MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Kreĭn spaces and applications to differential operators

17.2. bis 23.2.1991

Die Tagung fand unter Leitung von H. Langer (Regensburg), R. Mennicken (Regensburg) und J. Rovnyak (Charlottesville, Va.) statt; es nahmen 24 Mathematiker aus 11 Ländern teil. Die Vorträge behandelten Fragen zur Operatortheorie in Kreĭn-Räumen und deren Anwendungen in der Analysis, z.B. in der Spektraltheorie von Differentialoperatoren, in der Theorie der speziellen Funktionen, bei der Nullstellenbestimmung für Matrixpolynome und auf das Fortsetzungsproblem für positiv-definite Funktionen.

In einer Reihe von Vorträgen standen Colligations (oder, äquivalent, Julia-Operatoren oder Elementary Rotations) im Mittelpunkt. Dieser ursprünglich aus der Systemtheorie stammende Begriff fand durch Untersuchungen der letzten Jahre (zum Teil noch unveröffentlicht) seine natürliche Formulierung in Kreĭn-Räumen. In einigen Vorträgen wurde auch der Zusammenhang zur Complementation Theory und zu Randwertaufgaben für gewöhnliche Differentialgleichungen hergestellt.

Hervorzuheben ist ein Vortrag von J. Rovnyak (für die Teilnehmer der beiden gleichzeitig in Oberwolfach abgehaltenen Tagungen) über die von L. de Branges entwickelten Methoden (Hilbert-Räume und Kreĭn-Räume ganzer Funktionen, Entwicklungen nach speziellen Funktionen als Eigenfunktionsentwicklungen), die bei seinem Beweis der Bieberbachschen Vermutung eine Rolle spielten und die auch mit der Riemannschen Vermutung im Zusammenhang zu stehen scheinen.

Durch die Vielfalt der Vorträge und intensive Diskussionen in kleineren Kreisen ergab sich eine große Reihe von Anregungen, die die künftige Arbeit der Tagungsteilnehmer fruchtbar beeinflussen werden. Die Tagungsleiter sind der Meinung, daß das ursprüngliche Ziel, Mathematiker verschiedener Arbeitsrichtungen zusammenzuführen, voll erreicht wurde.

Vortragsauszüge

D. Alpay:

Some constructions of reproducing kernel Kreĭn spaces and reproducing kernel Hilbert spaces of pairs.

By a result of L. Schwartz, the one-to-one correspondence between positive functions [1] and reproducing kernel Hilbert spaces extends to a surjective correspondance between differences of positive functions and reproducing kernel Kreĭn spaces. It is in general difficult to decide if a given function can be written as a difference of two positive functions and in the talk we will present some criteria in terms of continuity and analyticity.
If one replaces the difference of positive functions by a linear combination of such functions, one obtains a surjective correspondance with "Hilbert spaces of pairs" with reproducing kernel. These objects are, roughly speaking, pairs of Hilbert spaces endowed with a sesquilinear form and admitting a left and a right reproducing kernel. Precise definition will be given during the talk as well as applications to a "non-hermitian" Schur algorithm.

T. Ja. Azizov:

Some applications of operator theory in a Kreĭn space to the decidability problem of nonlinear Hamiltonian systems.

Main theorem: Let $\mathcal{H}$ be a finite dimensional Hilbert space, let $f(t,x) : [0,\omega] \times \mathcal{H} \to \mathcal{H}$ be a function satisfying the Caratheodory condition, let $\mathcal{A}_i(t) = \mathcal{A}_i^*(t) \quad (t \in [0,\omega], i = 0,1)$ be a strongly measurable bounded operator function in the space $\mathcal{H}$. Suppose:

1) There exists a $\gamma > 0$ such that for each $t \in [0,\omega]$, $x, h \in \mathcal{H}$ we have:

$$\gamma(h,h) \leq (A_0(t)h,h) \leq (f(t,x+h) - f(t,x),h) \leq (A_1(t)h,h).$$

2) The numbers of eigenvalues $\lambda > 1$ of the linear pencils

$$L_0(\lambda)u = (i\lambda J\frac{d}{dt} + A_0(t))u \qquad \text{and}$$
$$L_1(\lambda)u = (i\lambda J\frac{d}{dt} + A_1(t))u,$$

where $u \in \mathcal{D} = \{u \in W_2^1([0,\omega];\mathcal{H}), u(0) = \rho u(\omega)\}$ and $J : H \to H$, $J = J^* = J^{-1}$, coincide.

Then the problem

$$\begin{cases} iJ\dfrac{dx}{dt} + f(t,x) = 0 \\ x \in \mathcal{D} \end{cases}$$

has a unique solution.

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[1] A function $K(\lambda,\omega)$ defined for λ, ω in Ω is positive if $\sum_{i,j=1}^r c_i c_j^* K(\omega_i,\omega_j) \geq 0$ for all choices of $r \in \mathbf{N}, c_1, \ldots c_r$ in $\mathbf{C}, \omega_1, \ldots, \omega_r$ in Ω.

P. A. Binding

Some variational principles, old and new.

Hilbert's interest in the connection between the calculus of variations and Sturm-Liouville (SL) theory, together with the Klein-Bôcher theory of two-parameter problems, (which include those of indefinite weight), led Richardson (1910) to give recursive variational principles for the left definite SL case. He also conjectured (1918) that similar formulae would hold for eigenvalues of large modulus. Recent work of several authors has led to various formulae of the type

$$\lambda_j = \max_{\operatorname{codim} S = j-1+d} \quad \inf_{x \in S, b(x)=1} \quad a(x)$$

for sufficiently large eigenvalues λ_j of the self-adjoint problem $Ax = \lambda\, Bx$, where $a(x) = (x, Ax)$ has finitely many negative squares, and $b(x) = (x, Bx)$ is either semidefinite or indefinite. The "index shift" d is the dimension of an appropriate "spectral" subspace of (A, B) (e.g. of the operator $B^{-1}A$ if B is $1-1$) and $j+d$ is also the number of nonpositive eigenvalues of $A - \lambda_j B$. Two-parameter eigencurves form a convenient tool to explain these results in a unified fashion.

J. Bognár:

Fundamental norms and spectral radius

If T is a continuous linear operator on a Kreĭn space K then each fundamental decomposition $K = K_+ + K_-$ induces a "fundamental norm" $p(T)$ of T. The non-negative number

$$q(T) = \inf\{p(T) : p \quad \text{a fundamental norm}\}$$

is an invariant of T. It is proved that if K has finite dimension and T is symmetric then $q(T) = r(T)$, the spectral radius of T. On the other hand, there is a non-symmetric T_0 such that $q(T_0) \neq r(T_0)$.

T. Constantinescu:

Selfadjoint extensions of symmetric operators.

A method of M.A. Naimark concerning selfadjoint extensions of symmetric operators is adapted in a nonstationary context and for extending factorizations in indefinite metric spaces.

Applications of this new formulation are given to various completion problems appearing in lifting of commutants, band extension method and interpolation theory in Kreĭn spaces.

B. Curgus:

Kreĭn spaces as completions.

Let $(S, [\cdot,\cdot]_S)$ be a Kreĭn space continuously embedded in a Hilbert space $(\mathcal{H}, \langle\cdot,\cdot\rangle_{\mathcal{H}})$. The adjoint of the inclusion $i : S \to \mathcal{H}$ is a bounded selfadjoint operator in $\mathcal{H}$. A bounded selfadjoint operator S in $\mathcal{H}$ is the adjoint of the inclusion $i : S \to \mathcal{H}$ if and only if $(S, [\cdot,\cdot]_S)$ is a Kreĭn space completion of $(\mathcal{R}(S), \langle\cdot,\cdot\rangle_S)$. A natural Kreĭn space completion of $(R(S), \langle\cdot,\cdot\rangle_S)$ is $(\mathcal{R}(|S|^{1/2}), \langle\cdot,\cdot\rangle_S)$. This completion is the unique Kreĭn space completion of $(\mathcal{R}(S), \langle\cdot,\cdot\rangle_S)$ which is continuously embedded in $\mathcal{H}$ if and only if for some $\varepsilon > 0$ at least one of the intervals $(-\varepsilon, 0), (0, \varepsilon)$ is in $\rho(S)$. Further we use the above propositions to study Kreĭn spaces continuously embedded in a Kreĭn space $(\mathcal{K}, [\cdot,\cdot]_{\mathcal{K}})$. We give a rather large class of selfadjoint operators A in $\mathcal{K}$ for which there exists a unique Kreĭn space $(\mathcal{A}, [\cdot,\cdot]_{\mathcal{A}})$ continuously imbedded in $\mathcal{K}$ such that the adjoint of the inclusion $i : \mathcal{A} \to \mathcal{K}$ coincides with A. This result extends and unifies some results of de Branges. We explain how the above results lead to a new proof of de Branges' complementation theorem in Kreĭn spaces.

A. Dijksma:

Holomorphic operators between Kreĭn spaces and the number of squares of associated kernels.

Report on joint work with Daniel Alpay, Jan van der Ploeg and Henk de Snoo.

Associated with a holomorphic mapping θ from a neighbourhood of 0 in the open unit disk to $\mathcal{L}(\mathcal{F}, \mathcal{G})$, the space of bounded linear operators from a Kreĭn space $\mathcal{F}$ to a Kreĭn space $\mathcal{G}$, are three kernels:

$$\tau_\theta(z,w) = \frac{I - \theta(w)^*\theta(z)}{1 - \overline{w}z}, \quad \tau_{\tilde\theta}(z,w) = \frac{I - \theta(\overline{w})\theta(\overline{z})^*}{1 - \overline{w}z},$$
$$S_\theta(z,w) = \begin{pmatrix} \tau_\theta(z,w) & \frac{\theta(w)^* - \theta(\overline{z})^*}{\overline{w} - z} \\ \frac{\theta(\overline{w}) - \theta(z)}{\overline{w} - z} & \tau_\theta(z,w) \end{pmatrix}.$$

We consider the relation between the negative squares of the three kernels: $sq_-(\tau_\theta)$, $sq_-(\tau_{\tilde\theta})$ and $sq_-(S_\theta)$.

Fix fundamental decompositions $\mathcal{F} = \mathcal{F}_+ \oplus \mathcal{F}_-, \mathcal{G} = \mathcal{G}_+ \oplus \mathcal{G}_-$, and write
$\theta = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} : \begin{pmatrix} \mathcal{F}_+ \\ \mathcal{F}_- \end{pmatrix} \to \begin{pmatrix} \mathcal{G}_+ \\ \mathcal{G}_- \end{pmatrix}$.

Theorem 1. (a) $sq_-(S_\theta) < \infty \Leftrightarrow$ (b) $sq_-(\tau_\theta) < \infty, sq_-(\tau_{\tilde\theta}) < \infty$. $(a), (b) \Rightarrow \theta_{22}$ invertible and the three numbers are equal.

Theorem 2. $\mathcal{F}, \mathcal{G}$ Pontryagin spaces $(\dim \mathcal{F}_- < \infty, \dim \mathcal{G}_- < \infty), u \in \mathbb{N} \cup \{0\}$.
(a) $sq_-(S_\theta) = u \Leftrightarrow$ (b) $sq_-(\sigma_\theta) = u, \dim \mathcal{F}_- = \dim \mathcal{G}_- \Leftrightarrow$ (c) $sq_-(\sigma_{\tilde\theta}) = u, \dim \mathcal{F}_- = \dim \mathcal{G}_-$.
$(a), (b), (c) \Rightarrow \theta_{22}$ invertible.

A. Dijksma:

Unitary colligations and boundary value problems.

Consider the system of $2n$ first order differential equations

$$(1)\qquad \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}' + H(t) \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = l\Delta(t) \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \Delta(t) \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix},$$

where, besides other conditions, $H(t) = H(t)^*$, $\Delta(t) \geq 0$ on $[0, \infty)$, together with the l-dependent boundary condition

$$(2)\qquad \mathfrak{A}(l)y_1(0) + \mathfrak{B}(l)y_2(0) = 0,$$

where $\mathfrak{A}(l)$ and $\mathfrak{B}(l)$ are certain holomorphic $n \times n$ matrix functions on $\mathbf{C}\backslash\mathbf{R}$. We assume that the system (1) is limit point at ∞ and regular at 0. Under some additional conditions, the problem (1), (2) can be linearized to one of the form $Au = lu + v$ where A is a selfadjoint operator in a Kreĭn space $\mathfrak{K}$ extending the minimal operator S associated with (1) in $L^2(\Delta)$. The Cayley transformation $C_\mu(A)$ defines a unitary colligation from $(\mathfrak{K} \ominus L^2(\Delta)) \oplus \ker(S^* - \overline{\mu})$ to $(\mathfrak{K} \ominus L^2(\Delta)) \oplus \ker(S^* - \mu)$. Its characteristic function can be expressed in terms of $\mathfrak{A}(l)$, $\mathfrak{B}(l)$ and the values of the Titchmarsh-Weyl coefficient associated with (1) at the points μ and $\overline{\mu}$. This formula gives a $1-1$ correspondence between (1), (2) and its linearization A.

M. A. Dritschel:

The uniqueness of Julia operators for continuous linear operators on Kreĭn spaces.

Let $T \in \mathcal{B}(\mathcal{H}, \mathcal{K})$, $\mathcal{H}$ and $\mathcal{K}$ Kreĭn spaces. A Julia operator is a continuous linear transformation $U = \begin{pmatrix} T & D \\ \tilde{D}^* & L \end{pmatrix}$ wich is unitary from $\mathcal{H} \oplus \overline{D}$ to $\mathcal{K} \oplus \overline{\tilde{D}}$. Here $\tilde{D}$ and D are defect operators (i.e., $\tilde{D}\tilde{D}^* = 1 - T^*T$ and $DD^* = 1 - TT^*$, and $\ker \tilde{D} = \{0\}$, $\ker D = \{0\}$). We say that the Julia operator U of T is essentially unique if for any other Julia operator U' there exist continuous Kreĭn space unitary operators V and W so that $U' = \begin{pmatrix} 1 & 0 \\ 0 & V \end{pmatrix} U \begin{pmatrix} 1 & 0 \\ 0 & W \end{pmatrix}$. Necessary and sufficent conditions are given for uniqueness. The proof depends on a careful examination of the properties of ranges of continuous operators on Kreĭn spaces. The connection of this result with extension theorems of the Parrott type and commutant lifting theorems will also be discussed.

H. Dym:

On the Hermite theorem for matrix polynomials.

A classical theorem of Hermite identifies the number of roots of a polynomial (with no conjugate pairs of or real roots) with the number of positive eigenvalues of an invertible Hermitian matrix which is specified in terms of the coefficients of the given polynomial.

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In this talk we shall use finite dimensional reproducing kernel Kreĭn spaces to establish an analogous theorem for nonsingular, column reduced matrix polynomials which admit a "reflection".

As applications of this result and of its analogue for the disc (which was obtained earlier in collaboration with N.J. Young) we shall establish 1) a refined version of the Hermite theorem for scalar polynomials and 2) a new proof of a theorem of Alpay-Gohberg-Lerer on the number of roots in the disc of a pair of matrix polynomials associated with an invertible Hermitian block Toeplitz matrix which is subject to some mild regularity conditions.

B. McEnnis:

Model theory in Kreĭn spaces.

We consider Hilbert space operators T with spectrum in the closed unit disk. When T is a contraction, models for T can be obtained from its characteristic function by either modelling the Hilbert space as a reproducing kernel space (the approach of de Branges and Rovnyak), or by modelling the space of the unitary dilation of T in terms of L^2 spaces (the approach of Sz.-Nagy and Foias).

When non-contractions are considered, the first approach still yields a Hilbert space model (the reproducing kernel is positive definite), but the L^2 spaces of the second approach need to be replaced with suitably defined Kreĭn spaces of analytic functions. We will look at some partial solutions to the problem of defining these spaces.

M. Faierman:

An applicationof Kreĭn space theory to a two-parameter eigenvalue problem.

We consider the simultaneous two-parameter systems

$$(p_1(x_1)y_1')' + (\lambda_1 A_1(x_1) - \lambda_2 B_1(x_1) - q_1(x_1))y_1 = 0,\ 0 \le x_1 \le 1,\quad ' = d/dx_1, \qquad (1)$$

$$y_1(0) = y_1(1) = 0, \qquad (2)$$

and

$$(p_2(x_2)y_2')' + (-\lambda_1 A_2(x_1) + \lambda_2 B_1(x_2) - q_2(x_2))y_2 = 0,\ 0 \le x_1 \le 2,\quad ' = d/dx_2, \qquad (3)$$

$$y_2(0) = y_2(1) = 0, \qquad (4)$$

where the p_r, A_r, B_r and q_r are real-valued smooth functions, the p_r and A_r are positive on their intervals of definition, and $\omega = A_1B_2 - A_2B_1$ assumes both positive and negative values on I^2 (the product of the intervals $0 \le x_r \le 1,\ r = 1,2$). If Ω denotes the interior of I^2, then we know that there is associated with the system $(1-4)$ a symmetric sesquilinear form $B(u,v)$ which is bounded from below and with domain $V = H_0^1(\Omega)$. We let γ denote the lower bound of B, A the selfadjoint operator in $\mathcal{H} = L^2(\Omega)$ associated with B, T the operator of multiplication in $\mathcal{H}$ induced by ω, and assume henceforth that $\gamma < 0$ and $0 \in \rho(A)$. Then V is a Pontrjagin space with respect to the inner product $B(\ ,\ )$

and $K = A^{-1}T$ is a compact selfadjoint operator in this space. Hence by appealing to the spectral properties of K, we are able to establish some basic facts about the system (1-4).

M. Faierman:

On an elliptic boundary value problem with a floating singularity.

In the lecture we present some recent results (joint paper with R. Mennicken) concerning the spectral properties of an elliptic boundary value problem containing the floating sigularity $(u(x) - \lambda)^{-1}$. Our results are arrived at through a study of the spectral properties of an associated λ-linear problem and in this study we use some basic facts from the theories of holomorphic operator-valued functions and definitizable operators in Kreĭn spaces.

A. Gheondea:

Completion problems in Kreĭn spaces.

Let K_1, K_1', K_2, K_2' be Kreĭn spaces and $T_r \in L(K_1[+]K_1', K_2)$ and $T_c \in L(K_1, K_2[+]K_2')$ be linear operators. It is required to determine (if any) a linear operator $T \in L(K_1[+]K_1', K_2[+]K_2')$ such that:

$$T|K_1 = T_c, \quad \kappa^-[I - T^\# T] = \kappa^-[I - T_c^\# T_c] \quad \dots$$

and

$$T^\#|K_2 = T_r^\#, \quad \kappa^-[I - TT^\#] = \kappa^-[I - T_r T_r^\#].$$

This completion problem is solved in certain conditions, its relations with other completion problems are explained and also its applications to interpolation theory, extension of operators and other concrete problems are considered.

P. Jonas:

Perturbation theory for selfadjoint operators in Kreĭn spaces.

Let $(\mathcal{H}, [\cdot,\cdot])$ be a Kreĭn space and let A and B be selfadjoint operators in $\mathcal{H}$ whose resolvent sets have a nonempty intersection. Assume that A is definitizable, i.e. $\rho(A) \neq \emptyset$ and there exists a polynomial q such that $[q(A)x, x] \geq 0$, $x \in \mathcal{D}(q(A))$, and that the difference of the resolvents of A and B belongs to some Schatten-von Neumann ideal $\mathfrak{S}_p, 1 \leq p < \infty$. If A is fundamentally reducible (i.e. A is selfadjoint w.r.t. some Hilbert scalar product on $\mathcal{H}$ compatible with $[\cdot,\cdot]$) then by a result of H. Langer the accumulation points of the set of spectral singularities of B are contained in $\mathcal{N}(q) \cup \{\infty\}$, $\mathcal{N}(q) := \{t \in \mathbf{R} : q(t) = 0\}$. Moreover B fulfils certain local definitizability conditions.

If we replace the fundamental reducibility condition of A by the condition that the endpoints of a bounded real interval $\Delta, \Delta \cap \mathcal{N}(q) = \emptyset$ are no accumulation points of

$\sigma(B)\backslash\mathbf{R}$, then the set of spectral singularities of B has no accumulation points in Δ and we have again local definitizability over Δ. A similar result holds for a connected neighbourhood Δ of ∞ if it is assumed, in addition, that B arises from A by a relative $\mathfrak{S}_p(1 \leq p < \infty)$ perturbation in the form sense. The results are applied to the spectral theory of perturbed Klein-Gordon and wave equtions.

H. Langer:

Some remarks about contractive and expansive operators.

An operator T in the Kreĭn space $\mathcal{K}$ is called contractive, if $[Tx,Tx] \leq [x,x]$, and expansive if $[Tx,Tx] \geq [x,x](x \in \mathcal{K})$. It is shown that the linear span of all root subspaces S_λ of an expansive operator T, corresponding to eigenvalues λ with $|\lambda| < 1$, is nonpositive, and the linear span of all S_λ with $|\lambda| < \|T\|^{-1}$ is negative. Further, the spectrum of a contraction on the unit circle is studied; similar to contractions in Hilbert space this spectrum is "very much like" the spectrum of a unitary operator. Finally, expansive operators in π_κ-spaces (κ is the number of negative squares) are considered and a certain canonical decomposition of such operators is given, which implies, e.g., a variant of an invariant subspace theorem.

S. Marcantognini:

A continuation problem for $\mathcal{K}$-indefinite generalized Toeplitz kernels$^{(*)}$.

A generalization of a continuation result due to M.G. Kreĭn and H. Langer is proved. More precisely, it is shown that every κ-indefinite generalized Toeplitz kernel defined on a bounded interval has a κ-indefinite generalized Toeplitz extension to all the real axis. For the Kreĭn-Langer continuation problem a κ-indefinite version of a Kovalishina-Potapov non-uniqueness criterion is obtained. For a special case of the generalized Kreĭn-Langer problem, a parameterization of the set of extensions is also given. The starting point is that to a given κ-indefinite generalized Toeplitz kernel a local semigroup (or a generalized semigroup) on a suitable π_κ-space can be associated.

(*) The talk concerns joint work with R. Bruxual.

R. Mennicken:

Expansions of analytic functions in series of special functions.

E. Hilb and H. Weyl established their well-known spectral theory of selfadjoint boundary eigenvalue problems on subintervals of $\mathbf{R}$ in order to identify certain expansions in terms of special functions, such as Fourier Bessel series or Fourier Dini series, as eigenfunction expansions. F.W. Schäfke had the object to identify expansions of analytic functions in series of special functions, such as Neumann series of the first and second kind, as eigenfunction expansions corresponding to certain differential operators in the complex domain.

The lecture gave a survey on Schäfke's theory, showed the limits of his method and outlined generalizations which permit to identify also such expansions in series of special functions which are related to differential operators depending nonlinearly on the eigenvalue parameter. The new method, due to H. Langer, M. Möller, A. Sattler and the speaker, was illustrated by identifying Berson series and Carlitz series as eigenfunction expansions related to certain first order systems of differential operators in the complex domain.

B. Najman:

Generalized eigenvalue problems.

Let A be a selfadjoint operator in a Kreĭn space $\mathcal{K}$ such that $0 \in \rho(A)$ and the form $[u|v]_A = [Au|v]$ has finitely many negative squares on $D(A)$. The completion $\mathcal{K}_A$ of $(D(A), [\cdot|\cdot]_A)$ is a Pontrjagin space and the realization $\tilde{A}$ of A in $\mathcal{K}_A$ is a selfadjoint operator with spectral properties closely related to those of A. The interplay between A and $\tilde{A}$ has been observed and used repeatedly in concrete situations but has not been pursued systematically. The applications of this general principle are discussed for the case of the Klein-Gordon equation and the generalized eigenvalue problem $-\Delta u - qu = \lambda wu$ on a domain Ω. Regularity of the critical point ∞ and its significance are discussed for these examples.

J. Rovnyak:

Sonine spaces of entire functions and Euler products for character zeta functions.

The lecture is a report on a seminar on a manuscript by L. de Branges on the Riemann hypothesis, held at the University of Virginia (Fall 1990). The manuscript develops an attack on the Riemann hypothesis based on the theory of Hilbert spaces of entire functions. The spaces are determined by entire functions $E(z)$ which satisfy $|E(\bar{z})| < |E(z)|$ for $y > 0$. For any such entire function, let $H(E)$ be the Hilbert space of entire functions $F(z)$ such that $F(z)/E(z)$ and $F^*(z)/E(z)$ are of bounded type and nonpositive mean type for $y > 0$ with

$$\|F\|^2 = \int_{-\infty}^{\infty} |\frac{F(t)}{E(t)}|^2 dt < \infty.$$

Hence $F^*(z) = \overline{F}(\bar{z})$.

An example comes from the theory of the Hankel transform

$$g(x) = \int_0^{\infty} f(t) J_\nu(xt)\sqrt{xt}\,dt$$

of any order $\nu > -1$. L. De Branges (1964) posed the problem to find all Hankel transform pairs in $L^2(0,\infty)$ which vanish a.e. in an interval $(0,a)$. In this case, the weighted Mellin transforms

$$F(z) = 2^{\frac{1}{2}\nu+\frac{1}{2}-iz}\Gamma(\frac{1}{2}\nu + \frac{1}{2} - iz) \int_0^\infty f(t)t^{-\frac{1}{2}+2iz}dt,$$

$$G(z) = 2^{\frac{1}{2}\nu+\frac{1}{2}-iz}\Gamma(\frac{1}{2}\nu + \frac{1}{2} - iz) \int_0^\infty g(t)t^{-\frac{1}{2}+2iz}dt$$

are entire functions which satisfy $G(z) = F(-z)$. A given entire function $F(z)$ arises in this way if and only if $c^{iz}F(z)/\Gamma(\frac{1}{2}\nu + \frac{1}{2} - iz)$ and $c^{iz}F^*(z)/\Gamma(\frac{1}{2}\nu + \frac{1}{2} - iz)$ $\quad (c = (\frac{1}{2}a^2)^{-1})$ are of bounded type and nonpositive mean type for $y > 0$ and

$$\|F\|^2 = \int_{-\infty}^{\infty} |\frac{F(t)}{\Gamma(\frac{1}{2}\nu + \frac{1}{2} - it)}|^2 dt < \infty.$$

The set of all such entire functions is a Hilbert space of the type $H(E)$ (but the choice of $E(z)$ is not obvious). This Hilbert space is called the ***Sonine space of order ν and parameter c.***

The structure problem for the Sonine spaces is to determine a choice of entire function $E(z) = E_\nu(c,z)$ for the space. The choice is to be made satisfying differential equations of a form given by the theory of Hilbert spaces of entire functions. The structure problem was solved in the case $\nu = 0$ by de Branges (1964), and in the case $\nu = 1, 2, \ldots$ by J. and V. Rovnyak (1969). While an explicit solution remains unknown in the general case, an asymptotic solution has been found. A special choice of $E_\nu(c,z)$ is made so that a functional equation relating $E_\nu(c, z+i), E_\nu(c,z), E_\nu^*(c,z)$ holds. If $\nu > 0$,

$$(*) \qquad \lim_{t\to\infty} t^{iz}E_\nu(t,z) = \Gamma(\frac{1}{2}\nu + \frac{1}{2} - iz)$$

uniformly on compact sets in the half-plane $y > -\frac{1}{2}$.

The connection with zeta functions

$$\rho_\chi(z) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^z} = \prod_{p\in P}(1 - \frac{\chi(p)}{p^z})^{-1},$$

Re $z > 1$, is shown. Assume, for example, that χ is a nonprincipal primitive character modulo ν. If χ is even, then $\xi_\chi(z) = (\frac{\nu}{\pi})^{\frac{1}{2}z}\Gamma(\frac{1}{2}z)\rho_\chi(z)$ is an entire function which satisfies a functional equation. The Sonine spaces are used to formulate a noncommutative version of the Euler product which is valid in the complex plane. This factorization, together with the asymptotic relation $(*)$, is the basis of de Branges' attack on the Riemann hypothesis.

J. Rovnyak:

Julia operators and complementation in Kreĭn spaces.

In joint work with Michael Dritschel, we examine L. de Branges' theory of complementation in Kreĭn spaces from an operator-theoretic point of view. It is shown how the main concepts may be conveniently derived by operator methods. Complementation theory is seen in this light to be closely related to recent work on Julia operators in Kreĭn spaces. Some new information is added, chiefly in the direction of uniqueness questions.

Z. Sasvari:

Functions with k negative squares.

The aim of the talk is to give a survey of the theory of functions with k negative squares and its connections to operator theory in Pontryagin spaces. Let S be a semigroup with identity and with involution $*$. A function $f : S \to \mathbb{C}$ is said to have k negative squares if $f(s^*) = \overline{f(s)}$ and the matrix $(f(s_j^* s_i))_{i,j=1}^n$ has at most k negative eigenvalues for any choice of $s_1, \ldots, s_n \in S$ and it has exactly k negative eigenvalues for some choice of $s_1, \ldots, s_n \in S$. These functions can be represented in the form $f(s) = (v, E_s v)$ where E_s is a certain linear operator in a π_k-space Π_k, and $v \in \Pi_k$. One of the important problems is to find an integral representation for these functions. This problem is closely related to the existence of common invariant, nonpositive, k dimensional subspaces of the operators E_s. Under some conditions on S or on f we can find such invariant subspaces and this leads to integral representation theorems generalizing results of Kreĭn, Iohvidov and Naimark.

A. Schneider

A generalization of Wielandt's theorem to Π_κ-spaces.

The following theorem is proved:
Let $(\mathfrak{R}, [\ ,\ ])$ be a π_κ-lineal and A a densely defined hermitian (and thus linear) operator in $\mathfrak{R}$ with the properties:

(i) For all $\mu \in \mathbb{C} \setminus \{0\}$ we have $\dim(\ker(A - \mu) = \operatorname{codim}((A - \mu)D(A)) < \infty$

(ii) $\sigma_p(A)$ is bounded and 0 is its only possible accumulation point.

If Π_κ is the completion of $\mathfrak{R}$ (unique up to isomorphism) and $\overline{A}$ is the hermitian closure of A in Π_κ, then $\overline{A}$ is selfadjoint and compact. Outside 0 the spectrum of $\overline{A}$ coincides with the eigenvalues $\mu_\nu \neq 0$ of A and if the corresponding root subspace $\mathfrak{L}\mu_\nu(A)$ is nondegenerate, then $\mathfrak{L}\mu_\nu(A) = \mathfrak{L}\mu_\nu(\overline{A})$. If $\overline{A}$ is injective and hence 0 not a critical point, then we get for Π_κ a Riesz basis consisting of the eigen and root functions. If $\kappa = 0$, Wielandt's theorem follows. Finally we show, how indefinite regular Sturm-Liouville problems can be reduced to the spectral theory of an operator A in a π_κ-lineal with the properties (i), (ii) and thus we get a Riesz basis consisting of eigen- and root-functions for the corresponding π_κ-space.

K. Veselić,

Some remarks on the use of minimax in perturbation theory.

Minimax fomulae are known to be the source of useful perturbation theorems. Relative perturbation bounds $|x^*Kx| \leq \eta x^*Hx, \quad \eta < 1$, $(H, K$ hermitean) produce relative perturbation estimates for the eigenvalues $|\lambda_i(H+K) - \lambda_i(H)| \leq \eta\lambda_i(H)$. On positive definite H this enables to account for floating point elementwise errors in matrix entries. Indefinite H are more difficult but if represented as $H = GJG^*, J = diag(I, -I)$, then the minimax formula for the equivalent pair $\widehat{H} = G^*G, J$ produces again equally good relative perturbation bounds – an example how "classical" hermitean problems can sometimes be better handled by "indefinite metric" methods.

K. Veselić:

Linear damped systems (some case studies).

We consider the damped system $M\ddot{x} + C\dot{x} + Kx = 0$, M, K positive definite, $C = \gamma uu^{\mathsf{T}}, \|u\| = 1$ (one dimensional damping). We show that this model can cover any eigenvalue configuration all possible eigenvalues being highest defective. Open problem: Do this in an ∞-dimensional Hilbert space thus generalizing the old result of Brodski & Livšic. The defectivity is a spectral pathology but may be very desirable for the stability. This is shown on the example of a properly damped vibrating string which can be managed to have no spectrum at all. We indicate some possible ways how to overcome this diffculty in practical calculations.

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