MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 39/1991

Niedrigdimensionale Topologie

8.9 bis 14.9.1991

The meeting was organised by M. Boileau (Toulouse), K. Johannson (Knoxville) and H. Zieschang (Bochum). The participants presented recent developments in low-dimensional topology and in particular in 3-manifold theory, mostly centered around topics such as hyperbolic structures and incompressible surfaces, representations of π_1, knot polynomials and Witten invariants, Heegaard splittings, complexity and algorithmic questions, and some related problem from combinatorial group theory.

Some of the talks were joint with the parallel meeting "Knoten und Verschlingungen". The interaction with the participants of this parallel meeting was interesting and fruitfull.

Vortragsauszüge

Wolfgang Heil:

Dehn fillings of Kleinbottle bundles

Let M be a fiber bundle over S^1 with fiber a once punctured Kleinbottle bundle. Pedja Raspopović (Ph.D. thesis 1990) classified all incompressible surfaces in M. This is used to obtain a classification of all manifolds obtained from M by Dehn fillings. Several applications are given, e.g. (1) we exhibit a large class of non-orientable closed manifolds, different from torus bundles, each of which contains exactly one incompressible surface, (2) we obtain homeomorphic manifolds from a fixed (closed) Kleinbottle bundle via distinct surgeries on a section, (3) for a closed Kleinbottle bundle there is a non-trivial knot k in M such that infinitely many distinct surgeries on k result in M.

Michael Heusener:

$SU_2(\mathbb{C})$-representation spaces for knot groups

Given a 2-bridge knot $b(\alpha,\beta) \subset S^3$ we are interested in studying $G_{\alpha,\beta} := \pi_1(S^3 - b(\alpha,\beta))$. Our approach is to consider the space of all representations of $G_{\alpha,\beta}$ into $SU(2)$ and $SO(3)$. The representation spaces are described by real plane algebraic curves $C_{\alpha,\beta} \subset \mathbb{R}^2$. $C_{\alpha,1}$ consists of $n = \frac{\alpha-1}{2}$ lines. In the case $\beta = 3\alpha - 2$ the curve $C_{\alpha,\beta}$ is rational. This leads to an exact description of the representation spaces. The set $C_{\alpha,\beta}$ is not an invariant of the knot. Given a homeomorphism h: $S^3 \to S^3$ (orientation preserving or not) with $h(b(\alpha,\beta')) = h(b(\alpha,\beta))$ $(\beta\beta' \equiv \pm 1 \, mod \, \alpha)$ the result is a birational transformation $\hat{h}$: $C_{\alpha,\beta} \to C_{\alpha,\beta'}$.

In the cases $\beta^2 \equiv \pm 1 \, mod \, \alpha$ we use this transformation to derive the following properties of $C_{\alpha,\beta}$:

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1. If $\beta^2 \equiv 1 \, mod \, \alpha$ it follows that $C_{\alpha,\beta}$ is reducible.

2. If $\beta^2 \equiv -1 \, mod \, \alpha$ there exists a knot $b(\alpha, \beta)$ such that $C_{\alpha,\beta}$ is irreducible and not rational.

This disproves the conjecture that $C_{\alpha,\beta}$ always consists of rational components. Some remarks about Casson's invariant are made.

W.B.R. Lickorish:

Pairs of distinct 3-manifolds with the same Witten invariants

The $(sl_2\mathbb{C})$-3-manifold invariant of Witten, corresponding to a root of unity, have a very simple description in terms of the Kauffman bracket. This leads to two methods of producing pairs of 3-manifolds having, for every root of unity, identical invariants. It can be seen that often these manifolds are distinct. One of these methods should produce such examples for invariants based on other Lie algebras.

Yves Mathieu:

On knot complements in 3-manifolds

When two knots have homeomorphic complements in an orientable 3-manifold M, we can ask if the knots are equivalent by homeomoprhism. In other words, is a knot determined by its complement? Gordon and Luecke (1988) have proved that nontrivial Dehn surgery on a nontrivial knot never yields the 3-sphere, and so knots are determined by their complements in the 3-sphere.

We give several examples of manifolds in which knots are *not* determined by their complements.

By two distinct particular Dehn surgeries on the trefoil knot we get the same closed manifold. The cores of the two surgeries have homeomorphic complements, but they are nonequivalent knot. In every compact manifold with boundary a torus we construct distinct knots with homeomorphic complements.

For homotopically trivial knots in M, compact orientable irreducible with π_1 infinite, if $M(k;\alpha) \cong M$ then either k is hyperbolic or a cable with winding number 2 around a hyperbolic knot. In particular, if $H_1(M, \partial M; \mathbb{Q}) \neq 0$, homotopically trivial knots are determined by their complement.

Yoshihiko Marumoto:

Stably equivalences of ribbon presentations of knots

A ribbon knot is obtained from a trivial link by connecting bands, and distinct "connecting manners" might define the same knot. We call this connecting manner a *ribbon presentation* of the knot. Our question is "How many ribbon presentations can a knot have?" For making this question reasonable, we need equivalence relations between ribbon presentations of a knot. And we define a *stable equivalence* between them. We construct a ribbon presentation of a superspun knot, which is a generalization of a spun knot, in terms of a classical knot diagram. Then we can prove that our constructed ribbon presentations of a knot are stably equivalent.

S.V. Matveev:

Complexity theory of 3-manifolds

A compact polyhedron P is called almost special if the link of each of its points can be embedded in Δ, where Δ ist the 1-dimensional skeleton of the standard 3-simplex. The points whose links are homeomorphic to Δ are said to be the vertices. The complexity $e(M)$ of a compact 3-manifold M equals k, if M possesses an almost special spine with k vertices and has no almost special spines with a smaller

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number of vertices. It turns out that the notion of complexity is naturally related to practically all the known methods of presenting manifolds and adequately describes complexity of manifolds in the informal sense of the expression.

Theorem. Let a 3-manifold M_1 be obtained from a 3-manifold M by cutting along an incompressible surface $F \subset M$. Then $c(M_1) \leq c(M)$.

Alexander D. Mednykh:

Automorphism groups of three dimensional hyperbolic manifolds

We investigate the discrete groups of isometries acting on three dimensional hyperbolic space with fixed points. We obtain an explicit lower bounds for distance between fixed points of different types. The results are applied to describe maximal discrete groups in hyperbolic space and to calculate the isometry groups of hyperbolic manifolds.

W. Metzler:

Simple h-type of 2-complexes, Andrews-Curtis conjecture and presentations of free products

1) Presentations of free products may need less defining relators than expected from the factors. This yields examples: a) with defect $(G * H) < def(G) + def(H)$; b) potential counterexamples to the relator-gap conjecture.

2) Using one-point unions with $\mathbb{Z}_2 \times \mathbb{Z}_4$-standard complexes, values of $Wh(\pi_1)$ can be realized by homotopy equivalences of 2-complexes, thereby distinguisting sh-tupe and h-type in dimension 2.

3) One point unions with $\mathbb{Z}_2 \times \mathbb{Z}_4$-standard complex likewise can be used to "improve" sh-equivalences of 2-complexes to Andrews-Curtis equivalences.

4) Presentations of free products also yield potential counterexamples to the Andrews-Curtis conjecture with $\pi_1 \neq 1$. For their treatment we present a) normal forms and normal Andrews-Curtis moves for presentations of free products and b) a theorem about "pushing up" quotients of corresponding defining relators (of sh-equivalent 2-complexes) in the commutator series of the relator subgroup.

Some of the results are contained in joint publications with C. Hog-Angeloni and M. Lustig.

J. Montesinos:

Arithmetic universal groups and 3-manifolds

(joint work with Hilden and Lozano).

In a recent meeting here in Oberwolfach I showed that the Borromean orbifold U (= Borromean rings with isotropy cyclic of order 4) is universal in the sense that every 3 - manifold (closed and oriented) is the underlying space of a hypherbolic orbifold covering U. We show the same for the 2-bridge links (not toric) with isotropy cyclic of order 12. We discuss the arithmeticity of these and related groups, using a new arithmeticity criterium.

Yoav Moriah:

Tunnels, bridges and K_1

(Joint work with Martin Lustig.)

In this work we use the torsion invariant $\mathcal{N}(G) = K_1(\mathbb{Z}G/I)/\pm G$ introduced in a previous paper, to compute the rank of the fundamental group G, the tunnel number and the bridge number for a large class of knots/links. For corresponding closed 3-manifolds (i.e. obtained by surgery) we determine the rank of the fundamental group and the Heegaard genus. Furthermore $\mathcal{N}(G)$ gives non-isotopic tunnel systems and non-isotopic Heegaard splittings of arbitrary large gnus. We derive an invariant $\bar{\mathcal{N}}(G)$ which distinguishes these, in fact, up to homeomorphism. Using $\mathcal{N}$-torsion we provide a lot of evidence for Johannson's claim "... uniqueness of Heegaard surfaces fails drastically and is a rather special phenomenon for 3-manifolds."

Masaharu Morimoto:

Equivariant Surgery on 3-dimensional Manifolds

(Joint Work with Anthony Bak)

Let G be a finite group. In dimensions $n \geq 5$, a G-normal map $f\colon X \to Y$ determines the surgery obstruction in a Witt group of quadratic forms or automorphisms of quadratic form, that ist, the Wall group $L_n(G, w)$ or the Bak group $W_n(Z[G], \Gamma, w)$, if Y ist simply connected and satisfies the gap hypothesis. We obtained a similar result in dimension three.

Let $G(2)$ be the set of elements in G of order two. Let $f\colon X \to Y$ be a degree one G-map. Suppose (1) Y is simply connected, (2) X satisfied gap hypothesis, (3) $\dim X^g = 1$ for any $g \in G(2)$, (4) $f_s\colon X_s \to Y_s$ is a homology equivalence. Then the G-surgery obstruction to converting f into a homology equivalence lies in the Bak group $W_3(Z[G], \max, w)$.

H.R. Morton:

Quantum $SU(k)$ link invariants and the Homfly polynomial

A account will be given of the way to realise the general quantum $SU(k)$ link invariant in terms of the Homfly polynomials of suitable chosen cables about the link.

Examples of special cases will be given, for $SU(3)$ ad $SU(4)$, where the invariants can be calculated in other ways, giving relations between certain link polynomials.

Mario Eduave Munõz:

Dehn surgery on simple knots in S^3

Let k be a knot in S^3. Denote by $M(r) = S^3(k,r)$ the 3-manifold obtained by performing Dehn surgery on k with coefficient $r \in \mathbb{Q} \cup \{1/0\}$. $\Delta(r,s)$ denotes the distance between the slopes r and s, i.e. its minimal intersection number.

Suppose k is a simple knot, then in general $M(r)$ will be a simple manifold, i.e. it will contain no incompressible torus. But there some examples for which $M(r)$ is nonsimple. The following conjecture was proposed by C. Gordon.

Conjecture: If k is simple, but $M(r)$ is not then $\Delta(r,u) \leq 2$ (u denotes a meridian of k).

We prove that the conjecture holds for strongly invertible knots, and also prove that if $M(r)$ is nonsimple and $\Delta(r,u) = 2$, then there is an incompressible torus in $M(r)$ which intersects the surgery torus twice.

We have some examples for which $\Delta(r,u) = 2$.

W. Neumann:

Rigidity of Cusps in Hyperbolic 3-Orbifolds

(Joint work with Alan Reid)

We give examples of a phenomenon conjectured not to exist by D. Cooper and D. Long and (independently) M. Kapovitch: two-cusped hyperbolic manifolds in which deformations induced by hyperbolic Dehn surgery on one cusp have the other cusp uneffected (we say the latter cusp is "geometrically isolated" from the former).

There are, in fact, three different degrees of isolation of cusps from each other, all of which occur in examples. These types of isolation all have convenient interpretation in terms of the analytic function on the character variety introduced by Zagier and myself to study volumes. This has interesting theoretical consequences. For example, two of the types of isolation are symmetric relations (this is non-trivial) but the other one – namely the one of the first paragraph of this abstract – is not. The strongest type of isolation forces a sum formula for volume.

Leonid Potyagailo:

The boundary of deformation space of some hyperbolic surface bundles over S^1

Let us consider the conformal group $M(3) \cong SO(1,4)$ of S^3 which is a subgroup of the isometry group of 4-dimenisonal hyperbolic space. A finitely generated subgroup $G \subset M(3)$ is Kleinian iff the domain of discontinuity $\Omega(G) \subset S^3$ is not empty.

Deformation space Def(Γ) consists of all admissible presentations ρ: $\Gamma \to M(3)$ (ρ is faithful, preserves the type of elements and is induced by a homeomorphism f: $\Omega(\Gamma) \to \Omega(\rho(\Gamma))$) modulo conjugation.

We say that $G \in \mathcal{L}$ iff there exists a Kleinian subgroup $F \subset G$ such that $\pi_1(\Omega(F)/F)$ is infinitely generated. Let's identify $[\rho] \in \mathrm{Def}(\Gamma)$ with $\rho(\Gamma)$. Our main result is

Theorem: Suppose Γ is the fundamental group of a closed hyperbolic 3-manifold which fibers over S^1, and Γ is commensurable with a reflection group G in the faces of a right angled polyhedron D. Then there exists a finite index subgroup $\Gamma_0 \subset \Gamma$ such that
$\partial\, \mathrm{Def}(\Gamma_0) \cap \mathcal{L} \neq \emptyset$.

T.H. Rubinstein:

An algorithm to recognise the 3-sphere

An effective algorithm is given to decide whether a closed orientable triangulated 3-manifold M is homeomorphic to the 3-sphere.

The fundamental proposition, proved by a minimax technique, is that if $M = S^3$ then there is an embedded 2-sphere $\wedge$ which is almost normal, i.e. is normal outside a small ball. Moreover these is a finite number of possibilities for $\wedge$ is this ball. So there is an effective procedure to find $\wedge$ by trying all possible pictures in all small balls and attempting to add normal disks constructed by Haken's method to complete $\wedge$.

Split M along a maximal collection of disjoint embedded normal 2-spheres into pieces $M_1, M_2, \ldots, M_k$. Some of these pieces are obviously 3-cells. If M_i is one of the other pieces and is homeomorphic to a punctured 3-ball, then there is an almost normal 2-sphere $\wedge_i$ in M_i. Push $\wedge_i$ off itself on either side. The number of intersections of the 2-spheres with the edges and faces of the triangulation can be reduced by standard moves to give normal 2-spheres in ∂M_i. Since $\wedge_i$ is a "barrier", an isotopic sweep out (possibly with surgeries) across M_i is constructed, proving homeomorphism to a punctured 3-call.

So M is S^3 if and only if each (non trivial) M_i contains a $\wedge_i$. This completes the algorithm.

Makoto Sakuma:

Incompressible Seifert surfaces for some arborescent links

Let L be an arborescent link represented by a weighted planar tree T, where each weight is non-zero and even. Then: (1) L has only "canonical" incompressible Seifert surfaces; and two such Seifert surfaces are isotopic, if and only if they are isotopic via "canonical" isotopies. (2) The simplicial complex $IS(L)$ defined by O. Kakimizu is homeomorphic to a ball.

Martin Scharlemann:

Heegaard splittings of product manifolds are standard

Theorem (S.-Thompson): The 3-manifold (surface) $\times$ I has a unique irreducible Heegaard splitting.

More generally, there is a hybrid theorem for arcs in (surface) $\times$ I which includes the above and Frohman's unknotting theorem as special cases.

Theorem (Schultens): The 3-manifold (surface) $\times$ S^1 has a unique irreducible Heegaard splitting.

Corollary: There are prime closed 3-manifolds with unique irreducible Heegaard splittings and non-abelian fundamental group.

Oleg Viro:

Quantum Sl_2-invariants: transition to the shadow world

After the discovery of the Jones polynomial for classical knots a lot of new invariants of low-dimensional topological objects were introduced. They are related to quantum groups. The invariants related to the quantum deformation of $sl_2(\mathbb{C})$ can be introduced in an elementary way starting with the Kauffman brackets. In the talk it is explained how to pass from the Kauffman brackets to phase models which can be used for constructing invariants of shadow links and shadow 3-manifolds, introduced recently by Turaev.

Andreas Zastrow:

Barycentres in hyperbolic geometry and their applications to hyperbolic manifolds

A barycentric operation on an arbitrary metric space (Y, ρ) is considered to be a continuous abelian semigroup-structure on the set $H := R_+ \times Y$ ("weighted points") satisfying additivity of distances, local additivity of weights and a uniformity-axiom ((i)-(iv)).

Theorem 1: If two more axioms are added to (i)-(iv) (ensuring finiteness of the dimension an an infinite length for each straight line) then each space permitting such an operation is either homothetic to the hyperbolic space H^n or is an affine space metrizised by a norm.

Theorem 2: There exists a canonical extension of the barycentric semigroup H to a barycentric group G. If $Y = \mathsf{H}^n$, then G is isomorphic to the vector-space of hyperbolic functionals which is generated by the functions $Y \to R$, $x \to \cosh(\rho(x, P))$.

Applications: (1) A slightly extended system of these axioms describes the Euclidean or hyperbolic geometry. (2) One obtains a natural definition for barycentric coordinates in hyperbolic geometry and for straightening chains. (3) One obtains a sheaf of hyperbolic functionals on any hyperbolic manifold.

Remark: There exists an interpretation for this barycentric operation in the model of H^n being embedded in the Minkowski-space.

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