MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Singularitäten der Kontinuumsmechanik: Numerische und konstruktive Methoden zu ihrer Behandlung

17. – 23.11.1991

Die Tagung fand unter Leitung von W.L. Wendland (Stuttgart) und J.R. Whiteman (Uxbridge, U.K.) statt. 39 Vorträge gaben den 45 Teilnehmern aus 9 Ländern (unter ihnen auch etliche aus den "neuen Bundesländern") Gelegenheit zu vielen Diskussionen und wissenschaftlichem Austausch.
Besonders erfreulich waren die vielfältigen Kontakte zwischen den Teilnehmern mit recht unterschiedlichen wissenschaftlichen Interessen, den theoretisch arbeitenden Mathematikern, den Ingenieuren und Mechanikern und den numerischen Mathematikern.
Gemäß Themenstellung der Tagung wurden vielfältige Singularitätenprobleme aus der Kontinuumsmechnik behandelt. Hier sind vor allem neue Beiträge zur Bruchmechanik zu nennen, die Ausbreitungskriterien für Risse, ihr Verzweigen, ihren Zusammenhang mit fraktalen Rißgeometrien, die Ausbreitung nichtglatter Risse in anisotropen Materialien, Thermoelastizität und Rißwachstum in viskoelastischen Materialien einschließlich zeitabhängiger Rißprobleme betreffen.
Aus der Strömungsmechnik gab es Beiträge zur Berechnung von Wasserwellen mit sehr dünner Grenzschicht und Singularität, die Untersuchung von Singularitäten bei zähen Strömungen und die Berechnung von Strömungen nicht-Newtonscher Fluide mit Singularitäten. Für die Beschreibung der Ausbreitung sowohl elastischer als auch elektromagnetischer Wellen ist die Untersuchung der Singularitäten bei verallgemeinerten Sommerfeldschen Streu- und Diffraktionsproblemen von zentraler Bedeutung. Für die dabei eingesetzten Wiener-Hopf-Methoden sind einige erhebliche Neuerungen auch im Zusammenhang mit Singulärentwicklungen erzielt worden.
Unter den vielen Fragestellungen ließen sich klar einige Schwerpunkte erkennen. So werden die lokalen Darstellungen der Lösungen und ihrer Asymptotik nahe Kanten und konischen Punkten konstruktiver, und neue von der Jordanschen Normalform der Keldysh-Darstellung unabhängige Formulierungen für die Singulärentwicklungen sind in mehreren Beiträgen vorgestellt worden. Dadurch sind auch neue Berechnungsverfahren für die Singulärentwicklungen sowohl bei zwei- als auch bei dreidimensionalen Problemen möglich geworden, die zusammen mit Abschätzungen und Fehleranalysis untersucht wurden.
Etliche Beiträge waren numerischen Verfahren im Zusammenhang mit Singularitäten gewidmet, insbesondere finite-Element- und Randelement-Methoden. Hier hat das Zusammenspiel von theoretischen Resultaten mit geeigneten Modifikationen der Algorithmen zu vielen Verbesserungen geführt. Bei den finiten Elementen sind spezielle Transformationen,

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Defekt–Korrekturmethoden, adaptive Mehrgittermethoden, die Verwendung anisotroper Elemente, geeignete Extrapolationsverfahren, Rückgewinnungsmethoden und die Kombination von Fourier– und finiten Elementen für Probleme mit Singularitäten behandelt worden. Bei den Randelementmethoden mit Singularitäten standen modifizierte Knotenwahl, spezielle Transformationen, graduierte Maschenverfeinerungen bei h, bei p und bei $h-p$–Verfeinerungen sowie Iterationsverfahren und ihre Fehleranalysis im Vordergrund. Erfreulich war auch die Verwendung dieser Resultate in neuen drei–dimensionalen Festigkeitsberechnungen der Festkörpermechanik.
Neben den vielen Fortschritten in den oben angeführten bekannten Problemstellungen wurden auch einige ganz neue Fragen gestellt, so die nach Fredholm–Eigenschaften singulärer Integralgleichungen auf Kurvensystemen mit Spitzen, nach Singularitäten bei der Kopplung verschiedener Modellierungen (Platte–Membran) oder verschiedener Materialien und nacj Singularitäten bei zeitabhängigen Gleichungen.
Eine ganze Reihe offener Fragen wurde in den Diskussionen angesprochen. So ist für einige stationäre dreidimensionale Risse bis heute das Singulärverhalten nicht geklärt, und die Lücke zwischen theoretischen Ansätzen und einer parktikablen konstruktiven Singularitätenbeschreibung ist immer noch zu groß. Die mathematische Beschreibung nichtglatter Risse steht erst ganz am Anfang, das Gleiche gilt für viskoelastische Materialien und Fluide. Die Konvergenzanalysis von numerischen Verfahren insbesondere von finite Element– und Randelementmethoden mit Singularitäten für höherdimensionale Probleme ist immer noch lückenhaft. Die wenigen aber z.T. überraschenden Resultate für zeitabhängige Probleme mit Singularitäten zeigen, daß hierfür in naher Zukunft ebenfalls interessante neue auch unkonventionelle Resultate erwartet werden dürfen.
Es gab viele anregende Diskussionen. Die angenehme Atmosphäre der Tagung, die nicht zuletzt der hervorragenden Betreuung durch die Mitarbeiterinnen und Mitarbeiter des Instituts zu danken ist, soll besonders hervorgehoben werden. Im Namen aller Tagungsteilnehmer danken wir allen Angehörigen des Instituts herzlich dafür. Herzlich danken wir auch für die hilfreiche Unterstützung aller und insbesondere der Teilnehmer aus Osteuropa und den neuen Bundesländern.

gez. W. Wendland, J. Whiteman

Vortragsauszüge

J. AALTO:

On mapping of finite elements around corner singularities of the Poisson and the quasi–harmonic equation

A geometrical mapping, called here *natural mapping*, is introduced which can be used in a subgrid (*patch*) of finite elements in the vicinity of a corner singularity point on the boundary of the domain of the Poisson and the quasi–harmonic equation. The equations of the mapping are based on the analytical singular solution of the problem. The geometry of the elements within the patch can be given by two techniques. In the first one, called here *isoparametric element geometry*, standard iso–parametric elements are used and only the nodes of the elements of the patch are located using the equation of the mapping. In the second one, called here *exact element geometry*, the geometry of all the elements within the patch is specified exactly according to the equations of the mapping. Outside of the patch, however, standard isoparametric elements are used. Some analytical and numerical results to compare are presented the two techniques and to show their efficiency. Finally, a special Zienkiewicz–Zhu–type error estimate for the problem is proposed amd justified by numerical examples.

[1] Aalto, J.: *Singularity elements for seepage analysis.* Int. J. Num. and Anal. Meth. Geomech. 9 (1985) 185-196.

[2] Aalto, J.: *Singularity elements for the quasi–harmonic equation.* Proc. Second World Congress on Computational Mechanics, August 1990, Stuttgart FRG, Extended Abstracts of Posters, 582-585.

E. BECACHE:

A boundary integral equation method for the scattering of transient elastic waves

The problem of the transient elastic wave scattering by a crack is investigated. This problem is formulated by means of a boundary integral equation (BIE) and a space–time variational formulation associated with this BIE. Some mathematical results as existence and uniqueness of the solution and the continuity of the operator are obtained for the time domain formulation, which are derived from the frequency domain formulation.

The kernel of the BIE is hypersingular. In order to get the explicit expression of the variational formulation, we have developped a regularization method; and finally the time–space bilinear form is expressed by means of weakly singular kernels satisfying the causality principle.

For the particular case of a rectangular crack, some numerical results are presented which show the accuracy and the stability of the method.

H. BLUM:

On defect correction schemes for elliptic problems with singularities

Local defect correction is a simple strategy for removing the pollution effect of reentrant corners in numerical approximation schemes. Its realization requires an accurate solution of local problems in (mutually disjoint) neighbourhoods of the singular points whereas globally the non-modified discretization is used. We discuss several possible implementations of this idea and show the optimal pointwise convergence.

M. COSTABEL:

General edge singularities for elliptic boundary value problems

Singularities of elliptic boundary value problems at edges of the boundary are, roughly speaking, of the form

$$\sum_{kqn} c_{kqn}(y) r^{\nu_k(y)} \log^q r \varphi_{kqn}(y,\omega),$$

where y is a parameter describing the edge, r is the distance to the edge, and ω describes the angular variable normal to the edge. At points y where some of the exponents ν_k coincide, the coefficients $c_{kqn}(y)$ and the shape functions $\varphi_{kqn}(y,\omega)$ may have poles. In this case, one can replace the functions $r^{\nu_k} \log^q r$ by divided differences of the function $\lambda \mapsto r^\lambda$, taken at some of the $\nu_k(y)$. The new coefficients so abtained allow regularity estimates in Soboloev spaces on the edge; also the remainder in the corresponding singularity expansion can be shown to be smooth.

M. DAUGE:

Singularities near curved edges for solutions of the Laplace equation

The vertex singularities of the Dirichlet problem for the Laplace operator in a 2–dimensional sector can be expressed as products of powers of ζ and $\bar{\zeta}$ with certain exponents, and of $\log \zeta$ and $\log \bar{\zeta}$ where ζ is the complex writing of the 2–dimensional coordinates. We extend such a description of singularities to the expression of the asymptotics of the solution of the Dirichlet problem near a curved edge in a 3–dimensional domain. This description not only involves powers of ζ and $\bar{\zeta}$ as above but also divided differences of the functions $\lambda \mapsto \zeta^\lambda$ calculated at the exponents which appear in 2–dimensional problems (for instance $\frac{\zeta^{\nu_1} - \zeta^{\nu_2}}{\nu_1 - \nu_2}$ with $\nu_1 = \nu_1(y) = \frac{\pi}{\omega(y)}$ and $\nu_2 = 2$).

A.R. DAVIES:

Reentrant corner singularities in non–Newtonian flow

Biorthogonal series expansions are used to study the creeping flow of a co–rotational Maxwell fluid in plane reentrant and non–reentrant sectors. The governing equations are written in terms of stream function and Airy stress function, the radial parts of which are determined by an infinite system of 4th–order nonlinear ordinary differential equations which are seen to be singular pertubations of the linear 2nd–order equations for Stokes flow. Exact formal solutions in logarithmic series are derived, but do not readily enable the asymptotics of the corner singularities to be studied. Instead, an iterative method combined with dominant component analysis is used to decouple the system, whence the singularities are studied in terms of generalized hypergeometric equations and Meijer G–functions. Certain conditions on roots of associated quartic equations are derived, the satisfaction of which would indicate the existence of lip vortices in a reentrant sector and the integrability of stress in a neighbourhood of the corner.

M. DOBROWOLSKI:

Anisotropic finite element interpolation

For the numerical approximation of anisotropic structures such as edges, boundary and interior layers it is a natural idea to use finite element meshes with different mesh sizes in different directions. We derive a simple algebraic condition for the finite element which ensures that the anisotropic mesh is superior to a convential isotropic mesh. These results are the theoretical justification of an r–finite element method in which anisotropic meshes occur in a natural way. Some numerical results are given.

R. DUDUCHAVA, T. LATZABIDZE, A. SAGINASHVILI:

Singular integral equations on curves with cusps

Classical singular integral operators with piecewise–continuous coefficients and complex conjugated unknown functions are studied in the Lebesgue space with weights. The contour of integration may have cusps where arcs touch each other with the order $\nu > 1$. The symbol is defined for $\nu = 2$ and $\nu = 1 + 1/n$ with $n = 2, 3, \ldots$. Criteria for the Fredholm property as well as index formulae are derived. The dependence of the symbol on the order ν is shown evidently.

R.V. GOLDSTEIN, A.B. MOSOLOV:

Singularities of fractal cracks

Although fracture surfaces are often idealized as flat it is now generally recognized that most of the crack surfaces have a very irregular structure. This irregularity possesses often the property of "self–similarity" and can be described in terms of fractal geometry. In the paper, the fractal model is derived for cracks whose roughness influence has a description on a megascale.

The self–similar structure of singularities in case of a fractal crack and scaling ideas lead to a cascade process of elastic energy release and transition from scale to scale. The analysis of this process allows us to obtain the renorm–group equation for a fractal fracture process description in brittle solids. It is determined by power–law asymptotics of the stress and displacement fields near the crack tip in dependence on the crack fractal dimension. Some generalized fracture criteria for fractal cracks are suggested, including the Barenblatt–Irvin, Griffith and the Barenblatt–Novogilov criterions. These criterions can be valid for "short" crack growth conditions and the determination and transfer to usual criterions for macro–cracks.

Some models of fractal crack behaviour in compression along the crack average line are derived.

It is shown that similar asymptotic consideration can be derived for the fractal punch–interaction problem.

I.G. GRAHAM:

The computation of water waves modelled by Necrasov's equation

Necrasov's nonlinear integral equation, describing water waves of almost extreme form, is solved numerically. The method consists of applying a simple quadrature rule to a rearranged version of the original equation. Strongly graded meshes are used to resolve an expected boundary layer in the solution. For methods based on the trapezoidal rule, we use global bifurcation theory to prove, for fixed disretization parameter n, the existence of a continuous branch of positive numerical solutions. These are parametrized by μ, a natural parameter occuring in the original integral equation. For fixed μ, collective compactness arguments then prove subsequential convergence of these solutions as the mesh is refined (i.e. as $n \to \infty$). Numerical experiments using higher order quadrature rules are reported. These reveal that the method is capable of detecting "Gibbs phenomenon" type oscillations of maximum height about 0.37^o in a boundary layer of width $O(70/\mu)$ for μ large (typically $\mu \in [10^{18}, 10^{20}]$).

D. GROß:

Crack tip singularities and fields in power law hardening materials: some recent results

Investigated are singularities and dominant fields for crack and notch tips in power law hardening materials. For longitudinal shear problems the hodograph transformation can be applied. It allows the determination of the singularities and the field quantities in closed form for different types of boundary conditions. Alternatively, a formulation in terms of a stress function leads to a nonlinear eigenvalue problem. A solution for the whole spectrum is found by the perturbation method. The accompanied eigenfunctions are determined numerically by use of the Levenberg–Marquardt algorithm. This technique also can be used to solve the equivalent problem in plane strain and plane stress boundary conditions even if high hardening exponents are considered.

B. HEINRICH:

Edge singularities in axisymmetric domains and their approximation by the Fourier–finite element method

We consider the Fourier–finite element method applied to the numerical solution of the Dirichlet problem for Poisson's equation in axisymmetric domains with edges. This method combines approximate Fourier analysis and synthesis (here, with respect to the rotational angle) with the finite element method for the approximation of Fourier coefficients being the solution of two–dimensional elliptic problems on the meridian plane. The behaviour of the solution near some edge is characterized by two different singularity functions of tensor product and non–tensor product type. Modified finite element approximations with local mesh refinement are proposed to obtain the same rate of convergence as for regular solutions.

K. HERRMANN:

The treatment of singular stress fields in composite mechanics using the method of caustics

An interesting problem in today's fracture mechanics research represents the crack path prediction of growing thermal cracks as a function of the geometrical configuration of a self–stressed body as well as of the applied thermal load distribution. Starting with experimental results of cooling experiments for two–phase composite structures, different types of singular stress states arising in these structures are characterized. Besides, the analytical treatment of special stress singularities in composite mechanics given by S.S. Wang and F. Erdogan is mentioned.

Furthermore, boundary value problems of plane thermoelasticity for uncracked and cracked two–phase solids are formulated, getting their solutions by means of the complex function theory as well as by using the finite element method. As a special problem, a Hilbert–problem for a curvilinear interface crack is considered where the singular stress state around the crack tip has been treated by applying the shadow optical method of caustics. Thereby the corresponding caustics have been determined for quasistically extending and fast moving straight and curvilinear cracks by consideration of nonhomogeneity as well as of the optical anisotropy of the material.

L. JENTSCH:

On boundary integral equations of thermoelasticity in domains with material discontinuities in corner points

A plane boundary value problem of thermoelasticity is called a bimetal problem, if the interface between the different materials is a straight line extending to the outer boundary. Singularities depending on the material parameters and the angles between S_0 and the boundary S occur at the endpoints P_1 and P_{-1} of the interface S_0. The indirect boundary integral method is applied for solving the problem. Potentials with the Green contact tensor instead of the fundamental solution satisfy a–priori the transmission condition on S_0. The boundary integral equation yields a system of singular integral equations with two fixed singularities. The Fredholm property and the index of the boundary integral operator as well as the asymptotics of the solution are determined by a Mellin symbol, which can be expressed by elementary transcendental functions.
In particular, the anti–plane deformation is discussed in detail.

V.A. KONDRATJEV:

The singularities of solutions of nonstationary problems in a neighbourhood of an edge

The nonstationary problems in domains with singularities are considered. First, the wave equation $u_{tt} = \Delta u + f(x,t)$ in the domain $Q = \Omega \times [0,1], \Omega \in \mathbb{R}^n$ under Dirichlet or Neumann boundary conditions on $\partial\Omega$ and initial conditions on Ω is studied. Here, $\partial\Omega$ contains a conical point $x = 0$. The smoothness of the solution as well as the asymptotic behaviour near an edge $\{(x,t) : x = 0, 0 < t \leq T\}$ are considered. It is proved that the singularities of the solution are concentrated on the edge.
Analogous problems are studied for the dynamical system of elasticity. It is proved that $u = c(t,x)|x|^\lambda + o(|x|^{\lambda+\epsilon})$, where λ is the smallest eigenvalue of some operator pencil connected with the stationary system of elasticity.

V.A. KOZLOV, V.G. MAZ'YA, C. SCHWAB:

On the spectral properties of operator pencils generated by the Lamé and Stokes systems in a cone

The spectral properties of operator pencils are considered which characterize the singularities of solutions of boundary value problems for a three–dimensional cone $K = (0,\infty) \times \Omega$. It is shown that the set

$$\{\lambda \,|\, (Re\lambda + \frac{1}{2})^2 - (Im\lambda)^2 < \frac{9}{4} + (1-2\nu)(5-4\nu)\}, \qquad (1)$$

where ν is the Poisson coefficient (for the Stokes system $\nu = \frac{1}{2}$), contains only real eigenvalues of the operator pencils. Furthermore, it is proved that real eigenvalues in the set

(1) do not admit associated generalized eigenfunctions; and therefore logarithmic terms in the asymptotics of solutions near the vertex of K do not occur for these eigenvalues. The real eigenvalues in (1) are characterized by a variational principle. For the Dirichlet problem, this principle yields that these eigenvalues depend monotonically on Ω.

U. LANGER, M. JUNG:

Adaptive multigrid methods for elliptic problems with boundary and interface singularities

The presence of boundary and interface singularities deteriorates the accuracy of the standard finite element approximation u_h compared with the solution u of the boundary value problem under consideration, e.g. $\|u - u_h\|_1 = O(h^\alpha)$ with some $\alpha < 1$ instead of $O(h)$ for linear triangular elements, and effects the convergence of standard multigrid methods as well. It is well known that one can obtain the optimal accuracy $O(h)$ in the H^1–norm $\|\cdot\|_1$ if the mesh is locally refined near the singularity points in accordance with the strength of the singularity, provided that the latter is explicitly known. We propose to use three multigrid solutions $u_{\ell-3}, u_{\ell-2}$ and $u_{\ell-1}$ obtained within a Full–Multigrid–Strategy on the last three meshes $\tau_{\ell-3}, \tau_{\ell-2}$ and $\tau_{\ell-1}$ of a sequence $\{\tau_q\}_{q=1,2,\ldots,\ell-1}$ of uniformly refined meshes in order to estimate the strength of the singularities and to adapt the next finer grid τ_ℓ to the behaviour of the solution. For solving the adapted finite element equations on the finest grid τ_ℓ, we use the conjugate gradient method preconditioned by a special hierarchical preconditioner which involves the multigrid method only on the uniformly refined meshes. The numerical experiments carried out for academic test problems where the exact solution is known as well as for real–life problems from industry show that our method works well.

D. LEGUILLON:

Numerical analysis of a crack branching in non–isotropic materials

In classical fracture mechanics, isotropy is invoked twice, for the elastic behaviour as well as for the fracture process. Accounting for a non–isotropic constitutive law modifies significantly the analysis. The two classical crack tip modes no longer enjoy symmetry properties and as a consequence many features have to be revised.

Accounting in addition for a non–isotropic fracture process leads to the definition of a modified Griffith criterion to predict propagation and kinking.

An asymptotic and numerical analysis of the revisited Williams series is proposed to examine the stability of a kink, with applications to a carbon fiber reinforced material.

M. LORENZ, SCHMUTZLER, UMNUß :

Elliptic equations in domains with edges

If four symbols (inner, boundary, edge and exit) are invertible, the boundary value problem is a Fredholm operator in some weighted Sobolev spaces. The general formula for the asymptotics can be given, which depends smoothly on the edge variable. Hörmanders reduction to the boundary is realized for some operators in a wedge.

J. MASON:

Boundary elements for singularities on curves

It is well known that square–root singularities in the solution of boundary element and finite element formulations of continuum mechanics problems may be accurately modelled by placing internal element nodes at special positions and adapting isoparametric elements. This approach is rooted in the "quarter–point" quadratic element, developed independently for an end–point singularity on a straight line boundary by Barsoum and Henshell and Shaw, and in the quadratic "transaction element" by Lynn and Ingraffea for incorporating an exterior singularity on a straight line boundary.

In the presented paper we extend these ideas to curved boundaries, developing a cubic element for an end–point singularity and a quartic element for an exterior singularity. The derivation of the appropiate internal nodes involves the solution of one or more nonlinear algebraic equations. In the case of the cubic element, a unique solution exists for continuous convex curves, an inclusion region is found for its determination by a convergent bisection method, and necessary and sufficient conditions are established for a one–to–one mapping. The quartic element involves the solution of two simultaneous functional equations, and Newton's method has been successfully applied in many cases. A wide variety of numerical results for both elements is presented. We have also developed quartic elements for end–point singularities, in which an additional constraint may be imposed, such as the specification of one internal node or prescription of the direction of the boundary of the singularity (e.g. the crack–tip direction).

E. MEISTER, F.S. TEIXEIRA :

Two–media scattering problems in a half–space

We consider mixed–boundary transmission problems for the Helmholtz equation in a half–space, taking different wave numbers in each quadrant. Dirichlet, Neumann or mixed type boundary conditions are imposed on the half–planes $\pm x_1 > 0, x_2 = 0, x_3 \in \mathbb{R}$ and transmission conditions are prescribed in the half–plane $x_1 = 0, x_2 > 0, x_3 \in \mathbb{R}$. These problems are seen to be well posed in the setting of finite energy norm spaces H_1; and explicit solutions are given for the Dirichlet and Neumann problems. The singular behavior of the solutions is discussed for the mixed problem.

N.F. MOROZOV:

The singular points and the problems of brittle fracture

It is discussed the influence of results of mathematical investigation of for using possibilities of different criterions of frature.

The properties of fracture near the cornerpoints were studied. The modification of the method Muskhelishvily is investigated.

It is proposed the method of count the delay of the crack's propagation.

S. NICAISE, A. MAGHNONJI:

We study interface problems on polygonal domains of the plane, where the order of the operators is different on each face. We investigated, wether the associated operator on appropiate Hilbert–spaces is a Fredholm operator or not. If it is, we give an expansion of the weak solution into a regular part and a singular one.

S. PRÖßDORF, W. Mc LEAN, W.L. WENDLAND:

On discrete collocation for the logarithmic kernel integral equation on an open arc

Consider the equation

$$(1) \qquad -\frac{1}{\pi}\int_{\Gamma} \log|t-\tau|v(\tau)ds_\tau = g(t) \text{ for } t \in \Gamma\,,$$

where v is the unknown solution, ds_τ the element of arc-length, and Γ a smooth open arc in the plane with transfinite diameter different from one. It is well known that in this case the (unique) solution of (1) may have singularities of the form $O(|t-c|^{-\frac{1}{2}})$ at the endpoints c of Γ. However, changing the variable by the cosine transformation, we remove the singularities and reduce (1) to a periodic equation. To the latter, and more general, to periodic pseudodifferential equations of arbitrary integer order, a combination of collocation with a Nyström–like quadrature method is applied, using trigonometric polynomials of degree n as the space of trial functions. For this method, the pointwise and Sobolev space rates of convergence are established. In particular, we show that the error in the maximum norm is $O(n^{-r}\log n)$ provided the solution is in C^r. This estimate is sharp since it is of the same order as for the interpolant. The method considered defines a fully discretized system of linear algebraic equations which is well suited for efficient solution via fast Fourier transform in combination with multigrid techniques.

A. RATHSFELD:

Quadrature methods for the double layer potential equation over a polyhedron

As it is well known, the integral operator of the boundary equation corresponding to the double layer potential over a polyhedron is neither compact nor strongly elliptic. In order to derive the convergence of numerical methods, one can apply the techniques developed for the one–dimensional case of Mellin convolution equations. For special triangulations, the stability of a simple quadrature method can be proved. However, an important assumption for its application is the convergence of the finite section method applied to the corresponding double layer equations over the tangent cones. Simultaneously with the stability analysis, one can prove the convergence of the two–grid iteration for the solution of the linear system of equations. The singular behaviour of the solution function suggests to choose a mesh refinement near the corners and edges of the polyhedral boundary.

A.-M. SÄNDIG:

Coefficient formulae and their stability

The structure of asymptotic expansions of solutions of elliptic boundary value problems near conical boundary points is investigated, if the right–hand sides are given in standard Sobolev spaces.

The coefficients in the expansions can be unstable if the angles are critical. Then a stabilisation procedure is necessary. Some boundary value problems for the Laplace equation and the Lamé system in a polygon or in a circular cone are handled as examples. Domains of different materials are also included.

E. SANCHEZ–PALENCIA:

Non–isotropic energy criteria in fracture of composites

The classical Griffith criterion for fracture stipulates that some specific energy is necessary to produce a new element of the crack surface. This energy is a characteristic of the material and corresponds to the energy necessary to break down the molecul links of the material. In anisotropic media, this energy depends also on the orientation of the surface element. For instance, in a fibered anisotropic material, the energy necessary to produce an element of a crack along the fibers is very much less than for a crack across the fiber. Moreover, an asymptotic analysis of the structure of the stress field in the vicinity of the crack tips allows us to get the direction of propagation of the crack under a given loading. Then, under appropriate anisotropic dates it appears that the propagation without change of direction (the smooth crack) is possible only in some specific directions. In other directions, the propagating cracks exhibit necessarily kinks and are not smooth curves.

H. SCHMITZ:

Penalty methods for Signorini problems

The Laplace equation with monotone boundary conditions is considered. The boundary–element–Galerkin approximation of the Steklov–Poincaré operator leads to a numerical scheme whose solutions converge with optimal order in the energy space $H^{\frac{1}{2}}(\Gamma)$. A globally convergent scheme is developed in order to solve the discrete equations. Using residual corrections in the energy space accelerates the convergence in special cases. These results are proved by monotone–operator–techniques.
The method is applied to mixed Dirichlet–Signorini problems by uing a penalty approach. A suitable balance of the meshsize h and the penalty parameter ε yields an asymptotic convergence of order h.

E. SCHNACK:

Numerical computation of 3–D singularities for elastic structures

At first the advantage of mixed variational formulations for computing stress intensity factors in fracture mechanics is shown. The method has limits with respect to reentrant corner problems. For this case, an algorithm has been developed on the existing analysis in order to compute the unknown eigenvalues. The results show that the dominant eigenvalue is more important to predict crack growth than stress intensity factors, especially if the crack front pushes the free surface unter an angle of more than 90°. In order to obtain a generalized algorithm for reentrant corners of crack fronts we intend to develop a method on the basis of the Galerkin BEM scheme.

C. SCHWAB, M. SURI:

Approximation properties of the p–version of the boundary element method on polyhedra

It is well known that the solutions of elliptic boundary value problems exhibit singularities near edges and vertices of the boundary. These govern in particular the rate of convergence of the p–version of the boundary element method.
We show sharp rates of convergence of the p–BEM, if the domain is a polyhedron in $\mathbb{R}^3$. The rate is twice that of the h–version (with uniform mesh).

F.-O. SPECK, F. PENZEL:

Asymptotic expansion of singular operators on Sobolev spaces

The topic of asymptotic expansion of pseudo–differential equations in the spirit of Eskin's work is extended to a more general situation. Taylor expansion of the Fourier symbol matrix functions is replaced by a series of generalized invertible operators, which act on vector Sobolev spaces. The fractional orders of these spaces are obtained from the jumps of the lifted symbol matrix function at infinity in a situation which is most interesting for applications. Asymptotic and regularity results for the solutions of corresponding systems of equations are direct consequences.

E.P. STEPHAN:

On some improved boundary element methods for elliptic problems with $2D$ and $3D$ singularities

We show convergence for the h, p and $h-p$ versions of Galerkin boundary element schemes applied to weakly singular and hypersingular integral equations of the first kind. Those integral equations result from Dirichlet, Neumann and interface problems for the Laplacian. The influence of mesh refinement towards corners and edges on the asymptotic error is given; e.g. for the $h-p$ version on geometric meshes we obtain exponentially fast convergence of the Galerkin error. The implementation of these numerical methods is discussed together with adaptive h and $h-p$ algorithms, and corresponding numerical experiments are presented for two– and three–dimensional problems.

F.S. TEIXEIRA:

A Sommerfeld–type diffraction problem with second–order boundary conditions

An operator–theoretic approach is used to study the problem of diffraction of time–harmonic waves by a metal–backed dielectric half–space. The correspondent boundary value problem for the two–dimensional Helmholtz equation is considered in a Sobolev space setting and is reduced by equivalent boundary integral equations of Wiener–Hopf type in $L_2^+(\mathbb{R})$. An explicit analytical solution is obtained for the particular case of having the same dielectric layer on both banks of the half–plane.

R.W. THATCHER:

Estimating the form of three–dimensional singularities

When using the finite element technique at a point singularity it is usual to adopt some sort of grid refinement technique to get a good approximation of the singular behaviour. Special elements or special functions are often used when the form of the singularity is known. In this contribution, a technique for estimating the behaviour of the singularity is described which takes the idea of a grid refinement to an infinite limit. This leads to an infinite sequence of equations and the local behaviour of the singularity is given by the solution of a recurrence relationship. This recurrence relationship is homogeneous when both the boundary conditions and the differential equation are homogeneous in the neighbourhood of the singularity; and the recurrence relationship is independent of inhomogeneous conditions remote from the singularity. Thus, the fundamental solution of the recurrence relationship approximates the behaviour at the singularity for all loadings remote from it. By analyzing the terms in the fundamental solution, estimates of the terms in the singular behaviour are obtained.

Two examples of the technique were presented. The first was at a singularity with known behaviour to test the method. The second example was the point singularity at the end of a slit orthogonal to a stress-free surface, this being an important but not fully resolved singularity in fracture mechanics.

K. VOLK, H. SCHMITZ, W.L. WENDLAND:

On a boundary element method for the computation of corner singularities of elastic bodies in $\mathbb{R}^3$

For computing the singular behaviour of an elastic field near a three-dimensional vertex subject to displacement boundary conditions, we use a boundary integral equation of the first kind whose unknown is the boundary stress. Localization at the vertex and Mellin transformation yield a one-dimensional integral equation on a piecewise circular curve γ in $\mathbb{R}^3$ depending holomorphically on the complex Mellin parameter. The corresponding spectral points and packets of generalized eigenfunctions characterize the desired singular behaviour of the stress field. We derive a decomposition into the regular part and edge and vertex singularities.

The edge singularities which are given implicitly by the algebraic eigenfunctions can be achieved analytically by applying the same Mellin technique as above to the one-dimensional parameter dependent integral equation. In order to compute the vertex singularities, the spectral problem with the integral equation of the first kind is solved by a spline-Galerkin method with graded meshes at the corner points of the curve γ. For this approximation we provide a rigorous asymptotic error analysis.

We present numerical results for various geometries, characterizing the leading singular term of the desired stress fields.

J.R. WALTON:

Stress singularities of accelerating crack tips in viscoelastic materials: methods for constructing their time dependent stress intensity factors

It is proposed to describe a new method for constructing solutions to accelerating crack problems in elastic and viscoelastic materials. This method yields for the first time exact solutions for dynamically accelerating cracks in viscoelasticity from which the near crack tip asymptotic stress and displacement fields can be constructed.

M.K. WARBY, J.R. Walton, J.R. Whiteman:

Finite element model of crack growth in a finite body in the context of Mode I linear viscoelastic fracture

A finite element method is described for modelling crack initiation and crack motion in the context of Mode I linear viscoelastic fracture. The numerical mode is based on modelling the crack tip region by a Barenblatt type failure zone. In our model this involves a constant failure load on an intervall length a_f from the crack tip on the crack faces with the length a_f determined so that the stress is finite at the crack tip. Mathematically this condition is expressed as an equation which makes use of correspondence principles and properties of J-integrals. With such a model crack initiation and crack motion are given in terms of a critical crack opening displacement. Numerical results are presented in order to investigate the conditions under which crack motion occurs and to show that when crack moves, the motion can be stable for a significant period.

W.L. WENDLAND, J. ZHU:

Viscous flows through an open pipe with arbitrary cross–section

We analize the velocity field of an incompressible viscous flow exterior to an open bounded surface in three dimensions, which is modelled by a system of integral equations of the first kind on the open surface. Existence and uniqueness of the solution of the integral equations can be shown with the help of the variational formulation of the boundary integral equations and the coerciveness of the corresponding bilinear form. Following previous work by Costabel and Stephan, Eskin's Wiener–Hopf technique can be used along the edge of the open surface for the analysis of the associated edge singularity, which is based on the explicit factorization of the matrix–valued principal symbol of the boundary integral operator.

For the construction of a boundary element approximation with Galerkin schemes for the integral equation, we introduce a Lagrangian multiplier in order to incorporate constraint conditions. Thus we use augmented boundary elements which simulate the singular behaviour near the edge and also admit the geometrical approximation of the surface and its edge. Here, asymptotic convergence results for the method and, in particular, for the velocity field are obtained.

[W.L. Wendland & J. Zhu: The boundary element method for three–dimensional Stokes flows exterior to an open surface. Math.Comp. Modelling **15** (1991) 19-41]

J.R. WHITEMAN

Superconvergence of recovered gradients of finite element approximations to some problems of solid mechanics

A review is first given of methods of recovery of gradients of finite element approximations of displacements in problems of linear elasticity so that superconvergence effects are produced. The techniques are described for meshes of triangles and tetrahedra, respectively for two– and threedimensional problems, and their application to produce adaptive methods is also dicussed. Some theoretical error estimates are described for the case of problems containing boundary singularities, where the solutions possess low regularity.

The extension of recovery techniques to time dependent problems is then presented, initially for simple parabolic problems, using Galerkin in space / finite differences in time. Again theoretical error estimates of these techniques are given.

All the above enables the numerical solution of problems of quasistatic linear viscoelesticity to be considered. Two algorithms based on Galerkin in space plus discretisation in time are given, together with error estimates; the recovery techniques can be applied in the space dimension. Thus we have provided machinery for modelling the behaviour of cracks in viscoelastic materials, as discussed by M.K. Warby in an associated lecture.

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