MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 40/1991

Knoten und Verschlingungen

8. 9. - 14. 9. 1991

The conference was organized by U. Koschorke (Siegen) and J. Levine (Waltham).

The topics of lectures and discussions comprised a broad spectrum of recent developments in the understanding of knotting and linking phenomena, e. g.: the link concordance problem and related techniques of localization and representation spaces; quantum-invariants of links in 3-manifolds.

Abstracts of the talks

C. Kearton

Branched Cyclic Covers of Knots.

(joint work with S.M.T.Wilson)

Let k be an n-knot and let K_m denote the m-fold branched cyclic cover of k. If K_m is a sphere, then we have a knot k_m which is the fixed point set of a $\mathbb{Z}_m$-action, and quotienting out by this action gives us back the knot k.

The *order* of a polynomial $f(t)$ is the *lcm* (possibly infinite) of the orders of its roots. The *Alexander invariant* of a knot module is its annihilator, which by work of Crowell is the quotient of the first Alexander polynomial by the second.

Theorem: Let k be a simple $(2q-1)$-knot, $q > 1$, whose Alexander invariant has finite order m and is not square-free. Then K_r is a sphere for all r coprime to m, and the corresponding k_r are all distinct.

In the next two results, $B(p)$ is a function arising from transcendental number theory.

Theorem: Let k be an n-knot, at least one of whose Alexander invariants has infinite order, and degree p. If K_r is a sphere then $r < B(p)$. In particular, there are only finitely many knots k_r covering k.

Theorem: Let l be a stable knot or a simple 3-knot at least one of whose Alexander invariants has infinite order, and degree p. If $l = k_r$ for some knot k, then $r < B(p)$. In particular, there are only finitely many such knots k_r, and hence only finitely many inequivalent $\mathbb{Z}_r$-actions on S^{n+2} with fixed point set l.

Uwe Kaiser
Alexander-modules of band constructions.

We derive short exact sequences which relate Alexander–modules of fusion links to covering modules of the original link. Here the fusion link is the result of attaching a 1–handle to a link in S^3 reducing the number of components. The strong fusion is the fusion completed by the unknotted circle which links the band. Alexander–moduls of strong fusions are related to the original link in a more rigid way. We give applications:

1) The Alexander–polymonial of a strong fusion does not depend on the choice of band (only on the components where the band is attached) and satisfies an easy relation with the polynomial of the original link.

2) An explicit formula for the Alexander–polynomial of any band-sum of two knots can be given depending on the polynomials of the knots and the band.

Daniel S. Silver
An Entropy–like Invariant for n–Knots.

Growth rates of group endomorphisms were introduced by Rufus Bowen in 1978 in order to study topological entropy of continuous maps. We use them to define an "entropy–like "invariant γ_K for any n–knot (spherical or disk n–knot) K, provided that the commutator subgroup of the knot group $\pi_1(X(K))$ is finitely generated. In the special case of a fibered hyperbolic 1–knot K, γ_K is the log of the stretching factor of the pseudo-Anosov monodromy.

In general γ_K is sensitive — capable of distinguishing many n–knots having the same Alexander module. Applications of the invariant include:

1. construction of new doubly slice fibered 1–knots;
2. description of a ribbon concordance obstruction for fibered 1–knots pairs;
3. detecting noninvertibility for certain higher–dimensional n–knots.

In the final part of the talk we discuss some published work of O. Kakimizu [Math. Ann. 284], and use these ideas to extend the definition of γ_K in a natural way for all 1–knots. We conclude with the observation that Kakimizu's 1–knot invariant $\mu(S)$ can be defined for any n–knot, suggesting that the invariant γ_K can be defined in a natural way for any n–knot, too.

P. Gilmer
Concordance of Classical Knots and Links.

I will describe a Witt group obstruction to knot cobordism which combines the obstructions of Levine and Casson-Gordon. Then I will discuss joint work with Livingston on link concordance and the question: when is a link concordant to a boundary link.

Pierre Vogel
Representations of Link Groups.

Let G be the fundamental group of the complement of a high dimensional link. A choice of meridiens induces a morphism from a free group F to G.

The only properties of this map are the following:

- it induces an isomorphism on H_1 and H_2 (with integral coefficients)
- and it is normally surjective.

If the link is a boundary link, the map $F \to G$ split, but in general it is not the case. Nevertheless the canonical map from F to its algebraic closure F' (in the sense of Levine), factors uniquely through G.

Let Γ be a connected compact Lie group.

Theorem: every representation of F to Γ extends to a representation of G.

This theorem is a consequence of:

Theorem: every representation from F to Γ extends to a representation of F'.

In the proof of this last theorem, we need to construct $\rho(a)$ where ρ is a representation from F to Γ which is near the trivial representation, and a is an element of F'. We define it as a sum of a series induced by the Magnus expansion of a and we prove the convergence of this series.

Ju. Drobotukhina
Links in $\mathbb{R}P^3$.

To links in $\mathbb{R}P^3$ the notions of diagram, alternating diagram und Reidemeister moves are extended. Jones-type polynomials for links in $\mathbb{R}P^3$ are defined. Kauffman-Murasugi theorems on relationship between Jones polynomial and crossing number are extended to the case of links in $\mathbb{R}P^3$. Criteria for non-affiness of projective links are formulated in terms of the generalized Jones polynomial. This polynomial plays the key role in classification of irreducible projective links up to 6–crossings.

Projective analogs of the Monteninos links are classified up to homeomorphism and isotopy.

Tim D. Cochran
Group Theoretic Invariants of Homology Cobordism of 3–manifolds.

We present some new invariants to detect when two compact oriented 3–manifolds X, Y with $\partial X \approx \partial Y$ are homology cobordant relative to ∂. When $X = S^3 - L$ these are invariants of link concordance. Specifically suppose $f : G \to \pi$ is a homomorphism of groups which induces an isomorphism on $H_1(\ ;\mathbb{Z}_p)$ and an epimorphism on $H_2(\ ;\mathbb{Z}_p)$. Suppose G is finitely-generated and π is finitely presented. For any epimorphism $\phi : \pi \twoheadrightarrow \mathbb{Z}_p(p \text{ prime})$ define $\tilde{\pi} = \ker\phi, \tilde{G} = \ker(\phi \circ f)$.

Theorem: $\tilde{f} : \tilde{G} \to \tilde{\pi}$ is a isomorphism on $H_1(\ ;Z_p)$ and an epimorphism on $H_2(\ ;Z_p)$ and for any $n \in \mathbb{Z}_+$ $\tilde{f}$ induces isomorphisms

$$\tilde{f} : \tilde{G}/\tilde{G}_n \otimes \mathbb{Z}_{(p)} \to \tilde{\pi}/\tilde{\pi}_n \otimes \mathbb{Z}_{(p)}$$

$(\tilde{G}_n/\tilde{G}_{n+1} \otimes \mathbb{Z}_{(p)} \cong \tilde{\pi}_n/\tilde{\pi}_{n+1} \otimes \mathbb{Z}_{(p)}, G_n = \text{lower–central series})$.

Victor Kobelškii

Relations between Alexander modules of a high-dimensional link and its sublink.

Let K be an n-component link of codimension 2 and let L be the sublink of K, obtained by removing the n–th component of K. The relation between Alexander polynomials $\Delta(K) \in \mathbb{Z}[t_1^{\pm1}, \dots t_n^{\pm1}] = \Lambda_n$ of K and $\Delta(L) \in \mathbb{Z}[t_1^{\pm1}, \dots t_{n-1}^{\pm1}] = \Lambda_{n-1}$ of L in classical case (i. e. $\dim K = 1$) is described by Torres formula. Recently Tuarev has proved that in (odd) high-dimensional case $\Delta(L) \doteq \Delta(K)(t_1, \dots t_{n-1}, 1)\lambda\bar{\lambda}$, where $\lambda \in \Lambda_{n-1}$, aug $(\lambda) = 1$, overbar denotes the conjugation in Λ_{n-1} and $\doteq$ is an equality up to multiplication by units of Λ_{n-1}. He also has conjectured, that there are no other restrictions on this polynomial λ. Unfortunately, it's not a true.

Theorem: $\lambda \doteq 1$.

Actually, it's possible to say more:

Theorem: Let $H_i(K)$ and $H_i(L)$ be i–th Alexander modules of K and L respectively and $2 \leq i \leq \dim L - 1$. Then $H_i(L) = H_i(K) \otimes_{\Lambda n} \Lambda_{n-1}$, where Λ_{n-1} is considered as Λ_n–module with trivial action of t_n.

Nathan Habegger

Quantum Homology based on the Kauffman bracket.

The Atiyah-Segal axiomatic system for topological quantum field theory has solutions in $2+1$ dimensions. One model, based on the Kauffman bracket invariant for banded links in the 3–sphere, leads to solutions precisely for even roots of unity of the parameter $q^{1/4} = A$. Precisely, one must take into account an extra structure (Pontryagin structure) for the axiomatic system to be satisfied in which case it is (essentially) unique. For roots of unity of q, these are the $SU(2)_q$ invariants predicted by Witten and verified by Reshetikin-Turaev. For odd roots of unity of A^2, these invariants were also discovered by Kirby and Melvin.

M. Farber

Noncommutative rational functions and links.

We show that rational functions on non-commuting variables (introduced first in the theory of languages) give an algebraic classification of link modules. More precisely, each link module M determines a rational function χ_M, and χ_M classifies M up to semi-simple equivalence. The properties of χ_M, and its relations to classical invariants will be described.

We show also (joint with P. Vogel) that the ring of rational functions coincides with the Cohn localizationt of the free group ring with respect to the augmentation.

Larry Smolinsky

The framed Braid group and 3–manifolds.

Let B_n be the braid group and $\pi : B_n \to \Sigma_n$ be the function given by $\pi(\sigma_i) = (i, i+1) \in \Sigma_n$ for $i = 1, \dots n-1$. The *framed braid group* is $\mathcal{F}_n = \mathbb{Z}^n \rtimes B_n$ where B_n acts on Σ_n via Σ_n.

Framed braids determine 3–manifolds via,

$$\{\text{Framed braids}\} \xrightarrow{\text{closing}} \{\text{oriented framed links}\} \xrightarrow[\text{}]{\substack{\text{forget orientation}\\ \text{do surgery}}} \{\text{3–manifolds}\}$$

Theorem: Two framed braids represent homeomorphic 3–manifolds if they are equivalent under an equvalence relation generated by 5 moves.

Every representation of $\mathcal{F}_n$ is constructed from representations of $\mathbb{Z}^n$ and certain subgroups of B_n (those that preserve some partition of the initial points of the braids). Examples via cabling and Jones' algebraic A_n will be given.

Victor Kobelśkii
Intersections of high-dimensional knots.

Let K_1 and K_2 be the n–dimensional knots in the sphere S^{n+2}, and let their (transversal) intersection be the sphere too. In such a situation we have four knots: $K_1, K_2, k \subset K_1$ (denoted by k_1), and $k \subset K_2(k_2)$. The question is to describe the connections between these knots. It's obvious that Seifert surfaces of k_1 and k_2 are cobordant. The main result is that for simple odd-dimensional knots this is the only restriction, even if we ask the knots K_1 and K_2 to be trivial:

Theorem: Let k_1 and k_2 be two arbitrary simple knots of dim $= 4k-3, k \geq 2$, or two simple knots of dim $= 4k-1, k \geq 1$ with equal signatures. Then there exist two intersected trivial knots K_1 and K_2 such that the knot $k \subset K_1$, is equivalent to k_1 and the knot $k \subset K_2$ is equivalent to k_2.

G. T. Jin
Polygonal Knots.

In the three dimensional Euclidean space, every knot is ambient isotopic to a polygonal knot, i. e., a simple closed curve obtained by joining finitely many vertices with straight line segments. The minimal number of vertices (or equivalently edges) of the family of polygonal knots which are ambient isotopic to a given knot is certainly a knot invariant. We'll call it the polygon index, and denote by $p(k)$, the polygon index of K. The superbridge index defined by N. Kuiper gives a good lower bound for p. For any knot $2sb(K) \leq p(K)$, where $sb(K)$ denotes the superbridge index of K. If $(r,s) = 1$ and $2 \leq r < s$, then $2\min\{2r, s\} \leq p(T_{r,s}) \leq r\min\{n | n > 2\frac{s}{r}\}$, where $T_{r,s}$ is the torus knot of type (r,s). Also, $p() - 1$ is strictly subadditive with respect to the connected sum of knots.

Kent Orr
The status of the link slice problem.

It follows from Ledimet's exact seqUce that the (codimension 2) link slice problem splits up into a homotopy problem and a surgery problem. Unfortunately, realization of homotopy obstructions is difficult, since there is both a lack of examples and invariants are hard to compute (taking values in *unknown* homotopy groups of "bad spaces"). Recently W. Mio gave a concordance classification of links $S^p \cup S^{2p-1} \subset S^{2p+1}$ in terms of knot concordance and Kojima's η–fruction (computable invariants). The following approach might give rise to new interesting examples: Associate to concordance classes of codimension two links, links with only one codimension two component. Two examples of those constructions are discussed. (reported by Uwe Kaiser)

Jerome Levine
Signature Invariants of Unitary Representations.

If λ is a hermitian matrix over $\mathbb{C}G$, the complex group algebra of a group G, and ρ is a unitary representation of G, then by considering the signature of $\rho(\lambda)$, one obtains a function σ_λ defined on the real algebraic variety of unitary representations of G. This can be (and has been) used to study the various surgery obstruction groups of G. For the homology surgery groups of Cappel-Shaneson, σ_λ is "piece-wise continuous"and well-defined on a Zariski-open set. σ_λ is related to manifold invariants obtained from the twisted eta-invariants of Atiyah-Patodi-Singer.

We are particularly interested in re-interpreting σ_λ as an invariant of the associated "torsion-pairing"on a dimension-one torsion $\mathbb{C}G$–module obtained from a theorem of Vogel. This is related to the problem of giving a direct topological definition of the Atiyah-Patodi-Singer invariant.

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