MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 28/1991

Elliptische Operatoren auf singulären und nichtkompakten Mannigfaltigkeiten

30.06. bis 06.07.1991

Die Tagung fand statt unter Leitung der Herren J.-M. Bismut (Paris), J. Brüning (Augsburg) und R. Melrose (Cambridge).

Trotz der kurzfristigen Absage einiger Mathematiker, deren Beiträge als sehr interessant angesehen wurden, gestaltete sich die Tagung mit 35 Teilnehmern erfolgreich und sehr fruchtbar für alle. Nachdem bereits 1987 eine erste Veranstaltung über "Elliptische Operatoren auf singulären und nichtkompakten Mannigfaltigkeiten" stattgefunden hatte, konnten nun die Fortschritte, aber auch die sich neu ergebenden Perspektiven resümiert werden. Die Schwerpunkte der Vorträge und Diskussionen lagen bei der Indextheorie, der Hodge-Theorie und schließlich der allgemeinen Spektraltheorie auf singulären Mannigfaltigkeiten. Im Bereich der Indextheorie wurden Aspekte der nichtkommutativen Differentialgeometrie zum Beweis von Indexsätzen behandelt (Connes, Moscovici), dann Indexsätze für Mannigfaltigkeiten mit spezifischen Singularitäten (Eskin, Getzler, Grubb, Mazzeo, Seeley), schließlich verschiedene Aspekte des singulären Riemann-Roch-Satzes (Bismut, MacPherson, Pardon, Sjamaar, Shubin). Für die feinere Fragestellung der Hodge-Theorie (MacPherson, Lesch, Stern) muß die Klasse der betrachteten Mannigfaltigkeiten weiter eingeschränkt werden, und für tieferliegende Untersuchungen in der Spektraltheorie (Froese, Grunewald, Hislop, Müller, Perry, Wolpert, Zworski) noch wesentlich mehr. Es konnte festgestellt werden, daß einige Fragen, die auf der vorausgegangenen Tagung herausgehoben bzw. explizit gestellt wurden (wie Gromovs Frage nach einem allgemeinen Riemann-Roch-Satz für elliptische Gleichungen, vgl. den Beitrag von Shubin), mittlerweile gelöst worden sind. Gleichzeitig wurde klar, daß das Zusammenspiel zwischen Analysis und algebraischer Geometrie (vgl. MacPherson) sich in den vergangenen Jahren verstärkt hat, daß aber eine weitere Intensivierung notwendig erscheint, um beide Gebiete wesentlich voranzubringen.

Alle Teilnehmer waren sich darin einig, daß eine Nachfolgekonferenz in etwa drei Jahren als sehr wünschenswert anzusehen wäre.

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VORTRAGSAUSZÜGE

A.V. BABIN: (nur schriftlich, weil Visumprobleme die Teilnahme verhinderten)

Smoothness at singular points of the boundary of solutions of elliptic equations degenerating at the boundary

Let Ω be a bounded domain with a piecewise-smooth boundary $\partial\Omega$. The boundary may have singularities like edges or even more complicated ones. In Ω the equation

$$\rho^2 u - \sum_{i,j\geq 1} \frac{\partial}{\partial x_i}(a_{ij}(x)\frac{\partial u}{\partial x_j}) + a_\infty(x)u = f$$

is considered. It is supposed that a_{ij} and f are analytic over $\bar{\Omega}$, and

$$\sum a_{ij}(x)\xi_i\xi_j \geq 0 \text{ for any } x \in \bar{\Omega},\ \xi = (\xi_1, ..., \xi_n) \in \mathbb{R}^n .$$

If $x \in \partial\Omega$ and $\nu(x) = (\nu_1, ..., \nu_n)$ is a vector normal to a smooth piece of $\partial\Omega$ at a point x then

$$\sum a_{ij}(x)\nu_i(x)\nu_j(x) = 0 .$$

Under some conditions imposed on the boundary (these conditions are not restrictive, they admit non-Lipschitzean boundaries) it is proved that $u(x)$ is smooth up to the boundary even at singular points, $u \in C^s(\bar{\Omega})$ where $s \geq k\rho$, $k > 0$, for large ρ. Obviously $s \to \infty$ as $\rho \to \infty$. These results are obtained by means of approximation theory.

JEAN-MICHEL BISMUT:

Complex immersions and Quillen metrics

In this lecture, I surveyed a few recent results on Quillen metrics. They involve

1. The construction of Quillen metrics

Take X a compact manifold, ξ a holomorphic vector bundle. Set

$$\lambda(\xi) = \otimes_{i=0}^m (\det H^i(X, \xi))^{(-1)^{i+1}} .$$

Let g^{TX}, h^ξ be Hermitean metrics on TX, ξ. Let $|\ |_{\lambda(\xi)}$ be the metric on $\lambda(\xi)$ induced by the Hodge theory of ξ. Let τ be the Ray-Singer analytic torsion of the Dolbeault complex. Then

$$\|\ \|_{\lambda(\xi)} = |\ |_{\lambda(\xi)}\tau .$$

2. The smoothness of $\|\ \|_{\lambda(\xi)}$

Let $\pi : M \to Y$ be a holomorphic submersion with compact fibre X. Let ξ be a holomorphic vector bundle on X. Let $\lambda^G(\xi)$ be the inverse of the determinant of the cohomology of $\xi_{|X}$. Introduce metrics on T^Y and ξ.

Theorem: (Bismut,Guillet,Soulé) *The metric* $\| \|_{\lambda^G(\xi)}$ *is smooth.*

3. The curvature theorem

Theorem: (Bismut,Guillet,Soulé) *Let* $\nabla^{\lambda^G(\xi)}$ *be the holomorphic Hermitean connection on* $(\lambda^G(\xi), \| \|_{\lambda^G(\xi)})$. *Then under "natural" Kähler conditions, the curvature* $\Omega^{\lambda^G(\xi)}$ *of* $\nabla^{\lambda^G(\xi)}$ *is given by*

$$[Td(TX, g^{TY})ch(\xi, h^{\xi})]^{(2)} .$$

4. The immersion formula

Let $i : Y \to X$ be an embedding of complex manifolds, which are compact. Let η be a holomorphic vector bundle on Y, let (ξ, v) be a chain complex on X which resolves $i_*\eta$. Let $\lambda^G(\eta)$, $\lambda^G(\zeta)$ be the inverses of the determinants of the cohomology of η, ξ. Then $\lambda^G(\eta) \cong \lambda^G(\xi)$.

Theorem: (Bismut,Lebeau) *If the metrics* g^{TX}, g^{TY} *are Kähler, then*

$$\frac{\| \|_{\lambda^G(\xi)}}{\| \|_{\lambda^G(\eta)}} = \textit{local explicit formula.}$$

ALAIN CONNES (joint work with H. Moscovici):

Higher Γ-indices

We shall extend the Γ-index theorem for Atiyah and Singer from the Γ-trace case to higher Γ-cocycles. Γ will denote a countable discrete group, acting properly and freely on a smooth manifold $\tilde{M}$, with compact quotient $M = \Gamma \setminus \tilde{M}$. It will be convenient to fix a Riemannian metric on M end endow $\tilde{M}$ with the lifted metric.

To begin with, let us introduce an algebra which is needed in the construction of the higer Γ-indices. This algebra, to be denoted $\mathcal{A}$, consists of all Γ-invariant, bounded operators A on $L^2(\tilde{M})$ whose Schwartz kernel $A(\tilde{x}, \tilde{y})$, *a priori* only a distribution on $\tilde{M} \times \tilde{M}$ satisfying the γ-invariance property

$$A(g \cdot \tilde{x}, g \cdot \tilde{y}) = A(\tilde{x}, \tilde{y}), \quad \forall \gamma \in \Gamma ,$$

is actually a C^∞ function with compact support modulo Γ. A more complete (and suggestive) notation for this algebra is $C_c^\infty(\tilde{M} \times_\Gamma \tilde{M})$.

We now proceed to construct certain homomorphisms from $\mathcal{A}$ to $\mathbb{C}\Gamma \otimes \mathcal{R}_M$ and $\mathbb{C}\Gamma \times \mathcal{L}^2_M$ where $\mathcal{R}_M$ (resp. $\mathcal{L}^2_M$) is the algebra of smoothing operators (resp. Hilbert Schmidt operators) in $L^2(M)$.

Let $\{B_1, ..., B_r\}$ be an open covering of M by small balls B_i, domains of smooth cross-sections $\beta_i : B_i \to \tilde{M}$ for the canonical projection $\pi : \tilde{M} \to M$. Let $(\chi_i)_{i=1,...,r}$ be a smooth partition of unity subordinate to the above covering. We shall assume, as we may, that each $\chi_i^{1/2}$ is a smooth function. The following formula defines a Γ-equivariant isometry U from $L^2(\tilde{M})$ into $L^2(M \times \{1, ..., r\} \times \Gamma)$:

$$(U\xi)(x, i, g) = \chi_i(x)^{1/2}\xi(g\beta_i(x)) \quad \forall g \in \Gamma, \ x \in M, \ i \in \{1, ..., r\}.$$

Let

$$\theta(A) = \sum_{g \in \Gamma} \rho(g) \otimes \theta_g(A),$$

where ρ denotes the right regular representation of Γ and $\theta_g(A) \in M_r(\mathcal{R}_M)$ is the matrix of smoothing operators given by:

$$(\theta_g(A))_{ij}(x, y) = \chi_i(x)^{1/2}\chi_j(y)^{1/2} A(\beta_i(x), g\beta_j(y)).$$

The compactness of $\overline{\bigcup \beta_j(B_j)}$ and of the support of A, together with the fact that Γ acts properly on $\tilde{M}$, ensure that $\theta_g(A) = 0$ except for finitely many g's. Thus $\theta(A)$ belongs to the *algebraic* tensor product $\mathbb{C}\Gamma \otimes M_r(\mathcal{R}_M)$ where $\mathbb{C}\Gamma$ denotes the complex representation ring of Γ.

Lemma. *(i) θ is an algebra homomorphism of $\mathcal{A}$ into $\mathbb{C}\Gamma \otimes M_r(\mathcal{R}_M)$.*
(ii) The induced homomorphism $\theta_* : K_0(\mathcal{A}) \to K_0(\mathbb{C}\Gamma \otimes \mathcal{R}_M)$ is independent of the choice of $\{B_j, \beta_j, \chi_j\}$.

Using an orthonormal basis of eigenfunctions for the Laplacian Δ on M associated to a given Riemannian metric one can identify the algebra $\mathcal{R}_M$ with the algebra $\mathcal{R}$ of matrices $(a_{ij})_{i,j \in \mathbb{N}}$ such that

$$\sup_{i,j \in \mathbb{N}} i^k j^\ell |a_{ij}| < \infty \quad \forall k, \ell \in \mathbb{N}.$$

This identification, together with the above lemma, gives rise to canonical homomorphisms

$$\Theta : K_0(\mathcal{A}) \to K_0(\mathbb{C}\Gamma \otimes \mathcal{R}).$$

With minor and obvious modifications, all of the above remains valid if we introduce bundles into the picture. Thus, $L^2(M)$ gets replaced by $L^2(M, E)$, where E is a (hermitian) vector bundle over M, $L^2(\tilde{M})$ by $L^2(\tilde{M}, \tilde{E})$, where $\tilde{E} = \pi^* E$, and $C_c^\infty(\tilde{M} \times_\Gamma \tilde{M})$ by $C_c^\infty(\tilde{M} \times_\Gamma \tilde{M}, \tilde{E} \otimes \tilde{E}^*)$, consisting of Γ-invariant, compactly supported mod Γ, smooth kernels A such that

$$A(\tilde{x}, \tilde{y}) \in \mathrm{Hom}(E_y, E_x) \simeq E_x \otimes E_y^*, \quad \forall (\tilde{x}, \tilde{y}) \in \tilde{M} \times \tilde{M}.$$

Let now $\tilde{D} : C^\infty(\tilde{M}, \tilde{E}^+) \to C^\infty(\tilde{M}, \tilde{E}^-)$ be a Γ-invariant elliptic differential operator. $\tilde{D}$ is invertible modulo $\mathcal{A} = C_c^\infty(\tilde{M} \times_\Gamma \tilde{M}, \tilde{E} \otimes \tilde{E}^*)$. More explicitly, one can lift almost local parametrices Q of D to Γ-invariant parametrices $\tilde{Q}$ and $\tilde{D}$, so that $\tilde{S}_0 = I - \tilde{Q}\tilde{D}$, $\tilde{S}_1 = I - \tilde{D}\tilde{Q} \in \mathcal{A}$. From such a parametrix $\tilde{Q}$ one can manufacture the idempotent

$$\tilde{P} = \begin{bmatrix} \tilde{S}_0^2 & \tilde{S}_0(I + \tilde{S}_0)\tilde{Q} \\ \tilde{S}_1\tilde{D} & I - \tilde{S}_1^2 \end{bmatrix} \in M_2(\mathcal{A}).$$

The corresponding reduced class

$$[\tilde{R}] = [\tilde{P}] - \left[\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}\right] \in K_0(\mathcal{A})$$

can be shown to be independent of the choice of the almost local parametrix Q.

Definition. *The K-theoretical index of a* Γ*-invariant elliptic operator* $\tilde{D}$ *on* $\tilde{M}$ *is the class:*

$$\operatorname{ind}_\Gamma \tilde{D} = \Theta([\tilde{R}]) \in K_0(\mathbf{C}\Gamma \otimes \mathcal{R}).$$

The higher Γ-indices of the elliptic operator $\tilde{D}$ will be obtained by pairing $\operatorname{ind}_\Gamma \tilde{D}$ with cyclic cocycles on $\mathbf{C}\Gamma \otimes \mathcal{R}$ constructed from group cocycles on Γ. We recall that the graded cohomology group $H^*(\Gamma) = H^*(\Gamma, \mathbf{C})$ of Γ is by definition the graded homology group associated to the complex $\mathcal{C}^*(\Gamma;\Gamma) = \{\mathcal{C}^*(\Gamma;\Gamma), d\}$, whose p-cochains are functions $c : \Gamma^{p+1} \to \mathbf{C}$ satisfying the invariance condition

$$c(g \cdot g_0, \ldots, g \cdot g_p) = c(g_0, \ldots, g_p)\,, \quad \forall g, g_0, \ldots, g_p \in \Gamma\,,$$

and with coboundary given by the formula

$$(dc)(g_0, \ldots, g_{p+1}) = \sum_{i=0}^{p+1} (-1)^i (g_0, \ldots, g_{i-1}, g_{i+1}, \ldots, g_{p+1})\,.$$

Since we only deal with real or complex coefficients, the above complex can be replaced by the subcomplex $\mathcal{C}^*_\alpha(\Gamma;\Gamma)$, where

$$\mathcal{C}^p_\alpha(\Gamma;\Gamma) = \{c \in \mathcal{C}^p(\Gamma;\Gamma); c(g_{\tau(0)}, \ldots, g_{\tau(p)}) = \operatorname{sgn}(\tau) c(g_0, \ldots, g_p) \quad \forall \tau \in S_{p+1}\}\,,$$

without altering the cohomology.

Let $c \in Z^p_\alpha(\Gamma;\Gamma)$ be a p-cocycle of the latter complex. It defines a cyclic p-cocycle $\tau_c \sharp \operatorname{tr}$, or abbreviated τ_c, on $\mathbf{C}\Gamma \otimes \mathcal{R}$, via the formula

$$\tau_c(f^0 \otimes A^0, \ldots, f^p \otimes A^p) = \operatorname{tr}(A^0 \ldots A^p) \sum_{g_0 g_1 \ldots g_p = 1} f^0(g_0) f^1(g_1) \ldots f^p(g_p)$$
$$\times\, c(1, g_1, g_1 g_2, \ldots, g_1 g_2 \ldots g_p)\,,$$

where $f^0, \ldots, f^p \in \mathbf{C}\Gamma$ and $A^0, \ldots, A^p \in \mathcal{R}$. Note that this cocycle extends to $\mathbf{C}\Gamma \otimes \mathcal{L}^2$if $p \geq 1$. This cyclic cocycle gives an additive map $[\tau_c] : K_0(\mathbf{C}\Gamma \otimes \mathcal{R}) \to \mathbf{C}$. Explicitly,

$$[\tau_c]([e] - [f]) = \tilde{\tau}_c(e, \ldots, e) - \tilde{\tau}_c(f, .., f)\,;$$

here e, f are idempotent matrices with entries in $(\mathbf{C}\Gamma \otimes \mathcal{R})^\sim$ = the unital algebra obtained by adjoining the identity to $\mathbf{C}\Gamma \otimes \mathcal{R}$, and $\tilde{\tau}_c$ is the canonical extension of τ_c to $(\mathbf{C}\Gamma \otimes \mathcal{R})^\sim$.

By definition, the (c, Γ)-index of $\tilde{D}$ is the number

$$\operatorname{Ind}_{(c,\Gamma)} \tilde{D} = [\tau_c](\operatorname{ind}_\Gamma \tilde{D})\,.$$

It only depends on the cohomology class $[c] \in H^*(\Gamma)$ and is linear with respect to $[c]$.

Theorem. *Let* M *be a compact smooth manifold,* Γ *a countable discrete group,* $\tilde{M} \to M$ *a* Γ*-principal bundle over* M *and* $\tilde{D}$ *a* Γ*-invariant elliptic differential operator on* $\tilde{M}$. *For any group cocycle* $c \in Z^{2q}_\alpha(\Gamma;\Gamma)$, *one has*

$$\operatorname{Ind}_{(c,\Gamma)} \tilde{D} = \frac{(-1)^{\dim M}}{(2\pi i)^q} \frac{q!}{(2q)!} < \operatorname{ch} \sigma_{pr}(D) \tau(M) \psi^*(i[c]), [T^*M] >\,,$$

where $\psi : M \to B\Gamma$ *is the map classifying the covering* $\tilde{M} \to M$.

GREGORY ESKIN:

Index formulas for elliptic boundary value problems in domains with wedges

Consider the boundary value problem

(1) $Au = f$ in Ω,

(2) $B_1 u|_{\Gamma_1} = h_1$, $B_2 u|_{\Gamma_2} = h_2$,

where $\Gamma = \partial\Omega$ consists of two smooth pieces Γ_1 and Γ_2, $\Gamma_0 = \bar{\Gamma}_1 \cap \bar{\Gamma}_2$ and Γ_1 and Γ_2 intersect at $\forall\, x_0 \in \Gamma_0$ under angles $\alpha(x_0)$, $0 < \alpha(x_0) < 2\pi$, A is an elliptic system of differential operators, B_1 and B_2 satisfy the Shapiro-Lopatinsky condition on $\bar{\Gamma}_1$ and $\bar{\Gamma}_2$, respectively.

Denote by $\kappa(a)$ the topological index of the symbol $a(x, \xi)$ on $S^*(M)$ where M is a smooth manifold without boundary.

It is proven that the problem (1), (2) is Fredholm in some Sobolev space $H_s(\Omega)$ (i.e. $u \in H_s(\Omega)$) and that $\operatorname{ind}(A, B_1, B_2) = \kappa(a) + \kappa(b)$, where a, b are some symbols on closed manifolds obtained from A, B_1, B_2 using homotopies.

RICHARD FROESE:

Eigenfunction expansion for the Laplacian on certain hyperbolic manifolds

The study of Eisenstein series for a discrete group of hyperbolic isometries with parabolic elements of non-maximal rank requires an analysis of the Laplace operator on a hyperbolic manifold whose boundary at infinity has singularities. In this talk we compared such a Laplace operator to two-body and N-body Schrödinger operators, whose boundaries at infinity are respectively less and more singular. We then showed how to use the idea of localization at infinity—borrowed from the study of N-body Schrödinger operators—and a geometric perturbation formula to determine the asymptotics of the Green's function. This leads to proof of the meromorphic continuation of the Eisenstein series. The work described in this talk was carried out jointly with Peter Hislop and Peter Perry.

EZRA GETZLER:

Cyclic cohomology and the Atiyah-Patodi-Singer theorem

If A is a Banach algebra with identity (for example $C^1(M)$ where M is a compact manifold), let

$$C^k(A) = \operatorname{Hom}(A, \underbrace{\bar{A}, \ldots, \bar{A}}_{k-\text{times}}; \mathbf{C})$$

be the space of continuous $k+1$-multilinear forms, where $\bar{A} = A \setminus \mathbf{C}$. For example, with $A = C^1(M)$ and μ a k-current, let

$$c_\mu(a_0, \ldots, a_k) = \frac{1}{k!} \int_\mu a_0\, da_1 \ldots da_k\,.$$

These are the cyclic cochains of Connes. If $b+B$ is the boundary of cyclic cohomology, then $(b+B)C_\mu = C_{\delta\mu}$, where $\delta\mu$ is the boundary of μ. Thus, cyclic cochains generalize currents.

Now let D be a b-Dirac operator in the sense of Melrose, where M is now an even-dimensional compact spin-manifold with boundary. Let

$$\mathrm{ch}(D)(a_0,\ldots,a_{2\ell}) = \int_{\Delta^{2\ell}}^{b} \mathrm{Str}(a_0 e^{-\delta_0 D^2}[D,a_1]e^{-\delta_1 D^2}\ldots[D,a_{2\ell}]e^{-\delta_{2\ell} D^2}).$$

We prove that $(b+B)\mathrm{ch}(D) = \mathrm{ch}(D_{\partial M})$, and a similar secondary result

$$(b+B)\tilde{\mathrm{ch}}(tD) = \mathrm{ch}(tD_{\partial M}) + \frac{d}{dt}\mathrm{ch}(tD),$$

which implies the Atiyah-Patodi-Singer theorem, and may be viewed as a higher degree generalization.

GERD GRUBB:

Heat traces and index for general (non-cylindrical) Atiyah-Patodi-Singer problems

In 1975, Atiyah, Patodi and Singer determined the index of a first order elliptic operator $P : C^\infty(E) \to C^\infty(F)$ (E and F being hermitean vector bundles over a compact n-dimensional manifold X with boundary Y) under the hypothesis that on a collar neighborhood $X_c = Y\times[0,c[$ of the boundary, E and F are pull-backs of $E|_Y$ resp. $F|_Y$, and

$$P = \sigma_0(\partial_{x_n} + A) \qquad (*)$$

for a self-adjoint operator A in $L_2(E|_Y)$, $\sigma_0 : E|_Y \to F|_Y$ a unitary morphism. P is given the boundary condition $\Pi^+(u|_Y) = 0$, Π^+ the orthogonal projection onto the nonnegative eigenspaces of A.

We treat the case where E and F are general, and P satisfies $(*)$ only on Y and only principally. Here we show trace expansions

$$\mathrm{Tr}\ \exp(-t(P_{\Pi^+})^* P_{\Pi^+}) = c_{1,-n}t^{-n/2} + \ldots + c_{1,-1}t^{-1/2} + c_{1,0} - \frac{1}{4}\eta_A + O(t^{3/8}),$$

$$\mathrm{Tr}\ \exp(-tP_{\Pi^+}(P_{\Pi^+})^*) = c_{2,-n}t^{-n/2} + \ldots + c_{2,-1}t^{-1/2} + c_{2,0} + \frac{1}{4}\eta_A + O(t^{3/8}),$$

with locally determined coefficients $c_{i,j-n} = a_{i,j-n} + b_{i,j-n}$ ($a_{i,j-n} = \int_X \alpha_{i,j-n}$, $b_{i,j-n} = \int_Y \beta_{i,j-n}$) and η_A denoting the (global) eta invariant. This implies

$$\mathrm{index}\, P_{\Pi^+} = a_0 + b_0 - \frac{1}{2}\eta_A\,, \quad a_0 = a_{1,0} - a_{2,0},\ b_0 = b_{1,0} - b_{2,0}\,,$$

where b_0 is a new boundary term, nonzero in general. For twisted Dirac operators (n even), b_0 vanishes for $n = 2$, but is nonzero for $n \geq 4$, and is determined by an explicit formula involving the second fundamental form of Y in X (developed from a sketch by L. Hörmander).

PETER D. HISLOP:

Geometrically finite infinite volume hyperbolic manifolds

This is joint work with R. Froese and P. Perry. Let M be a geometrically finite, infinite volume hyperbolic manifold of dimension $n \geq 3$. The manifold M has the form H^n/Γ, where Γ is discrete, torsion-free, geometrically finite and co-infinite. In particular, M may have cusps of maximal and non-maximal rank. For $n = 3$, we prove that the Eisenstein series (ES) for Γ have meromorphic continuations to **C**. This result is based on an analysis of the asymptotic behavior of the Green's function at infinity. The ES are obtained as weighted limits of the Green's function as one of the variables tends to the boundary of M at infinity. Using these asymptotics and a functional relation for the Green's function, the S-matrix is constructed. This operator is proved to be invertible with inverse meromorphic on the right side of the critical line. As the S-matrix relates ES with spectral parameters equidistant from the critical line (at s and $2-s$), this proves the meromorphic continuation: $S(s)^{-1}E(u;w';s) = E(u;w';2-s)$. In dimension $n \geq 4$, the geometric structure of non-maximal rank cusps is more complicated, but essentially the same analysis applies.

MATTHIAS LESCH (joint work with J. Brüning):

Some remarks on conic singularities

Let

$$0 \longrightarrow C_0^\infty(E_0) \xrightarrow{d_0} C_0^\infty(E_1) \xrightarrow{d_1} \cdots \xrightarrow{d_{N-1}} C_0^\infty(E_N) \longrightarrow 0 \tag{1}$$

be an elliptic complex on a Riemannian manifold M. If M is non-compact the operators d_j may have several closed extensions. Every choice of closed extension D_j with domain $\mathcal{D}_j$ in $L^2(E_j)$ such that

$$0 \longrightarrow \mathcal{D}_0 \xrightarrow{D_0} \mathcal{D}_1 \xrightarrow{D_1} \mathcal{D}_2 \longrightarrow \cdots \xrightarrow{D_N} \mathcal{D}_N \longrightarrow 0 \tag{2}$$

is a complex, is called an *ideal boundary condition* of the complex. An abstract complex of the form (2) is called a *Hilbert complex*.
In my talk I first discuss some abstract aspects of Hilbert complexes. Many notions known from elliptic complexes on compact manifolds can be expressed in terms of Hilbert complexes and surprisingly many proofs become more perspicuous. As an example I mention

Theorem (smoothing of homology) *Let $(\mathcal{D}, D)$ be an ideal boundary condition for the elliptic complex $(C_0^\infty(E), d)$. Then $C^\infty(E_j) \cap \mathcal{D}_j$ is a core for D_j and $(C^\infty(E_j) \cap \mathcal{D}_j, D_j) \hookrightarrow (\mathcal{D}, D)$ induces an isomorphism in homology, i.e.*

$$\operatorname{Ker} D_j / \operatorname{im} D_{j-1} \simeq \operatorname{Ker} D_j|_{C^\infty(E_j)\cap\mathcal{D}_j} / \operatorname{im} D_{j-1}|_{C^\infty(E_{j-1})\cap\mathcal{D}_{j-1}}.$$

If $(C^\infty(E), d) = (\Omega_0(M), d)$ is the de Rham complex, one has for the formal adjoint δ_k of d_k

$$\delta_k = (-1)^{Nk+1} *_{N-k}\, d_{N-k-1} *_{k+1} \tag{3}$$

and one can ask whether there is an ideal boundary condition $(\mathcal{D}, D)$ sucht that (3) holds in the strong operator sense

$$D_k^* = (-1)^{Nk+1} *_{N-k} D_{N-k-1} *_{k+1}\,; \tag{4}$$

this property is called Poincaré duality for $(\mathcal{D}, D)$.
It turns out that (4) can always be fulfilled if $N \not\equiv 1 \pmod 4$ and for $N \equiv 1 \pmod 4$ it can be fulfilled if and only if the symmetric operator

$$*_{\frac{N+1}{2}} d_{\frac{N-1}{2}} \tag{5}$$

has self-adjoint extensions. So the deficiency indices

$$n_\pm(M) := \dim\ker((*_{\frac{N+1}{2}} d_{\frac{N-1}{2}})^* \mp iI) \in \mathbb{Z}_+ \cup \{\infty\} \tag{6}$$

enter the scene. We have the following example

Theorem *Let M be a Riemannian manifold* $\dim M = 4k+1$, *with an open set $U \subset M$ such that*
(i) $M \setminus U$ is a complete manifold with compact boundary N,
(ii) $U \approx (0, \varepsilon) \times N$ and the metric on U is quasi-isometric to $dx^2 \oplus x^2 g_N$.
Then the deficieny indices $n_\pm(M)$ are finite and one has

$$n_+(M) - n_-(M) = \operatorname{sign}(N).$$

In the last part of my talk I turned to Kähler manifolds and proved the socalled "Kähler package" for Kähler manifolds with asymptotically cone-like singularities.

ROBERT MACPHERSON:

Comparing L^2 cohomology for different metrics

Let X be a Kähler manifold with metric g. It often happens that, for a compactification $\bar{X}$ of X, we have

$$IH^i(\bar{X}) = H^i_{(2,g)}(X)$$

where IH is intersection cohomology and $H_{(2,g)}$ is L^2-cohomology. Call $\bar{X}$ and g compatible if this happens.
Now suppose that $\bar{X}, g$ and $\bar{X}', g'$ are two compatible pairs, and that there is a complex algebraic map

$$f : \bar{X} \longrightarrow \bar{X}'$$

which is the identity on X. (For example, $\bar{X}$ may be a resolution of singularities of $\bar{X}'$.) Then the Decomposition Theorem for intersection homology gives

$$H^i_{(2,g)}(X) = H^i_{(2,g')}(X) \oplus \bigoplus_\alpha H^{i+n_\alpha}_{(2,g')}(Y_\alpha)$$

for certain pairs $Y_\alpha \subset \bar{X}' \setminus X$ and $n_\alpha \in \mathbb{Z}$.
Question Can this be understood by elliptic operator methods?

RAFE MAZZEO:

Edge operators and geometric applications

This talk reports on recent joint work with R.B. Melrose. On manifolds with asymptotically cylindrical ends (which we regard as manifolds with boundary endowed with exact b-metrics in their interiors) there is an extension of the ordinary trace functional from, say, smoothing operators with Schwartz kernels vanishing rapidly along the cylindrical ends to the residual operators in Melrose's pseudodifferential b-calculus. This "b-trace" is a Hadamard regularization of the logarithmic divergence of the ordinary trace. Using it we define a renormalized b-eta invariant by using the standard definition of eta as an integral of the trace of the heat kernel, only replacing the standard trace with the b-trace. This definition is well-defined on odd-dimensional spin manifolds with exact b-metrics. The notion of b-trace arose earlier in Melrose's "direct" proof of the Atiyah-Patodi-Singer index theorem, and is related to earlier work of Stern. The b-eta invariant also appears in an extension of this index theorem recently proved by Melrose-Singer in the context of manifolds with corners with exact b-metrics. This last result suggests that the b-eta invariant might be the limit of the ordinary eta invariant under deformations, and this is exactly what we prove. Thus let X be a compact odd-dimensional spin manifold with embedded hypersurface H (for convenience assumed to have oriented tangent and normal bundles, and to separate X). Letting x be a defining function for H, consider the family of metrics on X:

$$g_\varepsilon = \frac{dx^2}{x^2 + \varepsilon^2} + h$$

where h is some nondegenerate metric on X. For any $\varepsilon > 0$ this is an ordinary metric on X, but as ϵ tends to 0, two infinite cylindrical ends on either side of H are formed. Let D be the Dirac operator on X and E any hermitian bundle, and form the twisted Dirac operator $D_{E,\epsilon}$ corresponding to the bundle E and the metric g_ϵ. Our theorem, under the nondegeneracy assumptions that neither $D_{E,0}$ nor the associated twisted Dirac operator on H have nullspace, is that the eta invariant for $D_{E,\epsilon}$ converges to the b-eta invariant for $D_{E,0}$. The proof uses the analytic surgery pseudodifferential calculus recently developed by McDonald. Also the result may be modified when either of the nondegeneracy assumptions is dropped. Such singular deformations are closely related to earlier work of Seeley and Seeley-Singer.

HENRI MOSCOVICI:

Eta invariants and stationary phase on the free loop space

In a joint paper with R.J. Stanton [Invent. math. 95 (1989), 629 – 666], we showed that the eta invariant of a (generalized) Dirac operator D on a compact locally symmetric manifold $M = \Gamma\backslash G/K$ of non-positive sectional curvature is given by a special value (at $s = 0$) of the meromorphic continuation of a "geometric" zeta-function. The coefficients of this zeta function are "Lefschetz numbers" attached to the submanifolds of closed geodesics, parametrized by the nontrivial conjugacy classes of the fundamental group $\Gamma = \pi_1(M)$; they have a cohomological expression formally similar to the Lefschetz numbers occuring in the Atiyah-Singer Equivariant Index Theorem.

The purpose of this talk was to point out a different interpretation of these Lefschetz numbers. Namely, they are given by the leading coefficients of the stationary phase expansion of the "odd" heat kernel $\mathrm{Tr}(t^{1/2}De^{-tD^2})$, represented as a path integral on the free loop space ΛM. Thus, the eta invariant formula in [loc. cit.] can be regarded as an instance of an exact stationary phase expansion on ΛM. The interesting new feature of this expansion is that, unlike the Atiyah-Witten version of the Duistermaat-Heckman exact stationary phase, this one is localized at the non-constant critical points of the energy.

WERNER MUELLER:

Spectral geometry and scattering theory for complete surfaces of finite volume

We study complete surfaces (M,g) which have finite volume and hyperbolic ends. Let Δ be the Laplace operator on M associated to the metric g. We regard Δ as an unbounded operator in $L^2(M)$ with domain $C_0^\infty(M)$. Then Δ is essentially self-adjoint. The spectrum of $\overline{\Delta}$ consists of a sequence of eigenvalues $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots$ of finite multiplicity and an absolutely continous spectrum which is the interval $[1/4,\infty)$ with multiplicity equal to the number of ends of M.

A generic surface has only finitely many eigenvalues and all of them lie below the continuous spectrum. Besides of the eigenvalues, there is the scattering matrix $C(s)$ which contains important spectral information. The scattering matrix is a meromorphic matrix valued function of $s \in \mathbf{C}$. Let

$$\phi(s) = \det C(s).$$

The poles of the meromorphic function $\phi(s)$ are called resonances. We then consider the set $\sigma(M)$ which is the union of the following three sets:

(a) The set of all poles and zeros of $\phi(s)$ in the half-plane $\mathrm{Re}(s) < 1/2$.

(b) The set of all $s_j \in \mathbf{C}$ such that $s_j(1-s_j)$ is an eigenvalue of Δ.

(c) $\{1/2\}$.

Each point $\eta \in \sigma(M)$ occurs with a certain multiplicity $m(\eta)$. We call $\sigma(M)$ the resonance set. If B denotes the generator of the Lax-Phillips semi-group $Z(t)$, $t \geq 0$, associated to the hyperbolic wave equation on M, then $\sigma(M)$ is precisely the spectrum of $B + \frac{1}{2}I$.

The basic problem that we study is the following one:

To what extent does the resonance set $\sigma(M)$ determine the geometric structure of (M,g) and vice versa?

This may be compared with the forward and inverse problem of scattering on the real line.

One of our main tools is an analogue of the trace formula for the heat operator on a compact surface. The left hand side of this trace formula is a sum running over $\sigma(M)$. We also introduce two different determinants in the context of complete surfaces of finite volume. One of them is a relative determinant defined via the trace formula. The other one is defined through the resonance zeta function

$$\zeta_B(s) = \sum_{\eta \in \sigma(M) - \{1\}} (1-\eta)^{-s}.$$

These determinants are not equal but closely related.

Finally, for hyperbolic surfaces we study the inverse problem of scattering theory. It turns out that $\sigma(M)$ determines a hyperbolic surface of finite volume up to finitely many possibilities.

WILLIAM PARDON (joint work with M. Stern):

$L^2 - \bar{\partial}$-cohomology of projective algebraic varieties

Let V be a complex projective variety and give $V - \operatorname{sing} V$ the hermitian metric induced by any (algebraic) imbedding of V in a $\mathbb{P}^N$. Then $\|\omega\|_2^2 := \int_{V - \operatorname{sing} V} <\omega, \omega> d \operatorname{Vol}$ defines the L^2-norm of a (p,q) from ω and we set $A_{(2)}^{p,q}(V - \operatorname{sing} V) = \{\omega \mid \|\omega\|, \|\bar{\partial}\omega\| < \infty\}$. Then $(A_{(2)}^{p,*}, \bar{\partial})$ is a complex whose cohomology is denoted $H_{(2)}^{p,q}(V - \operatorname{sing} V)$, (the L^2-$\bar{\partial}$-cohomology). We seek a generalization of the Dolbeault Theorem: V smooth implies $H^{p,q}(V) \cong H^q(\Lambda^p(V))$.

Theorem A *For any complex projective variety*

$$H^{n,q}(V - \operatorname{sing} V) \cong H^q(\tilde{V}; \Lambda^n(\tilde{V}))$$

where $\tilde{V}$ is any resolution of V.

Theorem B *For any variety V with $\dim V \leq 2$,*

$$H^{0,q}(V - \operatorname{sing} V) \cong H^q(\tilde{V}; O(Z - |Z|))$$

where Z = scheme-theoretic (normal) exceptional divisor of the resolution $\tilde{V} \to V$.

Theorem A answers a question of R. MacPherson (does the L^2-$\bar{\partial}$-arithmetic genus equal the arithmetic genus of any resolution?) which motivated this work.

PETER A. PERRY:

Scattering theory and trace formulas for Kleinian groups

Suppose that Γ is a geometrically finite, torsion-free, convex co-compact discrete group of isometries of real hyperbolic n-dimensional space $\mathbb{H}^n$, so chosen that $\operatorname{vol}(\mathbb{H}^n/\Gamma) = \infty$. The Laplacian on $M = \mathbb{H}^n/\Gamma$ has at most finitely many discrete eigenvalues together with purely absolutely continuous spectrum of infinite multiplicity. In the case $n = 2$, the geometry of these manifolds is very explicit and a trace formula, due to Patterson, allows a very complete study of spectral geometry. We discuss Patterson's trace formula, Weyl's law for the scattering phase, and the distribution of scattering poles. We discuss why Patterson's strategy breaks down for $n \geq 3$ and discuss the distribution of scattering poles for $n \geq 3$.

ELMAR SCHROHE:

Fredholm criteria for boundary value problems on noncompact manifolds

Boutet de Monvel's calculus, established in 1971, made boundary value problems amenable to pseudodifferential methods. In particular, it gave necessary and sufficient conditions for the Fredholm property of boundary value problems on smooth compact manifolds. It has been an open problem to treat the noncompact case. A solution was presented for the model case where the manifold is the halfspace $\mathbb{R}^n_+$.

A Boutet de Monvel type calculus is established for boundary value problems on noncompact manifolds. It is based on a class of weighted symbols and weighted Sobolev spaces.

This leads to the following results:

(1) The algebra $\mathcal{G}$ of operators of order and type zero is a spectrally invariant Fréchet subalgebra of $\mathcal{L}(H)$, where H is a suitable Hilbert space, i.e.

$$\mathcal{G} \cap \mathcal{L}(H)^{-1} = \mathcal{G}^{-1},$$

(2) there is a holomorphic functional calculus for the elements in $\mathcal{G}$ in several complex variables,

and

(3) there is a necessary and sufficient criterion for the Fredholm property of boundary value problems of arbitrary order, type, and size, based on the invertibility of an operator-valued symbol.

BERT-WOLFGANG SCHULZE:

Mellin pseudo-differential operators and ellipticity on manifolds with corners

The basic question in the analysis of pseudodifferential operators on manifolds with singularities, e.g. edges, corners, are analogous to the compact closed C^∞ case or that with C^∞ boundary. The program is to construct an operator algebra with symbolic structures with the concept of ellipticity, the Fredholm property in (weighted) Sobolev spaces, and the parametrix within the algebra. Furthermore, the problem of evaluating the index is of the same importance as in the classical cases. The lecture did present the algebra for edges and corners, consisting (say in the case of a stretched manifold

$\mathbf{W}$ with edges Y and $J_+, J_- \in \mathrm{Vect}\,(Y)$) of matrices

$$\mathcal{A} = \begin{pmatrix} A+M+G & K \\ T & Q \end{pmatrix} : \begin{array}{c} \mathcal{W}^{s,\gamma}(\mathbf{W}) \\ \bigoplus \\ H^{s}(Y,J_-) \end{array} \rightarrow \begin{array}{c} \mathcal{W}^{s\mu,\gamma-\Gamma}(\mathbf{W}) \\ \bigoplus \\ H^{s\Gamma}(Y,J_+) \end{array}$$

$\gamma, s \in \mathbb{R}$, $\mu =$ad $\mathcal{A}$. The typical differential operators of order μ are of the form

$$A = t^{-\mu} \sum_{k+|\alpha| \leq \mu} a_{k\alpha}(t,y)\left(-t\frac{\partial}{\partial t}\right)^k (tD_y)^\alpha$$

in the coordinates $(x,t,y) \in X \times \mathbb{R}_+ \times \Omega$, X being the base of the model cone, $\mathbb{R}_+$ the cone axis, Ω an open subset of y. The operators $M+G$ are of smoothing Mellin and Green type, T is a trace, K a potential operator with respect to Y, and Q is a pseudo-differential operator along y.

ROBERT SEELEY (joint work with J. Brüning):

Resolvent asymptotics for manifolds with a singular stratum

We study heat traces for geometric elliptic operators near a "wedge singularity" i.e. a family of metric cones parametrized over a compact base. The separation of variables leads to certain normal forms, which allow to introduce Fredholm boundary conditions under some natural assumptions. These, in turn, are satisfied for e.g. the signature and the Gauß-Bonnet operator. The corresponding supertrace admits an asymptotic expansion which, in principle, leads to a local contribution to the index. Under certain invertibility assumptions on the "normal operator" it can be seen that these contributions are local in the base variables but, in general, nonlocal in the conic fiber. It remains to evaluate these formulas more explicitly.

M.A. SHUBIN (joint work with M. Gromov):

The Riemann-Roch Theorem for general elliptic operators

Let X be a compact closed C^∞-manifold, $\dim X = n \geq 2$, E, F complex vector bundles over X, $q = \dim_{\mathbf{C}} E_x = \dim_{\mathbf{C}} F_x$ is the dimension of their fibers, $A : \Gamma(X,E) \rightarrow \Gamma(X,F)$ an elliptic operator of order d, $\Omega(X)$ the bundle of complex densities on X, $E^* = \mathrm{Hom}(E, \Omega(X))$, $(\cdot,\cdot)$ is the natural bilinear pairing $\Gamma(X,E) \times \Gamma(X,E^*) \rightarrow \mathbf{C}$, $A^* : \Gamma(X,F^*) \rightarrow \Gamma(X,E^*)$ is the adjoint operator to A defined by the pairings $(\cdot,\cdot)$ in E and F. Let us introduce a divisor $\mu = x_1^{p_1} x_2^{p_2} \ldots x_m^{p_m}$, $x_i \in X$, $x_i \neq x_j$ if $i \neq j$, $p_i \in \mathbb{Z} \setminus \{0\}$, $p_i < 0$ if $i \leq \ell$, $p_i > 0$ if $i \geq \ell + 1$, $\mu^{-1} = x_1^{-p_1} x_2^{-p_2} \ldots x_m^{-p_m}$ is the dual divisor. The degree of μ is defined as

$$d(\mu) = \sum_{1 \leq i \leq m} \mathrm{sign}\, p_i \left[\binom{|p_i| + n - 1}{n} - \binom{|p_i| + n - 1 - d}{n} \right],$$

where $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ if $N \geq n$ and 0 otherwise. Denote

$$\begin{aligned} L(\mu, A) = \{u \mid u \in \Gamma(X \setminus \{x_1, \ldots, x_\ell\}),\ & Au = 0 \text{ on } X \setminus \{x_1, \ldots, x_\ell\}; \\ & u(x) = o(|x - x_i|^{d-n-|p_i|}) \text{ as } x \to x_i,\ 1 \leq i \leq \ell; \\ & u(x) = O(|x - x_i|^{p_i}) \text{ as } x \to x_i,\ \ell + 1 \leq i \leq m\}, \end{aligned}$$

$\Gamma(\mu, A) = \dim_{\mathbb{C}} L(\mu, A)$.

Theorem $\Gamma(\mu, A) = \operatorname{ind} A - qd(\mu) + \Gamma(\mu^{-1}, A^*)$.

The classical Riemann-Roch theorem is a particular case if we take $A = \bar{\partial} : C^\infty(X) \to \Lambda^{0,1}(X)$ where X is a non-singular algebraic curve. The case of the scalar Laplace-Beltrami operator on a Riemannian manifold was considered by N. Nadirashvili (1988).

Corollary $\Gamma(\mu, A) \leq \operatorname{ind} A - qd(\mu)$.

This inequality becomes an equality if X, A are real-analytic or A is a scalar second-order operator, provided $x_1, \ldots, x_m$ are fixed, $\sum\limits_{-\ell+1 \leq i \leq m} p_i \leq N_0$ with a fixed N_0 and $\sum\limits_{1 \leq i \leq \ell} |p_i|$ is sufficiently large.

REYER SJAMAAR:

Geometric quantization and symplectic reduction

Let M be a Kähler manifold with Kähler form ω and let L be a hermitian line bundle on M with connection ∇ such that $\operatorname{curv}(\nabla) = \frac{1}{2\pi i}\omega$. Suppose that G is a compact Lie group which acts on M by Kähler isometries and suppose that the action can be lifted to an action on L preserving the fibre metric and the connection. Then G acts on smooth sections of L in a natural way and the infinitesimal action of an element ξ of $\mathcal{G} = \operatorname{Lie}(G)$ on a section s is given by Kostant's formula,

$$(\nabla_{\xi_M} s)(m) + \phi^\xi(m) s(m),$$

where ξ_M is the Killing vector field on M associated to ξ and ϕ^ξ is the ξ-th component of a momentum map $\phi : M \to \mathcal{G}^*$ for the action of G. The space of holomorphic sections $\Gamma(M, L)$ of L is a representation space for G. Let $\mathcal{O} \subset \mathcal{G}^*$ be an integral coadjoint orbit and let $V_{\mathcal{O}}$ be the unitary irreducible representation of G associated to it by the Borel-Weil correspondence. Guillemin and Sternberg have calculated the multiplicity of $V_{r\mathcal{O}}$ in $\Gamma(M, L^{\otimes r})$, for sufficiently large r, in terms of geometric data on the reduced phase space,

$$M_{\mathcal{O}} := \phi^{-1}(\mathcal{O})/G,$$

under a genericity assumption that implies that $M_{\mathcal{O}}$ is a smooth manifold. The formula is:

$$\operatorname{mult}(V_{r\mathcal{O}}, \Gamma(M, L^{\otimes r})) = \int_{M_{\mathcal{O}}} e^{r\omega_0} \wedge \det{}_{\mathbb{C}} \frac{R}{1 - e^{-R}}.$$

Here ω_0 is the reduced symplectic form on $M_{\mathcal{O}}$ and R is the curvature tensor of the holomorphic tangent bundle of $M_{\mathcal{O}}$ (so $\det_{\mathbb{C}}(R/(1-e^{-R}))$ represents $\mathrm{Td}\,(M_{\mathcal{O}})$).
In case $M_{\mathcal{O}}$ is not a manifold, E. Lerman and I showed that it is a stratified symplectic space. Let $\Sigma \subset M_{\mathcal{O}}$ denote the union of all singular strata of $M_{\mathcal{O}}$. Invoking the Baum-Fulton-MacPherson-Hirzebruch-Riemann-Roch formula and a rationality result of Boutot, I show that the multiplicity is given by an integral over the top stratum,

$$\mathrm{mult}(V_{r\mathcal{O}}, \Gamma(M, L^{\otimes r})) = \int_{M_{\mathcal{O}}/\Sigma} e^{r\omega_0} \wedge \det_{\mathbb{C}} \frac{R}{1-e^{-R}}$$

generalizing the formula above. (Here r is still assumed to be sufficiently large.) The result implies that for large r the multiplicity does not depend on the complex structure of M but on the symplectic structure only. Moreover, for $r \to \infty$ the multiplicity is asymptotically equal to r^k times the symplectic volume of $M_{\mathcal{O}}/\Sigma$ (where $k = \dim_{\mathbb{C}} M_{\mathcal{O}}$).

MARK STERN:

Hodge theory for locally symmetric spaces

Let $X = G/K$ be a Hermitian symmetric space, $\Gamma \subset G$ a neat arithmetic group, and $X_\Gamma = \Gamma \setminus X$. Let $H \in \mathrm{Lie}(K)$ be the element which induces the complex structure operator on X. Let τ be an irreducible representation of K and V_τ the associated holomorphic vector bundle. Let $j : X_\Gamma \hookrightarrow X_\Gamma^*$ denote the inclusion of X_Γ into its Baily-Borel-Satake compactification. We announce the following theorems.

Theorem A *If $i\tau(H) \le 0$ or $i\tau(H) > \dim_{\mathbb{C}} X_\Gamma$, then*

$$H^{0,p}_{(2)}(X_\Gamma, V_\tau) \text{ is finite dimensional.}$$

Theorem B *If $i\tau(H) \le 0$, then*

$$H^{0,p}_{(2)}(X_\Gamma, V_\tau) = H^p(X_\Gamma^*, j_!\mathcal{O}(V_\tau)).$$

If $i\tau(H) > 0$, then

$$H^{0,p}_{(2)}(X_\Gamma, V_\tau) = H^p(X_\Gamma^*, R_{j_*}\mathcal{O}(V_\tau)).$$

We also give examples where the hypotheses of Theorem A are not satisfied and the cohomology is infinite dimensional.
Next we drop the assumption that X be Hermitian and we let E be a flat vector bundle. Then we discuss a proof of the following.

Theorem C *$H^*(X_\Gamma, \Sigma)$ can be represented by harmonic forms.*

GANG TIAN:

Complete Kähler-Einstein metrics on quasi-projective manifolds

We discuss the construction of complete Kähler metrics with prescribed Ricci curvature on a quasi-projective manifold. A theorem of S.T. Yau and myself is given. This states: if $X = \bar{X} \setminus D$ is a quasi-projective manifold satisfying: 1) both $\bar{X}$ and D are smooth; 2) for some m, the sections in $H^0(\bar{X}, [D]^{Dm})$ give a morphism from $\bar{X}$ into some projective space, which is an embedding in a neighborhood of D, then 1) any $(1,1)$-form in $C_1(\bar{X}) \setminus C_1([D])$ is the Ricci curvature form of some complete Kähler metric on X; 2) any $(1,1)$-form Ω in $C_1(\bar{X}) \setminus \beta C_1([D])$, where $\beta > 1$, is the Ricci curvature form of some complete Kähler metric if there is a Kähler metric g on D with $\mathrm{Ric}(g) = (\beta - 1)\omega_g + \Omega$, where ω_g is the Kähler form of g. As a corollary, we prove the existence of complete Ricci-flat Kähler metrics on certain quasi-projective manifolds of the above type and with the additional property: $C_1(\bar{X}) = \beta C_1([D])$ for some $\beta \geq 1$. We also discuss some generalizations of this theorem. Finally, we show that any Ricci-flat ALE Kähler manifold of dimension $n \geq 3$ has to be a minimal resolution of C^n/Γ, where $\Gamma \subset U(n)$. The analogue of this for $n = 2$ was previously obtained by P. Kronheimer.

SCOTT WOLPERT:

The spectral conundrum for hyperbolic surfaces

Consider R a Riemann surface with a hyperbolic metric ($K \equiv -1$) of finite area. We are interested in the spectrum (≥ 0) of the Laplace-Beltrami operator. If R is compact the spectrum is discrete and for R noncompact the spectrum consists of a band $[\frac{1}{4}, \infty)$ plus eigenvalues (possibly including embedded eigenvalues). Consider then the following questions: determine

1) the number of embedded eigenvalues for the generic surface with cusps,
2) the generic multiplicity of an eigenvalue,
3) the variation of eigenvalues, both in the small and the large (degeneration).

For a one-parameter real analytic family R_ℓ, each eigenvalue of R_{initial} has a real analytic branch along R_ℓ. In fact there is a real analytic branch of a basis for each eigenspace. Consider then R_ℓ a one-parameter analytic degenerating family (a pinching).

Theorem *Except possibly for a finite number of branches of eigenvalues $(\varphi_\ell, \lambda_\ell)$ we have*

<u>if R_ℓ is compact</u>: either i) $\lambda_\ell \searrow 1/4$
or ii) $\lambda_\ell \to \lambda_0 > \frac{1}{4}$, and φ_ℓ has a subsequence converging to φ_0, a nontrivial L^2-eigenfunction of R_0;

<u>if R_ℓ is noncompact</u>: either i) λ_ℓ disappears (as discovered by Phillips and Sarnak) or ii) as above.

Remarks 1) There are examples for each possibility.
2) Branches of eigenvalues cannot escape to infinity
3) (as shown in the proof) If $\lambda_\ell \to \frac{1}{4}$, then the generic multiplicity of λ_ℓ is at most 2.
An open problem is to find the relative frequency of i) and ii).

MACIEJ ZWORSKI (joint work with J. Sjöstrand):
Complex scaling and the distribution of scattering poles

We establish sharp polynomial bounds on the number of scattering poles for a general class of compactly supported self-adjoint perturbations of the Laplacian in $\mathbb{R}^n$, n odd. We also consider more general types of bounds which give sharper estimates in certain situations. The general conclusion can be stated as follows: the order of growth of the poles is the same as the order of growth of eigenvalues for corresponding compact problems. From the few known cases the exact asymptotics are, however, expected to be different.

The scattering poles for compactly supported perturbations were rigorously defined by Lax and Phillips. In a more general setting they correspond to resonances the study of which has a long tradition in mathematical physics. In the Lax-Phillips theory they appear as the poles of the meromorphic continuation of the scattering matrix and coincide with the poles of the meromorphic continuation of the resolvent of the perturbed operator. Because of the latter characterization they can be considered as the analogue of the discrete spectral data for problems on non-compact domains.

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