MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 30/1991

Dynamische Systeme

14.7. bis 20.7.1991

The meeting is dedicated to the memory of Andreas Floer

(1956 - 1991)

Die Tagung fand unter der Leitung von J. Moser und E. Zehnder (ETH Zürich) statt.

Im Zentrum standen die mannigfaltigen neuen Entwicklungen der symplektischen Geometrie, welche im engen Zusammenhang mit globalen Poblemen der Hamilton'schen Systeme stehen. Neue symplektische Invarianten (sogenannte symplektische Kapazitäten) führen zu überraschenden Starrheitsphänomenen und zu einer Fülle von geschlossenen Charakteristiken auf Hyperflächen. Variationstechniken ermöglichen, unter Benutzung der Floer Homologie, Ansätze einer symplek-

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tischen Topologie und auch globale Fixpunktsätze Hamilton'scher Abbildungen auf allgemeinen symplektischen Mannigfaltigkeiten.

Schwerpunkte waren zudem Ergodenfragen klassischer dynamischer Systeme, spezieller Fragen über das N-Körperproblem und die Himmelsmechanik, wie z.Bsp. Kollisionssingularitäten und periodische Bahnen. Behandelt wurden auch die neuen Resultate auf dem Gebiet der integrablen Systeme und die allgemeinen homoklinischen Phänomene in dissipativen dynamischen Systemen.

KAM-Iterationsmethoden führten zu neuen Anwendungen auf dem Gebiet der Systeme welche in der Nähe von integrablen Systemen sind, sowohl für endliche wie für unendliche Freiheitsgrade. Behandelt wurden aber auch andere unendliche dimensionale dynamische Systeme welche erzeugt werden durch nicht lineare partielle Differentialgleichungen. Schliesslich seien noch die Untersuchungen von Kreisabbildungen mit einem kritischen Punkt mit Hilfe einer Renormalisierungsgruppen Analysis erwähnt.

Vortragsauszüge

ALAN ALBOUY

Integral Manifolds of the n-body Problem

The manifolds obtained by fixing the values of the energy and the angular momentum in the *3-dimensional* n-body problem change the topology when these parameters vary (momentum non-zero, energy negative) not only for critical values corresponding to central configurations, but also for values corresponding to critical points at infinity. These critical points at infinity are exhibited by following paths on critical manifolds of a modified energy, where some terms of the potential are neglected.

SIGURD ANGENENT

Mean Curvature flow

In the first part *I* gave a survey of the results which have been obtained by Huisken, Gage and Hamilton, Grayson and others concerning the mean curvature flow (and curve shortening, i.e. the case "$n = 1$"). In the second part *I* described the approach of Evans and Spruck, and of Chen, Giga and Goto for constructing generalized solutions of the mean curvature flow, using "viscosity solutions" to parabolic PDEs. It turns out that there exist two ways to define the evolution by mean curvature of an arbitrary compact hypersurface $\Gamma = \partial\Omega$ (with $\Omega \subset \mathbf{R}^{n+1}$ any open set). These two evolutions satisfy a list of axioms with which one can "compute" what they are for any given $\Gamma = \partial\Omega$. I gave an example (the "figure eight") where the two different evolutions were not the same, and I stated the following conditions which guarantee that the two evolutions coincide:

(1) $\Gamma = \partial\Omega$ is smooth and has nonnegative mean curvature

or

(2) Γ is a hypersurface of rotation.

A. BAHRI

Around the Birkhoff-Lewis theorem

In recent years, a variational approach has been developed by P. Rabinowitz in order to find periodic orbits of Hamiltonian systems on a given energy surface. Variants of this approach also exist, for convex Hamiltonians or problems admitting a Lagrangian formulation. Often, this approach leads to the existence of infinitely many periodic orbits; and a legitimate question that has been raised is whether all these periodic orbits are not provided by the Birkhoff-Lewis theorem.

I have tried to answer partially this question. I have therefore connected the framework of the Birkhoff-Lewis theorem to the variational framework of P. Rabinowitz; and I have tried to see the tori T^n which are mapped radially in the proof of the Birkhoff-Lewis theorem as Morse relations for the infinite dimensional variational problem.

Part of these tori can be seen as Morse relations. How many of them depends on the remainder term after the Birkhoff normal form reduction. The condition defining these tori is very similar to the one needed in the Birkhoff-Lewis theorem. These tori have then a polynomial contribution in the growth of the Betti numbers of the underlying variational space. This is specially meaningful in the case of 3-body type problems, where the growth of the Betti numbers is exponential. Then, we can say that there are at least in some cases, and probably in all cases, other periodic orbits than those provided by the Birkhoff-Lewis theorem.

VICTOR BANGERT

Closed flats in Riemannian manifolds of non-positive curvature (joint work with V. Schroeder)

A k-flat F in a Riemannian manifold M is an isometric and totally geodesic immersion of an Euclidean k-space into M. F is called closed if F is periodic with respect to some cocompact lattice so that F induces an immersion of a k-torus. In particular, a 1-flat is a geodesic (parametrized by arc-length) and a closed 1-flat is a closed geodesic. In generic Riemannian manifolds k-flats with $k > 1$ will not exist. However the existence of 2-flats is a common phenomenon when one studies manifolds M with non-positive curvature which have a lot of zero curvature.

Theorem: Let M be a compact real analytic Riemannian manifold with non-positive sectional curvature. If M contains a k-flat then M contains also a closed k-flat.

The proof relies on

(i) synthetic geometry of manifolds with non-positive curvature
(ii) the theory of subanalytic sets
(iii) a shadowing lemma for normally hyperbolic systems which can be found in: Hirsch/Pugh/Shub: Invariant manifolds, LN 583.

THOMAS BARTSCH

Multiple periodic solutions of Hamiltonian systems on symmetric starshaped energy surfaces

We are interested in periodic solutions of the Hamiltonian system (HS) $\dot{x} = J\nabla H(x)$ which lie on a fixed energy surface $S = H^{-1}(c) \subset \mathbb{R}^{2N}$. We assume that S bounds a compact starshaped neighborhood of 0 and satisfies an additional geometric condition. Moreover we assume that a compact Lie group Γ acts orthogonally on $\mathbf{R}^{2N}$ such that $\gamma^t J \gamma \in \{\pm J\}$ and that S is invariant under Γ. Then there exist at least $N/(1 + \dim \Gamma - \dim T\Gamma)$ Γ-orbits of periodic solutions of (HS) on S where $T\Gamma$ is the maximal torus of Γ. This result is new even for finite Γ. If Γ is trivial we recover a theorem of Berestycki et al. and if $\Gamma = \mathbf{Z}/2$ or $\mathbf{Z}/2 \times \mathbf{Z}/2$ we recover and generalize results of Rabinowitz and Szulkin on the existence of brake orbits and of van Groesen on the existence of normal mode solutions of (HS). In the course of the proof we develop a relative equivariant Lusternik-Schnirelmann category which can be applied to strongly indefinite functionals, in particular to the Hamiltonian action functional. The computation of this category can be reduced to a non-standard Borsuk-Ulam type theorem for torus actions.

M. BIALY AND L. POLTEROVICH

Birkhoff's theory on symplectic manifolds

On a symplectic manifold endowed with a Lagrangian distribution one can define a special class of optical Hamiltonian flows which roughly speaking twist every Lagrangian subspace in the positive direction with respect to this distribution. It turns out that the time-one maps of these flows have surprising properties which are similar to the ones discovered by G. Birkhoff for area-preserving twist maps of the annulus.

In particular we prove that under certain topological and dynamical assumptions a Lagrangian submanifold invariant under such a map is transversal to the Lagrangian distribution. This fact is closely related to the Maslov class rigidity phenomena for Lagrangian embeddings.

OLEG BOGOYAVLENSKIJ

Theory of Breaking Solitons

A new method for constructing nonlinear integrable $n+1$-dimensional equations and equations with attractors is developed. An integrable system of equations of hydrodynamical type $(v_i = v_i(t, y))$

$$v_{it} + v_i v_{iy} = v_i v_{i-1} \sum_{k=1}^{i-2} v_{ky} v_k^{-1} - v_i v_{i+1} \sum_{k=1}^{i+1} v_{ky} v_k^{-1} + \beta v_i (v_{i+1} - v_{i-1})$$

is found. This system is closely related to the Volterra system and the Toda lattice.

The continous limit of this system is a new integrable $2+1$-dimensional equation

$$v_t = 4vv_y + 2v_x \partial_x^{-1} v_y - v_{xxy} + \beta(6vv_x - v_{xxx}),$$

which describes the interaction between Riemann breaking waves, travelling in the y-direction and KdV long waves travelling in the x-direction.

A modified $2+1$-dimensional equation

$$v_t = 4v^2 v_y + 2v_x \partial_x^{-1}(v^2)_y + sV_{xxy}$$

is derived, $s = \pm 1$. All equations constructed possess breaking solitons which in the x-direction are smooth. Breaking N-soliton solutions are constructed.

ANDRY BOLIBRUCH

The 21st Hilbert problem for linear fuchsian systems

We proved that the 21st Hilbert problem has, in general, a negative solution and described all 3-dimensional representations which cannot be realised as monodromy representation of any Fuchsian system.

WALTER CRAIG AND EUGENE WAYNE

Newton's method and periodic solutions of nonlinear wave equations

This talk addresses the question of the existence of periodic solutions to nonlinear wave equations

$$\partial_t^2 u = \partial_x^2 u - g(x,u), \quad u(0,t) = 0 = u(\pi,t).$$

This can be viewed as an infinite dimensional Hamiltonian system, which has $u = 0$ an elliptic stationary point under mild conditions on g. The construction of periodic or quasiperiodic solutions involves small divisor problems. We describe a version of a Nash Moser iteration scheme which proves that under generic conditons on g, families of solutions of the equation exist. These families form Cantor sets foliated by circles. The requirements on the nonlinearity g are that it is sufficiently linearly nonresonant, and as well is genuinely nonlinear.

VICTOR J. DONNAY

Positive Entropy and Convex Billiards

A major conjecture in the field of non-uniform hyperbolic dynamics is that a generic compact Hamiltonian system has positive measure entropy. An interesting subclass of Hamiltonian systems is that of planar convex billiards for which there is an analogous conjecture. We made a modest start on the problem by showing: "There exist smooth strictly convex billiards with positive topological entropy". We make such examples by an explicit perturbation of the ellipse. Knieper - Weiss have proven a similar result for the geodesic flow on a strictly convex sphere. We have found a new class of convex billiards with positive Lyapunov exponent a.e. (but non-smooth, non-strictly convex), generalizing work of Bunimovich, Wojtkowski and Markarion. This class consists of convex arcs that satisfy the property of "focussing". This property is open in the C^6 topology of curves. A special example of a focusing arc is given by the half-ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, x \geq 0$, provided $a<\sqrt{2}$.

JEAN-PIERRE ECKMANN

Space-Time Evolution in 1 dimension

We discuss two equations:

(1) $$\partial_t u = \partial_x^2 u + u - u|u|^2 \, , \; u(x,t) \in \mathbf{C}$$

(2) $$\partial_t V = (\alpha - (1 + \partial_x^2)^2)V - V^3 \, , \; V(x,t) \in \mathbf{R}, \alpha > 0.$$

We present results on stationary solutions, their stability; propagating solutions (fronts). Furthermore we present results for (1) for the initial value problem with prescribed behaviour at $x = \pm\infty$, and we describe a conjugation between (1) and (2) as well as the first steps of a theory of normal forms for equations of the form (3) $\partial_t u = \partial_x^2 u + u - u|u|^2 +$ higher order terms.

MOHAMED SAMI ELBIALY

Simultaneous Binary Collisions in the Collinear n-body Problem

1. We show that the simultaneous binary collision singularity in the collinear N-body problem is C^1 block regularizable. That is we show that there is a homeomorphism which is C^1 together with its inverse - from collision and near collision orbits to ejection and near ejection orbits.

2. We also show that in the planar problem the collection of collision

orbits and ejection orbits form a real analytic submanifold of the phase space when the Levi-Civita spatial variables are used.

CHRISTOPHE GOLÉ

Ghost Circles and the Theorem of Aubry-Mather

We presented a new proof of the theorem of Aubry-Mather, involving the "gradient flow" of an "energy functional" on a sequence space. This setting is best understood as a generalisation of maximizing the sum of the chords in a billiard trajectory. For quasiperiodic trajectories, we can still make sense out of the gradient flow and find critical points, corresponding to orbits of any rotation number for a given Twist Map.

A closer analysis of the gradient flow brings in the motion of Ghost Circles: flow invariant, well ordered sets in the sequence space which fill the gaps of the Aubry-Mather sets. They imply a monotonous fitting of these sets in $S^1 \times \mathbf{R}$.

BORIS HASSELBLATT

Regularity of the Anosov splitting

Es wird Folgendes vermutet:

Vermutung : Eine Riemannsche Metrik negativer Schnittkrümmung mit C^2 horosphärischen Blätterungen ist lokalsymmetrisch.

Man weiss:

Satz : Eine Riemannsche Metrik negativer Schnittkrümmung mit C^∞ horospährischen Blätterungen hat einen geodätischen Fluss, der zu dem eines lokalsymmetrischen Raumes C^∞ konjugiert ist (KANAI; FERZES, KATOK;BENOIST, FOULON, LA BOURIE). Vor zwei Jahren trug ich vor, dass eine offene dichte Menge Riemannscher Metriken nicht C^2 horosphärische Blätterungen hat. Heute zeigte ich:

Satz : Wenn die horosphärischen Blätterungen einer $\frac{4}{n^2}$-eingesperrten Riemannschen Metrik ($-1 <$ Schnittkrümmung $< -\frac{4}{n^2}$) aus C^n sind, dann sind sie C^∞.

Satz : Es gibt ein "Hindernis" für C^2 horosphärische Blätterungen. Es hat gute globale Eigenschaften. Dies mag mithelfen, zu zeigen, dass C^2 horosphärische Blätterungen tatsächlich C^∞ sind.

ANATOLE KATOK

Commuting maps

We obtain a number of results demonstrating that the rank of an Abelian group acting effectively on a compact manifold by diffeomorphism with certain hyperbolic properties is bounded from above by the dimension of the manifold and that in the case of rank greater than one such an action possesses certain rigidity properties.

Theorem 1 Any element of an effective real-analytic action of $\mathbb{Z}^2$ on a compact surface has zero topological entropy.

The analyticity condition in this theorem can be replaced by C^2 if effectiveness is replaced by a stronger condition.

Theorem 2 If an element of an effective area-preserving real-analytic action of $\mathbb{Z}^2$ on a compact surface has a hyperbolic periodic point p then there exists an invariant open set V containing p in its closure such that a subgroup of $\mathbb{Z}^2$ of finite index acts on V either in a completely integrable way or can be embedded into an analytic flow.

In higher dimensions there are counterparts of Zimmer-rigidity for cocycles for actions with non-zero Lyapunov exponents.

ANDREAS KNAUF

Planare Molekülstreuung (mit M. Klein)

Wir haben die klassische und semiklassische Streuung eines Teilchens in der Ebene an einem Potential mit n anziehenden Coulombsingularitäten ($\approx -1/r$) untersucht. Das klassische Streuproblem zeigt oberhalb einer potentielabhängigen Schwellenenergie ein qualitativ universelles Verhalten. Für $n \geq 3$ divergiert die Zeitverzögerung auf einer Cantormenge von Anfangsbedingungen. Grössen wie die Hausdorff-Dimension der gebundenen Orbits oder die topologische Entropie besitzen eine universelle Energie-Asymptotik. Trotz der Irregularität der Streuung ist der (klassische) differentielle Wirkungsquerschnitt in vielen Fällen glatt.

Quantenmechanisch wurde für $n = 2$ die Lage der Resonanzen und für $n \geq 3$ eine $\hbar$-abhängige Schranke für ihre Anzahl berechnet, wobei die Hausdorff-Dimension entscheidend eingeht.

SERGEJ KUKSIN

On interpolation of an analytic symplectomorphism by a Hamilton flow

It is proved that a nearly integrable analytic symplectomorphism may be represented as the Poincaré map of an analytic Hamiltonian vectorfield. The radius of analyticity of the vectorfield is estimated from below.

SERGEI KUKSIN

Perturbations of finite-gap solutions of the KdV equation

It is checked that all families of finite-gap periodic solutions of the KdV equation are nondegenerate. Due to the earlier results of the author it completes the proof of the fact that most of these solutions survive under Hamiltonian perturbations of the equation.

OSCAR E. LANFORD III

Renormalization group analysis for critical circle mappings in the zero rotation number limit

The renormalization group analysis for smooth circle mappings with critical points leads to a sequence of *renormalization operators* $J_r, r = 1, 2, 3, \ldots$, acting on commuting pairs (ξ, η) where

- η is defined on $[0, 1]; \eta(x) < x$
- ξ is defined on $[\eta(0), 0]$ and sends 0 to 1
- 0 is a critical point for both ξ and η.

The operator J_r send (ξ, η) to $(\eta, \eta^r \circ \xi)$ rescaled.

Numerical experiments and heuristic arguments strongly suggest that each J_r has a hyperbolic fixed point, where its derivative has just one expanding eigenvalue δ_r. The limit $r \to \infty$ is singular - in particular $\delta_r \sim$ const. r^3 - but has some comprehensible structure. By passing from the operator J_r acting on "points" in the space of commuting pairs to an associated "Sinai operator" J_r^* acting on one-parameter families, we replace the hyperbolic fixed point problem by an *attracting* fixed point problem. The operators J_r^* have a well-behaved limit as $r \to \infty$. Computer studies suggest that the limiting operator has an attracting fixed point (and hence that all J_r^* with sufficiently large r have attracting fixed points). Furthermore, the limiting operator admits some formal simplification which makes it more accessible to study than the operators for finite r.

V. LAZUTKIN

Hamiltonian dynamical systems

Global properities of the stable and unstable separatrices for the standard map

$$(x, y) \mapsto (x + y + \varepsilon \sin x, y + \varepsilon \sin x)$$

and the semistandard map

$$(u, v) \mapsto (u + v + \exp(u), v + \exp(u))$$

are investigated in the complex domain both analytically and by use of a computer. The regions of wild behaviour of iterates where found to form fern-like structures on unstable seperatrices, the tips of leaves being interpreted as homoclinic points. This suggests a new approch to the investigation of chaotic phenomena in conservative systems.

PATRICE LE CALVEZ

A dissipative version of the Conley-Zehnder theorem

Every lift $f : \mathbb{R}^2 \to \mathbb{R}^2$ of a diffeomorphism of $\mathbf{R}^2/\mathbf{Z}^2$, isotopic to the identity map, is a composition of twist maps. We can construct like in the Aubry-Mather theory, a vector field ξ on a manifold $\mathbf{T}^2 \times \mathbf{R}^N$ which permits us to give the following equivariant version of the Brouwer translation theorem.

Theorem : If f has no fixed point, there exists a continous vector field Θ, $\mathbb{Z}^2$-periodic and uniquely integrable such that $f(C)$ and $f^{-1}(C)$ are in the different components of $\mathbb{R}^2 \backslash C$ for every integral curve C of Θ.

If f preserves the center of mass and the area, ξ is a gradient flow and we obtain a proof of the Conley-Zehnder Theorem (f has at least 3 fixed points). Other properties of ξ permit us (with J.M. Gambaudo) to obtain the following results, proved also by J. Franks in a different way

Theorem :

If $F : \mathbf{T}^1 \times [0,1]$ is a C^1-diffeo. which preserves the area and has one fixed point (If $F : S^2 \to S^2$ is a C^2-diffeo. which preserves the area and has 3 fixed points)
then F has an *infinite* number of periodic orbits.

RICHARD MCGEHEE

Iteration of Relations

A relation f on a set X is a subset of $X \times X$. An orbit for a relation f is a sequence of points $p_i \in X$ satisfying $(p_i, p_{i+1}) \in f$. The composition of relations can be defined in the obvious way so that $(x, y) \in f^n$ if and only if there exists an orbit $(p_0, p_1, \ldots, p_n)$ satisfying $p_0 = x$ and $p_n = y$. If X is a compact Hausdorff space and if f is a closed subset of $X \times X$, then f^n is also closed.

Much of the standard theory of dynamical systems can be developed for closed relations on compact Hausdorff spaces. For example, the notions of chain recurrence and attraction can be extended to this setting and are related in the same way as they are for flows on compact metric spaces. The order relation defined by Conley (where two points are related if there exists, for arbitrarily small ε, an ε-pseudo-orbit from one to the other) can also be extended to this setting, where it has an interpretation as a limit of f^n as $n \to \infty$.

JÜRGEN PÖSCHEL

Nekhoroshev Estimate for Convex Hamiltonians

Let $H = h(I) + \varepsilon f(I, \vartheta)$ be a nearly integrable Hamiltonian system in action angle coordinates $I \in D \subset \mathbb{R}^n, \vartheta \in T^n$, real analytic in the complex neighborhood $\{I : \|I - D\| < r_\circ\} \times \{\vartheta : |\text{ Imag. } \vartheta|_\infty < s_\circ\}$. Suppose h is convex:

$$m\|\xi\|^2 \leq \langle Q(I)\xi, \xi\rangle \leq M\|\xi\|^2,$$

$Q(I) = \partial_I^2 h$. Then for $\varepsilon \leq \varepsilon_\circ = \frac{m\, r_\circ^2}{2^{10} A^{2n}}$, $A = 18\frac{M}{m}$ *every* orbit starting in $D \times T^n$ satisfies

$$\|I(t) - I(0)\| \leq R_\circ \left(\frac{\varepsilon}{\varepsilon_\circ}\right)^a$$

for

$$|t| \leq T_\circ \exp\left(\frac{s_\circ}{6}\left(\frac{\varepsilon_\circ}{\varepsilon}\right)^a\right)$$

except when $\|\partial_I h\,(I(0))\| \leq \frac{mr_\circ}{12}$ in which case one only has $\|I(t) - I(0)\| \leq r_\circ$. The parameters are

$$a = \frac{1}{2n}, R_\circ = \frac{r_\circ}{A}, T_\circ = A^2\frac{s_\circ}{\Omega},$$

$\Omega = \sup\|\partial_I h\,(I)\|$. Moreover with probability $1 - 0(r_\circ)$ one even has $\|I(t) - I(O)\| \leq R_\circ\sqrt{\frac{\varepsilon}{\varepsilon_\circ}}$.

YOSHIMASA NAKAMURA

Some generalization of the Moser-Toda equation and applications

There have been some systematic efforts toward generalizations of the Moser-Toda equation $\dot{x}_k = y_k, \dot{y}_k = e^{x_{k-1}-x_k} - e^{x_k - x_{k-1}}, k = 1, 2, \ldots n, x_0 = -\infty, x_{n+1} = \infty$, by O.I. Bogoyavlensky, B. Kostant, W.W. Symes, R. Goodmann, N.R. Wallach and others. The purpose of this talk is to describe some recent developments of generalizations of the Moser-Toda equation and applications.

First, the Moser-Toda equation is the Hamiltonian equation of Lax type. It is known that the level set is diffeormorphic to $\mathbf{R}^n$ which is a connected component of a space of rational functions. A generalization is presented from the view point of the complete parametrization of the space of rational functions. This new dynamical system has the level set being diffeomorphic to a certain cylinder.

Secondly, the Moser-Toda equation also describes the gradient flow for a quadratic form on a sphere. This enables us to get a generalization to symmetric spaces. The resulting equations have applications, e.g. to linear programming and matrix eigenvalue problems.

R. PEREZ-MARCO

Dynamics and symmetries of a holomorphic map near a fixed point

A holomorphic germ $f(z) = \lambda z + O(x^2)$ is said to be linearizable if it is locally conjugate to its linear part, that is, there is $h(z) = z + O(z^2)$ such that $h^{-1} \circ f \circ h\ (z) = \lambda z$. We study the qualitative dynamics and the symmetries of a non-linearizable germ for which $\lambda = e^{2\pi i \alpha}, \alpha \in \mathbf{R} - \mathbf{Q}$. If $(p_n/q_n)_{n \geq 0}$ are the convergents of α, for such a germ, if $\sum\limits_{n\geq 1} \frac{\log \log q_{n+1}}{q_n} < +\infty$ there are always an infinite number of periodic orbits near 0 and $\neq 0$. The above diophantine condition is also optimal to have the result. The symmetries of f are $\mathrm{cent}(f) = \{g(z) = \mu z + O(z^2); f \circ g = g \circ f\}$. There exists f such that $\mathrm{cent}(f)$ is uncountable, in particular the group of iterates of f is not of finite index on $\mathrm{cent}(f)$. Such an f can be chosen with or without periodic orbits near 0 (and $\neq 0$).

DIETMAR SALAMON

Floer homology and Novikov rings

The V. Arnold conjecture in the nondegenerate case states that the number of fixed points of a Hamiltonian symplectomorphism of a compact symplectic manifold (M, ω) can be estimated from below by the sum of the Betti numbers. Floer's proof for monotone symplectic manifolds generalizes to the case where the first Chern class $c_1(TM) = 0$. The resulting Floer homology groups form a module over Novikov's ring Λ_ω of generalized Laurent series which algebraically incorporates the period map $\varphi_\omega : \pi_2(M) \to \mathbf{R}$. The compactness proof is based on the observation that the holomorphic spheres form a set of codimension 4 and generically do not intersect the 3-dimensional set of connecting orbits with index difference 1 (joint work with H. Hofer).

K.F. SIBURG

Simultaneous Binary Collisions for the N-body Problem

In celestial mechanics it is known that single binary collisions are both branch- and blockregularisable, while triple collisions are neither. Also simultaneous binary collisions are branchregularizable in an analytic way. In contrast, the question of blockregularization is in general still open. In this lecture we presented a simple approach to this problem. In appropriate coordinates the equations of motion take the form of

Euler equations whose solutions can be written (by the variation-of-constants-formula) as fixed points of an integral operator. By studying this operator we get a C°-blockregularization at least for the collinear case; moreover the transition time is Hölder-continuous. Perhaps this new method could give a similar result for higher dimensions or an obstruction for the analyticity of blockregularizations of simultaneous binary collisions.

MARCELO VIANA

Homoclinic bifurcations, strange attractors and infinitely many sinks

The study of homoclinic bifurcations is a central problem in Dynamics and, in view of a recent conjecture of Palis, might lead to a global understanding of nonhyperbolic dynamics, at least for surface diffeomorphismus. We discuss several results on phenomena occurring during the unfolding of such a tangency, for a large (positive measure, second category, etc) set of parameter values: hyperbolic dynamics (Palis, Takens, Yoccoz), infinitely many sinks (Newhouse, Palis-V), strange attractors (Benedicks-Carleson, Mora-V). We also discuss the unfolding of saddle-node cycles, which exhibits positive density, at the saddlenode bifurcation value, of the set of parameters corresponding to existence of strange attractors (Diaz, Rocha, V).

CLAUDE VITERBO

Generating function and symplectic topology

The talk was an exposition of my paper "Symplectic topology as the geometry of generating functions" to appear in Math. Annalen. After defining the notion of generating function quadratic at infinity associated to a Lagrange submanifold in T^*M, we show how this can be used to prove that any Lagrange submanifold of T^*M, which is the time one image of the zero section by a Hamiltonian flow, intersects the zero section in $cl(M) \geq 2$ points (at least).

We then state a uniqueness theorem for generating functions, and use this to show that certain critical levels of it only depend on the associated Lagrange submanifold, thus yielding invariants of a Lagrange submanifold.

We apply this to the graph of a symplectic map with compact support in $\mathbf{R}^{2n}$, to get "capacities". We finally give applications to Gromov's theorem, and existence of *infinitely* many periodic points for compactly supported symplectic maps (the periodic points we find are of course in the support).

MACIEJ P. WOJTKOWSKI

Positive Lagrangian subspaces and Kalman-Bucy filters

Graphs of symmetric linear maps from $\mathbf{R}^n$ to $\mathbf{R}^n$ are Lagrangian subspaces in the standard linear symplectic space $\mathbf{R}^n \times \mathbf{R}^n$. We call a Lagrangian subspace positive if it is a graph of a positive definite linear map. Further we call a linear symplectic map monotone if it maps positive Lagrangian subspaces onto positive Lagrangian subspaces. Bougerol discovered that the symplectic matrices in Kalman-Bucy filtering theory are monotone. He shows that the action of any monotone map on the manifold of positive Lagrangian subspaces contracts the metric of the Riemannian symmetric space. It is the only (up to scale) Riemannian metric which has this property.

Nevertheless we introduce a natural Finsler metric in the manifold of positive definite matrices which, in addition to being contracted by the action of any monotone map, has striking geometric properties. In particular we obtain that the coefficient of least contraction is equal to the hyperbolic tangent of one half of the diameter of the image. This is the same relation which was obtained by Birkhoff for the Hilbert projective metric.

Berichterstatter: E. Zehnder

Tagungsteilnehmer

Dr. Alain Albouy
2 av. Pozzo-di-Borgo

F-92210 Saint Cloud

Dr. Thomas Bartsch
Mathematisches Institut
Universität Heidelberg
Im Neuenheimer Feld 288

W-6900 Heidelberg 1
GERMANY

Prof.Dr. Sigurd B. Angenent
Department of Mathematics
University of Wisconsin-Madison
617 Van Vleck Hall
610 Walnut Street

Madison , WI 53705
USA

Prof.Dr. Misha Bialy
Tel Aviv University
Raymond and Beverly Sackler
Faculty of exact Sciences
Ramat-Aviv

Tel Aviv 69978
ISRAEL

Dr. Marie-Claude Arnaud
U. E. R. de Mathématiques
T. 45-55, 5ème étage
Université de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Oleg I. Bogoyavlenskij
Steklov Mathematical Institute
MIAN
Academy of Sciences of the USSR
42, Vavilova str.

Moscow 117 966 GSP-1
USSR

Prof.Dr. Abbas Bahri
Dept. of Mathematics
Rutgers University
Busch Campus, Hill Center

New Brunswick , NJ 08903
USA

Dr. Andrei A. Bolibruch
Steklov Mathematical Institute
MIAN
Academy of Sciences of the USSR
42, Vavilova str.

Moscow 117 966 GSP-1
USSR

Prof.Dr. Victor Bangert
Mathematisches Institut
Universität Freiburg
Hebelstr. 29

W-7800 Freiburg
GERMANY

Prof.Dr. Leonid A. Bunimovich
c/o Prof. Ph. Blanchard
Forschungszentrum BiBoS
Universitätsstr. 25

W-4800 Bielefeld 1
GERMANY

Prof.Dr. Marc Chaperon
U. E. R. de Mathématiques
T. 45-55, 5ème étage
Université de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Prof.Dr. Jean-Pierre Eckmann
Physique Théorique
Université de Genève
32 Blvd. d'Yvoy

CH-1211 Genève 4

Prof.Dr. Alain Chenciner
U. E. R. de Mathématiques
T. 45-55, 5ème étage
Université de Paris VII
2, Place Jussieu

F-75251 Paris Cedex 05

Dr. Mohamed Sami El-Bialy
Dept. of Mathematics
University of Toledo

Toledo , OH 43606
USA

Dr. Walter L. Craig
35 Ridgmount Gardens

GB- London WC1E 7AS

Prof.Dr. Hakan Eliasson
Dept. of Mathematics
Royal Institute of Technology
Lindstedtsvägen 30

S-100 44 Stockholm

Jochen Denzler
Mathematik
ETH-Zentrum
Rämistr.101

CH-8092 Zürich

Albert Fathi
Dept. of Mathematics
University of Florida
201, Walker Hall

Gainesville , FL 32611-2082
USA

Prof.Dr. Victor J. Donnay
Mathematics Department
Bryn Mawr College

Bryn Mawr , PA 19010
USA

Dr. Christophe Gole
Forschungsinstitut für Mathematik
ETH-Zürich
ETH Zentrum
Rämistr. 101

CH-8092 Zürich

Dr. Boris Hasselblatt
Forschungsinstitut für Mathematik
ETH-Zentrum
Rämistr. 101

CH-8092 Zürich

Dr. Tyll Krüger
Physik
BIBOs Forschungszentrum
Universität Bielefeld
Universitätsstr. 1

W-4800 Bielefeld 1
GERMANY

Prof.Dr. Helmut W. Hofer
Institut f. Mathematik
Ruhr-Universität Bochum
Gebäude NA, Universitätsstr. 150
Postfach 10 21 48

W-4630 Bochum 1
GERMANY

Prof.Dr. Sergei B. Kuksin
Institute for Control Sciences
Academy of Sciences of the USSR
ul. Profsojuznaja 65

Moscow , GSP-7 117806
USSR

Prof.Dr. Anatole B. Katok
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Oscar E. Lanford III.
Mathematik
ETH-Zentrum
Rämistr.101

CH-8092 Zürich

Prof.Dr. Urs Kirchgraber
Mathematik
ETH-Zentrum
Rämistr.101

CH-8092 Zürich

Prof.Dr. Vladimir F. Lazutkin
Pestelya 13/15 kv. 68

Leningrad 191 028
USSR

Dr. Andreas Knauf
Fachbereich Mathematik - MA 7-2
Technische Universität Berlin
Straße des 17. Juni 136

W-1000 Berlin 12
GERMANY

Prof.Dr. Patrice Le Calvez
Mathématiques
Université de Paris Sud (Paris XI)
Centre d'Orsay, Bâtiment 425

F-91405 Orsay Cedex

Prof.Dr. John N. Mather
Department of Mathematics
Princeton University
Fine Hall
Washington Road

Princeton , NJ 08544-1000
USA

Prof.Dr. Leonid Polterovich
Tel Aviv University
Raymond and Beverly Sackler
Faculty of exact Sciences
Ramat-Aviv

Tel Aviv 69978
ISRAEL

Prof.Dr. Richard McGehee
School of Mathematics
University of Minnesota
127 Vincent Hall
206 Church Street S. E.

Minneapolis , MN 55455
USA

Dr. Jürgen Pöschel
Institut für Angewandte Analysis
Universität Bonn
Beringstr. 4-6

W-5300 Bonn 1
GERMANY

Prof.Dr. Jürgen Moser
Mathematik
ETH-Zentrum
Rämistr.101

CH-8092 Zürich

Prof.Dr. Paul H. Rabinowitz
Department of Mathematics
University of Wisconsin-Madison
Van Vleck Hall

Madison WI, 53706
USA

Dr. Yoshimasa Nakamura
Department of Mathematics
Fafculty of Education
Gifu University
Yanagido

Gifu 501-11
JAPAN

Prof. Dr. Dietmar Salamon
Mathematics Institute
University of Warwick

GB- Coventry , CV4 7AL

Dr. Ricardo Perez-Marco
Université de Paris-Sud
Bâtiment 425

F-91400 Orsay Cedex

Karl-Friedrich Siburg
Seminar für angewandte Mathematik
ETH
Fliederstr. 23

CH-8092 Zürich

Prof.Dr. Jean-Claude Sikorav
Université Paul Sabatier
U.F.R. M.I.G.
118, route de Narbonne

F-31062 Toulouse Cedex

Prof.Dr. Clarence Eugene Wayne
Department of Mathematics
Pennsylvania State University
218 McAllister Building

University Park , PA 16802
USA

Prof.Dr. Marcelo Viana
Instituto de Matematica Pura e
Aplicada - IMPA
Jardim Botanico ; CEP 22460
Estrada Dona Castorina, 110

Rio de Janeiro , RJ
BRAZIL

Prof.Dr. Maciej P. Wojtkowski
Dept. of Mathematics
University of Arizona

Tucson , AZ 85721
USA

Prof.Dr. Claude Viterbo
CEREMADE, Université de Paris
Dauphine (Université de Paris IX)
Place de Lattre de Tassigny

F-75775 Paris Cedex 16

Prof.Dr. Eduard Zehnder
Mathematik
ETH-Zentrum
Rämistr.101

CH-8092 Zürich