MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 51/1991

Numerische Methoden der Approximationstheorie

Nov 24. – 30, 1991

In the week November 24 - 30, 1991, a conference on *Numerical Methods in Approximation Theory* was conducted at the Mathematical Research Institute in Oberwolfach by Prof. Dr. Dietrich Braess, Ruhr-University Bochum and by Prof. Dr. Larry L. Schumaker, Vanderbilt University, Nashville, Tennessee. 49 mathematicians from nine countries participated and reported in 32 lectures on new results in this area.

Areas of particular interest were (among others):

multivariate spline spaces, basis and dimension problems

multivariate interpolation and approximation by univariate functions

radial functions

subdivision and recursion schemes

wavelets and applications

numerical problems in the solution of differential equations

numerical modelling of neural networks.

Many of these research activities were motivated by non-mathematical problems. Applications in the fields of medical statistics, meteorology, environmental research, brain research, holography, tomography, computer aided design, image processing and acoustics were mentioned by the speakers.

The atmosphere of the conference was very lively and productive. The time outside the lectures was extensively used for discussions and joint work. The hospitality and generous help of the institute's staff was greatly appreciated by all of the guests, and helped a lot to make this meeting a success. We would like to thank the director, all members of the staff, the lecturers, and the chairpersons for their valuable work.

A pioneer and very active researcher in applied mathematics with a particular interest in numerical approximation theory was sadly missed at the conference: Prof. Dr. Dr. h. c. mult. Lothar Collatz, Hamburg, who suffered a sudden lethal heart attack when visiting a conference last year. The organizers and all of the participants would like to show their respect to Lothar Collatz by dedicating this conference to his memory.

It is planned to isssue a proceedings volume in Birkhäuser's series *International Series of Numerical Mathematics* (ISNM).

Lecture Abstracts

P. ALFELD

Generic Dimension of Trivariate Spline Spaces

This is a report on joint work with Larry Schumaker and Walter Whiteley. We consider the linear space of globally differentiable piecewise polynomial functions defined on a three-dimensional polyhedral domain which has been partitioned into tetrahedra. Combining Bernstein-Bézier methods and combinatorial and geometric techniques from rigidity theory, we give an explicit expression for the generic dimension of this space for sufficiently large polynomial degrees ($d \geq 8$). This is the first general dimension statement of its kind.

G. BASZENSKI

Blending Interpolation with Sine Functions

For functions defined on the unit square, an interpolation scheme involving sine functions is constructed using Blending methods. Interpolatory properties are derived as well as asymptotic error estimates. The computational complexity in terms of floating point operations is also considered.

H. BERENS and M. FINZEL

Discrete linear Chebyshev Approximation

We examine approximations by subspaces U of $\mathbb{R}^n$, equipped with the maximum norm, by using Plücker-Graßmann coordinates of U. A classification of the indices by these coordinates allows to determine all extremal points of the intersection of the orthogonal complement $U^\perp$ of U with the dual unit ball $\bar{b}_1^1(0)$. These vertices of the polyhedron $Q = U^\perp \cap \bar{b}_1^1(0)$ play a central role: They determine the shadow line, the metric complement of U in $\ell^\infty(n)$; they furthermore determine the distance of arbitrary points to the subspace U as well as the set of characteristic points of a point $x \in \mathbb{R}^n \backslash U$. Moreover, they correspond to convex cones which partition $\mathbb{R}^n$. On each of these cones the metric projection has specific properties, for example it is linear for Chebyshev subspaces. It follows in this case that the metric projection is globally Lipschitz continuous. This property also occurrs with strict non Chebyshevian approximations: a finer subdivision of $\mathbb{R}^n \setminus U$ into finitely many convex cones yields domains of linearity. In particular it holds that strict approximations are globally Lipschitz continuous as well. — These results are mainly taken from the Ph. D. thesis of M. Finzel.

D. BRAESS

Interpolation by Ridge Functions

We consider the approximation of functions of n variables by functions of the form $f(x) = \sum_{i=1}^m g_i(a^i x)$. Here $a^1, \ldots, a^m$ are m given directions and $g_1, \ldots, g_m$ may be real valued functions which can be chosen according to the data. The interpolation problem for $m = 2$ directions has already been solved. We treat the case $m = 3$. It turns out that non-interpolation configurations are not characterized in a way which is a simple generalization of the case $m = 2$. Here, in addition to bricks, some special hexagons enter into the theory.

M. D. BUHMANN and C. A. MICCHELLI

Non-Stationary and Non-Uniform Subdivision

We consider necessary and sufficient conditions for the convergence of a non-stationary and non-uniform subdivision scheme induced by a sequence of 2-slanted bi-infinite matrices $A_1, A_2, \ldots$ which map $\ell_\infty(\mathbf{Z})$ into $\ell_\infty(\mathbf{Z})$. The 2-slantedness means that there exist integers ℓ and m such that $(A_r)_{ij} = 0$ unless $\ell \le i - 2j \le m$ for all i, j and r. In such a subdivision scheme, a sequence of control points $\lambda^{r-1} \in \ell_\infty(\mathbf{Z})$ is mapped to $\lambda^r := A_r \lambda^{r-1}$ at the r-th level, and we say that the scheme is convergent if there exists a continuous function f_{λ^0} so that $\sup_{j\in\mathbf{Z}} |\lambda_j^r - f_{\lambda^0}(2^{-r}j)| \to 0$ as $r \to \infty$. The scheme is said to be stationary if all A_r are the same, and it is uniform if there exist sequences $\{(a_r)_i\}_{i=-\infty}^{\infty}$ such that $(A_r)_{ij} = (a_r)_{i-2j}$. Both of these special cases have been studied in the literature, but in this talk we develop a theory that contains schemes which do not satisfy these requirements. Examples are given to illustrate our theory.

C. K. CHUI

Wavelets

While spline functions are very useful in computational mathematics, wavelets provide a powerful tool for analysing errors and hence, the original (unknown) function. We will be concerned with compactly supported wavelets that are expressed in terms of B-splines. The numerical analysis and implications of these wavelets will be discussed.

W. DAHMEN

Multilevel Preconditioning

A general multilevel framework for the numerical solution of elliptic boundary value problems by means of Galerkin methods is described, and estimates for the condition numbers arising from the corresponding multilevel preconditions are established. These results are applied to Galerkin methods based on wavelet expansions as well as to conforming finite elements on adaptively refined triangulations for second and fourth order problems. It is shown that in all these cases the resulting condition numbers remain uniformly bounded.

N. DYN, M. D. BUHMANN, D. LEVIN

On Quasi-Interpolation by Radial Functions with Scattered Centers

Approximation by radial basis functions with non-uniformly distributed centers is discussed. A construction of new algebraically decaying basis functions is presented, and the properties of the quasi-interpolation operator with these functions are investigated. It is shown that, under certain conditions on the distribution of the centers, the quasi-interpolant reproduces polynomials and gives approximation orders identical to those in the uniform square-grid case.

R. W. FREUND

Quasi-Kernel Polynomials and Applications in Matrix Iterations

In this talk, we introduce the general concept of complex quasi-kernel polynomials. Roughly speaking, quasi-kernel polynomials are approximations to true kernel polynomials obtained from a set of arbitrary basis polynomials, rather than orthogonal polynomials. Some general theory for quasi-kernel polynomials is developed, such as recurrence relations and a characterization of roots of quasi-kernel polynomials as generalized eigenvalues. If the basis polynomials satisfy short recurrences, then the corresponding quasi-kernel polynomials can also be generated by means of short recursions. As a result, matrix iterations based on such quasi-kernel polynomials can be implemented with short recurrences. We point out that two recently proposed quasi-optimal algorithms for solving non-Hermitian linear systems are based on particular instances of quasi-kernel polynomials. Finally, we demonstrate that quasi-kernel polynomials can also be used for solving other non-Hermitian matrix problems, such as approximating eigenvalues or pseudo-eigenvalues of large sparse non-Hermitian matrices.

M. HEILMANN

Linear Combinations of Operators of Baskakov-Durrmeyer-Type

We consider linear combinations M_n^r of operators of Baskakov-Durrmeyer-type which are defined by $(M_n f)(x) = \sum_{k=0}^{\infty} p_{nk}(x)(n-c)\int_I p_{nk}(t)\,f(t)\,dt$ in dependence of a parameter $c \in \{-1, 0, 1, \ldots\}$. The corresponding intervals are given by $I = [0,1]$ if $c = -1$ and $I = [0,\infty)$ otherwise. The weight functions are $p_{nk}(x) = \binom{n}{k}x^k(1-x)^{n-k}$ if $c = -1$, $p_{nk}(x) = \frac{(nx)^k}{k!}e^{-nx}$ if $c = 0$ and $p_{nk}(x) = \frac{x^k}{k!}(1+cx)^{-(n+kc)/c}\prod_{l=0}^{k-1}(n+cl)$ if c is positive. The linear combinations M_n^r are constructed in such a way that all polynomials of degree at most $r-1$ are reproduced. We will present global direct, inverse and saturation theorems and a general Voronovskaja-type result.

R. Q. JIA

Nonlinear Approximation with Multivariate Wavelets

In this talk we discuss the problem of wavelet decompositions in the L_p spaces ($0 < p \leq \infty$) and the related nonlinear approximation problem. Our results are an extension of the recent work of DeVore, Jawerth and Popov. Shift invariant spaces of functions are introduced as a convenient framework for multiresolution analysis. Nonorthogonal wavelet decompositions can be easily constructed on the basis of multiresolution analysis.

Concerning nonlinear approximation by wavelets we prove the direct theorem (Jackson estimates) and the inverse theorem (Bernstein estimates) in such a general setting. Possible extensions and applications of the results are discussed.

A. LE MÉHAUTÉ

$L^{m,s}$ Splines in $\mathbb{R}^d$

In order to generalize Duchon's thin plates, we first need to define some spaces of Beppo Levi type; then we introduce some general interpolation schemes in the distributional sense. — We investigate some properties of the spaces and of the scheme and define $L^{m,s}$ Spline in $\mathbb{R}^d$. Particular cases are Duchon's thin plates for Lagrange interpolation, for Hermite interpolation in $\mathbb{R}^d$ local averaging splines, thin plates under tension, can be studied in the same framework.

B. LENZE

Constructive Multivariate Approximation via Sigmoidal Functions

We show how to use sigmoidal functions in order to generate approximation operators for multivariate functions of bounded variation. We start with Lebesgue-Stieltjes type convolution operators, then — via numerical quadrature — we pass over to point-evaluation operators and give local and global approximation results for them. In the following we discuss an important application of our results to neural networks with one hidden layer consisting of so-called sigma-pi units. At the end we apply our operators to a special test function in order to get some visual idea of their behaviour.

W. A. LIGHT

Quasi-Interpolation by Thin-Plate Splines on $[-1,1]^2$

The thin plate spline function is of the form $\theta(x) = \|x\|_2^2 \ln\left(\|x\|_2^2\right)$, $x \in \mathbb{R}^2$. From this functions it is well-known that a second function Ψ of the form $\Psi(x) = \sum_{i \in N} a_i\, \theta(x - z_i)$ can be constructed such that $|\Psi(x)| \sim \|x\|^{-4}$ as $\|x\| \to \infty$. Here N is a finite set of indices, $a_i \in \mathbb{R}$ and $z_i \in \mathbf{Z}^2$. The function Ψ can now be used to construct a quasi-interpolant $(L^h f)(x) = \sum_{z \in \mathbf{Z}^2} f(zh)\, \Psi(\frac{x}{h} - z)$, $x \in \mathbb{R}^2$. Then this operator leads in a natural way to estimates of the type $\|f - L^h f\|_\infty = \mathcal{O}(h^k)$ for smooth $f : \mathbb{R}^n \to \mathbb{R}$. We show how to truncate and modify this quasi-interpolant so that only function values in $[-1,1]^2$ are used and $\|f - \tilde{L}^h f\|_\infty = \mathcal{O}(h)$.

R. A. LORENTZ

Discretization of Polyharmonic Operators by Polyharmonic Spline Wavelets

Jaffard has shown that the conditions number of stiffness matrices resulting from discretizing second order uniformly elliptic partial differential operators in the plane with wavelet bases are uniformly bounded independently of the mesh-size h. His wavelet bases are implicitly defined and have global support. We show that one can use wavelet bases consisting of polyharmonic (or thin-plate) splines to discretize polyharmonic operators Δ^k in any space dimension. The resulting stiffness matrices are (up to any desired tolerance ϵ) the identity matrix. Thus the computations required to obtain an approximate solution consist only of a wavelet decomposition and a wavelet reconstruction. Due to the local support of the bases, these computations can be carried out in $O(N)$ arithmetic operations, where N is the number of unknowns.

T. LYCHE

Spline Wavelets for Arbitrary Knots

In a joint work with Knut Mørken I consider spaces $S_{k,\tau} \subset S_{k,t}$ of univariate splines of order k with arbitrary knots τ and t. We derive an explicit formula for a minimal support basis for the orthogonal complement of $S_{k,\tau}$ in $S_{k,t}$. We give simple proofs of existence, zero structure, and linear independence.

K. MØRKEN and T. LYCHE

Wavelet Decompositions from Discrete Inner Products

In multiresolution analysis, a space $V_1 \subseteq L_2$ is decomposed as $V_1 = V_0 \oplus W_0$, where $V_0 \subseteq V_1$, and W_0 is the L^2-orthogonal complement of V_0 in V_1. We propose instead to measure the size of a function in V_1 by the ℓ^2-norm of its coeffcients (relative to the basis $\{\phi(2x-i)\}_{i\in\mathbf{Z}}$). In this way we obtain an ℓ^2-orthogonal decomposition of V_1, and a basic wavelet generating this complement turns out to have a very simple form. For example, in the spline case, its support is considerably smaller than the L^2-wavelet of Chui and Wang.

B. MULANSKY

Chebyshev Approximation by Spline Functions with Free Knots

This talk is concerned with the Chebyshev approximation of real continuous functions from the class $S_{n,k}$ of polynomial splines of degree n with k free knots. Using the notion of the tangent cone in an extended version and a sign rule for spline functions a necessary alternant condition for local best approximations from $S_{n,k}$ is derived. It shows that the corresponding error functions must have an alternant of a certain length with a prescribed sign on a subinterval. Aside a characterization of best approximation by fixed knots splines with coefficient constraints is obtained. Some ideas for the characterization of global best approximations from $S_{1,k}$ are also presented.

M. NEAMTU

Some Remarks on Multivariate Divided Differences and Simplex Splines

We investigate the connection between two different notions of multivariate divided differences. The first one has been introduced by Hakopian in 1981, and the second one by Neamtu in 1989. Some new properties of the Hakopian's divided differences are derived and some remarks about the relation between divided differences and multivariate simplex splines are given.

G. NÜRNBERGER

Bivariate Spline Interpolation

Methods are developed for constructing sets of points which admit Lagrange interpolation by spaces of bivariate splines of arbitrary degree and smoothness. The splines are defined on rectangular partitions adding one or two diagonals to each rectangle. Special emphasis is laid on selecting the grid points of the partition as interpolation points. The method is to construct a net of lines and to place points on these lines which satisfy the Schoenberg-Whitney-condition for univariate spline spaces such that a principle of degree reduction can be applied. The interpolation splines can be computed by solving several small systems of linear equations instead of one large system. Our approach is completely different from the known interpolation methods for splines of degree at most two. Numerical examples are given. The results were obtained jointly with Th. Riessinger.

J. PETERS

On Stability of m-Variate C^1 Interpolation

A simplicial mesh (triangulation) is constructed that generalizes the two-dimensional 4-direction mesh to $\mathbb{R}^m$. This mesh, with symmetric, (2-) shift-invariant values at the vertices, is shown to admit a bounded C^1 interpolant if and only if the alternating sum of the values at the vertices of any (1-) cube is zero. This implies that interpolation at the vertices of an m-dimensional , simplicial mesh by a C^1 piecewise polynomial of degree $m+1$ with one piece per simplex is not stable.

M. J. D. POWELL

Tabulation of Thin Plate Splines on a Very Fine Two-Dimensional Grid

A thin plate spline approximation has the form

$$s(x) = p(x) + \sum_{j=1}^{n} \lambda_j \|x-x_j\|_2^2 \log\|x-x_j\|_2, \quad x \in \mathbb{R}^2,$$

where p is a linear polynomial and where $\{\lambda_j \in \mathbb{R} : j = 1,2,\ldots,n\}$ and $\{x_j \in \mathbb{R}^2 : j = 1,2,\ldots,n\}$ are parameters. There exist several applications that require s to be tabulated at all the lattice points of a very fine square grid. For example, 10^8 grid points and $n=500$ can occur, and then the direct evaluation of s at every grid point would be impracticable. Fortunately each thin plate spline term is smooth away from its centre x_j, so it is possible to apply a scheme that subtabulates by finite differences provided that special attention is given to those terms whose centres are close to the current x. Thus the total work is bounded by a small constant multiple of the number of grid points plus a constant multiple of $n\epsilon^{-1/6}|\log h|$, where ϵ is a given tolerance on the calculated values of $s(x)$ and where h is the mesh size of the fine grid. Further, the exponent $-1/6$ is due to the order of the differences that are employed. An algorithm for this calculation will be described and discussed and some numerical results will be presented.

T. RIESSINGER

Bases for Bivariate Spline Spaces

In order to construct interpolation sets for bivariate splines on certain regular grids it is necessary to construct a suitable basis of so-called cone-spline spaces. In fact, we consider the well-known bivariate truncated power functions being supported on cones, and give a modification of these functions that allows the construction of interpolation points in certain cones. The method is to subdivide a given cone in suitable subcones and to multiply truncated power functions by polynomial factors depending on the considered subcone.

R. SCHABACK

A Multi-Parameter Algorithm for Nonlinear Discrete L_2 Approximation

To solve $H_0(x) = \sum_{i=1}^{m} f_i^2(x) \to$ Min! over $x \in \mathbb{R}^n$ for smooth functions $f_i : \mathbb{R}^n \to \mathbb{R}$ in case of multiple local minima we propose to minimize $H_\lambda(x_1, \ldots, x_m) = \sum_{i=1}^{m} f_i^2(x_i) + \lambda \sum_{i=1}^{m} \|x_i - \bar{x}\|_2^2$, $x_i \in \mathbb{R}^n$, $1 \le i \le m$, $\bar{x} = \frac{1}{m} \sum_{i=1}^{m} x_i$ by a multistage algorithm:
1) Minimize $H_0(x_1, \ldots, x_m)$ on $\mathbb{R}^{nm}$
2a) Minimize $H_\lambda(x_1, \ldots, x_m)$ on $\mathbb{R}^{nm}$ for small $\lambda > 0$.
2b) Minimize $H_\lambda(x_1, \ldots, x_m)$ on $\mathbb{R}^{nm}$ for certain strategies that adjust λ to ensure $\lambda \to \infty$ and $x_i \to \bar{x}$, $1 \le i \le m$.
3) Minimize $H_\infty(x)$, starting at $\bar{x}$ from 2b).
This multi-parameter algorithm has a computational complexity that is only by a factor worse than the complexity of classical algorithms for minimizing H_∞ on $\mathbb{R}^n$. Several strategies for 2b) based on work of Nottbohm and Jäger are presented. A number of examples with special emphasis on rational approximation demonstrates the applicability of the method. Generalizations to vector- or matrix-valued λ's and to additively decomposable optimization problems are possible.

W. SCHEMPP

The Hexagonal Resistive Network and the Circular Approximation

The computation of the circular approximation of the hexagonal resistive network can be reduced to the very efficient algorithm with error control of the arithmetic and geometric mean for computing complete elliptic integrals of the first and second kind, and the stepwise evaluation of a three-term recurrence relation. For computer vision and pattern recognition, the hexagonal resisitive network has been implemented in silicon by a VLSI retina. An extension of the hexagonal network to holographic fractals and their relation to adaptive wavelets are also pointed out.

J. W. SCHMIDT

Histogram Smoothing under Constraints

In this talk we consider the problem to approximate a histogram $F = \{f_1, \ldots, f_n\}$ given on the mesh $\Delta = \{x_0 < x_1 < \ldots < x_n\}$ by quadratic splines s under constraints like convexity, monotonicity, or positivity. For smoothing F, it is proposed to minimize the functional

$$K_2(s) = l \int_{x_0}^{x_n} s'(x)^2\, dx + \sum_{i=1}^{n} p_i \left(f_i - \frac{1}{h_i} \int_{x_{i-1}}^{x_i} s(x)\, dx \right)^2 .$$

Here $l > 0$ is a global parameter, $p_1 > 0, \ldots, p_n > 0$ are local parameters, and $h_i = x_i - x_{i-1}$. The feasible functions s are assumed to be quadratic C^1-splines on Δ, or on coarser meshes. In this way, we are led to quadratic programs of the partially separable structure

$$\text{minimize} \sum_{i=1}^{n} F_i(y_{i-1}, m_{i-1}, y_i, m_i) \quad \text{such that } (y_{i-1}, m_{i-1}, y_i, m_i)^T \in W_i,\ i = 1, \ldots, n,$$

where the variables are $y_i = s(x_i)$, $m_i = s'(x_i)$, $i = 0, \ldots, n$. To these programs belong duals of the same structure which have the advantage to be unconstrained. Thus, the general strategy is to solve the dual programs numerically and then to return to the constrained original program by means of an explicit formula. Some graphically illustated examples are given.

H.-P. SEIDEL

The Curry-Schoenberg Theorem for Multivariate B-Splines

The classical Curry-Schoenberg Theorem is generalized to the multivariate setting. It is shown that every C^k-continuous piecewise polynomial F of degree n over an arbitrary triangulation $T = \{\Delta(I) \mid I \in \mathcal{J}\}$ of $\mathbb{R}^s$ can be written as linear combination of the normalized multivariate B-splines $\{N_\lambda^I\}_{I \in \mathcal{J}, |\lambda|=n}$ over an appropriate knot sequence $\mathcal{K}$, where every vertex of the given triangulation T appears as knot of multiplicity $m = n - k$. The coefficients $\mathbf{c}_{I,\lambda}$ in the resulting representation $F(\mathbf{u}) = \sum_{I,\lambda} N_\lambda^I(\mathbf{u})\mathbf{c}_{I,\lambda}$ are given as $\mathbf{c}_{I,\lambda} = f_I(\mathbf{t}_{i_0,0}, \ldots, \mathbf{t}_{i_0,\lambda_0-1}, \ldots, \mathbf{t}_{i_s,0}, \ldots, \mathbf{t}_{i_s,\lambda_s-1})$, where f_I is the polar form of the restriction F_I of F to the simplex $\Delta(I) = [\mathbf{t}_{i_0}, \ldots, \mathbf{t}_{i_s}]$.

J. STÖCKLER

Multivariate Wavelets

The general concept of multivariate wavelets is described with emphasis on the construction procedure. Certain symbol matrices are useful to identify special properties of a wavelet basis, for example symmetry and the order of vanishing moments. An explicit construction of compactly supported symmetric wavelets based on the multiresolution analysis of box splines in any dimension is given.

C. R. TRAAS

Construction of Monotone Extensions to Boundary Functions

Let be given monotonically increasing smooth univariate functions along the edges of the unit square such, that the functions have the same values in common points, the point $(0,0)$ takes the global minimum and the point $(1,1)$ the global maximum. The problem is to construct a C^1 extension of these boundary functions to the interior of the square such that the resulting function $F(x,y)$ is monotone, in the sense that $\partial F/\partial x > 0$,, $\partial F/\partial y > 0$ for every point in the square. (Of course the boundary functions must be compatible with such F). It has been shown (Micchelli, Dahmen and deVore) that no linear method exists for constructing F. A nonlinear method is presented which defines F in terms of a set of level lines, each of which is represented as a cubic Bézier curve. Considering a sequence of these level lines, from the point $(0,0)$ up to the point $(1,1)$, the controlpoints b_0 and b_3 shift along the edges of the square, and b_1 and b_2 along certain adopted trajectories inside the square. These internal trajectories contain kinks in order to assure the C^1 property of F. The kinks compensate for the shifting of b_0 and b_3 around the corners $(1,0)$ and $(0,1)$, respectively.

F. I. UTRERAS

Convergence Rates for Smoothing with Radial Basis Functions

We consider the problem of smoothing of noisy data coming from evaluations of a "smooth" function f in the d-dimensional space, using radial basis functions. To do this, we study in detail the structure of the semi-Hilbert spaces that provide a variational characterization of the radial basis functions interpolation and smoothing. For the case of basis functions coming from semi-elliptic pseudo-differential operators, we prove that the spaces involved are indeed Besov spaces of the appropiate order. Moreover, we obtain that if the errors are random variables with zero mean i.i.d., the error can be given by

$$E[|f - S_{h,n,\lambda^*}|^2_{\theta,\Omega}] = O(n^{-\frac{\mu-2\theta}{\mu+d}}),$$

where μ is the order of the operator, h is the radial function, n is the number of data points, λ^* is the optimal smoothing parameter and $S_{h,n,\lambda}$ is the smoothing spline defined with h, attaining thus the same convergence rates as those given by the author in the case of thin plate splines of fractional order.

G. Wahba

Analysis of Variance in Function Spaces

Let $\mathcal{H}$ be a reproducing kernel Hilbert space of real-valued functions on $\tau = \bigotimes_{\alpha=1}^{d} \tau^{(\alpha)}$. Let $y_i = f(\psi_1(i), \ldots, \psi_d(i)) + \epsilon_i$, $i = 1, \ldots, n$ where the ϵ_i are independent zero mean random variables with common (possibly unknown) variance. Given an averaging operator $\mathcal{E}_\alpha$ on continuous functions on $\tau^{(\alpha)}$, that is $\mathcal{E}_\alpha f = \int f(x_\alpha)\,d\mu_\alpha$, where $d\mu_\alpha$ is a probability measure, and assuming that $\prod_{\alpha_l} \mathcal{E}_{\alpha_l} f \in \mathcal{H}$ where $\prod_{\alpha_l}$ is the product over any subset $\{\alpha_l\}$ of $\alpha_1, \ldots, \alpha_d$, is in $\mathcal{H}$, then we have the ANOVA decomposition of $f \in \mathcal{H}$ as $f = \mu + \sum_\alpha f_\alpha(x_\alpha) + \sum_{\alpha<\beta} f_{\alpha\beta} + \cdots + f_{1,2\ldots d}$ resulting from the decomposition $I = \prod_1^d \Big(\mathcal{E}_\alpha + (I - \mathcal{E}_\alpha)\Big)$. Here $\mu = \prod_1^d \mathcal{E}_\alpha f$, $f_\alpha = (I - \mathcal{E}_\alpha)\prod_{\beta\neq\alpha} \mathcal{E}_\beta f$, $f_{\alpha\beta} = (I - \mathcal{E}_\alpha)(I - \mathcal{E}_\beta)\prod_{\gamma\neq\alpha,\beta} \mathcal{E}_\gamma f$, etc. If $\mathcal{H}$ is the tensor product $\mathcal{H} = \bigotimes \mathcal{H}^\alpha$, the above decomposition is orthogonal. We discuss methods and computer programs for fitting smoothing generalized splines given the y_i in subspaces corresponding to the first few terms of the ANOVA decomposition.

G. Walz

Recursion schemes and B-Splines

In many branches of numerical analysis there appear linear transformations of the form $T_\nu^k = \sum_{i=\nu}^{\nu+k} \alpha_{i,\nu}^k T_i$, where $\{T_i\}$ is a given sequence of numbers or vectors, and $\{\alpha_{i,\nu}^k\}$ is a set of prescribed coefficients. A special case of such transformations are the so-called *triangular recursion schemes* of the type

$$T_\nu^m = \lambda_\nu^m T_\nu^{m-1} + \mu_\nu^m T_{\nu+1}^{m-1}, \qquad m = 1, \ldots, k$$

with $T_i^0 := T_i$. In our talk we analyse several properties of recursion schemes of this form and illustrate them by means of the recursion formula for polynomial B-splines, which is of the type under consideration. The results on which this talk is based on were worked out jointly with C. Brezinski (Lille).

Report by G. Baszenski, Dortmund

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