MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Convergence Structures in Topology and Analysis

27.10. bis 2.11.1991

The international conference on "Convergence Structures in Topology and Analysis" was held from October 10 to November 2, 1991 in Oberwolfach, Germany. The organizing committee consisted of E. Binz (Mannheim), H. Herrlich (Bremen) and G. Preuß (Berlin). The meeting was opened by G. Preuß who welcomed 24 participants from 7 countries. The last conference of this kind was realized in Oberwolfach in 1987. In contrast to the situation at that time the number of participants had to be reduced because simultaneously a conference on statistics took place in Oberwolfach. On account of this reason there was much more time for fruitful discussions besides the presented lectures. The special flavour of this meeting resulted from the fact that scientists working in so many different areas of convergence structures came together. Thus, e.g. pure topologists joined workers in the field of applications in analysis.

The talks and discussions dealt mainly with the following:

I) Filter convergence (Behling, Bentley, Herrlich, Kent, Lowen-Colebunders, Preuß, Weck-Schwarz, Schwarz)

II) Sequential convergence (Börger, Koutník)

III) Convergence and topological spaces (Giuli,Hušek, Richter)

IV) Convergence and completeness problems (Butzmann, Gähler, Kent, Koutník)

V) Convergence and order (Erné, Kent)

VI) Convergence and algebraic structures (Frič, Ivanov, Koutník, Porst)

VII) Applications of convergence structures in Analysis (Börger, Frölicher, Pumplün)

It turned out again that for several kinds of problems convergence structures are more suitable than topological spaces.

Besides other ones the following results are very remarkable:

1) a) It is known, that the intersection of two reflective subcategories of a given category may fail to be reflective. Within the realm of Cauchy space a natural (i.e. a non artificial) example of this fact can be found, namely the category of compact Hausdorff Cauchy spaces is the intersection of the category of totally bounded Hausdorff Cauchy spaces with the category of complete Hausdorff Cauchy spaces (Bentley/Herrlich).

   b) Closely related to this observation is the problem whether there exists a compactification γX for every topological space X such that every continuous mapping from X to a compact space Y can be extended continuously to γX. Such a compactification does not exist, if X contains a sequence of closed noncompact subsets X_n with $X_n \cap X_m$ compact for $n \neq m$ [e.g. if X is an infinite discrete space, or if $\beta X \backslash X$ is infinite] (Hušek).

2) There are $exp(exp(\omega))$ strict $\mathcal{L}_0^*$–ring completions of rationals (Frič).

3) a) The hereditary (product–stable, pullback-stable) quotient maps of any non-trivial, well-fibred monotopological construct $\mathcal{A}$ can be characterized by being quotients in the extensional topological hull (cartesian closed topological hull, topological universe hull) of $\mathcal{A}$ provided these hulls exist (Weck–Schwarz/Schwarz).

   b) The cartesian closed topological hull of the category of completely regular filter–merotopic spaces can be described by means of suitable axioms (Bentley/Lowen–Colebunders).

   c) It is an open problem, whether the topological universe hull of the category of Cauchy spaces can be described by means of suitable axioms (Preuß).

4) a) Mackey convergence plays a crucial role in the theory of convenient vector spaces which are used for getting a calculus with good closedness properties with respect to function spaces [in particular: cartesian closedness in the case of smooth maps] (Frölicher).

   b) If one calls a finitely additive continuous map $\mu : B \rightarrow E$ from a sequential Boolean algebra to a sequentially convex space E a measure, then universal measures exist, which give rise to the definition of an integral generalizing the usual Lebesgue integral (Börger).

   c) "Abstract" (super) convex structures arise quite naturally as the algebraic component of the theory of base–normed vector spaces (resp. base–normed real Banach spaces) (Pumplün).

The (female) chairman of the last session, E. Lowen-Colebunders, closed the meeting and expressed the gratitude of the participants for living together in such a nice and creative atmosphere. Finally, G. Preuß, who spoke in the name of the organizers, thanked the participants for coming and making this conference a success. He expressed the hope of all participants that such a meeting should take place again in Oberwolfach in 1995.

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ABSTRACTS

A. Behling:

Topological universes of semiuniform limit spaces

The category of uniform spaces fails to have two nice convenience properties – extensionability and cartesian closedness. A topological category having both properties is called a topological universe. So one is looking for topological universe extensions of Unif. A natural setting is the common generalization of the category SUnif of semiuniform spaces in the sense of Čech which is extensionable but not cartesian closed and of the category ULim of uniform limit spaces in the sense of Wyler which is cartesian closed but not extensionable. The resulting category SULim [for def. s. Wyler (74)] of semiuniform limit spaces turns out to be a topological universe. Building the final hull of Unif in SULim one yields the category UGLim of uniformly generated semiuniform limit spaces, which is also a topological universe, because Unif is a bireflective subcategory of SULim. Consider the initial hull in UGSULim of all objects of the form $[X, Y^{\#}]_{\underline{UGSULim}}$ with $X, Y \in$ Ob Unif, which is the topological universe hull of Unif. The objects of this hull are the saturated UGSULim-spaces (this definition is based on a similar one in a corrected version of a paper about this hull by Adamek/Reiterman [87]). Main parts of their proof of the construction in a different category can be carried over. Only their assumption that SUnif is the extensionable hull of Unif – which is not true – has to be replaced by correct arguments.

H.L. Bentley/ E. Lowen-Colebunders:

The cartesian closed topological hull of the category of completely regular filterspaces

Complete regularity is one of the most interesting notions in topology, since it is closely related to the real number system. In order to develop a theory about completely regular extensions of topological spaces, Bentley, Herrlich and Ori introduced a notion of complete regularity for merotopic spaces. In the setting of merotopic spaces they proved that a space is a subspace of some completely regular topological space if and only if it is a completely regular filterspace.

We show that completely regular filterspaces fit into the following diagram

$$\begin{array}{ll}
\underline{\text{Fil}} & \text{Here } \underline{\text{Fil}} \text{ is the cat. of filterspaces} \\
| & \\
\underline{\text{Chy}} & \underline{\text{Chy}} \text{ is the cat. of Cauchy spaces} \\
| & \\
\underline{\text{A}} & \underline{\text{A}} \text{ are exactly the subspaces of the spaces of Bourdaud } (\omega\text{-closed} \\
& \text{domained, } \omega\text{-regular, pseudotopological)} \\
| & \\
\underline{\text{CRegFil}} & \underline{\text{CRegFil}} \text{ is the cat. of completely regular filterspaces} \\
| & \\
\underline{\text{uChy}} & \underline{\text{uChy}} \text{ is the cat. of uniformizable Cauchy spaces}
\end{array}$$

We show that A is the smallest finally dense cartesian closed extension of CRegFil , i.e. it is the cartesian closed topological hull.

H.L. Bentley/ H. Herrlich:

Compactness = Completeness ∩ Total Boundedness
- A Natural Example of a Non-reflective Intersection of Epireflective Subcategories
It is known that the intersection of two reflective subcategories of a given category may fail to be reflective, but examples which have been given in which this pathology occurs are rather artificial. We give an example in the nice setting of the topological category of Cauchy spaces and Cauchy continuous maps: The full subcategory of complete Hausdorff Cauchy spaces is an epireflective subcategory of the Hausdorff Cauchy spaces and the full subcategory of totally bounded Hausdorff Cauchy spaces is bireflective, but their intersection, the subcategory of compact Hausdorff Cauchy spaces, is not reflective (is not even almost reflective).

R. Börger:

Measures on Sequential Boolean Algebras
For a Boolean σ-algebra B, σ-additivity of a map $B \to \mathbb{R}$ can be expressed as "finite additivity plus a continuity condition". More generally, for a sequential Boolean algebra B and a sequentially convex space E (i.e. a sequential $\mathbb{R}$-vector space with some conditions) we call a finitely additive continuous $\mu : B \to E$ a measure. Then universal measures exist, and they give rise to a defintion of an integral, which generalizes the usual Lebesgue integral. The universal measure is also a universal multiplicative measure, and this yields a Fubini-type theorem. Moreover, the target space of the universal measure can be described as the space L^∞ with some coarser topology, which has a nice concrete description.

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H.-P. Butzmann:

Categorical aspects of completions

Whereas there is a canonical way to construct the completion of a topological algebra, e.g. a topological group or a topological vector space, no such procedure is known for convergence algebras. Therefore, in the last 20 years, many different theories have been developed in order to prove the existence of completions, e.g. the (regular) completion of a convergence vector space or a (commutative) convergence group. Although they all followed more or less the same patterns, no common way was known.

The concept of a topological category makes it possible to put them all into a common framework. The required completion functor is then represented as the composition of two functors. The existence of one of them follows from the general theory, whereas that of the other one requires the existence of the "fixed point" of a special functor which can be proved under rather general assumptions, in particular in each of the above-named cases.

M. Erné:

Scott Convergence, Core Spaces and Distance Functions

A core space (alias C-space or locally supercompact space) is one in which every point has a neighborhood basis of (not necessarily open) cores, where the core of a point is the intersection of its neighborhoods. Alternatively, core spaces may be characterized by complete distributivity of their topologies. They arise in many algebraic, analytic, and ordertheoretical contexts. For example, the sober C-spaces are precisely the continuous posets endowed with their Scott topology, or equivalently, those up-complete posets whose Scott convergence is (pre-) topological [ME, Hoffmann, Lawson 79].

Examples of C-spaces are the Alexandrov-discrete spaces (A-spaces), the spaces with a minimal basis (B-spaces), all spaces with a linearly ordered basis,and every cube $[0,1]^I$ with the upper (=Scott) topology, but also $\mathbb{R}^n$ with this "one-sided" topology.

A further important class of C-spaces is obtained from so-called value monoids: these are pairs (A, P) consisting of a p.o. monoid with a "residuation" - such that $a - b \leq c \iff a \leq b + c$, and a "set of positives" P , that is, a dual ideal with infimum 0 and $P \subseteq \uparrow (P + P)$; the sets $a \prec = \{b \in A : a \leq b - p \quad \text{for some} \quad p \in P\}$ $(a \in A)$ form the basis of a C-topology $\mathcal{T}_P$. Every embedding f of a C-space $(X, \mathcal{T})$ in $(A, \mathcal{T}_P)$ gives rise to a distance function $d(x, y) = f(x) - f(y)$ inducing the given topology $\mathcal{T}$ (in the same way as classically metrics induce topologies). One such embedding is given by the following ingredients: take $A = \mathcal{P}X$, $P = \mathcal{F}^{co}X$ (the co-finite subsets), $f(x) = \{y \in X : x \in (\uparrow y)^0\}$. This shows that every C-space is induced in a natural way by a (non-symmetric) generalized distance function.

The category of C-spaces has finite products. More precisely, an arbitrary product of spaces is a C-space if and only if almost each factor is a supercompact space and all factors

are C-spaces (= locally supercompact). (The close analogy to the case of locally compact resp. connected spaces is not casual). Of course, arbitrary sums (=disjoint unions) of C-spaces are again C-spaces. Since every completely distributive lattice is isomorphic to some C-space topology, this provides an easy proof of the lattice-theoretical fact (due to Jakubík) that every completely distributive lattice has a unique product.decomposition into irreducible factors (corresponding to the partition into components.)
Problem: Is the category of C-spaces cartesian closed?

R. Frič:

$\mathcal{L}$-Rings: There are exp (exp ω) strict $\mathcal{L}_0^*$-ring completions of rationals
An $\mathcal{L}_0^*$–ring convergence on R (the real numbers) is a convergence $L \subset R^n \times R$ of sequences which has unique limits and satisfies the Urysohn axiom. Denote by $\mathbb{M}$ the usual metric convergence on R and by $\mathbb{M}_Q$ its restriction to Q (the rational numbers). The pair $(R, \mathbb{L})$ is said to be a strict $\mathcal{L}_0^*$-ring precompletion of $(Q, \mathbb{M}_Q)$ if $\mathbb{L}$ is not coarser than $\mathbb{M}$, and each Cauchy sequence in Q $\mathbb{L}$-converges. The main result is stated in the title.
The proof is based on properties of the algebraic independence in the field extension R of Q . Let $B \subset [0,1]$ be an algebraic basis (maximal algebraically independent set – the degree of transcendence is exp ω). Every infinite subset of B gives rise to a precompletion and if the symmetric difference of two subsets of B is infinite, then the corresponding precompletions are different. I do not know whether any of the precompletions is $\mathcal{L}$-complete. If not, then every incomplete precompletion can be completed and the completion will yield a (ring) hyperreal number system.

A. Frölicher:

The Mackey Convergence in Analysis
In order to get a calculus with good closedness properties with respect to function spaces (in particular: cartesian closedness in the case of smooth maps) one replaces Banach spaces by the more general so-called convenient vector spaces [A.F. & A. Kriegl: Linear Spaces and Differentiation Theory, J. Wiley 1988]. Mackey convergence plays a role.
1) One can start general differentiation theory by using convergence vector spaces. The reason: convenient vector spaces can be characterized by means of Mackey convergence as certain convergence vector spaces.
2) If among the convergence vector spaces one looks for the appropriate ones one should end up with convenient vector spaces. The reason: if a calculus of smooth maps upholds two basic properties of Banach space calculus one gets a category which is equivalent to a full subcategory of that formed by the convenient vector spaces.
3) Nevertheless the many approaches to calculus based on convergence vector spaces did not lead to convenient vector spaces. The reason: one required differentiable maps to be continuous and this eliminates Mackey convergence. In fact, C.-A. Faure gave an exam-

ple of a smooth map $E \to F$ between convenient vector spaces (E even Banach and F Fréchet) which is at no point continuous with respect to Mackey convergence.

W. Gähler:

Completion Theory

The notions of Cauchy space, limit space and limit group is extended to the level of functor structures. Some interesting examples are presented, in particular one, based on fuzzy filters. On this level completions are constructed – always using Cauchy objects.
In order to include more types of completions – e.g. completions of partial algebras to total ones – and even compactifications, a more general, new, weaker concept is considered, that of extension structure.
If $\varphi : SET \to SET$ is a covariant set functor, then a $\varphi-$*extension structure* on a set X is a triple $(t, s, \sim)$ consisting of a subset t of $\varphi X \times X$, a subset s of φX and an equivlence relation $\sim$ on s such that for all $(\mathcal{M}, x) \in t$ and $\mathcal{N} \in s$ we have $\mathcal{M} \sim \mathcal{N} \Longleftrightarrow (\mathcal{N}, x) \in t$. $(t, s, \sim)$ is called *complete* if for each $\mathcal{M} \in s$ there is an $x \in X$ with $(\mathcal{M}, x) \in t$. $(t, s, \sim)$ is *separated* if t is a partial mapping.
By a canonical restriction to admissible morphisms, non-universal completions change into universal ones and can be handled in this way by categorical technics. Examples are the one-point-compactification and the Richardson compactification, which can be defined on the level of functor structures.

E. Giuli:

Closure operators and co-wellpoweredness

In 1975 H. Herrlich produced the first example of an epireflective subcategory $\underline{A}$ of $\underline{\text{Top}}$ such that there is X in $\underline{A}$ and a proper class $\{X_\alpha\}_{\alpha \in Ord}$ of pairwise non homeomorphic spaces of $\underline{A}$ with the property that there is an $\underline{A}$-epimorphism $f_\alpha : X \to X_\alpha$ for each $\alpha \in Ord$. In categorical terminology : $\underline{A}$ is non co-wellpowered.
Since then an appropriate theory of closure operators was developed that gave rise to a general procedure, explained below, to produce new and more natural examples of non co-wellpowered subcategories of $\underline{\text{Top}}$. (Recall that a closure operator of $\underline{\text{Top}}$ assigns to each subset M of any space X a subset $C_X M$ of X such that 1) $M \subset C_X M$; 2) $M \subset N \Rightarrow C_X M \subset C_X M$ and $f(C_X M) \subset C_Y(f(M))$ for each $f : X \to Y$).
Step 1. Look for an additive and non bounded closure operator C having an additional "technical" property.
Step 2. Describe the epireflective subcategory $\Delta(C) = \{X \in Top \,|\, \Delta_X = C_{X \times X} \Delta_X\}$.
Step 3. Try to construct by transfinite induction a class $\{X_\alpha\}_{\alpha \in Ord}$ of pairwise non homeomorphic spaces belonging to $\Delta(C)$ such that there is an $\Delta(C)$-epimorphism $f_\alpha : X_0 \to X_\alpha$ for each α.
Following the previous procedure it was shown that the subcategories listed below are

non co-wellpowered.

1. Spaces in which every compact subset is Hausdorff.

2. Spaces in which every subspace which is a continuous image of a Hausdorff compact space is Hausdorff.

3. (For every fixed $n \geq 2$) Spaces X such that, for each $x, y \in X$ there is a chain of open neighborhoods of x, $U_1 \subset U_2 \subset \ldots \subset U_n$ such that $\overline{U}_i \subset U_{i+1}$, $i = 1, 2, \ldots, n$ and $y \notin \overline{U}_n$.

M. Hušek:

Convergence structures in topology and analysis

PROBLEM: Does there exist a compactification γX for every topological space X such that every continuous mapping from X to a compact space Y can be extended continuously to γX ?

If Y is required to be compact Hausdorff or compact regular, then Čech-Stone-compactification βX and Wallman compactification wX solve the problem. (There are also other compactifications of such kind, e.g. generated by a four-point space, which was shown by S. Watson and A. Dow.)

We show, that for general compact spaces Y , such a compactification γX need not exist. It does not exist if X contains a sequence of closed noncompact subsets X_n with $X_n \cap X_m$ compact for $n \neq m$ (e.g. if X is an infinite discrete space, or if $\beta X - X$ is infinite.)

There are situations, when the requested compactification γX exists, namely for those topological spaces X with the finite Wallman remainder $wX - X$.

The above negative result remain true if we require all the spaces under consideration to be T_1-spaces.

A.A. Ivanov:

Problems of the theory of bitopological spaces

The talk was on problems of the theory and on its progress. This lecture was a continuation of the lecture given at the preceding meeting on Convergence Structure and its applications to Topology and Algebra. There were considered some problems partly connected with Kelly's theory but mostly with the general theory. Two questions are especially important namely applications of the theory to the theory of manifolds and the theory of groups. The theory of piece–linear manifolds for example can be considered as a part of the theory of bitopological manifolds. On the other hand there are different definitions of bitopological groups and the future will show what is the best.

D.C. Kent:

Continuity Multispaces

A continuity space $\chi = (X, d, V, P)$ (defined by Kopperman in 1988) consists of a set X, a value quantale V, a set of positves P in V, and a "generalized metric" $d : X \times X \to V$ which satisfies: (1) $d(x,x) = 0$; (2) $d(x,y) \leq d(x,z) + d(z,y)$. A continuity multispace $\xi = (X, d, V, \mathcal{P})$ is a generalization of a continuity space in which the set P of positives is replaced by a family $\mathcal{P}$ of sets of positives in V.
Every convergence space is "metrizable" relative to some continuity multispace, and every precauchy space is "metrizable" relative to some symmetric continuity multispace. For a large class of symmetric continuity spaces, we obtain a completion with a nice universal property. In addition, we show that all Cauchy spaces have a "generalized metric completion" obtained by completing an associated symmetric continuity multispace.

V. Koutník:

Completion of Abelian $\mathcal{L}_0^*$-groups

The structure $(L, \mathcal{L}, +)$ is said to be an $\mathcal{L}_0^*$-group if $(L, +)$ is an algebraic group and $\mathcal{L}$ is a single-valued maximal convergence such that the algebraic operation $+$ is sequentially continuous. J. Novák has shown that every Abelian $\mathcal{L}_0^*$-group has a completion by introducing a suitable convergence into the underlying algebraic quotient group. However, the Novák completion is the minimal completion and, e.g., for the $\mathcal{L}_0^*$-group of rational numbers it gives the real line with a much finer convergence than the usual metric one.
We try another approach by introducing a convergence into the group C of all Cauchy sequences. The group C_0 of all sequences converging to zero is then a closed subgroup of C. Hence we can take the quotient convergence in C/C_0. Thus we obtain for every Fréchet $\mathcal{L}_0^*$-group an extension which yields for the rationals (and, in fact, for every normed group) the standard metric completion.

H.-E. Porst:

Semi-uniform products and free topological groups

Based on the observation that a topological group is nothing but a group equipped with a uniformity , such that the multiplication is uniformly continuous as a map $G * G \to G$, where $- * -$ indicates the semi-uniform product of uniform spaces in the sense of Isbell [Isbell, Uniform spaces 1967], an alternative description of the free topological group $F\underline{X}$ over a completely regular space $\underline{X}$ is given.
It is shown that the right uniformity of $F\underline{X}$ can be obtained as the quotient uniformity on $F\underline{X}$ of the canonical map $M^*(\underline{X} + \underline{X}) \to F\underline{X}$, where $M^*\underline{Y}$, for some uniform space $\underline{Y}$, denotes the free semi-uniform monoid (i.e., monoid w.r.t. $- * -$) for $\underline{Y}$. Existence of

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the latter is proved; its uniformity is compared with various other uniformities canonically arising in this context. The uniform topology of $M^*\underline{Y}$ is shown to be $\coprod_n (T\underline{Y})^n, T\underline{Y}$ denoting the underlying topological space of $\underline{Y}$.
Finally, the question is asked, how to describe explicitely the uniformity of $M^*\underline{Y}$.

G. Preuß:

Improvement of Cauchy spaces

Over many years Cauchy spaces have been studied as a useful tool for constructing completions. Though the category $\underline{\text{Chy}}$ of Cauchy spaces has nice categorical properties, namely it is a cartesian closed topological category, in $\underline{\text{Chy}}$ quotient maps are neither countably productive nor hereditary as has been pointed out by Bentley, Herrlich and Lowen–Colebunders (1981). Thus, the question arises whether it is possible to find a better behaved supercategory in which $\underline{\text{Chy}}$ is nicely embedded. A very good candidate is the category of filtermerotopic spaces introduced by Katětov (1968). $\underline{\text{Fil}}$ is a topological universe, i.e. a topological category being a quasitopos in the sense of Penon. Thus, in $\underline{\text{Fil}}$ quotient maps are hereditary. Furthermore, in $\underline{\text{Fil}}$ quotient maps are productive. These properties of $\underline{\text{Fil}}$ have been proved by Bentley, Herrlich and Robertson (1976). If $\underline{\text{Prox}}$ denotes the category of proximity spaces (in the sense of Efremovič) and $\underline{T_{2w} - \text{Lim}}$ the category of weakly Hausdorff limit spaces, then one obtains the following diagram

$$\begin{tikzcd}
& \underline{Fil} & \\
& \underline{Chy} \arrow[u, "r"'] & \\
\underline{Prox} \arrow[ur, "r"] & & \underline{T_{2w} - Lim} \arrow[ul, "c"']
\end{tikzcd}$$

where r (resp. c) stands for embedding as a bireflective (resp. bicoreflective) subcategory. It can be proved that every filtermerotopic space is a quotient object (formed in $\underline{\text{Fil}}$) of some Cauchy space. Thus, $\underline{\text{Chy}}$ is finally dense in $\underline{\text{Fil}}$. Using results of Adámek, Reitermann and Schwarz (1986 and 1989), there is a least finally dense topological universe extension of $\underline{\text{Chy}}$, the so-called topological universe hull TUH ($\underline{\text{Chy}}$) of $\underline{\text{Chy}}$. In particular, $|TUH(\underline{\text{Chy}})| = \{X \in |\underline{\text{Fil}}| :$ there exists an initial source $(f_i : X \to X_i)_{i \in I}$ with $X_i \in \mathcal{K}$ for each $i \in I\}$, where $\mathcal{K}$ consists of exactly all power-objects $(K_2^*)^{K_1}$ with $K_1, K_2 \in |\underline{\text{Chy}}|$ and $*$ stands for forming the one-point extension (one-point extensions and power-objects exist in every topological universe). Thus, the following questions arise:

Question 1: $TUH(\underline{\text{Chy}}) = \underline{\text{Fil}}$?

Question 2: Does there exist a nice description of $TUH(\underline{\text{Chy}})$ by means of suitable axioms provided that the answer to question 1 is no.

D. Pumplün:

Convex sets

A convex combination (in a linear space) is represented by a sequence $(\alpha_1, \alpha_2, \cdots, \alpha_n)$, $\alpha_i \geq 0$ for $1 \leq i \leq n$, $n \in \mathbb{N}$ and $\sum_{i=1}^{n} \alpha_i = 1$. The set of all convex combinations is $\Omega_c := \{\alpha_* \,|\, \alpha_* = (\alpha_1, \cdots, \alpha_n),\ n \in \mathbb{N},\ \text{a convex combination}\}$. The set of *superconvex combinations* is $\Omega_{sc} := \{\alpha_* \,|\, \alpha_* \in \mathbb{R}_+^{\mathbb{N}}, \sum_{i=1}^{\infty} \alpha := 1\}$. A *superconvex setX* in a linear space E (with some convergence structure) is a subset $X \subset E$, such that for any $x^* \in X^{\mathbb{N}}$ and any $\alpha_* \in \Omega_{sc}$ the limit $\sum_{i=1}^{\infty} \alpha_i x^i$ exists and lies in X . Superconvex sets have been very successfully used by G. Jameson (Ordered linear spaces, LNM 141, 1970) under the name of *CS-compact sets.*

A *(super) convex space is a set* C together with a mapping $\Omega_c \times C^{\mathbb{N}} \longrightarrow C(\Omega_{sc} \times C^{\mathbb{N}} \longrightarrow C)$, which is written as $(\alpha_*, c^*) \mapsto \sum_{i=1}^{\mathbb{N}} \alpha_i c^i (\sum_{i=1}^{\infty} \alpha_i c^i)$, s.th. the following equations hold:

(C1)
$$\sum_{i=1}^{n} \delta_{ik} c^i = c^k \quad \left(\sum_{i=1}^{\infty} \delta_{ik} c^i = c^k\right)$$

for any $c^* \in C^n$, $n \in \mathbb{N}(c^* \in C^{\mathbb{N}})$.

(C2)
$$\begin{aligned} \sum_{i=1}^{n} \alpha_i \sum_{k=1}^{m} \beta_k^i c^k &= \sum_{k=1}^{m} \left(\sum_{i=1}^{n} \alpha_i \beta_k^i\right) c^k \\ \Big(\sum_{i=1}^{\infty} \alpha_i \sum_{k=1}^{\infty} \beta_k^i c^k &= \sum_{k=1}^{\infty} (\sum_{i=1}^{\infty} \alpha_i \beta_k^i) c^k\Big) \end{aligned}$$

for any $\alpha_*, \beta_*^i \in \Omega_c(\Omega_{sc}), i \in \mathbb{N}$ and any $c^* \in C^m(C^{\mathbb{N}})$, $m \in \mathbb{N}$.

(Super) convex spaces are a canonical generalization of (super) convex subsets of linear spaces. They are used in physics (colour vision, quantum mechanics), chemistry (theory of mixtures) and mathematical economy (utility spaces). It has been shown by the speaker and H. Röhrl that mathematically (super) convex spaces constitute the *algebraic component* of the *theory of base-normed real vector spaces (Banach spaces).*

G. Richter:

Axiomatizing algebraically behaved categories of Hausdorff spaces

One of the fundamental differences between topology and algebra is that continuous bijections need not be isomorphic. Therefore, and for related reasons, an arbitrary full subcategory $\underline{X}$ of the category $\underline{\text{Top}}$ of topological spaces fails badly to be equivalent to a category of algebras of some type, in general. Nevertheless there are remarkable ex-

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ceptions, some of them given by topological properties like discretness, indiscreteness, or compactness, respectively. Moreover, the product closure of every strongly rigid, small, full subcatergory $\underline{R} \subseteq$ $\underline{\mathrm{Top}}_2$, the category of Hausdorff spaces, together with the empty space is nicely algebraically behaved. Its underlying set functor is even monadic, i.e. the corresponding Eilenberg–Moore comparison functor is an isomorphism of categories.
If the latter is only a full embedding or fully faithful, i.e. in case of descent type, there exists a whole realm of examples. In fact, every right adjoint restriction of the underlying set functor of $\underline{\mathrm{Top}}$ to a full subcategory $\underline{X}$ with left adjoint F, unit η , and counit ε yields various full subcategories $\underline{D}$ of descent type between the image $F(\underline{\mathrm{Set}})$ of F and $\underline{X}$, , by looking for certain $D \in \underline{X}$ with final counit εD. Note, that every $\varepsilon FT, T$ a set, is a retraction, hence final.
The case of pointwise dense units η allows a purely categorical description of all such $\underline{D} \subseteq \underline{\mathrm{Top}}_2$, which specializes to $\underline{D} \subseteq \underline{\mathrm{Comp}}_2$, the category of compact Hausdorff spaces. This enables a new axiomatic approach to Stone-duality as well as Gelfand-dualtity.

F. Schwarz:

Cartesian Closed Categories of Uniform Limit Spaces
It is well-known that the category Unif of uniform spaces is not cartesian closed. A natural extension – introduced by Cook & Fischer (1967) for different reasons – has the same deficiency. Slightly generalizing Cook & Fischer's condition, Wyler (1974) obtained the cartesian closed category ULim of uniform limit spaces, which however, turns out to be "too big" an extension of Unif: Unif is not finally dense in ULim. We discussed several other cartesian closed topological extensions of Unif – including the smallest one – all of them contained in Wyler's category. In particular, we gave a new characterization of the cartesian closed topological hull of Unif using only $\mathbb{R}$ in its usual uniformity and the double construction of Bourdaud.

S. Weck–Schwarz/F. Schwarz:

Pullback-stable quotient maps
The pseudo–open maps of topology have been characterized as those quotient maps in the category Top of topological spaces which are hereditary, i.e. the property of being a quotient map is preserved by pullbacks along embeddings (Archangelskii 1963). On the other hand, they can be described as quotient maps, between topological spaces, in the category of pretopological spaces (Kent 1969). In a similar way, the biquotient maps (Hájek 1966, Michael 1968) have a categorical description, as well as a description in terms of convergence: They are the pullback–stable quotients in Top (Day/Kelly 1970), and are also characterized by being quotients in the category of pseudotopological spaces (Kent 1969). Another connection to category theory is given by the observation that the pretopological spaces (resp. pseudotopological spaces) form the extensional topological

hull (resp. topological universe hull) of the category Top (Herrlich 1988, Wyler 1976). We show that that these topological phenomena follow a general categorical pattern: The hereditary (product-stable, pullback-stable) quotient maps of any non-trivial, well-fibred monotopological construct $\underline{A}$ can be characterized by being quotients in the extensional topological hull (cartesian closed topological hull, topological universe hull) of $\underline{A}$, provided these hulls exist. The result has nice applications to the epireflective subcategories of the category of pretopological spaces.

Berichterstatter: G. Preuß

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