

MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Kombinatorik geordneter Mengen

29. 9. bis 5. 10. 1991

Die Tagung fand unter der Leitung von M. Aigner (Berlin) und R. Wille (Darmstadt) statt. Auf der Grundlage von 36 Vorträgen diskutierten die Teilnehmer über ein breites Spektrum von Fragen aus der Ordnungstheorie und angrenzender Gebiete. Besonders intensiv wurden auf dieser Tagung Intervallordnungen, sowie Ordnungen in Booleschen Verbänden und Partitionsverbänden behandelt. Zu den Fragestellungen gehörten Ketten- und Antikettenzerlegungen, „scheduling“, Parameter wie die Dimension und die „jump number“, ordinale Strukturen, Automorphismengruppen, sowie Themen aus der Graphentheorie, beispielsweise Färbungsprobleme. Zusätzlich fand eine Problemsitzung statt, in der offene Probleme vorgestellt wurden. Eine Liste dieser Probleme findet sich im Anschluss an die Vortragsauszüge.

## Abstracts

*Martin Aigner*

### The game is over

Some remarks are made on recursion trees, in particular game trees and majority trees. Let  $\tilde{G}$  be an acyclic directed graph with a unique source. Assign 0, 1 to the sinks with equal probability, and process the values towards the root by  $v(x) = 1 - \min\{v_1, \dots, v_s\}$ , where  $v_1, \dots, v_s$  are the values of the sons of  $x$ . What is the average number of sinks that have to be inspected so that the value of the source is correctly determined. Sample result:  $P_n = \text{Boolean lattice} - \{0\}$ . Then average cost( $P_n$ ) =  $(n+1)(1 - \binom{n}{\lfloor n/2 \rfloor} / 2^n) \sim n - c\sqrt{n}$ .

*Gerhard Behrendt*

### Homogeneity in finite ordered sets

We give some results and problems about constructions and classification of finite ordered sets  $(X, \leq)$  which have a high degree of symmetry given by transitivity properties of their automorphism groups. Amongst the homogeneity conditions considered are the following: (1) every order isomorphism between any two order ideals is induced by an order automorphism of  $(X, \leq)$ ; (2) for every  $x \in X$  the stabilizer of  $x$  in  $\text{Aut}(X, \leq)$  acts as the full symmetric group on the set of elements covering  $x$ , and on the set of elements covered by  $x$ .

*Kenneth P. Bogart*

### Threshold representations of ordered sets

A *threshold representation* of an ordering  $P$  of a set  $S$  consists of two functions, a function  $C : S \rightarrow R_0$  the non-negative real numbers, called a *coordinate function* and a function  $T : S \times S \rightarrow R_0$  called the *threshold function* such that

$$(x, y) \in P \quad \text{if and only if} \quad c(x) > c(y) + T(x, y).$$

Every order has a threshold representation satisfying the triangle inequality, and every pair of functions  $c : S \rightarrow R_0$  and  $T : S \times S \rightarrow R_0$  in which  $T$  satisfies the triangle

inequality represents an ordering. Semiorders have threshold representations with constant thresholds, interval orders have threshold representations with the sums of half-lengths of representing intervals as thresholds, etc. Other thresholds arise naturally in applications and in the study of tolerance graphs. This talk will discuss some of these results.

*Walter Deuber*

### Ordering discrete sets by wobbling injections

Let  $M$  be a metric space. For  $X, Y \subset M$  let  $X \leq Y$ , if there exists an injection  $\varphi : X \rightarrow Y$  satisfying  $\sup_X d(\varphi(x), x) < \infty$ . We discuss this quasiorder on discrete sets and give criteria for comparability. We also describe the paradoxical case, i.e. those  $X$  for which there exists a decomposition  $X = X_1 \dot{\cup} X_2$  with  $X \leq X_1, X \leq X_2$ . The main tool are marriage theorems from graph theory, which yield local-global characterizations.

*D. Duffus*

### Partitioning maximal chains

It is evident that for finite ordered sets (without isolated elements) one can always define a 2-coloring of the elements so that each maximal chain receives both colors. Just color the minimal elements red and the remaining elements blue. In fact, this is possible for all countable ordered sets - there is a partition into an up-set and a down-set so that each maximal chain intersects both segments. However, the product  $\omega \times \omega_1$  shows that this is not always possible in the uncountable case. This is more difficult to answer.

Question: Can every ordered set be 2-colored so that all non-trivial maximal chains receive both colors?

Duffus, Rodl, Sauer and Woodrow have an example based on products of interval orders that gives a negative answer of cardinality  $(2^{2^\omega})^+$ . It remains open to determine the answer for smaller uncountable cardinals.

Konrad Engel

### On the filter inequality in the partition lattice

The following question is motivated by the investigation of the asymptotic Sperner property of posets: Is it true (or not) that for every filter  $F \neq \emptyset$  in the partition lattice  $P_n$  the following inequality holds:

$$\frac{1}{|F|} \sum_{\pi \in F} r(\pi) \geq \frac{1}{|P_n|} \sum_{\pi \in P_n} r(\pi) \quad (1)$$

Here  $r(\pi)$  is as usual the rank of the element  $\pi$ . We provide partial results: Let  $A$  be an antichain in  $P_n$  and  $\Gamma(A, \lambda)$  the number of colourings  $c: \{1, \dots, n\} \rightarrow \{1, \dots, \lambda\}$  such that for every element  $\pi$  of  $A$  there exists at least one block of  $\pi$  which is not monochromatic. The antichain  $A$  is called colouring monotonous if for all  $\lambda \in \mathbb{N}_+$

$$\frac{\Gamma(A, \lambda)}{|A|^n} \leq \frac{\Gamma(A, \lambda + 1)}{(\lambda + 1)^n}$$

We prove a necessary and sufficient criterion for (1) which has as consequence that a filter generated by a colouring monotonous antichain satisfies (1). Finally we study colouring monotonicity.

Ulrich Faigle

### k-decompositions of interval orders and an online-scheduling problem

Let  $P$  be a finite order and assume that the elements can be listed  $x_1, x_2, \dots, x_n$  such that  $y > x_i$  holds whenever  $y > x_{i+1}$  is true. Then the following greedy algorithm finds  $k$  chains in  $P$  that cover as many elements as possible:

“Assign  $x_i$  to that chain obtained so far whose top element has the largest possible index if such an assignment is feasible” ( $i = 1, \dots, n$ ).

The algorithm yields an optimal solution for assigning clients  $i$  online to one of  $k$  (identical) servers without buffers so that the number of “losses” is minimized.

The more general problem with servers working at different speeds is unsolved even in the case where the individual arrival times of the clients are known in advance. (For fixed  $k$ , a polynomial solution for the not-online model is known). (Joint work with W. Nawijn).

*Stefan Felsner*

### Colorings of interval order diagrams and $\alpha$ -sequences of sets

For a nonnegative integer  $k$ , let  $I_k$  be the interval order defined by the open intervals with endpoints in  $\{1, \dots, 2^k\}$ . The diagram of this order is the shift graph  $G(2^k, 2)$  and it is known that  $\chi(\text{Diag } I_k) = \lceil \log[\text{height } I_k] \rceil = k$ . This motivates the question: How large can the chromatic number of the diagram of an interval order with height  $k$  be? We transform the problem into the following: How long can a sequence  $(C_i)_i$  of subsets of  $\{1, \dots, n\}$  be if we require that

$$C_j \not\subseteq C_i \cup C_{i-1} \quad \text{for all } i < j$$

We give a construction of sequences of length  $2^{n-2} + n$  and prove an upper bound of  $2^{n-1} + \lfloor (n+1)/2 \rfloor$ . We conjecture that this upper bound is always tight. As an application of the techniques developed we give a construction of cycles of length  $1/4N$  in the graph consisting of the middle two levels of a Boolean Lattice  $B_{2k+1}$ . Until now only cycles of length  $\Omega(N^c)$  with  $c \sim 0.85$  and  $N$  the number of vertices of the graph have been known.

*Bernhard Ganter*

### Finding closed sets under symmetry

In applications of lattice theory to data analysis, closure systems occur naturally that are very large and also have a large group of automorphisms. We describe methods and algorithms to work with such systems effectively. This is joint work with K. Reuter and M. Zickwolff.

*Jerry Griggs*

### Chains partitions in $B_n$ and $\Pi_n$

We discuss the problem of existence of partitions of the Boolean lattice  $B_n$  into chains of given sizes, including the remarkable recent proof by Lonc of my conjecture that for fixed  $c$  and  $n > n_0(c)$ ,  $B_n$  can be partitioned into chains of size  $c$ , except for at most  $c-1$  elements, which also form a chain.

We also present very recent work of Canfield on the partition lattice  $\Pi_n$  which extends his earlier disproof of Rota's conjecture that  $\Pi_n$  is Sperner. His results concern when

it is possible to find a matching from one level (rank) of  $\Pi_n$  into an adjacent one.

*Hans-Dietrich Gronau*

### On an inequality of Sperner

Let  $\mathcal{F} \subseteq \binom{N}{k}$  be a family of  $k$ -element subsets of an  $n$ -element set  $N$ . The shadow  $\Delta\mathcal{F}$  is defined by  $\Delta\mathcal{F} = \{X : X \in \binom{N}{k-1}, X \subset Y \text{ for some } Y \in \mathcal{F}\}$ . Sperner's inequality  $|\Delta\mathcal{F}| \geq k/(n-k+1)|\mathcal{F}|$  is the crucial result in several techniques in extremal set theory. Unfortunately, this bound is not best possible if  $|\mathcal{F}| < \binom{n}{k}$ . In the talk best inequalities of type  $|\Delta\mathcal{F}| \geq c|\mathcal{F}|$  for all  $0 \leq |\mathcal{F}| \leq d$ , where  $c = c(d)$  are presented. Generalizations to the poset of multisets are given, too. The results are mainly due to my student A. Rausche.

*Michel Habib*

### Parallelism and order

We discuss some problems yielded by new developments in the area of distributed processing. Namely the vector clocks proposed by Mattern and Fidge are related to an incremental (on line) computation of the dimension of an ordered set. We also present some results related to minimal internal extensions of an order which appear naturally in this context.

*K. O. H. Katona*

### Cycle length of the dual antichain operation

Let  $X$  be a finite set,  $|X| = n$ . If  $\mathcal{F} \subseteq 2^X$ , define  $\hat{\mathcal{F}} = \{A : \exists F \in \mathcal{F} \text{ s.t. } F \subseteq A\}$ . Let  $d(\mathcal{F})$  denote the maximal elements of  $\hat{\mathcal{F}}$ . If  $\mathcal{F}$  is an antichain,  $d(\mathcal{F})$  is an antichain, again.  $d$  has an inverse. Consequently any antichain  $\mathcal{F}$  defines a cycle  $\mathcal{F}, d(\mathcal{F}), d^2(\mathcal{F}), \dots, d^r(\mathcal{F})$  of antichains and a cycle length  $c(\mathcal{F})$ . Duchet (1974) constructed an  $\mathcal{F}$  with  $c(\mathcal{F}) = 2$  for even  $n$  (Brouwer (1975) for odd  $n$ ). Kleitman and the speaker were looking for  $\mathcal{F}$  such that  $c(\mathcal{F}) = 2$ ;  $\mathcal{F}, d(\mathcal{F}) \subseteq \binom{X}{k}$ . We proved that such  $\mathcal{F}$  does not exist for small  $k$  but exists for larger  $k$ . We also found a large class of  $\mathcal{F}$ 's with  $c(\mathcal{F}) = n + 2$ .

David Kelly

## Ferrers Dimension

For a binary relation  $R$ , we write  $\text{Gal}(R)$  for

$$\{(A, B) \mid B = R[A], A = R^{-1}[B]\},$$

which is the Galois correspondence determined by  $R$  (considered as a set of ordered pairs).  $\text{Gal}(R)$  is a lattice with the ordering determined by " $\subseteq$ " on the first component, or " $\supseteq$ " on the second.  $\text{Gal}(R)$  is called the "Galois lattice" or "concept lattice". We discuss some of the history and consider Ferrers relations (those relations  $R$  for which  $\text{Gal}(R)$  is a chain). We consider a generalization of the result that Ferrers dimension and interval dimension coincide for strict order relations. The proof uses a new characterization of a minimal Ferrers extension of a relation. Although no finiteness assumption is made, much of the proof has a finite character.

W. Kern

## On the game chromatic number of graphs and orders

Consider two players jointly coloring a given graph  $G$  with  $k$  colors in the following way: The two players move in turns, with player 1 moving first. Each move consists in coloring (feasibly) one of the yet uncolored nodes of  $G$  with one of the available  $k$  colors. The aim of player 1 is to finally get the graph completely colored, while player 2 tries to obstruct, i.e. to create a situation where none of the players can move anymore, while the graph is not yet fully colored. Thus player 1 wins, if the graph becomes completely colored, otherwise player 2 wins. Let  $\gamma(G)$  denote the minimum  $k$  such that player 1 has a winning strategy. We present results on  $\gamma(G)$  for some classes of graphs such as trees ( $\gamma = 4$ ), unions of two trees ( $\gamma \approx \log n$ ), interval graphs ( $\gamma = 2\omega$ ) and comparability graphs of series-parallel orders, i.e. cographs ( $\gamma \geq 2^\omega$ ).

H. Kierstead (with S. Penrice and W.T. Trotter)

## On-line coloring of co-comparability graphs

An on-line vertex coloring algorithm receives the vertices of a graph one at a time in some externally determined order. Whenever a new vertex is presented, the algorithm also leaves to which of the previously presented vertices the new vertex is adjacent.

At this time the algorithm must assign the new vertex a color without knowledge of futur vertices. A class of graphs  $\Gamma$  is said to be *on-line  $\chi$ -bounded* if there exists an on-line algorithm  $\mathcal{A}$  and a function  $f$  such that  $\chi_{\mathcal{A}}(G) \leq f(\omega(G))$  for all  $G \in \Gamma$ , where  $\chi_{\mathcal{A}}(G)$  denotes the number of colors that  $\mathcal{A}$  uses to color  $G$  and  $\omega(G)$  denotes the clique number of  $G$ .

**Theorem:** For any radius two tree  $T$ , the class of graphs  $\text{ForG}(T)$ , which do not induce  $T$ , is on-line  $\chi$ -bounded.

Since co-comparability graphs do not induce the subdivision of  $K_{1,3}$  we obtain:

**Corollary:** The class of co-comparability graphs is on-line  $\chi$ -bounded.

*Zbigniew Lonc*

### Chain and antichain partitions of ordered sets

We consider partitions of ordered sets into chains (respectively antichains) with bounded sizes. We settle several complexity problems for such chain partitions. Analogous antichain problems are related to the  $k$ -machine unit execution time problem and seem to be much harder.

As a special case we deal with partitions of Boolean lattices into chains and antichains. We establish, for fixed  $k$  and  $n$  sufficiently large, the minimum number of chains of size at most  $k$  into which the Boolean lattice  $2^n$  can be partitioned. This leads to a solution of a conjecture by Griggs. An analogous problem of partition of  $2^n$  into antichains is considered too.

*Christoph Meinel*

### Möbiusfunction and communication complexity

The Graph Accessibility Problem ( $GAP_n$ ) plays a key role in complexity theory as a paradigmatic problem for nondeterministic log-space computations. In order to prove lower bounds for the communication complexity of ( $GAP_n$ ) the rank of certain communication matrices has to be estimated. Due to an approach of Lovasz et al. we construct a finite lattice  $\mathcal{L}_{GAP_n}$  whose join problem ("decide for two given elements  $x, y \in \mathcal{L}_{GAP_n}$  whether  $x \vee y = 1$ ") is computational equivalent to  $GAP_n$ . In order to compute the rank of the corresponding join matrix it suffices to determine the number of Möbiuselements  $m_{GAP_n}$  of  $\mathcal{L}_{GAP_n}$ .

*Jutta Mitas*

### **The jump number problem of interval orders is NP-complete**

Although the jump number problem for ordered sets is NP-complete in general, there are some special classes of ordered sets for which polynomial time algorithms are known.

Here we transform the problem for interval orders into a subgraph packing problem. This problem contains as special case the triangle packing problem which is NP-complete. Therefore we are able to develop a NP-completeness proof for the jump number problem for interval orders.

In addition, we describe a  $3/2$ -approximation algorithm for this problem.

*Rolf H. Möhring*

### **On the interplay between interval dimension and dimension**

In the first part (jointly with S. Felsner and M. Habib) we investigate a transformation  $P \rightarrow Q$  between partial orders  $P, Q$  that transforms the interval dimension of  $P$  to the dimension of  $Q$ , i.e.  $\text{idim}(P) = \text{dim}(Q)$ . Such a construction has been shown before in the context of Ferrer's dimension (Bouchet, Cogis). Our construction results in the same partial order  $Q$ , but has the advantage of

- (1) being purely order-theoretic,
- (2) providing a geometric interpretation of interval dimension similar to that of Ore for dimension, and
- (3) revealing several connections to other order-theoretic results.

For instance, the transformation  $P \rightarrow Q$  can be seen as an inverse of the well-known split operation, it provides a theoretical background for the influence of edge subdivision on dimension (e.g. the results of Spinrad), and it is invariant under changes of  $P$  that do not alter its comparability graph, thus providing another proof of the comparability invariance of interval dimension (in the finite case).

In the second part (jointly with M. Habib and D. Kelly) we give a simple, purely geometric proof of the comparability invariance of dimension and interval dimension.

*J. Nešetřil*

### **Boolean Dimension**

Boolean representation and the related notion of Boolean dimension presents a convenient tool for a concise representation of a given poset. We give examples and derive bounds which show the striking efficiency when compared with the (Dushnik-Miller) dimension.

*Richard Nowakowski*

### **Pagenumber, Posets and Planarity**

The pagenumber of a graph is the least number of pages needed to embed the graph so that the vertices are on the spine and the edges are on pages (infinite half planes all intersecting at the spine). The edges cannot cross the spine and each page is planar. For a poset the vertices form a linear extension and the edges are from the diagram.

**Theorem 1**  $pn(G) = 1 \iff G$  is outerplanar;  
 $pn(P) = 1 \iff$  the covering graph of  $P$  is a forest.  
 $pn(G) = 2 \iff G$  is subhamiltonian.

- Questions: (i) Characterize those  $P$  with  $pn(P) = 2$ .  
(ii) Does there exist  $k$  such that every planar poset can be embedded in  $k$  pages ( $k \geq 5$  if it exists)?  
(iii) Same for planar lattices ( $k \geq 3$  if it exists).  
(iv) Same for height 1, planar posets ( $k \geq 3$  if it exists).

*Maurice Pouzet*

### **A projection property for ordered sets and graphs**

SMALL CHANGES - BIG EFFECTS, or a strengthening of the Arrow impossibility theorem.

Oliver Pretzel

### Cycle lengths, graph orientations and matroids

Let  $G = (V, E)$  be a finite graph and give each edge a reference direction  $t(e) \xrightarrow{c} h(e)$ . A circuit is a sum  $K = \sum a_i e_i$ ,  $e_i \in E$ ,  $a_i \in \mathbb{Z}$ , such that  $\sum a_i h(e_i) - \sum a_i t(e_i) = 0$ . The length of  $K = \sum |a_i|$ . An orientation  $R$  is a map  $E \rightarrow \{\pm 1\}$ . The flow-difference  $R \cdot K$  is  $\sum a_i R(e_i)$ .

Old Theorem (Pretzel): The orientation  $S$  can be obtained from  $R$  by "pushing down" iff  $R \cdot K = S \cdot K$  for all circuits  $K$ .

Theorem (Youngs+Pretzel 1990): Given a function  $f$  from circuits to  $\mathbb{Z}$ , there exists an orientation  $R$  of  $G$ , s.t.  $f(K) = R \cdot K$  for all circuits iff: (1)  $f$  is linear (2)  $f(K) \equiv |K| \pmod{2}$  (3)  $|f(K)| \leq |K|$  for all  $K$ .

Theorem (Woodall; Youngs+Pretzel 1991): If  $G$  is 3-edge-connected, then the length function on circuits determines the edge-circuit incidence structure and hence the graph matroid.

Corollary: If  $G$  is 3-connected, the length function on circuits determines  $G$  up to isomorphism.

Klaus Reuter

### The linear dimension of lattices, orders and contexts

We introduce the concept of the linear dimension of an ordered set  $P$  as follows: The linear dimension of an ordered set  $P$  with respect to a field  $K$ ,  $\text{ldim}_K P$  is the minimal dimension of a vectorspace  $V$  over  $K$  such that  $P$  can be order preserving embedded into the lattice  $\mathcal{U}(V)$  of linear subspaces of  $V$ .

For a relation  $I \subseteq G \times M$  we define  $\text{ldim}_K(G, M, I)$  to be the minimal dimension of a vectorspace  $V$  over  $K$  such that  $(G, M, I)$  can be embedded into  $(V, V^*, \perp)$ .

If  $|K|$  is large enough, then these concepts are related as follows:

$$\begin{aligned} \text{ldim}_K \mathfrak{B}(G, M, I) &= \text{ldim}_K(G, M, I) \\ \text{ldim}_K P &= \text{ldim}_K(P, P, \leq), \end{aligned}$$

where  $\mathfrak{B}(G, M, I)$  denotes the concept lattice of  $(G, M, I)$ .

We give several basic results on this new parameter and describe connections to the multilinear Algebra method of Lovász, to the Nešetřil-Pultr dimension of graphs and to the jump number of ordered sets.

*Ivan Rival*

### Order, invariants and surfaces

An  $n$ -element connected covering graph has many orientations (usually at least  $2^{n/2}$ ). As a consequence it is not surprising that the search for a nontrivial orientation invariant has not been easy.

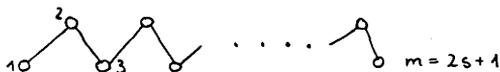
**THEOREM** [Ewacha, Li, Rival (1991)]: The order genus of an upward drawing = the graph genus of its covering graph.

**COROLLARY:** Order genus is an orientation invariant.

*Aleksander Rutkowski*

**On order-preserving selfmappings of an ordered set. How many of them are there?**

Let  $F_m$  be a fence of the following form



**Theorem.** a) There are exactly  $(m+1)2^{m-2} - 2(m-1)\binom{m-2}{\lfloor(m-1)/2\rfloor}$  strictly increasing (i.e. order- and level-preserving) selfmappings of  $F_m$ .

b) There are exactly  $b_m = A - (B+C+D)$  order-preserving selfmappings of  $F_m$ , where

$$\begin{aligned}
 A &= 2(s+1) \sum_{r=0}^s \binom{r+s}{2r} 4^r = \frac{m+1}{2\sqrt{2}} [(1+\sqrt{2})^m - (1-\sqrt{2})^m] \\
 &= (m+1) \sum_{i=0}^s \binom{m}{2i+1} 2^i \\
 B &= 2s \sum_{r=1}^s \binom{r+s}{r} \binom{s-1}{r-1} \\
 C &= 4s \sum_{r=0}^{s-1} \binom{r+s}{r} \binom{s-1}{r} \\
 D &= \sum_{r=0}^{s-1} (2r+1) \binom{r+s-1}{r} \binom{s-1}{r}
 \end{aligned}$$

*Selma Strahinger*

### Dimensionality of ordinal structures

$(S, (\leq_n)_{n \in \mathbb{N}})$  is called an *ordinal structure* if  $S$  is a set and  $(\leq_n)_{n \in \mathbb{N}}$  a family of quasi-orders (reflexive, transitive relations) on  $S$ . Ordinal structures are used as models for ordinal data. Since interpretations of data are always based on concepts and their relations, we assign to each ordinal structure a canonical conceptual structure, a so-called concept lattice. Now, the following question is basic:

What is the minimal number of quasi-orders on  $S$  which determines the same conceptual structure as the concept lattice of a given ordinal structure  $(S, (\leq_n)_{n \in \mathbb{N}})$ . Answers to this question are presented.

*Maciej M. Sysło*

### The jump number problem on interval orders

In 1981, I. Rival proved that the greedy algorithm solves the jump number problem on  $N$ -free posets. If  $P$  is not  $N$ -free then the behaviour of the greedy algorithm depends on the number of  $N$ 's in the diagram of  $P$  and their distribution.

The author has made a successful use of arc-diagrams of posets in studying non  $N$ -free posets with respect to their jump number. Two new types of greedy chains have been introduced which then led to a new type of linear extensions, called semi-strongly greedy (ss-g, for short). When the number of dummy arcs in arc-diagrams is bounded then an optimal ss-g linear extension can be generated in polynomial time (the number of dummies bounds from the above the number of  $N$ 's in the diagram).

For interval posets, we completely characterize their arc-diagrams and show that every ss-g linear extension contains at most 50% more jumps than an optimal one.

*Eberhard Triesch*

### Searching for acyclic orientations of graphs

Suppose  $G = (V, E)$  is a graph (finite, undirected, without loops or multiple edges). We consider the following search problem:

The search domain is the set of acyclic orientations of  $G$  and for each edge  $e \in E$ , we

may test whether  $e = uv$  is directed from  $u$  to  $v$  or from  $v$  to  $u$  in the unknown acyclic orientation. The worst case complexity  $c(G)$  of this search problem was studied. We showed that, for chordal graphs,  $c(G) = |E|$  if and only if  $G$  does not contain a  $K_4$  nor a path  $P_4$ , all of whose points are joined to one vertex. Furthermore, all covering graphs of partial orders satisfy  $c(G) = |E|$ .

Main Theorem: For all  $t, s \in \mathbb{N}$  there exists some graph  $G$  with girth at least  $s$  and  $c(G) \leq |E| - t$ .

Hence, the property " $c(G) = |E|$ " cannot be characterized by a finite list of forbidden subgraphs in general.

*William T. Trotter*

### Interval orders and shift graphs

We discuss four closely related combinatorial problems:

1. What is the least integer  $t = f_1(n)$  so that the subset lattice  $2^n$  has  $n$  antichains? (Dedekind's problem. See ORDER's problem list.)
2. What is the dimension  $f_2(n)$  of the set of all 1-element and 2-element subsets of  $\{1, 2, \dots, n\}$  ordered by inclusion?
3. What is the maximum value  $f_3(n)$  of the dimension of an interval order of height  $n$ ?
4. What is the chromatic number  $f_4(n)$  of the double shift graph?

It is well known that  $f_1(n) = f_4(n) = \log_2 \log_2 n + (1/2 + o(1)) \log_2 \log_2 \log_2 n$ . It is easy to see that  $f_2(n) \geq f_1(n)$  and  $f_3(n) \geq n$ . Sperner's work (1972) shows an upper bound for  $f_2(n)$ . Culminating 15 years of work since Rabinovitch first established the existence of  $f_3(n)$ , Z. Füredi, P. Hajnal, V. Rödl and I have just shown an asymptotically tight upper bound on  $f_3(n)$  so that all four functions are asymptotically equal to  $\log_2 \log_2 n + (1/2 + o(1)) \log_2 \log_2 \log_2 n$ .

*Zsolt Tuza*

### Coloring problems on comparability graphs and on their complements

We investigate the Precoloring Extension problem (PrExt), introduced recently in our joint paper with M. Biró and M. Huijter. Suppose that some vertices of a graph

$G = (V, E)$  are assigned to some colors from the set  $\{1, \dots, k\}$ . Can this “precoloring” of  $G$  be extended to a proper  $k$ -coloring  $f : V \rightarrow \{1, \dots, k\}$  of the entire graph? (In a proper coloring, adjacent vertices must have distinct colors.) We survey results and open problems concerning the algorithmic complexity of PrExt on some graph classes, with emphasis on those related to partial orders.

Originally, we introduced PrExt in order to solve a practical problem in scheduling, but it turned out that the extendibility of colorings is closely related to many concepts in various other areas as well, including VLSI, network flows, partial Latin squares, the bipartite matching problem, etc.

*Frank Vogt*

### Subgroup lattices of finite Abelian groups: Structure and cardinality

Subgroup lattices of finite Abelian groups can be decomposed using tolerance relations. It turns out that, considering a certain tolerance relation, the blocks of this relation are subgroup lattices of elementary Abelian groups. The factor lattice, the so called *skeleton*, is a subgroup lattice of a “smaller” finite Abelian group. This recursive structure allows to determine the number of subgroups using a counting formula which holds on finite lattices  $L$  and involves the Möbius function of the factor lattice of  $L$  by a tolerance relation.

*Douglas B. West*

### Snevily’s results on Chvátal’s Conjecture

Let  $I$  denote an order ideal in the lattice of subsets of  $[n]$ , and let  $S_x(I)$  denote the set of elements in  $I$  that contain  $x$ . Chvátal [1974] conjectured that the maximum size of a pairwise intersecting family of elements in  $I$  equals  $\max_{x \in [n]} |S_x(I)|$ , for any  $I$ . We present Snevily’s proof of this for ideals  $I$  having an element  $x$  such that  $A - a \cup x \in I$  whenever  $a \in A \in I$ . We also mention Snevily’s new proof of the conjecture in the case where maximal elements of  $I$  pairwise have at most one common element, which was originally proved by Stein [1983].

Rudolf Wille

### Coordinatization of ordinal structures

$(S, \leq_0, \leq_1, \dots, \leq_n)$  is called an *ordinal structure* if  $\leq_0, \leq_1, \dots, \leq_n$  are quasi-orders on the set  $S$ . Ordinal structures have been used to analyse compositional dependencies on the basis of the following axioms (cf. R. Wille, U. Wille: On the controversy over Huntington's equations, Math. Social Sciences, to appear):

- (A<sub>0</sub>)  $x \leq_i y$  for all  $i \in \{1, \dots, n\} \Rightarrow x_0 \leq y_0$ ,  
(A<sub>i</sub>)  $x \leq_0 y$  and  $y \leq_j x$  for all  $j \in \{1, \dots, n\} \setminus \{i\} \Rightarrow x \leq_i y$  ( $i = 1, \dots, n$ ),  
(P<sub>ij</sub>)  $\forall x, z \exists y : x \Theta_{ij} y \Psi_{ij} z$  ( $i, j \in \{0, \dots, n\}$  with  $i \neq j$ )  
where  $\Theta_i := \leq_i \cap \geq_i$  and  $\Psi_{ij} := \bigcap_{k \neq i, j} \Theta_k$ .

**Representation Theorem:** For an ordinal structure  $(S, \leq_0, \leq_1, \dots, \leq_n)$  with  $n \geq 3$  satisfying  $(A_0), (A_1), \dots, (A_n), (P_{ij})$  for  $i, j \in \{0, \dots, n\}$  with  $i \neq j$ , and  $\bigcap_{i=1}^n \Theta_i = id_s$ , there exists an ordered abelian group  $(A, +, \leq)$  and a bijection  $\lambda : S \rightarrow A^n$  such that  $x \leq_i y \iff \pi_i \lambda x \leq \pi_i \lambda y$  for  $i = 1, \dots, n$  and  $x \leq_0 y \iff \sum_{i=1}^n \pi_i \lambda x \leq \sum_{i=1}^n \pi_i \lambda y$ . (In case  $n = 2$ , one still gets an ordered loop.)

Questions of uniqueness and meaningfulness in the sense of measurement theory can also be answered.

Nejib Zaguia

### Adding or removing a comparability and the extension lattice

The elements of the extension lattice  $\text{Ext}(P)$  of an ordered set  $P$  are the extensions of  $P$ , that is, all ordered sets on the same underlying set as  $P$  in which  $x < y$  whenever  $x < y$  in  $P$ . Then  $\text{Ext}(P)$  is itself ordered: For  $Q, R \in \text{Ext}(P)$ ,  $Q < R$  if  $R$  itself is an extension of  $Q$ . These lattices are still largely unexplored. Let  $Q$  be an order. An interesting suborder of  $\text{Ext}(A_n)$ , where  $A_n$  is an  $n$ -element antichain, is  $\mathcal{P}(Q) = \{Q \cap L : L \text{ is a linear extension of } A_n\}$ .

**Theorem:** The suborder  $\mathcal{P}(Q)$  is cover-preserving in  $\text{Ext}(A_n)$ . As an easy consequence of Theorem 1, we have the following removal result.

**Corollary:** Let  $P$  be a finite ordered set. There is a comparability whose removal will not increase the dimension (if  $P$  is not a chain or antichain), and there is a comparability whose addition to  $P$  will not increase its dimension (if  $P$  is not a chain).

## Problem Session

Gerhard Behrendt: Given a (finite) ordered set  $(X, \leq)$ , which is rigid (i.e. has no non-trivial automorphism); does there always exist  $x \in X$  such that  $(X \setminus \{x\}, \leq)$  is rigid?

Kenneth P. Bogart: What is the order dimension of the lattice of subspaces of a  $k$ -dimensional vector space over a  $q$ -element field? In particular (how) does it depend on  $q$ ?

Walter Deuber: In the lattice  $2^n$  of subsets of  $\{1, \dots, n\}$  you want to cover the interval between ranks  $k, l$  by as few intervals as possible. It is known that  $\max\binom{n}{i}, \binom{n}{k}$  intervals suffice. What is the result for  $GF(q)^n$  instead of  $2^n$ ?

Status: Problem was around in Bielefeld for a few months. Richard Stanley did not immediately see a solution.

D. Duffus: A fibre in an ordered set is a subset which intersects each maximal antichain. It has been conjectured by Lonc and Rival [J. Combin. Th. A 44 (1987) 207-288] that the smallest fibre in the Boolean lattice  $2^n$  is the set of subsets and supersets of an  $\lceil n/2 \rceil$ -set. Duffus, Sands and Winkler [SIAM J. Disc.Math. 3 (1990) 197-205] show that every fibre in  $2^n$  has size  $\Omega(5/4)^n$ . Thus minimum sized fibres in  $2^n$  have size between  $(5/4)^n$  and  $(\sqrt{2})^n$ .

Problem: Find  $c$  so that the minimum fibre size in  $2^n$  is  $\Theta(c^n)$ .

U. Faigle: Is the following statement true?

"If  $L$  is a finite distributive lattice of cardinality  $2^n$ , then  $L$  is a Boolean lattice whenever the width of  $L$  is at least  $\binom{n}{\lceil n/2 \rceil}$ ." (Hereby "width" denotes the maximal size of a subset of pairwise incomparable elements.)

Jerry Griggs: Monochromatic chain partition of  $B_n$ .

$[n] := \{1, \dots, n\}$  is  $k$ -colored, where  $n_i$  elements are color  $i, 1 \leq i \leq k$ . Let  $S =$

$S(n_1, \dots, n_k)$  denote the maximum size of a  $k$ -color Sperner family for the coloring, i.e.,  $S = \max |F|$  where  $F \subseteq 2^{[n]}$  has the property that whenever  $A, B \in F$ ,  $A \subset B$ , then  $B \setminus A$  is not monochromatic. Clearly we have  $S \geq \binom{n}{\lfloor n/2 \rfloor}$ , and generally the inequality is strict.

Prove that  $B_n = (2^{[n]}, \subseteq)$  can be partitioned into  $S$  monochromatic chains, i.e., into collections  $\{A_1 \subset A_2 \subset \dots \subset A_r\}$  where  $A_r \setminus A_i$  is monochromatic.

Reference: Z. Füredi, J. Griggs, A. Odlyzko, J. Shearer "Ramsey-Sperner theory" Disc. Math. 63 (1987), 143-152.

Jerry Griggs, El Bender (Math. Dept., Univ. of California, San Diego): Chains and antichains in  $L_n(q)$ .

Duffus, Sands, and Winkler (SIAM J. Disc. Math. 3, (1990), 197-205) have shown for the Boolean lattice  $B_n = (2^{\{1, \dots, n\}}, \subseteq)$  that every red-blue coloring of the elements induces either a red maximal chain or a blue maximal antichain.

What other posets have this property? In particular, does it hold for the " $q$ -analogue" of the Boolean lattice, the subspace lattice,  $L_n(q)$ ? In general it fails for a product of chains, e.g., for  $2 \times 3$ .

M. Habib: When manipulating an order with a computer an important question remains the choice of its representations namely: Transitive closure, transitive reduction (Hasse diagram) or any acyclic digraph in between. Unfortunately these representations are not equivalent since it could cost the complexity of a transitive closure algorithm (between  $O(n^2)$  and  $O(n^3)$ ) to go from one to the another.

Fortunately, there exist some classes of orders for which they are all equivalent (see Ma und Spinrad 1990) namely: 2-dimensional, interval orders, ... We showed (Habib, Morvan, Rampon 91) that the Goralcikova and Koubek's algorithm runs linearly on other particular cases.

But it remains open the following problem: Let  $G$  be an acyclic digraph whose transitive closure is a lattice. What are the costs to go from one representation to others? (linear, quadratic, ...)

References: A. Goralcikova, V. Koubek, "A reduct and closure algorithm for graphs", Mathematical Foundations for Computer Science, (1979) 301-307.

T. H. Ma, J. Spinrad, "Transitive closure for restricted classes of partial orders", Proc. WG'90.

M. Habib, M. Morvan, J. X. Rampon, "On the calculation of transitive reduction-closure of orders", 1990, to appear in Discrete Math.

Zbigniew Lonc:

1. What is the complexity of the problem:

Instance: A sequence of  $3n$  positive integers.

Question: Is there a partition of the sequence into  $n$  3-term increasing subsequences?

This is an equivalent formulation of the problem of the existence of a partition of a 2-dimensional ordered set into antichains of size 3. It is also a special case of a problem by Möhring [1] who asked, for  $k \geq 3$  fixed, on the complexity of a problem of finding the minimum number of antichains of size at most  $k$  into which an ordered set can be partitioned.

[1] Möhring R., Problem 9.10, Graphs and Order (ed. Rival I.), Reidel, Dordrecht 1985.

2. Let  $B_n^-$  be the ordered set obtained from the Boolean lattice with  $n$  atoms by deleting both the greatest and the least elements. Define  $f(n)$  to be the minimum number of almost equal (i.e. equal within 1) antichains into which  $B_n^-$  can be partitioned. Problem: Find the asymptotic behavior of  $f(n)$ . It is known (see [1]) that  $c_1 n \leq f(n) \leq c_2 n^2$ , for some constants  $c_1$  and  $c_2$ .

[1] Lonc Z., Partitions of large Boolean lattices, Preprint Nr. 31, Instytut Matematyki, Politechnika Warszawska, Warsaw 1990.

R.H. Möhring: A *semi-order* is an order that does not contain  $2 + 2$  and  $1 + 3$  as induced suborders.



The *semi-order dimension*  $\text{sdim}(P)$  of an order  $P$  is the smallest number of semi-orders whose intersection is  $P$ .

Problem: Is semi-order dimension a comparability invariant, i.e., if  $P$  and  $Q$  have the same comparability graph, does this imply  $\text{sdim}(P) = \text{sdim}(Q)$ ?

J. Nešetřil: Poset  $P$  is *trivial* if it is a chain. Poset  $P$  is *asymmetric* if  $\text{Aut}(P) = \{\text{id}\}$ . Poset  $P$  is *minimal asymmetric* if it is nontrivial, asymmetric while no nontrivial subposet is asymmetric. Are there only finitely many minimal asymmetric finite

posets? Does every nontrivial infinite asymmetric poset contain a finite non trivial asymmetric poset?

R. Nowakowski: Let  $P$  be an ordered set and  $L$  a distributive lattice.

- (i) Find  $\min\{|L| : P \hookrightarrow L\}$ .
- (ii) Find  $\min\{|L| : P \hookrightarrow L, \dim(P) = \dim(L)\}$ .
- (iii) Let  $\{\underline{T}_i\}_{i \in R} = \{\{L_1^i, L_2^i, \dots, L_d^i\} | i \in R\}$  be the set of realizers for  $P$  where each  $L_j^i$  is a linear extension of  $P$ . Let  $f_i(P) \hookrightarrow \prod_{j=1}^d L_j^i$  be the canonical embedding of  $P$  into  $\prod_{j=1}^d L_j^i$ . Find the realizer  $\underline{T}_i$  which achieves the minimum for  $\min_{i \in R}\{|L| : f_i(P) \subseteq L \text{ and } L \text{ is a cover-preserving distributive lattice of } \prod_{j=1}^d L_j^i\}$ .

Werner Poguntke: The following is motivated by routing problems in communication networks.

Given a graph  $G = (V, E)$ , for each  $v \in V$ , orient a subset  $E' \subseteq E$  where an edge may be oriented in both directions. The resulting digraphs  $G(\rightarrow v)$  should satisfy the following:

- $v$  has outdegree 0;
- each  $w \neq v$  has outdegree 2;
- no cycles exist except 2-cycles on single edges.

The problem is to find a family  $G(\rightarrow v), v \in V$ , of such digraphs such that several criteria are met (e.g., high reliability).

Question: If the digraphs are such that there is even load distribution (in an appropriate model), does  $G$  have to admit a non-trivial automorphism?

Maurice Pouzet: Let  $G$  be an uncountable graph with no infinite independent set. Is it possible to make it a comparability graph by deleting some edges, without creating any infinite independent set? The conclusion holds if  $G$  is countable. There is an oriented version too, for which the countable case is completely settled. (This is not published yet.)

1.104891746 is the number of posets, up to an isomorphism, on 12 vertices. This result has been obtained by Nik Lygeros in Lyon 9/30/1991. The number  $P_n$  of posets, up to 11 vertices ( $P_{11} = 46749427$ ) has been computed by Culberson and Rawhins (order, vol 7, n°4, 1990-91)). Numerical facts collected so far support the unimodality of  $P_n(k)$  (number of posets on  $n$  vertices and the  $k$  comparabilities), the *Erné conjecture*  $P_{n-1} \cdot P_{n+1} \geq P_n^2$ , and that  $\max_k P_n(k)$  is attained for  $k_n \leq n(n-1)/4$ , e.g. Lygeros got  $\max P_{12}(k) = 76042383$  at  $k = 31$ . Question: How behave the difference  $\frac{n(n-1)}{4} - k_n$ ?

**Oliver Pretzel: Adám's Conjecture.**

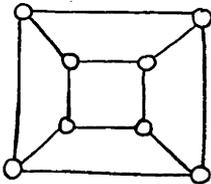
This conjecture is now 25 years old and somebody should settle it.

Adám proved that if a simple graph has an orientation with  $n$  monotonic cycles then there exist  $\leq n$  edges whose reversal produces an acyclic orientation. His conjecture is that there exists 1 edge whose reversal reduces the number of monotone cycles. Is this true?

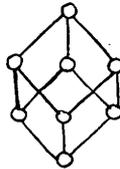
A. Adam: Bemerkungen zum graphentheoretischen Satze von I. Fidrich. Acta Math. Acad. Hung. 16 (1965), p. 9-11.

**Comment from Nešetřil:** For non simple directed graphs disproved independently by C. Thomassen (1987) and E. Grinberg (Riga) (1977, published posthumously 1988). Adám's theorem follows from a result of Dambit and Grinberg (1968) that the reversal and deletion are equivalent procedures for destroying cycles in directed graphs.

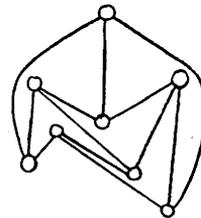
**Ivan Rival:** Does every triangle-free planar graph have a planar upright drawing? It is well-known that any triangle-free, planar graph is three-colourable and, therefore, it is a covering graph. But, does this covering graph always have a planar orientation, that is, a planar diagram - a planar upright drawing?



A triangle-free planar graph



A nonplanar orientation



A planar orientation

Tom Trotter:

1. Is the following question NP-complete?

Instance: A finite  $n$ -element poset  $(X, P)$ .

Question:  $\dim(X, P) < \text{width}(X, P)$ ?

2. Let  $n$  be a positive integer. Does there exist a listing  $S_1, S_2, \dots, S_{2^n}$  of the subsets of  $\{1, \dots, n\}$  so that

(1) for each  $i = 1, 2, \dots, 2^{n-1}$ , either

(a)  $S_i \subset S_{i+1}$ ,  $S_{i+1} = S_i \cup \{x\}$  or

(b)  $S_{i+1} \subset S_i$ ,  $S_i = S_{i+1} \cup \{x\}$ , and

(2) If  $1 \leq i < j \leq 2^n$  and  $S_j \subset S_i$ , then  $j = i + 1$ .

(with Stefan Felsner)

3. Are there infinitely many finite posets  $(X, P)$  satisfying:

a.  $(X, P)$  is an interval order,

b.  $\dim(X, P) = 4$ ,

c.  $\dim(X - \{x\}, P(X - \{x\})) = 3 \quad \forall x \in X$ .

Zs. Tuza: For  $n > k \geq 2$ , denote by

$K_k^n$  the collection of all  $k$ -subsets of an  $n$ -set

(= the complete  $k$ -uniform hypergraph of order  $n$ );

$T(n, k+1, k) = \max\{|\mathcal{H}| : K_k^{k+1} \not\subset \mathcal{H} \subset K_k^n\}$

(= the Turán number for  $K_k^{k+1}$ );

and, assuming  $\mathcal{H} \subseteq K_k^n$ , the *shadow*  $\partial\mathcal{H}$  of  $\mathcal{H}$  is defined as

$$\partial\mathcal{H} = \{F \in K_{k-1}^n : F \subset H \text{ for some } H \in \mathcal{H}\}.$$

Conjecture: If  $K_k^{k+1} \not\subset \mathcal{H} \subset K_k^n$ , then

$$\frac{|\partial\mathcal{H}|}{\binom{n}{k-1}} \geq \frac{|\mathcal{H}|}{T(n, k+1, k)}$$

The inequality is valid for  $k = 2$ . If true, it would extend a LYM-type theorem on hypergraphs satisfying the Helly property. Reference: Zs. Tuza, Helly property in finite set systems, J. Combin. Theory A, in print.

Douglas West:

1. Two chain decompositions of a poset  $P$  are *orthogonal* if every pair of elements appears on distinct chains in at least one of the decompositions. What is the maximum number of pairwise orthogonal decompositions of the Boolean algebra  $B_n$  into  $\binom{n}{\lfloor n/2 \rfloor}$  chains? Kleitman & Shearer constructed a family of size 2 and observed that  $\lceil (n+1)/2 \rceil$  is an easy upper bound.

Reference: Daniel J. Kleitman and James Shearer, Probabilities of independent choices being ordered, *Studies in Appl. Math.* 60(1979), 271-276.

2. Obtain a fast algorithm for checking whether an arbitrary binary relation  $R \subseteq V \times V$  is the intersection of two Ferrers relations whose union is all pairs. The motivation is that these are precisely the relations  $R$  (called *interval digraphs*) representable by assigning each element  $v \in V$  a pair of real intervals  $(S_v, T_v)$  such that  $(u, v) \in R$  if and only if  $S_u \cap T_v \neq \emptyset$ . If the condition on union is dropped, then the algorithm of Cogis checks for Ferrers dimension 2.

Reference: O. Cogis, A characterization of digraphs with Ferrers dimension 2. *Rap. Rech.* 19, G.R. CNRS 22 Paris (1979).

M. Sen, S. Das, A.B. Roy, D.B. West, Interval digraphs: an analogue of interval graphs, *J. Graph Theory* 13 (1989), 189-202.

Rudolf Wille:

1. How many essentially different Hasse diagrams exist for the Boolean lattice  $B_4$  consisting of edges in only 4 directions?
2. Let  $(A_t, b_t)_{t \in T}$  be a family for which  $A_t \subseteq \{1, 2, \dots, n\}$  and  $b_t \in \{1, \dots, n\} \setminus A_t$ . How can one construct a closure system  $\mathcal{H}$  on  $\{1, 2, \dots, n\}$  of minimal length so that for each  $t \in T$  there exists an  $H_t \in \mathcal{H}$  satisfying  $A_t \subseteq H_t$  but  $b_t \notin H_t$ ?

Nejib Zaguia: Given two ordered sets  $P$  and  $Q$  on the same underlying set. Say that  $P$  and  $Q$  are *perpendicular* if the only common order-preserving maps for  $P$  and  $Q$  are the trivial ones, (that is the identity and the constant maps). There are orders which do not have a perpendicular (antichain, four-element cycle, ...).

Is it true that every ordered set with no autonomous set has a perpendicular?

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