}y'^2 - ky^3 = 0 .$$

   Explicit solutions to this o. d. e. were given.

2. I also sketched a proof of my theorem with P. Ryan stating that a 3-dimensional affine sphere with Pick invariant $J \neq 0$ and constant affine sectional curvature is flat and is affinely equivalent to an open subset of

$$(x_1^2 - y_1^2)(x_2^2 - y_2^2) = 1$$

$$\text{or } (x_1^2 - y_1^2)(x_2^2 + y_2^2) = 1$$

$$\text{or } (x_1^2 + y_1^2)(x_2^2 + y_2^2) = 1 .$$

   I also conjectured that with the same hypotheses in the n-dimensional case the possiblities are

$$(x_1^2 \pm y_1^2)\ldots(x_m^2 \pm y_m^2)\, z = 1$$

$$\text{or } (x_1^2 \pm y_1^2)\ldots(x_m^2 \pm y_m^2) = 1 .$$

F. Martinez:

Surfaces with constant affine mean curvature and surfaces with flat affine metric

First I look at connected non-degenerate surfaces M in A^3 with constant affine mean curvature H and constant Pick invariant J, and prove:
THEOREM 1: If $J \neq 0$ then either $3J + 2H = 0$ or M is in an affine 2-sphere with flat affine metric.
THEOREM 2: If $J=0$ then either M is in a quadric or M is a ruled surface.
Second we study the second variation of the affine area of an affine-maximal surface with flat indefinite affine metric.

K. Nomizu and T. Sasaki:

A new model of unimodular-affinely homogeneous surface

Our main result is the discovery of the last model in the following THEOREM: Any non-degenerate surface in $\mathbb{R}^3$ which is homogeneous under unimodular affine transformations is a quadric or is affinely congruent to one of the following surfaces:

(i) $xyz = 1$,

(ii) $(x^2 + y^2)\, z = 1$,

(iii) $x^2 (z - y^2)^3 = 1$,

(iv) $x^2 (z - y^2)^3 = -1$,

(v) $z = xy - 1/3\, x^3$,

(vi) $z = xy + \log x$.

K. Nomizu and T. Sasaki:

Classification of projectively homogeneous surfaces

We classify locally projectively homogeneous surfaces, other than ruled surfaces, in the 3-dimensional real projective space $\mathbb{R}P^3$. Our approach to projective differential geometry makes use of the formalism of projectively flat equiaffine connections and affine immersions. For a non-degenerate surface M^2 in $\mathbb{R}P^3$ we first discuss the choice of (D, ω, ξ) satisfying a number of conditions, where D is an affine connection belonging to the projective structure of $\mathbb{R}P^3$, ω a D-parallel volume element and ξ a transversal vector field to M^2.
If the Fubini-Pick invariant is non-vanishing, we have a unique choice of such (D, ω, ξ) for which the Fubini-Pick invariant is equal to a given nonzero constant, say, 2. We then find an orthonormal frame field $\{X_1, X_2\}$ relative to which the cubic form is simplified. If M^2 is locally projectively homogeneous, the Christoffel symbols for ∇ and the components of S relative to $\{X_1, X_2\}$ are all constants. By the equations of Gauß and Codazzi we determine all possible values of these constants. Integrating the resulting systems we obtain all the surfaces in question.

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B. Opozda:

Affine locally symmetric surfaces in $\mathbb{R}^3$

Let $f: M \to \mathbb{R}^3$ be a connected oriented non-degenerate surface and let ∇ be the induced Blaschke connection. The surface is called locally symmetric if $\nabla R = 0$. Affine locally symmetric surfaces with affine shape operator S of rank 1 are classified as follows.

1) If $S^2=0$, then every point has local coordinates (u,v) such that for $U = \frac{\partial}{\partial u}$, $V = \frac{\partial}{\partial v}$:

$$\nabla_U U = \nabla_U V = 0,\ \nabla_V V = u\,U\,;$$

$$h(U,U) = 0,\ h(V,V) = k(v),\ h(U,V) = E \text{ (nonzero constant)};$$

$$S(U) = 0,\ S(V) = \frac{1}{E} U\,.$$

2) If S is diagonalizable, then every point has locol coordinates (u,v) such that:

$$\nabla_U U = (\log\psi)_u\, U,\ \nabla_U V = (\log\psi)_v\, V,\ \nabla_V V = -\varepsilon_1\varepsilon_2 (\log\psi)_u\, U\,;$$

$$h(U,U) = \varepsilon_1\,\psi,\ h(V,V) = \varepsilon_2\,\psi,\ h(U,V) = 0\,;$$

$$S(U) = \psi^{-1}\, U,\ S(V) = 0\,,$$

where $\psi=\psi(u,v)$ is a positive solution of:

$$\varepsilon_1\,\psi_{uu} + \varepsilon_2\,\psi_{vv} = -\psi, \text{ where } \varepsilon_1, \varepsilon_2 = \pm 1.$$

U. Pinkall:

Affine plane curves and the KdV-equation

It is shown that the KdV-equation

$$\dot{p} = -\frac{p'''}{2} - 3pp'$$

arises naturally from a certain flow on the space M of closed star-shaped curves in $\mathbb{R}^2$ with volume π. This flow, given by

$$\dot{\gamma} = -\frac{p'}{2}\gamma - p\gamma',$$

where p is the central affine curvature, can be motivated as follows: M carries a canonical symplectic form ω and the above flow comes from the Hamiltonian

$$H = \oint p\,.$$

F. Podesta:

Projective automorphisms of bounded strongly convex domains

We have studied the group of all projective transformations of a bounded strongly convex domain $\Omega\subset\mathbb{R}^n$, proving a fixed-point theorem:
If Γ is any subgroup of projective automorphisms such that one orbit of Γ through same point $x_0\in\Omega$ is relatively compact in Ω, then Γ has a fixed point in Ω.
When the boundary of Ω is smooth we could give some upper bounds for the dimension of the automorphisms group, using some results in the theory of projective hypersurfaces.

U. Simon:

Connections and global uniqueness

In part I of the lecture we give a structural set-up to the relative differential geometry of hypersurfaces, discussing equivalent approaches to the theory. In part II we give characterizations of special relative normalization corresponding to special transformation groups (Euclidean, equiaffine, centroaffine).
In part III we prove global results of the following type:

THEOREM 1: Let $x,x^{\#}:M\to A^3$ be ovaloids with relative normalizations $\{Y,y\}$ and $\{Y^{\#},y^{\#}\}$ (Y conormal, y relative normal). Assume that the connections induced from y, $y^{\#}$, resp., coincide: $\nabla(y)=\nabla(y^{\#})$, and that the volumes of the relative metrics coincide: $\omega=\omega^{\#}$. Then $\{x,Y,y\}$ and $\{x^{\#},Y^{\#},y^{\#}\}$ are affinely equivalent.

THEOREM 2: Let $x,x^{\#}$ be as above and assume that the dual connections coincide: $\nabla^*(Y)=\nabla^*(Y^{\#})$. If $\omega=\omega^{\#}$ (as above), then the tripels $\{x,Y,y\}$ and $\{x^{\#},Y^{\#},y^{\#}\}$ are affinely equivalent.

EQUIAFFINE COROLLARIES: One can drop the assumptions about $\omega=\omega^{\#}$.

EUCLIDEAN COROLLARIES: Results of Liebmann - Cohn-Vossen and Minkowski.

A relative version of Grove's theorem is given. The results are discussed in terms of statistical manifolds (see Kurose).

E. Teufel:

Kinematic contact formulas in the equiaffine geometry

In the seventies P. McMullen, W. J. Firey, R. Schneider, W. Weil studied kinematic contact measures and kinematic contact formulas for convex bodies in euclidean spaces. Two years ago we investigated kinematic contact situations for smooth submanifolds in Riemannian spaces of constant curvature. Today we prove analogous results for some kinematic contact situations in the equiaffine geometry:
Given two curves in the real equiaffine plane, we define a contact measure on the subset consisting of those affinities, which cause third-order contact between the fixed and the transformed curve. A kinematic formula expresses this contact measure in terms of affine length and affine curvatures of the given curves.
Parallel supporting planes of closed convex surfaces in affine spaces are treated in a similar way.

L. Verstraelen:

Closed affine curves of finite type

Let $x: M^n \to \mathbb{E}^m$ be an isometric immersion of a Riemannian manifold M^n in a Euclidean space $\mathbb{E}^m$. Let

$$(*) \quad x = x_0 + \sum_t x_t, \qquad \Delta x_t = \lambda_t x_t,$$

be the spectral decomposition of the position vector field x with respect to the Laplace operator Δ of M^n. Then M^n is said to be a submanifold of finite type (FT) if Σ in $(*)$ is finite. Otherwise M^n is said to be a submanifold of infinite type (∞T). A FT submanifold is said to be of k-type ($k \in \mathbb{N}$) when Σ in $(*)$ contains exactly k nonzero terms x_t which belong to distinct eigenvalues λ_t. For closed curves of FT in $\mathbb{E}^m$ this spectral decomposition is nothing but its Fourier series expansion with respect to the arclength. A survey was given of examples of FT curves in $\mathbb{E}^m$ and of their properties. Similarly, a closed curve in affine space $\mathbb{A}^m$ is said to be of FT, in particular of kT, when its Fourier series with respect to the affine arclength is finite, in particular when it contains exactly k different arguments in its sinus- and cosinus-terms appearing in it.
Partly in analogy with the situation in Euclidean spaces, but also significantly different with it, is the following classification result of the closed affine curves of FT in affine spaces.

THEOREM: For every $k \in \mathbb{N}$ there exist kT closed curves lying fully in $\mathbb{A}^{2k}$ and which are affinely equivalent to a closed W-curve of rank 2k in $\mathbb{E}^{2k}$. Moreover, these are the only closed affine curves of finite type.
A submanifold $x: M^n \to \mathbb{E}^m$ is said to be of restricted type (RT) if and only if for every tangent X at any point of M^n: $A_H X = (AX)^T$ for a fixed endomorphism $A \in \mathbb{R}^{m \times m}$, where A_H is the Weingarten map of M^n with respect to the mean curvature vector field H and $(\ )^T$ denotes the tangential component of $(\ )$. The full classification of hypersurfaces of RT in Euclidean space was discussed, including in particular the Euclidean planar curve of RT.
It is hoped that this talk might lead to a systematic study of FT and of RT submanifolds in affine spaces.

K. Voss:

Variation of Curvature Integrals

For a hypersurface $X: M^m \to \mathbb{R}^{m+1}$ in equiaffine space the curvature integrals $C_\nu = \int H_\nu \, dA$ $(0 \le \nu \le m)$ are considered, S_ν being the elementary symmetric functions of the affine principle curvatures. Formulas for the first variations of the S_ν are derived in a simple manner. The case $\nu = 0$ $(S_0 = 1)$ has been known for a long time; the formulas for $\nu \ge 1$ have first been found by Li, An-Min in 1988.
It is shown, why there appear rather complicated additional terms (coming from the variation of the affine normal), which are zero in Euclidean case.

L. Vrancken:

Affine spheres with constant sectional curvature and related results

The study of affine spheres with constant sectional curvature metric started several years ago by, amongst others, A.-M. Li, G. Penn, M. Magid, P. Ryan and U. Simon. They obtain a complete classification of surfaces in $\mathbb{R}^3$ which satisfy the above conditions.
In the locally strongly convex case, i. e. in the case that the affine metric is positive definite, hypersurfaces satisfying the above conditions were determined by A.-M. Li, U. Simon and the author as follows:

THEOREM: Let M^n be a locally strongly convex affine hypersphere in $\mathbb{R}^{n+1}$ with constant sectional curvature. Then M is an open part of a quadric or M is affine equivalent with $x_1 x_2 \dots x_{n+1} = 1$.

However, so far, little is known about affine hypersurfaces with constant sectional curvature which are not affine hyperspheres. In the special case that the hypersurface is flat and quasi-umbilical (i. e. the affine shape operator S has an eigenvalue of dimension $(n-1)$), we obtain:

THEOREM: Let M^n, $n \geq 3$, be a locally strongly convex, quasi-umbilical hypersurface in $\mathbb{R}^{n+1}$ which is flat with respect to the affine metric. Then M is affine equivalent with one of the following hypersurfaces:

(i) the elliptic paraboloid,

(ii) $x_1 x_2 \dots x_{n+1} = 1$,

(iii) $x_1 = \int^u \cosh^{-\frac{n}{n+1}} (n+1)v \, dv, \quad x_2 \dots x_{n+1} = \cosh^{\frac{n}{n+1}} (n+1)u$,

(iv) $x_1 = \int^u \sinh^{-\frac{n}{n+1}} (n+1)v \, dv, \quad x_2 \dots x_{n+1} = \sinh^{\frac{n}{n+1}} (n+1)u$,

(v) $x(u_1, \dots, u_n) = \left(u_1, u_2 u_1, \dots, u_n u_1, \frac{1}{2}\left(\sum_{i=2}^{n} u_i^2\right) u_1 + \frac{n+1}{2(2n+3)} u_1^{2n+3}\right)$,

(vi) $x_1 = \int^r s \left(\frac{1}{n} s^{-n} + c\right)^{-\frac{n}{n+1}} ds, \quad x_2^2 + \dots + x_{n+1}^2 = r^2 \left(\frac{1}{n} r^{-n} + c\right)^{\frac{2}{n+1}}$.

R. Walter:

Compact centroaffine spheres of codimension 2

First, there is given another characterization of the Lopsic normal in central affine differential geometry of codimension 2 which reveals its invariance under the full linear group. Then, as the main result, the following global theorem is proven: Any centroaffine sphere $x: M^m \to V^{m+2}$ with compact parameter manifold M^m embeds M^m as an m-dimensional ellipsoid in an affine hyperplane of the vector space V^{m+2}.

C. P. Wang:

Local theory of affine 2-spheres in $\mathbb{R}^3$

This is a joint work with U. Simon. We study the local theory of non-degenerate affine 2-spheres using complex representations. In particular, we prove the following results:
For any affine 2-sphere there exists an associate 1-parameter family with the property that all affine spheres of this family have the same Blaschke metric and constant equiaffine mean curvature. The metric of an affine sphere admits at most two different constant values for the mean curvature.
We also study the case where the Riemannian metric on a simply connec-ted surface can be realized as the Blaschke metric of an affine sphere. We prove that an affine 2-sphere is uniquely determined by the diffe-rence of the two induced connections.
Finallly we give a PDE whose solutions give all affine 2-spheres in $\mathbb{R}^3$.

Berichter: Christine Scharlach

Conjectures and open problems

The Conference Proceedings [Ed: U. Simon, Affine Differentialgeometrie, Proc. Conf. Math. Forschungsinstitut Oberwolfach, 1986, Druck Birkhäuser Basel, Vertrieb TU Berlin, ISBN 3 7983 1192 7, 1988] from 1986 contain on pp. 192 - 198 a list of "Conjectures and Open Problems". In Part I we give a short survey about the solutions with respect to this, and add in Part II new conjectures and problems.

Part I:

Problem 1 (Leichtweiß): See Leichtweiß' lecture at this conference.

Problem 2 (Svec): An ovaloid contains at least 6 points (counting multiplicities) with vanishing Pick invariant. (See Wang, Some Examples of Complete Hyperbolic Affin 2-Spheres in $\mathbb{R}^3$, Proc. Conference Diff. Geom. Global Analysis, TU Berlin 1989.)

Problem 7 (Voss): See the lecture of K. Voss at this conference.

Problem 9 (The affine Bernstein problem): Progress is made, but here is no final solution of the problem. See the following papers:

(a) E. Calabi: Convex affine maximal surfaces, Conference Proceedings 1986, 199 - 223.

(b) E. Calabi: Affine differential geometry and holomorphic curves, Preprint Univ. Pennsylvania 1988.

(c) E. Calabi: On affine maximal surfaces, to appear.

(d) A.-M. Li: Some Theorems in Affine Differential Geometry, Acta Math. Sinica. N. S. 5 (1989), 345 - 354.

(e) A.-M. Li: Affine Maximal Surface and Harmonic Functions, Lec. Notes in Math. 1369 (1988), 142 - 151.

(f) A. Martinez, F. Milan: On the Affine Bernstein Problem, Geom. Dedicata, to appear.

(g) A. V. Pogorelov: Complete affine-minimal hypersurfaces, Soviet Math. Dokl. 38 (1989), 217 - 219.

(h) C. M. Yau: Affine conormal of convex hypersurfaces, Proc. AMS 106 (1989), 465 - 470.

Part II:

Problem 1 (F. Dillen): Classify equiaffine homogeneous hypersurfaces.

Problem 2 (F. Dillen): Find examples of (non convex) affine spheres with constant sectional curvature different from the ones given by Magid (see his lecture).

Problem 3 (A.-M. Li): Conjecture: Let M be an Euclidean complete, locally strong convex, affine maximal hypersurface in A^{n+1} with

$$\lambda_1^2 + \lambda_2^2 + \lambda_n^2 < N,$$

where $\lambda_1, \ldots, \lambda_n$ are principal affine curvatures and N is a constant, then M is a paraboloid. (This is true for n=2. A similar result for n=2 and affine complete case was obtained by A. Martinez - F. Milan)

Problem 4 (A.-M. Li): Conjecture: Let M be an ovaloid in A^{n+1}. If the Pick invariant J=constant everywhere, then M is an ellipsoid. (This is true for n=2.)

Problem 5 (A.-M. Li): For an ovaloid M in A^{n+1}: Give a lower bound for the number of points where $J=0$? The solution is known for n=2. (See Svec's problem No. 2 in 1986.)

Problem 6 (M. Magid): Are affine hyperspheres with $J \neq 0$ and constant affine sectional curvature always members of the following families:

$$(x_1^2 \pm y_1^2) \ldots (x_m^2 \pm y_m^2)\, z = 1$$

or $$(x_1^2 \pm y_1^2) \ldots (x_m^2 \pm y_m^2) = 1 .$$

Problem 7 (A. Martinez): Classify affine complete surfaces in A^3 with flat affine metric.

Problem 8 (A. Martinez): Classify surfaces in A^3 with constant affine mean curvature H and constant affine curvature κ, $\kappa = H/3 \neq 0$.

Problem 9 (L. Vrancken): Do there exist locally strongly convex, flat (with respect to the equiaffine metric) affine hypersurfaces in $\mathbb{R}^{n+1}$ which have constant affine mean curvature but which are not affine spheres? (dim M > 2; for n=2 see Martinez' lecture and Vrancken: Affine surfaces with constant affine curvature metric. Part I. The positive curvature case. Math. Nachrichten, to appear)

Information about recent publications of participants, in particular contributions to the conference which already appeared (are in print/submitted) in other journals

Bokan, N., Nomizu, K. and U. Simon: Affine hypersurfaces with parallel cubic form, Tohoku Math. J. 42 (1990), 101 - 108.

Deszcz, R.: Certain curvature characterizations of affine hypersurfaces, Colloquium Math., in print.

Dillen, F. and L. Vrancken: 3-dimensional hypersurfaces of $\mathbb{R}^3$ with parallel cubic form, submitted.

Heil, E.: There are ovaloids without vertices, to appear.

Kozlowski, M.: Some improper affine spheres in A_3, Proceedings Conference TU Berlin, June 1990, to appear.

Lutwak, E.: Selected affine isoperimetric inequalities. A survey article, to appear.

Martinez, A. and F. Milan: On affine-maximal ruled surfaces, to appear.

Simon, U.: Local classification of 2-dimensional spheres with constant curvature metric, Differential Geometry and its Applications (Brno), vol. 1 (1991), to appear; Elsevier Publ. (North Holland)

Simon, U. and C.-P. Wang: Local theory of affine 2-spheres, submitted.

Simon, U.: Global uniqueness for ovaloids in Euclidean and affine differential geometry, submitted.

Simon, U. and V. Oliker: Affine geometry and polar hypersurfaces, to appear.

Teufel, E.: Kinematische Berührung im Äquiaffinen, Geom. Ded. 33 (1990), 317 - 323.

Vrancken, L., A.-M. Li and U. Simon: Affine spheres with constant sectional curvature, Math. Zeitschrift, to appear.

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