MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 53/1991

Stochastic Geometry, Geometric Statistics, Stereology

8.12. bis 14.12.1991

Die Tagung fand unter der Leitung von R.E. Miles (Queenbeyan), E.B. Vedel Jensen (Aarhus) und W. Weil (Karlsruhe) statt. Sie hatte 42 Teilnehmer, die alle einen Vortrag hielten. Gegenüber früheren Veranstaltungen über das gleiche Thema fiel diesmal das große Interesse an der Tagung auf, das dazu führte, daß eine Reihe von Interessenten nicht eingeladen werden konnte.

Die Vorträge beschäftigten sich fast ausschließlich mit aktuellen Forschungsergebnissen und dokumentierten die ganze Breite der drei Gebiete Stochastische Geometrie, Geometrische Statistik und Stereologie. Dabei wurden auch benachbarte Gebiete wie die Integralgeometrie oder die Geometrie der Fraktale gestreift. Erwähnenswert ist, daß diesmal verstärkt Vorträge aus dem Bereich Stereologie gehalten wurden, die direkt Anwendungen in der Medizin und den Materialwissenschaften betrafen.

Der internationale Teilnehmerkreis unterstreicht die wachsende Bedeutung, die der Stochastischen Geometrie und ihren Anwendungsgebieten zukommt.

Vortragsauszüge

R.V. Ambartzumian:

Measure generation in the space of lines in $\mathbb{R}^3$

In the book by the author "Combinatorial Integral Geometry" (Wiley, 1982) several functionals have been proposed which under certain conditions admit continuation to measures in the spaces of hyperplanes in $\mathbb{R}^n$. Some of these ideas can be also applied to construction of measures in the spaces of flats less than $n-1$ in dimension. The paper presents a number of theorems concerning the case of lines in $\mathbb{R}^3$. As a corollary the following result is obtained:

To any linearly additive smooth metric in $\mathbb{R}^3$ corresponds a measure in the space of lines in $\mathbb{R}^3$ whose image under the map

line $\longrightarrow$ point of its intersection with a plane

on each plane generates a measure on the same plane born by the restriction of the original metric to the plane in question.

A.J. Baddeley:

Hausdorff metric for capacities

We generalise the Hausdorff metric to spaces of capacities (increasing, outer-regular set functions mapping the empty set to zero) and verify that it generates the appropriate topologies (sup-vague and sup-weak). Under natural identifications it coincides with the Hausdorff metric for compact sets, the Lévy-Prohorov metric for weak convergence of probability measures, and a quantity used in defining Skohorod's M_2 topology for $D[0,1]$.

Applications to stochastic geometry are sketched.

I. Bárány:

Approximation by random polytopes

Given a convex compact body $K \subset \mathbb{R}^d$ with vol $K = 1$, the random polytope K_n is defined as the convex hull of n points chosen randomly, independently, and uniformly from K. For large n, K_n approximates K with high probability. We measure this approximation by the expectation of the volume of $K \setminus K_n$, which we denote by $E(K,n)$. When $d = 2$ and $n = 3$, Blaschke proved that

$$E(\Delta^2,3) \leq E(K,3) \leq E(B^2,3)$$

where Δ^2 and B^2 are the triangle and the ball (of area 1), resp. The right hand inequality was extended by Groemer to any d and n, but the conjecture $E(\Delta^d,n) \leq E(K,n)$ is open. We prove that

$$\liminf_n \frac{E(K,n)}{E(\Delta^d,n)} \geq 1 + \frac{1}{d+1}$$

unless K is a simplex. The proof is based on the following asymptotic formula. If $P \subset$

$\mathbb{R}^d$ is a convex polytope with volume one, then

$$E(P,n) = \frac{T(P)}{(d+1)^{d-1}(d-1)!} \frac{(\log n)^{d-1}}{n} (1 + o(1))$$

where $T(P)$ stands for the number of towers of P, i.e., the number of chains $F_0 \subset F_1 \subset F_2 \subset \ldots \subset F_{d-1}$ where F_i is an i-dimensional face of P.

(Joint work with C. Buchta)

V. Beneš:

Anisotropy in systems of particles

The lecture presents some ideas concerning the use of marked point processes in the modelling of anisotropic particle structures. It includes processes of ellipsoidal marks [Møller, 1988; Beneš, 1989a], simultaneous distribution of anisotropy of particle orientation and of spatial dispersion [Beneš, 1989b]. Anisotropic segment processes are recently investigated from the point of view of weighted mark correlation analysis and nearest neighbour orientation analysis [Stoyan, 1990; Stoyan and Beneš, 1991].

Beneš V. [1989a] Acta Stereol., 8/2, 701.
Beneš V. [1989b] Geobild '89, ed. Hübler A. et al., Akademie–Verl. Berlin, 135.
Møller J. [1988] J. Appl. Probab., 25, 332.
Stoyan D. [1990] Statistics, 3, 449.
Stoyan D., Beneš V. [1991] J. Microscopy, to appear.

A. Cabo:

Convex hulls II

See P. Groeneboom.

R. Coleman:

Vertical sections

The decomposition of the invariant random (IR) measure for lines in R_3 into an IR measure for lines in a horizontal plane and a sinα-weighted IR measure in the vertical plane is demonstrated. (α is the latitude of the line with respect to the horizontal.) This gives a procedure for the stereological sampling by IR lines (Baddeley, Oberwolfach 1983).

If a specimen is within a sphere, then an IR line section of the sphere which intersects the specimen is IR within the specimen. We can therefore restrict our considerations to spheres. The projection of an IR line section of a sphere gives a line in the horizontal plane which is from a length–weighted density. This apparently contradicts the decomposition result.

R. Cowan:

Topological aspects of cell divisions

We postulate an imaginary organism which grows in a monolayer on a planar medium. It starts from one cell which has a convex polygonal shape with k sides. The cell divides by connecting two sides by a division line. In the 'wild–type', each of the $\binom{k}{2}$ choices is equally likely. After the first division, the two polygonal daughter cells divide synchronously according to the same mechanism. Cell divisions proceed in this manner with synchronous but independent divisions.

Let X_n be the number of sides of a randomly chosen polygon from generation n (the first cell is generation 0). It is easily shown that $\mathbb{E}\, X_n \longrightarrow 4$ as $n \longrightarrow \infty$, but most interestingly: $X_n - 3 \xrightarrow{distrib.}$ Poisson with mean 1 . Methods of proof involve the recognition of a multitype branching process of cell types.

The talk also examined many variations of dividing rule, with equilibrium distributions given in each case. These 'mutant' versions included x–philic mutants (where a x–daughter is always created where possible), x–phobic mutants (where x–daughters are avoided) and x–selfish mutants (where x–type cells refuse to divide to ensure there own survival). Subtle competition between 'cliques' of cell types were explored.

R. Cowan & V.B. Morris (1988) J. Theor. Biol. **131** 33–42

R. Cowan (1989) Adv. Appl. Prob. **21** 233–234

L.M. Cruz–Orive:

The star–volume distribution: an application to duplex stainless steel

The motivation of this study was how to characterize the microstructure of a two-phase material (duplex stainless steel). The non–metallic inclusions of the material are rather anisotropic, and they can be modelled by a stationary process $\Phi = \{Y\}$ of bounded particles in $\mathbb{R}^3$. (It is however suggested that only 'star–boundedness' is necessary.) A useful, well-known descriptive measure for Φ is its 'star volume' $\overline{v}_V^*$. The 'star-volume distribution' $\overline{v}_V^*(u)$, ($u \in$ unit hemisphere) is defined by not averaging the relevant integrand over directions.

It is shown how to estimate $\overline{v}_V^*(u)$ from vertical sections. From the empirical $\overline{v}_V^*(u)$, a final descriptive measure is estimated, namely the ellipsoid of inertia, with its principal axes along the natural axes of anisotropy of Φ. Steels processed in three different ways are thereby compared, with a final interpretation difficult to guess before seeing the data.

An open problem is how to predict error variances from systematic observations of a bounded function defined on the unit sphere – even on the unit circle!

(Joint work with L.M. Karlsson, S. Larsen, and F. Wainschtein)

R. Dwyer:

On the convex hulls of random balls

While the convex hull of n d-dimensional balls in $\mathbb{R}^d$ is not a polytope, it does have an underlying combinatorial structure similar to a polytope's. In the worst case, its combinatorial complexity can be of order $\Omega(n^{\lfloor d/2 \rfloor})$. The thrust of this work is to show that its complexity is typically much smaller, and that it can therefore be constructed more quickly on average than in the worst case. To this end, three models of random d-balls are developed, and the expected combinatorial complexity of the convex hull of n independent random d-balls is investigated. For one model, this expectation is $O(1)$ as n grows without bound. For another, it is $O(n^{(d-1)/(3d)})$. The third model is analyzed only for $d = 2$; the expected combinatorial complexity is $O(1)$.

(Joint work with F. Affentranger)

W.F. Eddy:

A convex core algorithm

Problem: Given n points in the interior of a circle, find the largest (in area) empty convex subset. The solution set consists of arcs of the circle and straight lines. Each straight line is determined by one or two points. A simple variational argument shows that for edges determined by a single point, the point must lie at the midpoint of the edge. An explicit construction of the solution is given for $n = 1,2,3$ and for $n = 2$ a map is given describing the structure of the solutions. For $n = 4$ it is shown that the solution is a quadrilateral only for very special configurations of the points.

E. Enns:

<u>Some nearest neighbor families</u>

Randomly generated points in $\mathbf{R}^d$ are connected to their nearest neighbors. These points (called individuals) form connected clusters (called societies). If one has n points generated in such a way that nearest neighbors are uniquely defined, then let:

$$M_n = \text{the number of societies formed, } 1 \leq M_n \leq [n/2] ,$$

and if $M_n = m$, $\quad K_j$ = the number of individuals in the j^{th} society, $\sum_{j=1}^{m} K_j = n$.

Form an enclosure process, such as a convex hull or smallest enveloping sphere about each society. What is the distribution of the volume of an enclosure, or inhabited region? If $n \longrightarrow \infty$ so one has a constant density p of points in $\mathbf{R}^d$, then what fraction of space is inhabited? If M' is the number of societies per unit volume, then $M' = p \lim M_n/n$. For an enclosure process there is a possibility that a society of size $K_j = k$ embeds another. What is this probability?

Within a society of size $K_j = k$, let V_i = the number of individuals that consider the i^{th} individual as their nearest neighbor. We classify an individual by his number V_i, so that:

		Individual classified as	set of such individuals	number of individuals in set	
if	$V_i=0$	Lonely	$\mathscr{L}$	L_k	$\sum_{\mathscr{L}} V_i = 0$
	$V_i=1$	Normal	$\mathscr{N}$	N_K	$\sum_{\mathscr{N}} V_i = N_k$
	$V_i \geq 2$	Friendly	$\mathscr{F}$	F_k	$\sum_{\mathscr{F}} V_i \geq 2F_k$

Then $\sum_{i=1}^{k} V_i = k = L_k + N_k + F_k \geq N_k + 2F_k$ or $L_k \geq F_k$.

Also $V_i \leq f(d)$ where $f(1) = 2$, $f(2) = 5$, $f(3) = 11$ etc.

The presentation gave more specific results for $d = 1$ when points are generated on a line from either a uniform distribution or a Poisson process. (These are equivalent, see Pyke 1965 JRSS, B.) Results include the distributions of M_n and F_k, for example

$$\sum_{n=2}^{\infty} z^n \sum_{m=1}^{[n/2]} s^m \, P(M_n = m) = \frac{sz}{\sqrt{1-s}\,\coth(z\sqrt{1-s}) + 1} .$$

P. Goodey:

Determination of convex sets from random sections

We consider random k-dimensional sections of a convex body K in d-dimensional Euclidean space. The classical Crofton formula expresses the size of K as an average of the size of sections of K. We shall investigate the possibility of retrieving the shape of K from the "average shape" of its sections. We show that, in case $k = 2$, the shape of K is determined by that of its sections, whereas, for $k = 1$, this is not true.

(Joint work with W. Weil)

P. Groeneboom:

Convex hulls I

A. Cabo:

Convex hulls II

We study the limiting behavior of functionals of convex hulls of samples in $\mathbb{R}^d$. An example of such a functional is the number of vertices of the convex hull. It is shown that in the plane one can obtain limit results for these functionals by studying a Markov process, generating the extreme points of a Poisson point process, which is the local limit of a part of the original sample process.

Furthermore, it is shown that in $\mathbb{R}^3$ similar techniques can be used. For example, the study of the local limiting behavior of the convex hull of a random sample of points in the interior of a ball leads in a natural way to the study of extreme points of a Poisson point process inside a paraboloid. In this case one can associate with each realization of the Poisson point process a random tessellation of the plane, where the insides of the polygons of the tessellation correspond to directions of planes of support hitting only one point of the Poisson process. A sketch is given of a method for recovering properties of the convex hull, walking along a test line through the tessellation.

Finally we apply the method mentioned above to a uniform sample from the interior of a convex polygon in the plane. It is shown that it allows us to obtain limiting results for the remaining area and length of the boundary. For the remaining area we prove a central limit theorem. However, the length of the boundary has a different behavior. A characterization of the limiting behavior is given in terms of a functional of a Poisson process and the first two moments of the limiting distribution are explicitly determined.

H.J.G. Gundersen:

Estimation of connectivity

The stereological estimation of the Euler–Poincaré characteristic or the Euler number, χ, seems traditionally to have been made through the relationship to the integral of Gaussian curvature, C : $C = 4\pi\chi$. However, all estimators of curvature naturally require isotropy, which seems an unnecessary complication for the estimation of the integer–valued, total Euler number. Moreover, the traditional approach

$$\text{est } \chi = \text{est } \chi_V \cdot V$$

with a separate estimator of the density χ_V and of the total specimen volume, V, is met with several problems in finite and inhomogeneous specimens.

A more direct approach is the traditional estimator

$$\text{est } \chi = \sum \chi_i \cdot \prod_j f_j^{-1}$$

where the specimen is split or partitioned in a completely arbitrary but known way, one then samples in j steps a uniform fraction f_k of the pieces. In each of the small pieces in the final sample, one evaluates the Euler number. The contribution from the artificial surfaces, edges, and corners is always known for a known partition. If the partition is the simple one produced by three roughly orthogonal set of planes, with no intersection within a set, the real Euler number of a little slab, s, is

$$\chi_s = \chi_3 - \frac{\chi_2}{2} - \frac{\chi_1}{4} - \frac{\chi_0}{8}$$

where χ_2 through χ_0 are the Euler numbers of the artificial surfaces, edges and corners, respectively. An alternative correction under translation is given by Prassad et al., Acta Stereologica **8**, 101–106 (1989). With the additivity of the Euler number and preserving strictly the contribution from each piece to the total Euler number, the fractionator estimate is unconditionally unbiased. Biological examples of capillary and bone trabicular networks are presented.

J. Hüsler:

Convex hulls of random points

Consider the convex hull C_n of a sample of n randomly placed points in a unit circle. To construct efficiently the convex hull, not all points are needed. We show that asymptotically only the points belonging to the ring $K_1 \setminus K_r$ are used to compute the convex hull, where K_r denotes a circle with radius r and where $r = r(n) \longrightarrow 1$ as $n \longrightarrow \infty$. We prove the relation between the rate of $r(n) \longrightarrow 1$ and the error probability of not correct construction of the convex hull C_n by the random subset of points, tending to 0. A simpler method is also discussed which can be easily used for more general cases, where the points are not uniformly distributed in K_1.

K. Kiêu:

A stereological formula involving non-uniform sampling

The structure of interest is supposed to be a surface φ (with integer dimension d) in the Euclidean space $\mathbb{R}^n$. The observed sample is the intersection $\varphi \cap \psi$, ψ being a random p-dimensional surface. The random surface ψ is supposed to be such that there exists a function f with

$$\mathbf{E}\, \lambda^p(\psi \cap A) = \int_A f(x)\, \lambda^n(dx) \quad \text{for all } A \subset \mathbb{R}^n,$$

λ^i, $i \leq n$, being the i-dimensional Hausdorff measure.

The following formula is discussed

$$\mathbf{E} \int_{\varphi\cap\psi} h(x,\psi)\, \lambda^{d-n+p}(dx) = \int_{\varphi} \mathbf{E}_x[h(x,\psi) G(\varphi,\psi,x)]\, f(x)\, \lambda^d(dx)\,,$$

where $\mathbf{E}_x$ denotes the mean operator under the Palm distribution of ψ at x and $G(\varphi,\psi,x)$ is the $(n-p)$-dimensional volume of the projection onto the orthogonal of the tangent of ψ at x of a unit cube in the tangent of φ at x.

The case where $f(x) \equiv 1$ has been considered in Zähle (1982). The proof for a non stationary ψ is discussed in the case where ψ is parametrized by a point of a surface.

Also, the use of the formula is discussed in the case where φ is the product of a surface $\tilde{\varphi}$ with itself and ψ is the product of a random surface $\tilde{\psi}$ with itself. In general, ψ cannot be assumed to be stationary. Then the formula provides results of interest for second-order stereology. In particular, the stereological formulas for second-order properties of planar curves, presented in Ambartzumian (1981) and Stoyan (1981), appear to be particular cases of this use of the general formula.

D. Mannion:

Products of 2×2 random matrices and sequences of random triangle shapes

A sequence of random triangle shapes is obtained by iterating: choose three points at random in the interior of a triangle to form the next generation triangle. This process may be represented by a product of i.i.d. 2×2 random matrices. It is also possible to represent a 2×2 matrix by a triangle, and thus to define the 'shape' of a matrix. Thus products of i.i.d. 2×2 random matrices may be represented by a sequence of random triangle shapes. This gives a new way of exploring the asymptotic behaviour of products of random matrices. In particular, a more tractable formula is derived for the upper Ljapounov exponent. This shape approach also holds in higher dimensions.

S. Mase:

On asymptotic equivalence of grand canonical MLE and canonical MLE for Gibbsian point process models

Consider a random point pattern X_Λ on a bounded region Λ and we want to fit the Gibbsian point process model to this pattern. For each fixed potential function $\Phi(r)$, we parametrize the local energy of X_Λ as

$$z\cdot\#X_\Lambda + \alpha\sum\{\Phi(|x-y|);\ x,y\in X_\Lambda,\ x\neq y\},$$

where z is the chemical potential and α is the inverse temperature (in the physical context). Basically it is natural to consider that the point number $\#X_\Lambda$ is random and varying and we need to estimate both z and α (the grand canonical Gibbsian model). But if our main interest is in the parameter α, we can take the conditional point of view by fixing the point number and get the canonical Gibbsian model which includes only the parameter α.

Let $(\hat{z},\hat{\alpha})$ be the grand canonical MLE of (z,α) and let $\tilde{\alpha}$ be the canonical MLE of α. We can show that the asymptotic variances of $\hat{\alpha}$ and $\tilde{\alpha}$ are the same (at least if z and/or α are small enough).

T. Mattfeldt:

Number and spatial arrangement of particles within biological membranes

Freeze fracture is a preparative laboratory method by which membranes of biological structures (cells, mitochondria, mitoplasts) can be split parallel to their surface. In the electron microscope, such preparations show particles on a flat smooth background. These intra-membraneous particles (IMPs) presumably consist of proteins and constitute important functional elements of the membrane, whereas the background represents largely lipids. It is difficult to judge for the human mind whether the IMPs are arranged purely at random, in clusters, or in a pattern with mutual repulsion.

It was the aim of this study to develop methods for the quantitative analysis of IMPs under experimental conditions. An algorithm for the automatic detection of IMPs using an image analyzer is described. This algorithm provided the coordinates of the IMPs. From the empirical data, we determined the number of IMPs per unit area of membrane. In addition, the pair-correlation function $g(r)$ and the reduced second-order moment measure function $K(r)$ of the IMPs were determined, where r represents the Euclidean distance between different IMPs. The empirical estimates of $g(r)$ and $K(r)$ were checked versus the null hypothesis of a stationary Poisson point process in the plane, where $g(r) = 1$ and $K(r) = \pi r^2$. As estimates were available with replication (different cells and individuals), confidence intervals were calculated directly from the empirical data, which obviated the need for Monte Carlo simulations of point patterns.

(Joint work with H. Frey, I. Pavenstädt-Grupp, and O. Haferkamp)

J. Mecke:

Extremal properties of flat processes

Stationary Poisson k-flat processes in the d-dimensional Euclidean space are considered ($d/2 \leq k \leq d-1$). The mean k-content of the process per unit volume is said to be the intensity λ, and the mean $(2k-d)$-content per unit volume of the 2-intersection manifold is called intersection density σ. The problem is to maximize σ for given λ by a suitable choice of the directional distribution of the process. The maximal σ is known for all pairs (d,k) where $d-k$ is a factor of d, in some cases also the corresponding directional distribution can be described.

R.E. Miles:

Homogeneous rectangular tessellations

Homogeneous random tessellations (HRT's), in which every cell is a rectangle, are considered. Vertices are of T– or X–type. An anthropological study by H. McEldowney (Hawaii) shows that the former may throw light on the chronology of the formation of rectangle boundaries. Beyond homogeneity it is natural to also suppose isotropy, i.e. stochastic invariance under both $x \longleftrightarrow y$ and $x \longleftrightarrow -x$. Homogeneity $\Rightarrow$ $\bigcup$ (rectangle sides) = $\alpha \cup \beta$ where $\beta = \bigcup$ (lines) and $\alpha = \bigcup$ (bounded segments). The unions of collinear connected rectangle sides in α are called I–segments. Deletion of β yields a HRT, with $\bigcup$ (rectangle sides) = $\bigcup$ (I–segments). A first order theory shows that all 1st order moments of interest are expressible in terms of just 3 quantities: C_I = I–segment intensity, $\bar{I}$ = mean I–segment length and $\bar{N}_x$ = mean number of T–vertices on a random I–segment.

Specific models for HRT's are presented. Gilbert's needle model is specialized to this case, but seems intractable due to the 'blocking effect'. However, it has been simulated by M. Mackisack (QUT, Brisbane); bunches of close parallel I–segments therein may be avoided by starting with $\top$– rather than $-\bullet-$–type 'seeds'. The corresponding closely approximating Gilbert penetration model admits a full analysis of rays and offsets.

A final model for HRT is that of 'growing squares', the rectangles of which contain either 1 or 0 of the initial square centre–points. This too has been simulated by M. Mackisack.

I.S. Molchanov:

Statistics of random sets: empirical capacities approach

The approach to statistics of random sets based on empirical capacity functionals is proposed. Its mathematical ground is formed by the Glivenko–Cantelli theorem and the functional limit theorem for empirical capacities. It is shown that this approach allows to unify many previously obtained estimators and to derive new estimators for the Boolean model parameters. In particular, new estimators for the shape of a non–random grain and the size distribution for the randomly scaled typical grain are discussed. The empirical capacities approach is effective in handling with noisy or spatial censored observations (in case the observed image is modified by another random closed set). The notion of quantiles of random sets is introduced and their estimators are considered. General properties of set-valued estimators (bias, consistency, variance) and the maximum likelihood method for random sets are also discussed.

J. Møller:

Johnson-Mehl tessellations: exact and numerical results and simulations

A unified exposition of random Johnson-Mehl tessellations in d-dimensional Euclidean space is presented. In particular, Johnson-Mehl tessellations generated by time-inhomogeneous Poisson processes and nucleation-exclusion models are studied. The 'practical' cases $d = 2$ and $d = 3$ are discussed in detail.

The talk consists of two points based on [1] and [2], respectively.

Part I. Analytic results: Several new results are established including first and second order moments of various characteristics for both Johnson-Mehl tessellations and sectional Johnson-Mehl tessellations.

Part II. Simulations: An efficient simulation procedure which generates 'typical' crystals is discussed and some empirical results which illuminate the effect of nucleation-exclusion conditions is presented.

[1] Møller, J. (1992): Random Johnson-Mehl tessellations. Adv. Appl. Prob. To appear.

[2] Møller, J. (1992): Generation of Johnson-Mehl crystals and comparative analysis of models for random nucleation. In preparation.

F. Montes:

Random sets and coverage measures

It is well known that a random set determines its coverage measure. The talk gives a necessary and sufficient condition for the reverse implication. We introduce the concept of random closed support for any random measure and, using it, an equivalent formulation of the above condition. This alternative formulation constitutes a first step in the search for a way to recognize a random measure as being the random coverage measure of a random set.

The talk is completed with a proposition allowing to construct, for any random set, smoothed version verifying the condition and final considerations about the extension of the results and a "natural" generalization of the definition of coverage measure.

(Joint work with G. Ayala and J. Ferrandiz)

W. Nagel:

Covariograms of convex polygons

For compact subsets X of the euclidean space $\mathbb{R}^n$, Matheron's covariogram $C(X,\cdot)$ is defined by

$$C(X,h) = V(X \cap (X+h)),\ h \in \mathbb{R}^n,$$

where V denotes the volume. The covariogram is (up to the factor $V^2(X)$) the density of $x-y$, where x and y are random points which are independent and uniformly distributed on X. If X is convex, then $C(X,\cdot)$ is related to the family of orientation dependent chord length distributions (or the joint distribution of direction and length of random chords resp.).

Theorem: Let $X_1, X_2 \subset \mathbb{R}^2$ be compact convex sets, X_1 a polygon. If $C(X_1,\cdot) \equiv C(X_2,\cdot)$, then there is a $v \in \mathbb{R}^2$ such that $X_1 = X_2 + v$ or $X_1 = -X_2 + v$ (i.e. C determines convex polygons up to translation and reflection).

Two ways of generalizing the covariogram were discussed.

T. Norberg:

Ordered couplings of random sets

Let φ and ψ be two given random sets. By a coupling of φ and ψ we understand a pair $(\hat{\varphi},\hat{\psi})$ of random sets based on the same probability space and such that $\hat{\varphi} \stackrel{d}{=} \varphi$ and $\hat{\psi} \stackrel{d}{=} \psi$. Couplings always exist. Take, e.g., $\hat{\varphi}$ and $\hat{\psi}$ independent. A coupling is ordered, if $\hat{\varphi} \subseteq \hat{\psi}$ a.s., and we indicate the existence of an ordered coupling of φ and ψ by writing $\varphi \subseteq_{st} \psi$.

In the talk we describe a necessary and sufficient condition for the existence of an ordered coupling of two given random closed sets. The basic topological space in which the random closed sets live is assumed to be locally compact and second countable.

There are similar results in the cases when the random sets are compact saturated or compact or compact convex.

V.K. Oganian:

On configurations generated by random chords of a planar convex domain

Let D be a bounded convex domain in $\mathbb{R}^2$ and ∂D be its boundary. Denote by G the space of lines in $\mathbb{R}^2$, $[D] = \{g \in G : g \cap D \neq \emptyset\}$ and $C_n = \{(g_1,\dots,g_n)$: there exists a pair of lines g_i, g_j for which $g_i \cap g_j \cap \partial D \neq \emptyset\}$. The set $[D]^n \setminus C_n$ is the union of a finite number of connected subsets from G^n, i.e. $[D]^n \setminus C_n = \bigcup_k B_{nk}$, where $B_{ni} \cap B_{nj} = \emptyset$, $i \neq j$. There are only finitely many such sets B_{nk}, some being equivalent up to permutation of the lines. Let A_{nk} be the union of all equivalent B_{nk}. The A_{nk} will be called components. Denote by $\mu_n = \underbrace{\mu\times\cdots\times\mu}_{n \text{ times}}$, where μ is the measure on G which is invariant with respect to the group of Euclidean motions of $\mathbb{R}^2$. As $\sum_k \mu_n(A_{nk}) = |\partial D|^n$, we can say about the probabilities of A_{nk} : $p_{nk} = \mu_n(A_{nk})\cdot |\partial D|^{-n}$. The main result is to obtain expressions for p_{nk} $(n = 3,4)$. These expressions have a form of linear combinations of some integral parameters which depend on D. An example of the integral parameters is $I_n = \int_{[D]} \chi^n(g)\, d\mu(g)$ (n-th moment of the chord $\chi(g) = g \cap D$, introduced by W. Blaschke). The question arises: What information carry the probabilities p_{nk} about the convex domain D? This problem had been solved after some so-called additional relations between the integral parameters.

Y. Ogata:

Space-time evolution of magnitude frequency distribution inferred from earthquake catalogs

Lists of earthquakes are published regularly by the seismological services of most countries where earthquakes occur at all frequently. These lists give epicenter of each shock, focal depth, origin time and instrumental magnitude. Empirical law for magnitude frequency suggests the exponential distribution for shocks above a threshold magnitude where almost shocks are detected. The parameter of the exponential distribution is known to vary dependent on time and space. On the other hand, detection rate of shocks varies depending on the magnitude, time and space. In the talk, a model is given which simultaneously analyzes these dependencies by the likelihood-based inferences. Applications of the model and method to the earthquake data in and around Japan are also presented.

J. Ohser:

Variances of different estimators for the specific line length

The usual method to estimate the specific line length L_A of stationary and isotropic fibre processes is to measure its total line length $U(W)$ in a sampling window W and to divide $U(W)$ by the area of the sampling window. This simple method can be modified in several ways. Such modified estimators are used e.g. in image analysis.

On the base of the second order theory of stationary and isotropic random measures the estimation variance is computed for general classes of estimators of L_A. Analytical results can be obtained in the case of isotropic Poisson line processes. From these results improved estimators of L_A are derived.

D. Pfeifer:

Time dependent pattern processes

A statistical analysis of random spatial patterns in marine or terrestrial ecosystems often requires the simultaneous consideration of time, space and migration. For instance, the spatial distribution of birds or geese in a certain observation area is varying over time due to flights (incoming/outgoing) and individual movement on the ground; similarly, the spatial distribution of sand worms in the wadden sea depends on death, birth and migration of larvae. Here we discuss the most simple case of modelling such phenomena as a time-dependent (Markovian) spatial birth-death-Poisson process for which limiting distributions over time are readily available.

J.P. Rasson:

Clustering and discriminant analysis based on Poisson point processes

The clustering rule based on maximum likelihood estimations under Poisson process hypothesis is this one: find the partition into k clusters such that the sum of the Lebuesgue measures of their convex hulls is minimal. A discrimination rule based on the convex hulls is also proposed. The similarity between an individual and a population is measured by the difference between the Lebesgue measures of the convex hulls of the training set with and without the individual. This procedure satisfies most of Fisher and Van Ness admissibility conditions. Interesting results are obtained for the error rate estimation by "resubstitution" and "leaving one out" methods. The decision surface is shown to be piecewise linear. Several real examples in remote sensing are analyzed.

K. Sandau:

An estimation procedure for the joint distribution of spatial direction and thickness of flat bodies

In practice cracks of soil, membranes or walls of cells are the objects of consideration which are summarized in the following under the term flat bodies. To each point of the body's surface there is assigned a normal direction and a thickness. Considering the selection of a point on the surface as a random event a joint probability distribution of direction and thickness is implied. However, in practice the data cannot be observed directly, only the profiles in plane sections can be examined. A further constraint is that these sections must be vertical for practical reasons. The visible thickness and the visible direction shall be surveyed along test lines situated in the vertical sections at the points where the test lines hit the flat bodies. If test lines are available in all possible spatial directions the joint probability distribution can be estimated. Otherwise further assumptions are necessary. In an application a special rotation-symmetric case is considered where only vertical and horizontal test lines are used. In this case a parametric approach is proposed and a parameter estimation is added.

R. Schneider:

Random projections of regular simplices

If T^n is the regular n-simplex in $\mathbb{R}^n$ and if Π_d denotes the orthogonal projection from $\mathbb{R}^n$ onto an isotropic random d-subspace of $\mathbb{R}^n$, then $\Pi_d T^n$ is a random polytope. Let $f_k(\Pi_d T^n)$ denote the number of its k-dimensional faces ($0 \leq k < d \leq n-1$). For the expectation of this random variable, it is shown that

$$\mathbf{E}\, f_k(\Pi_d T^n) \sim \frac{2^d}{\sqrt{d}} \binom{d}{k+1} \beta(T^k, T^{d-1})\, (\pi \log n)^{\frac{d-1}{2}}$$

as n tends to infinity. Here $\beta(T^k, T^{d-1})$ denotes the internal angle of the simplex T^n at one of its k–faces. The result contributes to a question posed by Goodman and Pollack.

(Joint work with F. Affentranger)

M. Stoka:

Hitting probabilities for random ellipses and ellipsoids

Let $\mathcal{R}$ denote a rectangular lattice in the Euclidean plane E_2 generated by $(a\times b)$–rectangles. In this paper we consider the probability that a random ellipse having main axes of length 2α and 2β, with $2\beta \leq 2\alpha < \min\,(a,b)$ intersects $\mathcal{R}$. We assume namely that the lattice $\mathcal{R}$ is the union of two orthogonal sets $\mathcal{R}_a$ and $\mathcal{R}_b$ of equidistant lines and evaluate the probability that the ellipse intersects $\mathcal{R}_a$ or $\mathcal{R}_b$. Moreover, we consider the dependence of the events that the ellipse intersects $\mathcal{R}_a$ and that the ellipse intersects $\mathcal{R}_b$. We study further the case when the main axes of the ellipse are parallel to the lines of the lattice and satisfy $2\beta = \min\,(a,b) < 2\alpha = \max\,(a,b)$. In this case, the probability of intersection is one, and there exist almost surely two perpendicular segments in $\mathcal{R}$, within the ellipse. We evaluate the distribution function, density, mean and variance of the length of these segments. We conclude by a generalization of this problem in dimension three.

F. Streit:

Statistical tests for comparing different stochastic models in stochastic geometry

After a summary of the relevant methodology of statistical inference for marked point processes the following particular problems are investigated:

1. How can one decide by means of a statistical test and based on a spatially restricted observation of a germ–grain model whether interpenetration of the grains is possible?

2. Selection among different Stienen–models by means of a statistical test.

H.S. Sukiasian:

On metrics generated by flag functions

One of the versions of the Hilbert's 4-th problem in $\mathbf{R}^3$ can be as follows: in $\mathbf{R}^3$ we have a continuous and linearly additive metric $F(P_1;P_2)$. Is it true that this F can be represented as

$$(1) \qquad F(P_1;P_2) = \mu(P_1/P_2)\,,$$

where μ is some measure in the space of planes, P_1/P_2 is the set of planes which separate the points P_1 and P_2 ?

We consider metrics, for which exist so-called flag densities. We have solved the following problem: what condition guarantees that a flag function generates a measure (or a signed measure) by means of (1).

M. Tanemura:

Random packing, tessellations and statistics on the sphere

The surface of the sphere is a non-Euclidean space and shows interesting aspects different from the usual Euclidean spaces. We consider a random sequential packing of spherical caps and Voronoi tessellation of objects on the unit sphere with some applications. The discussion will refer to the method of obtaining configurations with certain optimal properties.

Each point on the unit sphere is also regarded as a vector which represents the 3-D direction of non-isotropic objects. We present a likelihood procedure of estimating parameters of directional interaction for the special case of configurations, i.e., time series of unit vectors, on the sphere.

E.B. Vedel Jensen:

Stereological estimation based on isotropic slices through fixed points

In the present talk, stereological estimators of number, length, surface area and volume in $\mathbf{R}^3$, based on measurements in an isotropic slice through the origin 0, are presented. Measurements of 3-D angles are not needed. The estimators depend only on distance measurements. The estimation principle is generalized, using isotropic r-slices through the origin 0 in $\mathbb{R}^n$ and an isotropic grid of q-subspaces.

(Joint work with K. Kiêu)

R.A. Vitale:

L_p metrics and Gaussian processes

The Hausdorff metric between a pair of convex bodies in $\mathbb{R}^d$ can be regarded as the L_∞ metric between their support functions. Accompanying distances of L_p type, $1 \leq p < \infty$, can be defined in a natural way. In earlier work, these were shown to satisfy upper and lower inequalities with respect to the Hausdorff metric. Recently it has become apparent that the system of lower bounds can be generalized and re-cast as a class of companion inequalities for Gaussian processes. A key tool is the isonormal map, which identifies such processes with subsets of Hilbert space. In this new setting, a natural passage to infinite dimensions can be done.

W. Weil:

Support densities of random sets

The anisotropy of stationary random sets X and particle point processes Y in $\mathbb{R}^d$ has to be expressed by directional characteristics. Since the centred support function $h(K,\cdot)$ of a convex body $K \subset \mathbb{R}^d$ is an additive and translation invariant functional, it has an additive extension to the convex ring. For random sets X in the extended convex ring (i.e. locally finite unions of convex bodies), a support density h_X can be introduced (in analogy to the classical quermass densities) as

$$h_X = \lim_{r \to \infty} [V_d(rB)]^{-1}\, \mathbb{E}\, h(X \cap rB, \cdot)$$

(where V_d is volume and B the unit ball). h_X is a continuous function on the unit sphere and fulfills

$$h_X = \mathbb{E}\, [h(X \cap W, \cdot) - h(X \cap \partial^+ W, \cdot)]$$

(where W is the unit cube and $\partial^+ W$ the 'upper right' boundary). In the plane, the following formula holds for a sampling window K in the convex ring

$$\mathbb{E}\, h(X \cap K, \cdot) = A(K)\, h_X + A_A\, h(K,\cdot)$$

($A(K)$ is the area of K and A_A the area density of X), higher dimensional analogs involve densities of mixed functionals (of X and K). As a consequence, for a (planar) Boolean model X (with underlying Poisson particle process Y of intensity γ), it is shown that

$$h_X = e^{-\gamma \overline{A}} \gamma h(\overline{K}, \cdot),$$

where $\overline{K}$ is a convex body representing the 'mean shape' of Y (and $\overline{A}$ is the mean area of the particles in Y). Consequently, the mean shape of Y can be estimated from measurements of the union set X.

M. Zähle:

Random fractal sets and measures

In order to give a rigorous mathematical definition of statistical self-similarity as used in physics, Palm distributions of random fractal measures have to be introduced. They also appear in form of random fractional tangent measures of strictly self-similar sets in the sense of Moran and Hutchinson. The corresponding invariance relations and ergodic theorems lead to the concept of fractional densities of such measures which may explicitly be computed for Cantor type sets.

H. Ziezold:

Statistical decisions based on mean figures and mean shapes

Figures and shapes are equivalence classes of k-ads $x \in C^k$. Means are defined in the sense of Fréchet with respect to suitable metrics. For m independent realizations of a figure distribution $\mathbf{P}$ and n independent realizations of a figure distribution $\mathbf{Q}$ a decision rule for testing $H_0 : \mathbf{P} = \mathbf{Q}$ is proposed.

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