MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Topologie

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The meeting was organized by
M.Kreck (Mainz), A.Ranicki (Edinburgh) and L.Siebenmann (Orsay).
The 19 talks mainly dealt with the application of algebraic topology in differential and algebraic geometry and vice versa.

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Vortragsauszüge

S.Bauer

Nonreduced Moduli Spaces and Donaldson Invariants

Let $V(p,q)$ denote an algebraic elliptic surface with Euler characteristic 12 and with two multiple fibers of multiplicity p and q, respectively. The moduli space of stable vector bundles with Chern classes $c_2 = 1$, $c_1 = 0$ in general is not smooth. It may be described the following way:
Theorem. Any component M of this moduli space is a locally complete intersection. The generic point is of length $a(p-a)$ for a positive integer $a < p$. It embeds as an infinitesimal neighborhood of the zero section in the Whitney sum $NF_p \oplus NF_p$ of two copies of the normal bundle NF_p of one of the reduced multiple fibers F_p in $V(p,q)$.
Using this theorem it is possible to compute Donaldson's Gamma-invariant for $V(p,q)$. If the gcd of p and q is less than 3, the number $(p^2-1)(q^2-1)$ turns out to be a diffeomorphism invariant. In particular, those $V(p,q)$ with fundamental group of order two have the set $\{p,q\}$ as a diffeomorphism invariant.

Cernarvskij

Topology of Fuchsian Systems (Riemann–Hilbert problem)

The report concerns results of a Moscow seminar on the Riemann–Hilbert problem: The existence of an integrable Fuchsian system on CP^n with a given monodromy. The main result is of A.A Bolibruch, who gave a first example of negative solution of the classical case $(n = 1)$, and also found some situations where the solution is positive. For $n > 1$ there is the question about integrability, but the generic case is simplified, because of commutativity of the monodromy. But there are very simple and very important in physics and the braid and knot theories nongeneric cases, where the problem is not trivial. The topological conditions for the integrability are given, and a theorem was proven by V.P. Lexin about the existence of integrable Fuchsians systems with given monodromies close to identity.

S.C.Ferry

Finiteness of Homeomorphism types in Gromov-Hausdorff space

Definition Let X be a metric space. A function $\rho : [0,R) \to [0,\infty)$ is a *contractibility function* for X if ρ is continuous at 0 and if for each $x \in X$, the ball of radius t around x contracts to a point in the ball of radius $\rho(t)$ around x.
Theorem. Let $n \geq 4$ and $\rho : [0,R) \to [0,\infty)$ be given.If S is a precompact subset of Gromov–Hausdorff space such that each element of S is a closed n-manifold with contractibility function ρ, then S contains only finitely many homeomorphism types.

This corrects an error in a paper of Grove-Petersen-Wu (See Invent.Math. 1990 and correction 1991 for details).
We also show how to estimate explicitly the number of simple/homotopy types in such a subset S. It seems likely that the technique can be extended to estimate the number of homeomorphism types in S.

R.E.Gompf
Irreducible 4-manifolds need not be complex

(Joint work with Thomasz Mrowka.) An old conjecture asserts that every smooth, closed, simply connected 4-manifold is a connected sum of algebraic surfaces. In fact, this is false. Infinite families of counterexamples can be constructed by generalizing the construction of elliptic surfaces. Specifically, one can construct counterexamples by simultaneously applying logarithmic transforms to several nonhomologous tori in an elliptic surface. For example, we obtain an infinite family of homotopy $K3$ surfaces which are counterexamples. These are distinguished by a Donaldson invariant which Mrowka has analyzed by cutting the manifolds apart along 3-tori.

J.-C.Hausmann
Gauge theory and symmetries of bundles

Let $\xi\colon P \xrightarrow{p} M$ be a smooth principal G-bundle (G a compact Lie group). Let us assume that M is equipped with a smooth action α of a compact Lie group T. Question: Does α come from an action on the bundle ξ? Besides easy necessary conditions we solve the problem (completely if G is abelian , partially in general) using a gauge theoretic necessary condition: the action α induces an action of T on the moduli space A/G of ξ and this action must have a fixed point. Interesting applications occur.

J.Klein
Higher Reidemeister Torsion

This is a report of joint work with Kiyoshi Igusa. The idea is to define (higher) algebraic K-theory valued invariants for a fibre bundle over a finite CW complex. Such an invariant is best thought of as a parametrized analogue of the classical Reidemeister-Franz torsion.
Specifically, let $p{:}E \to B$ be a fibre bundle with compact smooth fibre M and with finite connected CW base B. Suppose additionally that $\rho : \pi_1(E) \to U_r(\mathbb{C})$ is a representation such that $H_*(M;\mathbb{C}^r) = 0$,where coefficients here are taken with respect to the local system on M defined by restricting the local system on E defined by ρ.

Theorem. The pair (p, ρ) gives rise to an element $\tau(p, \rho)$ of the group of homotopy classes

$$[B, \Omega K(\mathbb{C})],$$

and this element is an invariant of the isomorphism class of p. Moreover, if B is a point, then this element coincides with classical Reidemeister-Franz torsion.

That this theorem has any interesting consequences is settled by examples we have constructed which show it to be non-trivial. Let $h_n \colon L(n,1) \to S^2$ be the circle bundle of Chern class n, and let $\rho_n \colon \mathbb{Z}_n \to U_1(\mathbb{C})$ be defined by choosing an n^{th} root of unity ζ_n. Also, let $b \colon K_3(\mathbb{C}) \to \mathbb{R}$ be the Borel regulator.

Theorem. The value of b at $\tau(p_n, \rho_n)$ is

$$n \cdot Im \sum_{k=1}^{\infty} \frac{\zeta_n^k}{k^2} \quad .$$

In particular, the values $\tau(p_n, \rho_n)$ are distinct for distinct n.

M.Kontsevich
Homotopical Algebra and Low-Dimensional Topology

We call a *graph with metric* a pair (X, l) where X is a finite CW-complex, l is a map from the set of 1- cells X_1 to R_+. Denote by G_n the moduli (orbi) space of connected graphs with metrics such that i) the Euler characteristic of graph is $1-n$, ii) the valency of each vertex is greater then 2. We endow G_n by Gromov-Hausdorff topology. A *ribbon graph* is a graph with a fixed cyclic order on the set of edges coming to each vertex. Denote by R_n an analogous moduli space of ribbon graphs, R_n is a noncompact PL-manifold. The obvious proper map $R_n \to G_n$ induces maps

$$H^*(G_n) \to H^*(R_n) \xrightarrow[\text{Poincaré}]{\simeq} H^{closed}_{3n-3-*}(R_n, \det) \to H^{closed}_{3n-3-*}(G_n, \det),$$

det denotes the local one-dimensional system equal to $\Lambda^n(H^1(X, Q))$.

Denote by C_1, C_2 and C_3 the three graded spaces from the previous formula.

Theorem. $C_1 \to C_2 \to C_3$ is a 3-term complex. Conjecturally C_* is exact in the middle term.

Geometrical meaning. There are homotopy equivalences $G_n \simeq$ BOut (Freegroup(n)) (M.Culler, K.Vogtmann),

$$R_n \simeq \coprod_{g,k:\ 2-2g-k=1-n, k>0} \mathrm{BDiff}_+(S_{g,k}),$$

where $S_{g,k}$ is an oriented surface with genus g and k holes (R.Penner,J.Harer). From V.Drinfeld's work on quasi-Hopf algebras it follows that elements of some modification of $H^{\text{closed}}_{3n-3}(G_n, \det)$ give universal nilpotent knot invariants. Conjecturally homology groups with closed support of G_n map to cohomology groups of $BDiff_+(M)$

for any oriented 3-manifold M.
Relation with homotopical algebra. There is a construction which associates to any differential Z_2- graded commutative, associative or Lie algebra with nondegenerate scalar product on it an even nonhomogeneous element in C_1, C_2 or C_3 respectively. This construction uses machinery of homotopical algebra. The idea is to construct from algebras with duality using Massey operations finite spin models on graphs.Partition functions of all graphs define (co)chains in corresponding complexes.

M.P.Latiolais

Obstruction Theory for the Homotopy Classification of 2-Complexes with Finite Fundamental Group

Given a finite group G satisfying Eichler (No quotient is generalized quarternion or binary polyhedral of order 24, 48 or 120) an unpublished theorem of W. Browning gives the total obstruction to the existence of a homotopy equivalence $f_\alpha \colon K \to L$, where $f_* = \alpha \colon \pi_1(K) \to \pi_1(L)(= G)$ for a specific isomorphism α. K and L are finite 2- complexes with $\chi(K) = \chi(L)$. The proof of Browning's theorem may be generalized to get
Theorem (Gutierrez, Latiolais): Let K and L be finite 2-complexes with fundamental group G. Let $0 \to N \to G \to Q \to 0$ be exact with Q satisfying Eichler and $H_2(N)$ free. Then there exists an equivariant homology equivalence between the finite covers of K and L corresponding to Q if and only if the obstruction element $\langle K, L\rangle \in K_1(Z_u Q)/K_1(ZQ)$ is congruent to zero modulo the u-self-equivalences on K.
Browning's obstruction depended on the particular isomorphism, as does the obstruction in the above theorem. This restriction may be eliminated.
Theorem (Latiolais): Given a finite group G satisfying Eichler's condition and given 2-complexes K and $L, \pi_1(K) \simeq \pi_1(L) \chi(K) = \chi(L)$, then there is a total obstruction $\langle K, L\rangle$ to K and L being homotopy equivalent to L. The obstruction and the obstruction group are independant of a particular identification of fundamental groups.
The above mentioned obstruction group is, for the most part, computable. Consequently, the question arises as to whether all of the elements of the group are realizable as obstructions to specific 2-complexes being homotopy equivalent. For finite abelian groups, this obstruction group can be computed, using constructive results of Sieradski. The result is a complete homotopy classification of 2-complexes with finite abelian fundamental group. These constructive techniques can be generalized to various semi-direct products. In particular, these techniques give that the homotopy classification of 2-complexes with dihedral fundamental group depends only on the Euler charcteristic, if the order of the dihedral group is divisible by 4 (Latiolais). New results of Hambleton and Kreck, together with a simple homotopy classification done by Jajodia and Magurn, gives that the simple homotopy classifi-

cation of 2-complexes with fundamental group a finite subgroup of SO_3 depends on the Euler characteristic only.
Comparing the finite abelian results of Browning with algebraic bounds for numbers of possible second homotopy groups computed by Dyer and Sieradski, we get the following result:
Theorem (Latiolais, Waller): The number of distinct second homotopy groups of 2-complexes of minimal Euler characteristic with respect to the fundamental group $Z_{m_1} \times \cdots \times Z_{m_n}$, where $m_i \mid m_{i+1}$, is precisely the order of the group

$$\frac{Z^*_{m_1}}{(Z^*_{m_1})^{n(n-1)/2}(Z^*_{m_1})^{n-1}}$$

and the number of such 2-complexes which are homotopically distinct, but have the same second homotopy group is the order of the group

$$\frac{(Z^*_{m_1})^{n(n-1)/2}(Z^*_{m_1})^{n-1}}{\pm(Z^*_{m_1})^{n-1}}$$

In fact, the above groups act as the obstruction groups for the two distinctions.

W.Lück

Torsion Invariants, von Neumann algebras and 3-manifolds

Given a compact Riemannian manifold M possibly with boundary we define a torsion type invariant

$$\rho(M) \in K_1^w(N(\pi))$$

where π is the fundamental group, $N(\pi)$ the von Neumann algebra of π and $K_1^w(N(\pi))$ the K_1-group of weak automorphisms of finitely generated Hilbert modules over $N(\pi)$. It is independent of the Riemannian metric if and only if the L^2-homology of M vanishes. The last condition is equivalent to the statement that there are no smooth harmonic L^2-integrable differential forms on the universal covering of M. The invariant satisfies some useful properties like glueing formula, product formula, Poincaré duality. It is a generalization of the classical notion of Reidemeister torsion for finite fundamental groups to infinite fundamental groups. It encompasses other invariants like the Alexander polynomial of a knot and the Lefschetz Zeta-function of a self map of a compact manifold.
In particular the invariant seems to be interesting for 3- manifolds. One can obtain a real number

$$\tilde{\rho}(M) \in R$$

by applying the Kadison Fuglede determinant which gives a homomorphism from the relevant K_1-group to the real numbers. In the sequel let M be a prime Haken 3-manifold with infinite fundamental group. Then following Jaco-Shalen, Johannsen and Thurston M can be cut into pieces along a family of incompressible tori such

that each piece is Seifert or hyperbolic. The invariant is additive under this glueing process and vanishes for Seifert pieces. Hence $\tilde{\rho}(M)$ is the sum of the $\tilde{\rho}(M_i)$ for $M_1, \ldots, M_r$ the hyperbolic pieces. There is strong evidence for the conjecture that $\tilde{\rho}(M_i)$ is just the volume up to a non-zero factor. Hence $\tilde{\rho}(M)$ measures the size of the hyperbolic pieces in M and vanishes if and only if there are no such pieces.
If the conjecture above is true, $\tilde{\rho}(M)$, the analytic L^2-torsion of M as well as Gromov's simplicial volume of M would agree. The analytic torsion of a hyperbolic manifold is known to be the volume and agrees with $\tilde{\rho}(M)$ in a setting where one does not use $bl^2(\pi)$ but finite- dimensional representations. The analytic L^2-torsion is defined in terms of the heatkernel of the universal covering, Gromov's simplicial volume is the norm of the fundamental class in the bounded homology whereas $\rho(M)$ can be read off from a presentation of the fundamental group of deficiency 0 or -1 depending on whether M is closed or has boundary.

A.Marin, joint work with R.Benedetti

Optimal Characteristic Surfaces in 3-manifolds and the Nash Conjecture in Dimension 3

Nash conjectured (Annals 1952) that any smooth closed manifold is homeomorphic to a real rational algebraic set. One defines smooth substitutes for the algebraic modifications by blowing-up and down. The so generated equivalence relation is called $m-$ (for modification) **equivalence** .
For proving Nash's conjecture in dimension n, you can try first to prove that any n-manifold is m-equivalent to the sphere S^n, and then algebraize the equivalence.
In 1989 Benedetti (and independently Akbulut and King) made that programm work in dimension 3. In the talk we gave an improved version of the topological part of Benedetti's approach.
The first remark is that a Dehn-surgery between two 3- manifolds V and V' (along links L and L') extend to a homeomorphism between the blowing-ups $\sigma_L V$ and $\sigma_{L'} V'$ if and only if the Dehn-surgery preserves the mod 2 classes of the meridians. Such a Dehn-surgery is called a **tear.** The only invariant for the classification of orientable 3-manifolds up to tears is the first Betti number with coefficients mod 2 but in the non orientable case tears preserve the multiplicative structure of the ideal in $H^*(V)$ generated by $w_1(V)$ and in the case where $w_1(V)^2 = 0$ tears preserve also the invariants (nullity) or not on the radical and Arf invariant when it is defined) of a **rolling-up quadratic form** q defined on :

$$\mathcal{B}_F = \ker(H_1(F; \mathbb{Z}/2) \to H_1(M; \mathbb{Z}/2))$$

where F is a characteristic surface in V (i.e. dual to $w_1(V)$).
This is a full set of invariants and non-singular rational modells are constructed in each class except one which has nevertheless a singular rational modell. The main point in the proof, beside the definition of the form q, is that any 3-manifold is

tear-equivalent to a manifold V having an optimal characteristic surface, i.e. such that $\dim(\mathcal{B}_F) = 1, 0$ or 2 according to when $q(\mathcal{B}_F^{\perp}) \neq 0, q(\mathcal{B}_F^{\perp}) = 0$ and Arf $(q) = 0$ or 1 respectively.

Nikita Netsvetaev

On the topological structure of hypersurfaces in $\mathbb{CP}^{n+1}$ with quadratic singularities (n even, ≥ 4)

Notations. Let $X \subset \mathbb{CP}^{n+1}$ be an algebraic hypersurface of even complex dimension $n \geq 4$ and degree d. We assume that X has only quadratic singular points; let their number equal k. The Lefschetz Hyperplane Section Theorem implies that $H_1(X) = 0$ for i odd and $H_i(X) = \mathbb{Z}$ for i even, $i \leq 2n$ and $i \neq n$. Let η be the generator of $H^2(X) = \mathbb{Z}$, and $h \in H_n(X)$ the homology class dual to $\eta^{n/2}$.
If X is non-singular, i.e. $k = 0$, then its topological type is uniquely determined by the dimension n and the degree d: e.g.,X is diffeomorphic to the so called Fermat hypersurface $X_n(d)$ defined by the equation $z_o^d + z_1^d + \ldots + z_{n+1}^d = 0$. We point out that in this case

$$(*)\text{Torsion}(H_n(X)) = 0, \quad \text{and} \quad (**) h \text{ is indivisible.}$$

Furthermore, non-singular hypersurfaces admit a connected sum decomposition:

$$X_n(d) \cong M_n(d) \# a(S^n \times S^n), \text{ where } b_n(M_n(d)) - | \operatorname{sign}(M_n(d)) | = 1 - (-1)^{n/2},$$

see R.S. Kulkarni/J.W. Wood, Topology of Nonsingular Complex Hypersurfaces, Advances in Math. 35(1980)239-263.
Theorem 1. If the conditions $(*)$ and $(**)$ are fulfilled, then the topological type of X is uniquely determined by n, d and k: $\quad X \cong: X_n(d; k)$.
Theorem 2. Assume that $n \geq 6$ and d is sufficiently large: $d \geq D(n)$. Then, with $(*)$ and $(**)$ fulfilled, X can be decomposed into a connected sum:

$$X \cong X_n(d; k) \cong M_n(d) \# (a - k)(S^n \times S^n) \# k(S^n \times S^n / \Delta),$$

where $S^n \times S^n/\Delta$ is obtained by contracting the diagonal $\Delta \subset S^n \times S^n$ to a point.
Remark. The conditon $(**)$ is always true for d odd.
Conjecture. The hypotheses $n \geq 6$ and $d \geq D(n)$ in Theorem 2 are superfluous.

D. Notbohm

Homotopy uniqueness of classifying spaces of compact connected Lie groups

Let p be an odd prime. Let G be a compact connected Lie group, T_G the maximal torus and W_G the Weylgroup. We say that a p-complete space X has the mod-p type of BG, if $H^*(BG; \mathbb{Z}/p) \cong H^*(X; \mathbb{Z}/p)$ as algebras over the Steenrod algebra.

Under these conditions, Dwyer, Miller and Wilkerson constructed a maximal torus of X. That is a map $BT^{\wedge}_{X_p} \to X$ of the classifying space of a torus T_X, such that the homotopy fiber has finite *mod*-p cohomology and that the rank $(T_X) = \mathrm{rank}(G)$. $BT^{\wedge}_{X_p}$ denotes the p-adic completion. Moreover they got an action of W_G on BT_X and a map $BT^{\wedge}_{X_p} \to BT^{\wedge}_{G_p}$, such that $BT_X \to X$ is W_G-equivariant (up to homotopy) and such that the diagram

$$\begin{array}{ccc} H^*(X;\mathbb{Z}/p) & \longrightarrow & H^*(BG;\mathbb{Z}/p) \\ \downarrow & & \downarrow \\ H^*(BT_X;\mathbb{Z}/p) & \longrightarrow & H^*(BT_G;\mathbb{Z}/p) \end{array}$$

commutes and is equivariant. We say that a space X with the *mod*-p type of BG has the p-adic type of BG, if $BT^{\wedge}_{X_p} \simeq BT^{\wedge}_{G_p}$ as W_G-spaces, i.e. there exist a W_G-equivariant map $BT_G \to X$, such that

$$\begin{array}{ccccc} & & H^*(BT_G;\mathbb{Z}^{\wedge}_p) & & \\ & \nearrow & & \nwarrow & \\ H^*(BG;\mathbb{Z}^{\wedge}_p) & & \rightarrow & & H^*(X;\mathbb{Z}^{\wedge}_p) \end{array}$$

commutes.

Theorem 1. Let G be a compact connected Lie group, such that $H^*(BG;\mathbb{Z})$ is p-torsion free, and let X be a p-complete space with the *mod*-p type of BG.

1. There exists a compact connected Lie group H, such that BH has the same mod-p type as BG and X, and X is of the $p-$ adic type of BH.
2. If in addition G is simply connected or a product of unitary groups, then X has the p-adic type of BG.

Theorem 2. Let G be compact connected Lie group, such that $H^*(BG;\mathbb{Z})$ is p-torsion free. If X is of the p-adic type of BG, then X and $BG^{\wedge}_p$ are homotopy equivalent.
Theorem 1 and 2 together imply the following corollary.
Corollary 3. Let G be simply connected and $H^*(BG;\mathbb{Z})$ be p-torsion free, or let G be a product of unitary groups. If X has the *mod*-p type of BG, then X and $BG^{\wedge}_p$ are homotopy equivalent.

E.Rees

Applications of parametrized Morse theory

When M is a closed, connected, smooth submanifold of Euclidean space, its focal set $F(M)$ is the set of critical values of the endpoint map of the normal bundle $e\colon N(M) \to R^k$. If $F(M)$ does not meet M the embedding is called non-focal. Examples of non-focal embeddings are transnormal submanifolds and these in turn include the class of isoparametric submanifolds. Principal orbits of the adjoint representation of a compact Lie Group are specific examples.

In 1967, S.Carter and S.A.Robertson proved that even dimensional real projective spaces do not admit non-focal embeddings and asked for similar results for other manifolds. In joint work with Duan Haibao, we prove that none of $CP(n)$, $HP(n)$, $OP(2)$, or the quotient of a sphere by a non-trivial free action by a finite group (if its dimension is not $1, 3, 7$ or 15) admits nonfocal embeddings. No example of a non-focal embedding of a quotient of a sphere is known in dimensions 7 and 15. The non-existence result also holds for many other homogeneous spaces.
The proof involves constructing a parametrized Morse function on the product $M \times M$ essentially following J.Milnor' book on Morse theory. An analysis of the critical graphs together with the use of fixed point theory yields the result for the even dimensional projective spaces. For the odd dimensional quotients of spheres it is proved that, under the given hypothesis, the tangent bundle of the sphere has subbundles of all possible dimensions. It is a consequence of J.F.Adams's solution of the vector fields problems that this happens if and only if the dimension is equal to $1, 3, 7$ or 15. For a transnormal embedding, the function on $M \times M$ has extra geometric properties which can be exploited and this is work still being carried out.

S.Rees

Automatic groups and the isomorphism problem.

In this talk I describe a computer program, written by myself and Derek Holt in Warwick, designed to test for isomorphism between any two finitely presented groups. This problem is of course undecidable for finitely presented groups in general, but we have found this program (which was originally written to deal with automatic groups) to be reasonable fast and effective on many examples arising out of topology (such as knot groups).
Given two finitely presented groups G and H, the program attempts to prove isomorphism or lack of it by running alternately (and with gradually increasing run parameters) two different processes, one attempting to explicitly construct an isomorphism as a map from the generators of one group into the other, the other attempting to prove non-isomorphism by constructing and comparing finite quotients of the two groups.
To make the systematic search for an isomorphism efficient, the program uses finite machinery similar to that which exists for automatic groups to generate representatives of the elements of each group as words in a "normal form" and also to reduce to that form. An efficient ordering of the generators of the two groups together with the consideration of the effects of partial maps between them on various small finite quotients speeds up the search considerably.
The program carries data about various small permutation groups, and checks for the existence of homomorphisms from G and H onto each of these groups. Where such a group R appears as a finite quotient of only one of the two groups, non-isomorphism is clearly proved, whereas if R is a quotient of both groups, the quotient can often be extended downwards, by elementary abelian sections on selected primes,

to produce a tree of finite quotient information. The various trees of this type are then compared, and differences between them, which show the existence of a finite quotient of one of G and H that does not exist for the other, prove non-isomorphism. The construction of finite quotient trees may well have applications outside the isomorphism problem. In any case the whole procedure can clearly be much faster if it is under user control, when the user can decide with which permutation groups, and by which primes, to build the trees downwards. A new program, which displays the quotient trees graphically and is interactively controlled by the user, is currently being developed. An early version is already running.
This software is freely available, and copies can be obtained by contacting either myself or Derek Holt (*dfh at maths. warwick, ac.UK*). Stable versions are installed in the Mathematics Institute of Warwick (together with other programs to deal with automatic groups), and should soon be obtainable via anonymous ftp.

S.Stolz and M.Kreck
Non-connected Moduli Spaces of Positive Sectional Curvature Metrics

Let M be $4k-1$ -dimensional spin manifold for which the rational Pontrjagin classes vanish. Given such a manifold and a positive scalar curvature metric g on it we define an invariant $s(M,g) \in \mathbb{R}$ depending only on the path component of g in the space of positive scalar curvature metrics.
Theorem. The conjugacy class of a S^1-subgroup of $SU(3)$ with quotient $N = SU(3)/S^1$ is determined by the order of $H^4(N;\mathbb{Z})$, and $s(N,g)$, where g is the normal metric on N.
Now Aloffs and Wallach have shown that these quotients carry positive sectional curvature metrics arbitrarily close to the normal metric and in previous work the authors proved that there are non-conjugate S^1-subgroups of $SU(3)$ with diffeomorphic quotients. It follows that the moduli space of positive sectional curvature metrics modulo the diffeomorphism group on such quotients is not connected.

U.Tillmann
Relation of the Van Est spectral sequence to K-theory and cyclic homology.

Abstract: We study how the smooth cohomology of the infinite general linear group GLA for a Banach algebra A relates to cyclic cohomology, Lie algebra cohomology and Dennis trace. Our main result is as follows.

Theorem A. The following diagram is commutative:

$$\begin{array}{ccc} HH_c^*A & \xrightarrow{B} & HC_c^{*-1}A \\ D_{sm}\downarrow & & A\downarrow \\ H_{sm}^*GLA & \xrightarrow{\lambda} & H_{Lie}^*gla \end{array}.$$

Here B is the boundary map in Connes' long exact sequence relating continuous cyclic cohomology to continuous Hochschild cohomology.A denotes the dual of the alternation operation that induces an isomorphism between the primitive elements in the Lie algebra homology of $gla = MA$ and the cyclic homology of A. λ is the classical map from the smooth cohomology of a group to its Lie algebra cohomology, which can be identified with one of the edge homomorphisms in the van Est spectral sequence. The definition of D_{sm} will rest on the observation that the dual of the Dennis trace map factors through the smooth group cohomology of GLA.

We incorporate the above diagram into a bigger commutative diagram to show its relation with the van Est spectral sequence and the various other well-known cohomology groups associated with a topological group.

Diagram B.:

$$\begin{array}{ccccccc} HC_c^*A & \xrightarrow{I} & HH_c^*A & \xrightarrow{B} & HC_c^{*-1}A & \xrightarrow{S} & HC_c^{*+1}A \\ & & D_{sm}\downarrow & & A\downarrow & & \\ H^*BGLA & \longrightarrow & H_{sm}^*GLA & \xrightarrow{\lambda} & H_{Lie}^*gla & \longrightarrow & H^*GLA \\ \| & & \downarrow & & \downarrow & & \| \\ H^*BGLA & \longrightarrow & H^*(BGLA^\delta) & \longrightarrow & H^*(GLA/GLA^\delta) & \longrightarrow & H^*GLA \end{array}$$

The top row is Connes' exact sequence. The second row is associated to the van Est spectral sequence. While the bottom row we consider as the cohomology sequence induced by the fibration.

$$(GLA/GLA^\delta)^+ \to (BGLA^\delta)^+ \to BGLA.$$

(A^δ denotes the algebra A with discrete topology. The definition of the quotient space GLA/GLA^δ is made precise as a quotient of two simplicial sets. + denotes Quillen's plus construction which does not alter the cohomology of a space.) Its homotopy sequence is by definition the long exact sequence of relative, Quillen's algebraic, and periodic K-theory groups

$$\to K_*^{rel}A \to K_*^{alg}A \to K_*^{top}A \to$$

The columns in Diagram B find their interpretation as follows. The left one is dual to the Dennis trace map $D: K_*^{alg}A \to H_*(BGLA^\delta) \to HH_*A$. In particular, the dual factors through the smooth cohomology of GLA. The right column is the dual of Karoubi's relative Chern character $ch^{rel}: K_*^{rel}A \to H_*(GLA/GLA^\delta) \to HC_{*-1}^cA$ which is shown to factor through $H_*^{Lie}gla$ as a map of chain complexes.

M Weiss

A-theory Euler characteristics. (Joint work with B. Williams, Notre Dame)

In [1] we defined

$$\Phi\colon \mathrm{TOP} \longrightarrow Q\underline{A}(*)_{hZ/2}$$

where $\underline{A}(*)$ is Waldhausen's A-theory spectrum, equipped with Vogell's involution, and the subscript $hZ/2$ denotes the homotopy orbit spectrum, and $Q = \Omega^\infty$. In [2], we defined

$$\Xi\colon \underline{L}.(Z) \longrightarrow \underline{A}(*)^{h\hat{Z}/2}$$

where $\underline{\mathrm{L}}.$ is L-theory and the superscript $h\hat{Z}/2$ indicates the cofiber of the norm map $(\ldots)_{hZ/2} \to (\ldots)^{hZ/2}$. Problem: Show that

$$\begin{array}{ccc}
\Omega(G/Top) & \xrightarrow{\Omega\Xi} & \Omega Q\underline{A}(*)^{h\hat{Z}/2} \\
\downarrow\delta & & \downarrow\delta \\
TOP & \xrightarrow{\Phi} & Q\underline{A}_{hZ/2}
\end{array}$$

commutes (up to homotopy). Method of proof: a) Philipps- Gromov submersion theory, b) a rule associating to a finite CW-space X (or an ENR) an "Euler characteristic" in $Q(X_+ \wedge \underline{A}(*))$.

Refs.:

[1] Weiss-Williams, Automorphisms... I, K-theory 1, 1988, 575-626.

[2] Weiss-Williams, Automorphisms... II, J.of pure and appl.Alg.62, 1989, 47-107. (Automorphisms... III is available as Aarhus University preprint.)

Sh.Wang

Gauge theory and real structures.

Abstract We discuss gauge theory in the presence of involutions, especially real structures. The fixed part of the moduli space under a lifting involution is compared with the moduli space on the quotient manifold. A couple of examples are given to illustrate such a comparison.

We also show some possible definitions of new invariants arising from moduli spaces, which fits particularly the above picture.

Berichterstatter: P.Teichner

Tagungsteilnehmer

Dr. Stefan Alois Bauer
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. Steven C. Ferry
Dept. of Mathematical Sciences
State University of New York
at Binghamton

Binghamton , NY 13902-6000
USA

Prof.Dr. Michel Boileau
Mathématiques
Laboratoire AsV
Université Paul Sabatier
118 route de Narbonne

F-31062 Toulouse Cedex

Dr. Thomas Fiedler
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. Aleksei V. Cernavskij
Institute for Information
Transmission Problems
Ac. of Science of the USSR
19 Ermolova Str.

Moscow 101 447 GSP-4
USSR

Prof.Dr. Robert Gompf
Dept. of Mathematics
University of Texas at Austin

Austin , TX 78712
USA

Prof.Dr. Tammo tom Dieck
Mathematisches Institut
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Prof.Dr. Jean-Claude Hausmann
Section de Mathématiques
Université de Genève
Case postale 240

CH-1211 Genève 24

Prof.Dr. Julia Drobotuchina
St.Petersburg Branch of Steklov
Mathematical Institute - LOMI
USSR Academy of Science
Fontanka 27

191 001 St.Petersburg
USSR

Prof.Dr. Klaus Johannson
Dept. of Mathematics
University of Tennessee at
Knoxville
121 Ayres Hall

Knoxville , TN 37996-1300
USA

Prof.Dr. John Klein
Lehrstuhl für Mathematik V
FB 6 - Mathematik
Universität Siegen
Hölderlinstr. 3

W-5900 Siegen 21
GERMANY

Dr. Wolfgang Lück
Dept. of Mathematics
University of Kentucky
735 Patterson Office Tower

Lexington ,Kentucky 40506
USA

Prof.Dr. Maxim Kontsevich
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

W-5300 Bonn 3
GERMANY

Prof.Dr. Alexis Marin
Dept. de Mathématiques, U.M.P.A.
Ecole Normale Supérieure de Lyon
46, Allée d'Italie

F-69364 Lyon Cedex 07

Prof.Dr. Ulrich Koschorke
Lehrstuhl für Mathematik V
FB 6 - Mathematik
Universität Siegen
Hölderlinstr. 3

W-5900 Siegen 21
GERMANY

Prof.Dr. Wolfgang Metzler
Fachbereich Mathematik
Universität Frankfurt
Robert-Mayer-Str. 6-10
Postfach 111932

W-6000 Frankfurt 1
GERMANY

Prof.Dr. Matthias Kreck
Fachbereich Mathematik
Universität Mainz
Postfach 3980

W-6500 Mainz
GERMANY

Prof.Dr. Nikita Netsvetaev
St.Petersburg Branch of Steklov
Mathematical Institute - LOMI
USSR Academy of Science
Fontanka 27

191 011 St.Petersburg
USSR

Prof.Dr. M. Paul Latiolais
Mathematics Dept.
Portland State University

Portland, Oregon 97207-0751
USA

Prof.Dr. Walter David Neumann
Department of Mathematics
Ohio State University
231 West 18th Avenue

Columbus Ohio 43210-1174
USA

Dr. Dietrich Notbohm
Mathematisches Institut
SFB 170
Universität Göttingen
Bunsenstr. 3-5

W-3400 Göttingen
GERMANY

Dr. Sarah Rees
Department of Mathematics
and Statistics
University of Newcastle

GB- Newcastle Upon Tyne NEI 7RV

Prof.Dr. Andrej Pazhitnov
Vavilov 55/7, 99

117312 Moscow
USSR

Dr. Roland Schwänzl
Fachbereich Mathematik/Informatik
Universität Osnabrück
PF 4469, Albrechtstr. 28

W-4500 Osnabrück
GERMANY

Prof.Dr. Volker Puppe
Fakultät für Mathematik
Universität Konstanz
Postfach 5560

W-7750 Konstanz 1
GERMANY

Prof.Dr. Laurent C. Siebenmann
Mathématiques
Université de Paris Sud (Paris XI)
Centre d'Orsay, Bâtiment 425

F-91405 Orsay Cedex

Prof.Dr. Andrew A. Ranicki
Dept. of Mathematics
University of Edinburgh
James Clerk Maxwell Bldg.
Mayfield Road, King's Building

GB- Edinburgh , EH9 3JZ

Prof.Dr. Wilhelm Singhof
Mathematisches Institut
Heinrich-Heine-Universität
Universitätsstraße 1

W-4000 Düsseldorf 1
GERMANY

Prof.Dr. Elmer G. Rees
Dept. of Mathematics
University of Edinburgh
James Clerk Maxwell Bldg.
Mayfield Road, King's Building

GB- Edinburgh , EH9 3JZ

Prof.Dr. Stephan Stolz
Max-Planck-Institut für Mathematik
Gottfried-Claren-Str. 26

W-5300 Bonn 3
GERMANY

Peter Teichner
Fachbereich Mathematik
Universität Mainz
Postfach 3980

W-6500 Mainz
GERMANY

Prof.Dr. Rainer Vogt
Fachbereich Mathematik/Informatik
Universität Osnabrück
PF 4469, Albrechtstr. 28

W-4500 Osnabrück
GERMANY

Ulrike Tillmann
Dept. of Pure Mathematics and
Mathematical Statistics
University of Cambridge
16, Mill Lane

GB- Cambridge , CB2 1SB

Prof.Dr. Shuguang Wang
Dept. of Mathematics
Michigan State University

East Lansing , MI 48824-1027
USA

Prof.Dr. Oleg J. Viro
St. Petersburg Branch of Steklov
Mathematical Institute - LOMI
USSR Academy of Science
Fontanka 27

191 011 St.Petersburg
USSR

Prof.Dr. Steven H. Weintraub
Institut für Mathematik
Universität Hannover
Welfengarten 1

W-3000 Hannover 1
GERMANY

Dr. Wolrad Vogell
Fakultät für Mathematik
Universität Bielefeld
Postfach 8640

W-4800 Bielefeld 1
GERMANY

Dr. Michael Weiss
Matematisk Institut
Aarhus Universitet
Ny Munkegade
Universitetsparken

DK-8000 Aarhus C