MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 41/1991

Geometrie der Banachräume

15.9. bis 21.9.1991

This conference on the "Geometry of Banach Spaces" was organized by Professor Hermann König (Kiel), J. Lindenstrauss (Jerusalem) and A. Pełczynski (Warsaw). 50 mathematicians from 12 countries attended the conference.

The meeting was devoted to most aspects of current research in Banach space theory and its applications. A great majority of the leading expects in the field as well as some very promising young researchers attended the meeting. There were 33 formal talks, about half of them were one hour talks.

The hilight of this meeting was the presentation of an example (or more precisely a class of examples) which were discovered just in the last month. These examples provided the solution of many famous open problems in infinite-dimensional Banach space theory. Perhaps the best known one is the "hyperplane-problem" which was formulated by Banach in 1929. Is every infinite dimensional Banach space isomorphic to its hyperplanes? The answer is negative and it is shown that even a "rather nice" separable space provides a strong counterexample. The new examples will certainly have many further consequences in Banach space theory, as well as Banach algebras and Operator theory. Some properties of these new examples were classified only during the conference. Three one hour lectures were devoted to the presentation of these really outstanding results.

Among other talks many were devoted to applications to other areas of mathematics. There are by now deep and well established connections between the local theory of Banach spaces and the theory of convex bodies in R^n, between

Banach space theory and harmonic analysis, as well as the study of concrete function spaces in analysis. In all these directions new and sometimes striking results were presented in the meeting. Recently ideas and results from modern Banach space theory were used to obtain stronger abstract results in non linear analysis than those currently available in this area. These new results yielded considerably simpler proofs (and sometimes also solution to problems) of Nirenberg, Brezis, P. L. Lions and others. A few talks at the meeting were concerned with new results in this direction.

In view of the striking new examples mentioned, above it is very likely that infinite-dimensional Banach space theory will change its direction and simultaneously become much more active in the near future.

The local theory of Banach spaces continues to grow at a rapid pace. New theorems based on the rich machinery already available are being proved and continue to reveal surprising phenomena in geometry of R^n. These results have applikations in many directions (including geometry of numbers, numerical analysis and eigenvalue distribution problems).

Also the study of the concrete function spaces of analysis is likely to continue its progress in a brisk pace. As some talks in the meeting show many difficult and central results from hard analysis can now be proved in simpler and often more general forms by the tools of Banach space theory.

It is likely that the applications of Banach space theory in non linear analysis (in particular non linear P.D.E.) will become much more central in this area.

Abstracts

D. ALSPACH

Level sets and the uniqueness of measures

A theorem of Nymann asserts that a finite positive measure with range an interval is determined up to a constant by its level sets. We extend this to the σ-finite case and in some cases for signed measures. We also present examples that show that if the range is a finite union of intervals, level sets may not determine the measure and that in the signed case a measure may be determined by its level sets but neither its positive nor negative part is.

K. BALL

Markov chains in uniformly smooth spacers

There is (for each $K > 1$ and $\Theta \in (0,1)$) a constant M so that if $(X_t)_{t \geq 0}$ is a symmetric Markov chain in a normed space with 2-uniform smoothness constant K, then, for all $t \geq 0$,

$$E \| X_t - X_o \|^2 \leq (t + M t^{1+\Theta}) \frac{d}{ds} E \| X_s - X_o \|^2 |_{s=0} .$$

The question is motivated by its relationship to extension/factorization problems for Lipschitz maps. The interesting question is whether the same holds for $\Theta = 0$, at least in L_p for $2 < p < \infty$. A reverse estimate of this type does hold if the space is 2-uniformly convex.

W. BANASZCZYK

Transference theorems in geometry of numbers

Given a lattice L in R^n, by $\mu(L)$ and $\lambda_i(L)$, $i = 1,\dots,n$, we denote the covering radius and the successive minima of L with respect to the euclidean unit ball. It had been known that

1) $\mu(L)\,\lambda_1(L^*) \le C_1 \cdot n^{3/2}$,

2) $\lambda_i(L)\,\lambda_{n-i+1}(L^*) \le C_2 \cdot n^2$ $(i = 1,\dots,n)$.

for every lattice L in R^n. Here C_1, C_2 are some numerical constants and L^* denotes the dual lattice. Recently, a proof has been found that the right sides in 1) and 2) can be replaced by $C \cdot n$. This result is best possible, up to the constant C. Generalizations to arbitrary symmetric convex bodies are discussed.

E. BEHRENDS

Points of symmetry of compact convex sets in $\mathbb{C}^2$

Let J be a closed subspace of a complex Banach space X. For $n \in \mathbb{N}$, J is said to have the n-ball-property (n-b.p. for short) if $J \cap \bigcap B_j \neq \emptyset$ whenever $B_1,\dots,B_n$ are open balls in X with $\bigcap B_j \neq \emptyset$ and $B_j \cap J \neq \emptyset$ for all j. A systematic investigation of such subspaces started with the work of Alfsen and Effros (1972) and Lima (1977). Since that time, one problem remained open, namely whether or not the 2-b.p. implies the 3-b.p. D. Yost was able to prove that the answer is yes iff the following holds: whenever $K \subset \mathbb{C}^2$ is a compact convex set such that f(K) is a disk for every linear $f: \mathbb{C}^2 \to \mathbb{C}$, then K has a center of symmetry.

In the present talk it is indicated how a counterexample can be constructed. The construction makes use of a complex version of the Legendre transform (technical details can be found in the paper of the speaker in Math. Annalen 290, 1991). It is also noted that the counterexample solves a problem in the theory of analytic multivalued mappings.

J. BOURGAIN

Uniform approximation in L^p- planes

Approximation and uniform approximation are poorly understood, even in a classical context. We discuss the rather surprising result:

(*) For any $p \neq 2$, L^p does not admit a polynomial uniform approximation function.

For a fixed $\lambda > 1$ and $n = 1,2,\ldots$ define (given a Banachspace X)

$$a(n,\lambda) = \sup \min \{\operatorname{rank} T \mid T: X \longrightarrow X,\ T|_Y = Id_Y,\ \|T\| \leq \lambda\},$$

where the supremum is taken over all n-dimensional subspaces Y of X.

If X is the space L^1 or L^∞, $a(n,\lambda)$ is growing exponentially in n (fixing any λ). Weak Hilbert spaces X are characterized by the property $a(n,\lambda) \sim n$ (Johnson/Pisier). The statement (*) may be made more precise, deducing it from following fact:

> There is (given $b > 0$ and N sufficiently large) a subspace X of L^p_N codim $X < N^s$, such that any operator T from L^p_N into X fulfills
>
> $$\operatorname{trace} T < N^{1-\delta^{c_p}},$$
>
> where c_p depends on $p \neq 2$ (T is assumed "well bounded").

The proof uses harmonic analysis on the Cantor group.

A. DEFANT

Complexification of operators between L_p-spaces

For a linear and continuous operator $T: L_q^{\mathbb{R}} \longrightarrow L_p^{\mathbb{R}}$ (p,q arbitrary, real functions) define its complexification $T^{\mathbb{C}}: L_q^{\mathbb{C}} \longrightarrow L_p^{\mathbb{C}}$ by $T^{\mathbb{C}}(u+iv) := T(u) + iT(v)$. Let $k_{q,p}$ be the best constant such that $\|T^{\mathbb{C}}\| \leq k_{q,p} \|T\|$ for all possible T. By a result of Figiel, Iwaniecz and Pełczynski $k_{q,p} = 1$ whenever $1 \leq q \leq p \leq \infty$, and by a result of Krivine $k_{\infty,1} = \sqrt{2}$. We give a formula for $k_{q,p}$ in terms of L_p-factorable and p-summing norms and calculate the exact value of these constants in the cases $1 \leq p \leq q \leq 2$ and $2 \leq p \leq q \leq \infty$ (for example: $k_{2,1} = \frac{\sqrt{2}}{4}\pi$). Joint work with Klaus Floret.

Ref.: A. Defant - K. Floret: Tensor norms and operator ideals (sections 26,28); to appear in: North Holland Math. Studies.

R. DEVILLE

A mountain pass principle for non differentiable mappings

We prove a general mountain pass principle for non differentiable mappings that extends a theorem of N. Ghoussoub and D. Preiss. Applications are given to an extension of a result of Brezis-Nirenberg and to the Resolution of elliptic partial differential equations with discontinuous non linearities.

T. FIGIEL

Best constants in Rosenthal's inequalilty

Some results from a joint paper with P. Hitczenko, W. B. Johnson, G. Schechtman and J. Zinn were presented. Rosenthal's inequality says that for each $p>2$ there is $R_p<\infty$ such that, if $X_1,\ldots,X_n$ are independent symmetric real random variables, then

$$1 \le \frac{\|\sum_{i=1}^{n} x_i\|_p}{\max(\|\sum_{i=1}^{n} x_i\|_2, (\sum_{i=1}^{n}\|x_i\|_p^p)^{1/p})} \le R_p .$$

It was known (Johnson, Schechtman, Zinn '83) that $R_p \le Cp/\log p$ for large p. In fact, the best value of R_p for $p \ge 4$ was found by S. A. Uter ('85) who obtained that, if $4 \le p < \infty$, then

$$R_p = \|P(\varepsilon)\|_p$$

where

$$P(\varepsilon) = \sum_{i=1}^{N} \varepsilon_i , \quad P(N = n) = \frac{1}{en!}, \quad n = 0,1,2,\ldots$$

and $\varepsilon_1,\ldots,\varepsilon_n$ are i.i.d. Rademacher variables.

In the paper some related extremal problems are solved. In particular, if $2<p<4$, then the best value of R_p is $(1+\mathbb{E}|g|^p)^{1/p}$, where g is a real Gaussian random variable, $\mathcal{L}(g) = N(0,1)$.

In the talk an approach was outlined, using Uter's result, which can be generalized to cover e.g. the cases where the X_i's are independent random vectors in $\mathbb{R}^d$ with radial symmetry.

K. FLORET

Trace duality and accessibility for operator ideals

The notion of accessible and totally accessible quasi-Banach operator ideals was introduced. As for tensornorms, these notions turn out to be quite fruitful under the absence of the metric approximation property; in particular, they allow to understand better the trace duality for infinite-dimensional Banach spaces. It was not known before this conference whether non-accessible operator ideals exist*! A suitable characterization of p-Banach ideals being accessible implies that all minimal p-Banach operator ideals are accessible. Excellent use of accessibility can be made when investigating composition ideals:

CYCLIC COMPOSITION THEOREM: Let $\mathfrak{A},\mathfrak{B},\mathfrak{C}$ quasi-Banach operator ideals, $\mathfrak{A}$ and $\mathfrak{B}$ accessible, such that $\mathfrak{A} \circ \mathfrak{B} \subset \mathfrak{C}$, then $\mathfrak{B} \circ \mathfrak{C}^* \subset \mathfrak{A}^*$ and $\mathfrak{C}^* \circ \mathfrak{A} \subset \mathfrak{B}^*$ (with the same constants), when $\mathfrak{A}^*$ is the adjoint operator ideal $\mathfrak{A}$.

This "rotation" transforms a result of Carl-Defant about mixing operators into the following:

Every q-summing operator on L_p (with values in an arbitrary Banach space) factors through a Hilbert-space if and only if $1/q \geq |1/2 - 1/p|$ where $q,p \in [1,\infty]$.

(Joint work with Andreas Defant)

Ref.: A. Defant - K. Floret: Tensor norms and operator ideals (sections 21,25,34); to appear in: North Holland Math. Studies.

*G. Pisier constructed during the conference a non-accessible normed maximal operator ideal.

V. FONF

Injections and linear-topological properties of Banach spaces

In geometry of Banach spaces the following problem is well known: does every Banach space contain isomorphic copy c_o or a boundedly complete basic sequence?

There is in a report approach to this problem. The idea is based on the study of topological boundary of bounded closed convex solid (with nonempty interior) subsets of given Banach space.

Obtained results on this way are interesting in some parts of the Banach space theory: G_δ-embeddings, which gave an idea of this approach, study properties of Banach spaces not containing c_o, polyhedral Banach spaces and so on.

Introduced two new conceptions: thick-set and thin-set and with help of this notions it is proved theorems about space not containing c_o and boundedly complete basic sequence.

D. J. H. GARLING

Complex martingales and unconditional martingale convergence

We use the following elementary equality, essentially due to Yudin:

Theorem 1. If $f,g \in L^{2k}(X,\Sigma,\mu)$, where $k \in \mathbb{N}$, and $\int f^k \bar{g}^k \, d\mu = 0$, then
$\|f\|_{2k} \le (\int (|f|^{2k} + |g|^{2k}) d\mu)^{1/2k} \le k\|f+g\|_{2k}$. (*)

Apply this to Π^ℓ, with Haar measure μ.

Corollary 2. If $\wedge, M$ are disjoint cones in $\mathbb{Z}^\ell$, and $f \in L^{2k}_\wedge$, $g \in L^{2k}_M$ (i.e. $\hat{f}(n) = 0$ for $n \notin \wedge$, $\hat{g}(n) = 0$ for $n \notin M$), then (*) holds.

Take $\Omega = \Pi^{\mathbb{N}}$, with usual filtration. A martingale (m_n) is a Hardy martingale if $\mathbb{E}(d_{n+1} e^{ik\Theta_{n+1}} | J_n) = 0$, for $k \ge 0$.

Theorem 3. If $1 < p < \infty$ and $f \in L^p(\Omega)$, there exist unique Hardy martingales $(m_n), (p_n)$ with $p_o = 0$ such that $f_n = m_n + T_n$ (where $f_n = \mathbb{E}(f | J_n)$). Let $m = \lim m_n$: The map $f \longrightarrow m$ is bounded.

A similar argument gives

Theorem 4. If (m_n) is an L^p bounded Hardy martingale $(1 < p < \infty)$ then $m_o + \Sigma(m_{n+1} - m_n)$ converges unconditionally in L^p.

Combining this Theorem 3, we obtain a short proof of the standard result:

Theorem 5. If (m_n) is an L^p bounded martingale $(1 < p < \infty)$ then $m_o + \Sigma(m_{n+1} - m_n)$ converges unconditionally in L^p.

N. GHOUSSOUB

Variational principles with second order conditions

In the problem of minimizing a C^2-function φ that is bounded below on a Hilbert space H, Ekeland's variational principle yields minimizing sequences $(\chi_k)_k$ (i.e. $\varphi(\chi_k) \longrightarrow \inf \varphi$) that are also Palais-Smale sequences (i.e. $\varphi'(\chi_k) \longrightarrow 0$). The smooth variational principle of Borwein-Preiss yields the additional second order condition: $\lim \inf_k \langle \varphi''(\chi_k)\omega,\omega\rangle \geq 0$ for all $\omega \in H$. If now the hypothesis of the "Mountain Pass Theorem" are satisfied, then we construct Palais-Smale sequences that also verify the following second order condition:

If $\langle \varphi''(\chi_k)\omega,\omega\rangle < -\frac{1}{k}\|\omega\|^2$ for every ω in a subspace E of H then $\dim(E) \leq 1$.

As already noticed by P. L. Lions, this additional information is sometimes useful in the proof of the convergence of such sequences.

G. GODEFROY

Unconditional ideals

In this joint work with N. Kalton and D. Saphar, we investigate the following situation: a closed subspace X of Banach space Y is called a u-ideal (resp. h-ideal in the complex case) if there exists $P: Y^* \longrightarrow Y^*$ with $P^2 = P$, Kern $P = X^\perp$ and $\|1-2P\| = 1$ (resp. $\|1-(1+\lambda)P\| = 1$ for every λ of modulus one). Examples of this situation include: a) M-ideals. b) ideals in Banach lattices. c) K(X) in L(X), if X is reflexive with a I-unconditional basis. We show in particular the following results.

Theorem 1: If X is u-ideal in Y and X does not contain $c_o(\mathbb{N})$, then X is u-complemented in Y.

Theorem 2: Let X be a separable complex space which is h-ideal in its bidual X^{**}. Then X has Pełczynski's property (u) with constant one, and there exists an hermetian projection T from X^{**} onto the space Ba(X) of first Baire class elements of X^{**}.

Theorem 2 means in particular that the structure of spaces X wich are h-ideals in X^{**} is quite similar to the structure of order-continuous Banach lattices.

Y. GORDON

Some geometric properties of p-normed spaces $(0 < p \leq 1)$

(Joint work with Nigel Kalton)

Wo proved the following:

Theorem 1: Let $0 < r \leq 1$, $0 < \delta < 1$. There exists $N(r,\delta) < \infty$, such that whenever X is an r-normed space of dimension $n \geq N(r,\delta)$, there is a quotient of a subspace of X, E, with $\dim E \geq \delta n$, whose distance to a Hilbert space, d_E, is smaller than $C(r) \left(\frac{1}{1-\delta} \log\left(\frac{2}{1-\delta}\right)\right)^{\frac{2}{r}-1}$.

Theorem 1 generalizes the well-known qotient-subspace theorem of V. Milman for Banach spaces (i.e. $r=1$), to the class of r-normed spaces, hence to quasi-normed spaces as well. We also stated the generalizations of the cotype-2 theorem for the class of r-normed apaces, and the volume-ratio theorem of G. Pisier, and Dvoretzky's theorem.

These results indicate that convexity is **not essential** assumption for many of the famous positive results in local theory.

T. GOWERS

The hyperplane problem

A question by Banach is whether every Banach space X is isomorphic to its codimension-one subspaces, or hyperplanes. Equivalently, must $X \cong X \oplus \mathbb{R}$? It turns out that a variant of the space X from the abstract of B. Maurey is a counterexample. In addition, it has a 1-unconditional basis. The proof that the variant is not isomorphic to its hyperplanes is similar to the proof that X has no unconditional basic sequence.

R. HAYDON

<u>Trees in renorming theory</u>

A recent paper (Bull. L. M. S. 1990) showed that if Υ is a full uncountably branching tree of uncountable height then then the space $C_o(\Upsilon)$ is an Asplund space that admits no equivalent norm that is either strictly convex or Gateaux-differentiable. A closer study of spaces obtained from trees in this way allows us to answer some other questions about renormings:

(i) if Υ is a full dyadic tree of uncountable height then $C_o(\Upsilon)$ admits a renorming with the Kadec property (the weak and norm topologies coincide on the sphere) but no locally uniformly convex renorming ;

(ii) for any tree Υ there is a Fréchet differentiable "bump" function on $C_o(\Upsilon)$ even though no differentiable renorming need be possible.

A by-product of the techniques used enables as to answer a question of Talagrand:

(iii) There is a Fréchet differentiable equivalent norm on $C_o([0,\Omega) \times [0,\Omega))$ for any ordinal Ω.

S. HEINRICH

<u>Gelfand numbers of tensor products and complexity</u>

Numerical complexity theory is concerned with the following task: Given a numerical problem and a $\delta > 0$, determine the minimal numbers of arithmetic operations needed to compute a δ-approximation to the solution. In the talk an open complexity problem for integral equations is discussed and it is shown that its solution is closely tied to the following problem on s-numbers:

Let $n \in \mathbb{N}$ and let $I_n : l_\infty^n \longrightarrow l_1^n$ be the identity. What is the asymptotic order of

$$c_m(I_n \otimes I_n : l_\infty^n \otimes_\varepsilon l_\infty^n \longrightarrow l_1^n \otimes_\varepsilon l_1^n) \ ?$$

Here $\otimes_\varepsilon$ denotes the injective tensor product and c_m the m-th Gelfand number. In particular, it would be of interest to know if

$$c_{\frac{n^2}{2}}\left(I_n \otimes I_n : l_\infty^n \otimes_\varepsilon l_\infty^n \longrightarrow l_1^n \otimes_\varepsilon l_1^n\right) \asymp n^2 \ .$$

W. HENSGEN

On weak sequentional completeness of vector valued L^1/H^1_o

Let $H^1_o(X) := \{f \in L^1(X) : \hat{f}(n) < 0 \,\forall n \leq 0\}$. Generalizing the well known Khavin-Mooney theorem (L^1/H^1_o w.s.c.), Bukhvalov and Petrenko had proved that $L^1(X)/H^1_o(X)$ is w.s.c. if X is a reflexive B space or a w.s.c. B lattice and asked for the general case (X w.s.c. B space). Their proof is of a functional analytic character as developed by Godefroy. Going back to the original function theoretic proof of Khavin, I am proving that $L^1(X)/H^1_o(X)$ is w.s.c. if X is w.s.c., RNP, compl. in X'' and X' has ARNP. In general, $L^1(X)/H^1_o(X)$ need not be w.s.c. for w.s.c. X (counterexample of Pisier; note that $L^1(X)/H^1_o(X) \cong L^1/H^1_o \hat{\otimes} X$).

M. JUNGE

Calculating (p,q)-summing norms with few vectors

(Joint work with Martin Defant)

Motivated by connected problems in the local theory of Banach space geometry T. Figiel asked whether there exist a construct $c_{pq} \geq 1$ such that for all operator T with rank (T) $\leq$ n one has

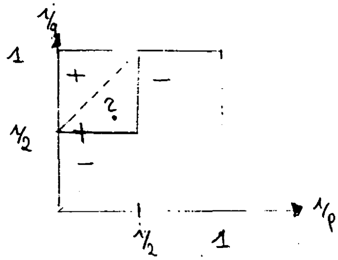
$$\Pi_{pq}(T) \leq c_{pq}\, \Pi^n_{pq}(T). \quad ?$$

Positive answers where given by Tomczak-Jaegermann for $p = q = 2$ and König for $p > 2 = q$. Using this König and Tzafriri gave a positive answer for $p > 2$, $q = 1$.

Negative answers where found by Pełczynski and Figiel for $p = q = 1$. Using information about the limit order of Π_{pq}, established by Carl, Maurey and Puhl, one has negativ answers also for $2 < q \leq p < \infty$.

M. Defant and the author developped quotient formulas, which allow to transfer known informations to unknown cases. For instance they proved a positive answer in the case $1/q > 1/p + 1/2$ using the König/Tzafriri result. Vice versa the negative information for $2 < q \leq p < \infty$ is transfered to the area $1 \leq p,q < 2$.

The question is cleared up apart from the area where $q < 2$ and $1/q \leq 1/p + 1/2$.



V. M. KADETS

Generlzations of the Liapunov convexity theorem

Let Σ be a σ-algebra of subsets of some set Ω, X is a Banach space, $\mu : \Sigma \to X$ is a σ-additive measure. Let

$$\mathrm{at}\,(\mu) = \sup\{\|\mu(A)\| : A \text{ is an atom of } \mu\}$$

and

$$\varepsilon(\mu) = \sup\{d\left(\tfrac{x+y}{2}, \mu(\Sigma)\right) : x, y \in \mu(\Sigma)\}$$

characterise the size of atoms and degree of non-convexity of the measure values set respectively.

Theorem. The following two conditions for the Banach space X are equivalent:

1) X is B-convex

2) $\sup\{\varepsilon(\mu) : \mu$ is X-valued measure, $\mathrm{at}\,(\mu) \leq a$, variation of μ is less then $1\} \xrightarrow[a \to 0]{} 0$.

Some estimates for $\varepsilon(\mu)$ in finite-dimensional case are given:

$$\varepsilon(\mu) \leq \dim X \cdot \mathrm{at}(\mu) \quad \text{for arbitrary } X$$

$$\varepsilon(\mu) \leq \sqrt{\dim X} \cdot \mathrm{at}(\mu) \quad \text{for euclidian } X .$$

S. KISLIAKOV

Interpolation of weighted H^p

Real and complex interpolation methods, when applied to the couple $\left(H^{p_o}(E_o; W_o), H^{p_1}(E_1; W_1)\right)$ give what is expected if E_o and E_1 are quasi-Banach lattices of measurable functions satisfying certain mild conditions and

$\log\left(W_o^{1/p_o} W_1^{-1/p_1}\right) \in$ BMO (W_o, W_1 being weights on the unit circle). The BMO-condition is in fact necessary. (It is expected, of course, that the resulting spaces coincide with the subspaces of analytic functions in the corresponding interpolation spaces for the couple $(L^{p_o}(E_o; W_o), L^{p_1}(E_1; W_1))$).

The results have been obtained jointly with Q.Xu.

J. LINDENSTRAUSS

<u>Approximating the sphere with zonotopes, another look</u>

This work is done with J. Bourgain.

<u>Theorem.</u> For every $n \geq 3$ and $\varepsilon > 0$ there are

$$(*) \qquad N = c(n) \left(\varepsilon^{-2} |\log \varepsilon|\right)^{(n-1)/(n+2)}$$

segments of **equal length** $\{I_j\}_{j=1}$ so that

$$B_n \subset \sum_{j=1}^{N} I_j < (1+\varepsilon) B_n$$

where B_n is the Euclidean ball.

It is known that up to the $|\log \varepsilon|$ factor (*) is the best estimate. The same result without the "equal length" assertion was proved by us several years ago. Surprisingly the passage to segments with equal length requires delicate arguments. For $n \leq 6$ this was done by the late Gerold Wagner. We show now how to do this for general n.

W. LUSKY

<u>On Banach spaces with the commuting bounded approximation property</u>

We consider finite rank operators R_n on a given separable Banach space X such that $R_n R_m = R_{\min(n,m)}$ whenever $n \neq m$ and $\lim_n R_n x = x$ for all $x \in X$

We prove the following:

If $R_n - R_{n-1}$ factors through l_p for same $1 \leq p < \infty$ (or c_o) then $X \oplus l_p$ (or $X \oplus c_o$) has a basis. As an example we mention the disc algebra where the assumptions of the theorem can be easily verified. Moreover the theorem holds for all weighted spaces

$$\{f:D\to\mathbb{C}\mid f \text{ holomorphic (harmonic)} \lim_{|z|\to 1} f(z)\vartheta(|z|)=0\}$$

where we take the norm $\|f\| = \sup_{z\in D} |f(z)|\vartheta(|z|)$. Here D is the unit disc and $\vartheta : [0,1]\to\mathbb{R}$ is continuous, decreasing with $\vartheta(1)=0$. Since all these spaces contain c_o they have bases.

B. MAUREY

<u>The unconditional basic sequence problem</u>

Tim Gowers and the author of this abstract independently constructed very similar examples of an infinite-dimensional Banach space X wich does not contain any infinite unconditional basic sequence. Actually, the properties of this space X are better described by the fact that X is **Hereditarily Indecomposable**, which means that whenever Y,Z are in finite dimensional subspaces of X and $\varepsilon > 0$, there exists $y\in Y$ such that $\|y\| = 1$ and $d(y,Z) < \varepsilon$.

A complex H. I. Banach space has few operators: every $T\in\mathfrak{L}(X)$ can be written as $T = \lambda\,\mathrm{Id} + S$, where S is strictly singular and $\lambda\in\mathbb{C}$.

A. PIETSCH

<u>Orthonormal systems and Banach space geometry</u>

We start a systematic treatment of such results and methods in geometry of Banach spaces which involve orthonormal systems. The basic idea is the following:

Let $\emptyset_n = (\varphi_1,\dots,\varphi_n)$ be any orthonormal system in a Hilbert space $L_2(M,\mu)$, and let H be a Hilbert space. Then, for $x_1,\dots,x_n\in H$, we have Parzeval's equation

$$\int_M \|\sum_1^n x_k\varphi_k(t)\|^2 d\mu(t) = \sum_1^n \|x_k\|^2$$

In arbitrary Banach spaces this is no longer true. However, we may ask for the smallest constant $c\ge 1$ such that

$$\int_M \|\sum_1^n x_k\varphi_k(t)\|^2 d\mu(t) \le c^2 \sum_1^n \|x_n\|^2$$

whenever $x_1,\dots,x_n \in E$. This yields a measure of the non-Hilbertness of E. There are many other ways to define similar quantities. This approach results in a unification of different theories which are concerned with Rademacher types and cotypes, the vector-valued Fourier transform and the vector-valued Hilbert transform.

G. PISIER

Interpolation between H^p spaces and non-commutative generalizations

We give an elementary proof that the H^p spaces over the unit disc (or the upper half plane) are the interpolation spaces for the real method of interpolation between H^1 and H^∞. This was originally proved by Peter Jones. The proof uses only the boundedness of the Hilbert transform and the classical factorisation of a function in H^p as a product of two functions in H^q and H^r with $1/q + 1/r = 1/p$. This proof extends without any real extra difficulty to the non-commutative setting and to several Banach space valued extensions of H^p spaces. In particular, this proof easily extends to the couple $H^{p_o}(l_{q_o}), H^{p_1}(l_{q_1})$, with $1 \le p_o,p_1,q_o,q_1 \le \infty$. In that situation, we prove that the real interpolation spaces and the K-functional are induced (up to equivalence of norms) by the same objects for the couple $L_{p_o}(l_{q_o}), L_{p_1}(l_{q_1})$. In an other direction, let us denote by C_p the space of all compact operators x on Hilbert space such that $tr(|x|^p) < \infty$. If $p = \infty$, C_p is just the space of all compact operators. Our proof allows us to show for instance that the space $H^p(C_p)$ is the interpolation space of parameter $(1/p,p)$ between $H^1(C_1)$ and $H^\infty(C_\infty)$. We also extend a recent result of Kaftal-Larson and Weiss, we prove that the distance to the upper triangular matrices in C_1 and C_∞ can be essentially realized simultaneously by the same element.

C. T. READ

Strange measures and quantum field theory:
what do applied Mathematicians want?

In this talk I discuss the difficulties involved in constructive quantum field theory, and the constraints on the measures required to do the "construction", especially the strange one, reflectian positivity. I discuss divergences, briefly indicating the difference between an 'infrared' and 'ultraviolet' one, and exhibit some scalar field theories without ultraviolet divergence, but which may have infrared divergences. All the work involved was done in collaboration with Noah Linden (Cambridge); but the lecture is mainly intended as an introduction to the subject for these working in Banach spaces. A peculiarity of the Linden-Read construction is its dimension-independence; it works in $\mathbb{R}^d$ for any dimension d.

W. SCHACHERMAYER

Equivalent martingale measures

We present the following theorem which is motivated by some problems in financial mathematics.

Theorem. Let $(X_n)_{n=1}^{\infty}$ be a stochastic process modelled on $(\Omega,(\mathcal{F}_n)_{n=1}^{\infty},P)$ such that $X_n \in L^{\infty}(\Omega,\mathcal{F}_n,P)$ for all $n \in \mathbb{N}$.

The following assertions are equivalent

(i) There exists a measure Q equivalent to P such that $(X_n)_{n=1}^{\infty}$ is a martingale eith respect to P.

(ii) Letting $Z = \text{span}\{(X_t - X_s)\chi_{A_s} : t \geq s,\ A_s \in \mathcal{F}_s\}$ there does not exist $k \in L_+^{\infty}$, $k \neq 0$, which may be approximated by a sequence $(f_i)_{i=1}^{\infty}$ of elements of Z in the following sense:

(a) $f_i(\omega) \geq -1$ for $n \in \mathbb{N}$, P-a.e. $\omega \in \Omega$

(b) $\liminf_{i\to\infty} f_i(\omega) \geq k(\omega)$ for P-a.e. $\omega \in \Omega$.

We also discuss the interpretation of condition (ii) in economic terms as the absence of "a free lunch with deterministically bounded risk".

E. SEMENOV (Joint work with T. APPELL)

On Lorentz functional spaces

Given a measurable function x on [0,1], we denote by x_n the truncation of x. Let $\Lambda(\varphi)$ be the Lorentz space generated by a concave positive function φ. $M(\varphi) = \Lambda^*(\varphi)$, $\overline{\varphi}(t) = t/\varphi(t)$. The set

$$\Gamma(x) = \left\{\varphi : \lim_{n\to\infty} \frac{\|x_n\|_{M(\overline{\varphi})}}{\|x_n\|_{\Lambda(\varphi)}} = 0\right\}$$

is investigated.

E. WERNER (Joint work with C. SCHÜTT)

The convex floating body of almost polytopal bodies

We ask what kind of functions can occur as the volume difference $\mathrm{vol}\,K - \mathrm{vol}\,K_\delta$ where K_δ is the convex floating body of a convex body in $\mathbb{R}^2$, i.e. K_δ is the intersection of all half-spaces whose defining hyperplanes cut off a set of volume δ of K. If K has smooth boundary it is known that $\lim_{\delta\to 0} \frac{\mathrm{vol}\,K - \mathrm{vol}\,K_\delta}{\delta^{\frac{2}{3}}}$ exists and is a positive real number. If K is a polytope, then $\lim_{\delta\to 0} \frac{\mathrm{vol}\,K - \mathrm{vol}\,K_\delta}{\delta \ln \frac{1}{\delta}}$. exists and is a positive real number.

Here we obtain the following:

> Let h be a positive differentiable function with $h(o) = 0$ and $\delta \ln\frac{1}{\delta} \leq h(\delta) \leq \delta^{\frac{2}{3}}$. Put $f(\delta) = 3\, h'(\delta^3) - 3\, \frac{h(\delta^3)}{\delta^3}$. If $\delta f(\delta)$ is decreasing and convex then there is a convex body K and a constant c such that for all small δ
>
> $$\frac{1}{c} h(\delta) \leq \mathrm{vol}\,K - \mathrm{vol}\,K_\delta \leq c\, h(\delta) .$$

W. WERNER

Smooth points in spaces of bonded operators

Let T be an operator between two Banach spaces X and Y. We present various cases in which the condition

> $\|T\|_e < \|T\|$, there is exactly one $y^* \in B_{Y^*}$ where T attains its norm, and $T^* y^*$ is smooth

is equivalent to T being a smooth point of $L(X,Y)$. This characterization e.g. holds whenever Y is a subspace of c_0 with the metric approximation property and X is arbitrary. Smooth points of $L(Y^*,X)$, where X and Y are as before, can be shown to behave similarly.

In proving these results use is made of certain M-ideals of compact operators as well as of sufficently many complemented subspaces with a basis in some of the involved Banach spaces.

As a byproduct, it turns out that in all cases in question operators not attaining their norms on B_X (or, respectively, B_{Y^*}) are nowhere dense in $L(X,Y)$.

M. WOJCIECHONSKI

Translation invariant projections on anisotropic Sobolev spaces on Tori.

We consider the translation invariant projection in Sobolev spaces $W_s^1(\pi^d)$ and in the spaces of smooth functions $C_s(\pi^d)$. Symbol S denote smoothness i.e. finite set of partial devivatives, which can be identified with a finite set of nonnegative lattice points of $\mathbb{R}^d$. We say that smoothness S satisfy condition (0) if it containes two points-one corresponding to partial derivative of odd order, second to a partial derivative of even order and there exist a hyperplane pasing through these points which supports the convex hull of S and is not parallel to any axis of $\mathbb{R}^d$, or the same property has one of the lower dimensional smoothnesses being the intersection of S with same number of coordinate hyperplane.

The following dychotomy holds:

Theorem 1. If S satify (0) then space $W_s^1(\pi^d)$ has a translation invariant complemented subspace isomorphic to infinity dimensional Hilbert space. Else the translation invariant projections are characterized by coset ring of $\mathbb{R}^d$ (it means that supp $\hat{R}$ belongs to the coset ring of $\mathbb{R}^d$ for every t. i. projections on W_s^1).

In the case of spaces $C_s(\pi^d)$ and $A(\pi^d)$ (= the polidysk algebra) the situation is more simple:

Theorem 2. Translation invariant projections in the spaces $C_s(\pi^d)$ and $A(\pi^d)$ are characterized by coset ring of $\mathbb{Z}^d$.

The proofs of theorems 1 and 2 use the Helson-Rudin-Cohen characterization of idempotent measures, some combinatorial consideration, Riesz product construction (or Rudin-Shapiro construction) and Mc Gehee-Pigno-Smith inequality.

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