MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12/1992

Teichmüller – Theorie und Modulräume Riemannscher Flächen

22.3. bis 28.3. 1992

Die Tagung fand unter der Leitung von Scott Wolpert (College Park) und Georg Schumacher (Bochum) statt.
Es handelte sich um die erste Tagung zum Thema Teichmüller – Theorie und Modulräume in Oberwolfach. Es wurden die neuesten Entwicklungen auf diesem sehr umfangreichen Gebiet dargestellt und die Beziehungen zwischen den einzelnen Arbeitsschwerpunkten wurden verstärkt. Das Spektrum der Vorträge umfaßte insbesonders die Theorie holomorpher quadratischer Differentiale, die Theorie Kleinscher Gruppen, differentialgeometrische Untersuchungen (Teichmüller – und Petersson – Weil – Metrik) und hyperbolische Geometrie.

Vortragsauszüge

L. Alessandrini

EXTENSION OF PLURISUBHARMONIC CURRENTS ACROSS ANALYTIC SUBSETS

Let Ω be an open subset of $\mathbf{C}^n$ and A an analytic subset of Ω. A classical theorem of Grauert and Remmert asserts that if f is a plurisubharmonic function on $\Omega \setminus A$, locally bounded from above in Ω, then f extends to a unique plurisubharmonic function f^0 on Ω. We provide an analogous result for currents.
A real current T on Ω of bidimension (p,p), $(1 \leq p \leq n)$ is called plurisubharmonic if the current $i\partial\bar{\partial}T$ is positive. Assume that $\dim A < p$ and T is a plurisubharmonic current on $\Omega \setminus A$ of bidimension (p,p); then:

Theorem 1. *If T is negative, T extends to a plurisubharmonic current on Ω.*
Theorem 2. *If T is of order zero and there exists a plurisubharmonic extension T' of order zero, T' turns out to be the simple extension.*
Theorem 3. *If T is positive or negative and has a plurisubharmonic extension R across A, then there exists also the simple extension T^0 and $i\partial\bar{\partial}T^0 = i\partial\bar{\partial}R$.*

C.–F. Bödigheimer

INTERVAL EXCHANGE MAPS AND MODULI OF RIEMANN SURFACES

The moduli space $\overrightarrow{\mathcal{M}}(g)$ of conformal equivalence classes of triples $[F, P, \chi]$ with F a closed Riemann surface of genus g, P a point on F, and χ a tangential direction at P has a parametrization as a configuration space $PS\mathbf{C}(g)$ of $2g$ pairs of horizontal, semi – infinite slits in $\mathbf{C}$.

Theorem. *The group generated by translations and dilations of* $\mathbf{C}$ *acts freely on the manifold* $PS\mathbf{C}(g)$, *and the orbit space is homeomorphic to* $\overrightarrow{\mathcal{M}}(g)$.

An interval recombination (of rank $2n$) is a rearrangement of $2n - 1$ subintervals of the unit interval, where we allow only permutations whose commutator with the cyclic permutation has order 2. This space is denoted by $Rec(2n)$. There is a map

$$\Psi : PS\mathbf{C}(g) \simeq \overrightarrow{\mathcal{M}}(g) \longrightarrow Rec(4g)$$

which assigns to $[F, P, \chi]$ the Poincaré first – return – map of the flow with dipole at P in direction of χ. By admitting certain degenerate Riemann surfaces (with multiple cone points) one constructs a finite complex $K(g)$ of dimension $6g - 3$, containing the manifold $\overrightarrow{\mathcal{M}}(g)$. $K(g)$ is a new compactification of the moduli space.

Theorem. Ψ *extends to a homotopy – equivalence* $\widehat{\Psi} : K(g) \xrightarrow{\simeq} Rec(4g)$.

G. Gonzàles – Diez

MODULI SPACES OF GALOIS COVERINGS OF $\mathbf{P}^1$ (λ AND j MODULAR FUNCTIONS OF SEVERAL VARIABLES)

The wellknown fact that for $g \geq 3$ the moduli space $\mathcal{M}_g$ of compact Riemann surfaces can be compactified by adding an analytic space of codimension 2 (Satake's compactification) kills any hope of constructing holomorphic functions on the Teichmüller space T_g, invariant under the action of the modular group Mod_g (or subgroups of finite index of it).
However if we restrict ourselves to submanifolds of T_g representing surfaces which are Galois coverings of $\mathbf{P}^1$ of prescribed geometric type, then one can define functions which in a natural way generalise the classical λ function of the elliptic modular theory.
In particular this allows us to describe, for any prime number p, the subset $\mathcal{M}_g^p \subset \mathcal{M}_g$ (consisting of points representing surfaces of genus g that are p – fold Galois coverings of $\mathbf{P}^1$) as a disjoint union of irreducible varieties each of which is a quotient of $\mathbf{C}^r \setminus \{\text{diagonal set}\}$ by the action of a subgroup of the symmetric group Σ_{r+3}.

C. Earle

COMPLEX GEODESICS IN TEICHMÜLLER SPACES

This is a description of joint research with Irwin Kra and Samuel Krushkal. We study holomorphic isometric embeddings of the Poincaré disk Δ into Teichmüller space, with its Teichmüller metric. Our prinzipal tool is:

Theorem 1. (Lifting Theorem) *Let* $\Psi : Belt(X) \longrightarrow Teich(X)$ *be the natural projection from the Beltrami forms on the Riemann surface* X *onto the Teichmüller space of* X. *For every holomorphic map* $f : \Delta \to Teich(X)$ *there is a holomorphic* $g : \Delta \to Belt(X)$ *so that* $\Psi \circ g = f$.

Theorem 2. *Let ρ_Δ and ρ_T be the Poincaré and Teichmüller distances on Δ and Teich(X), and let F_Δ and F_T be the infinitesimal metrics. Let $f : \Delta \longrightarrow$ Teich(X) be holomorphic. Suppose either that $F_T(f(0), f'(0)) = F_\Delta(0,1)$ $(= 1)$ or that $\rho_T(f(0), f(t)) = \rho_\Delta(0,t)$ for some $t \in \Delta \setminus \{0\}$. Then f is an isometry.*

Theorem 3. *Let μ be an extremal Beltrami form on X. The following are equivalent:*
(i) $|\mu| = ||\mu||_\infty$ a.e. and μ is unique extremal.
(ii) The geodesic in Teich(X) joining $\Psi(0)$ to $\Psi(\mu)$ is unique (up to parametrization).

F.P. Gardiner (joint work with D. Sullivan)

UNIFORMLY ASYMPTOTICALLY CONFORMAL DYNAMICAL SYSTEMS

The series $\varphi_j(x) = \sum_{k=0}^{\infty} 2^{-k} \sin(2^k jx)$ for odd values of $j \geq 3$ correspond to a basis for the holomorphic quadratic differentials acting as a tangent space to a certain Teichmüller space. This Teichmüller space is the space $\mathcal{UAC}$ of *uniformly asymptotically conformal* expanding degree 2 mappings with the unit circle as repellor. The Teichmüller metric on this space is the boundary dilatation metric and this Teichmüller space is a closed complex submanifold of the Teichmüller space QS mod S. Here, QS is the group of quasisymmetric homeomorphisms of the unit circle and S is the closed subgroup of symmetric homeomorphisms.
For a quadratic – like element P in $\mathcal{UAC}$, the quadratic differentials φ are holomorphic functions satisfying an approximate automorphy condition:

$$(\forall \epsilon > 0)\,(\exists U)\,(\forall n \geq 0)\,(\forall z \in P^{-n}(U)) \quad |\varphi(P^n(z))(P^n)'(z)^2 - \varphi(z)| < \epsilon\lambda^2(z)$$

Here U is an annular neighborhood of the repellor and λ is the Poincaré metric. There are two norms for these automorphic forms:

$$||\varphi||_{B(P)} = \lim_{U\searrow} \sup_{z\in U} |\varphi(z)\lambda^{-2}(z)|$$

$$||\varphi||_{A(P)} = \lim_{U\searrow} \sup\{\text{area}(\beta)^{-1} \int_\beta |\varphi(z)|\, dz \;\;|\;\; \beta \text{ is Carlson box in } U\}$$

The dual of the $A(P)$ – norm is equivalent to the $B(P)$ – norm and there is a theta – series operator. The principle of Teichmüller contraction applies and renormalization of unimodal mappings with quadratic singularity leads to exponential convergence of scaling ratios.

D.H. Hamilton

SIMULTANEOUS UNIFORMISATION

We sketch the proofs of:

Theorem 1. *Let $\Phi : R_1 \longrightarrow R_2$ be an orientation reversing homeomorphism of Riemann surfaces. Then there exists a Kleinian group Γ with disjoint Γ – invariant domains A_1, A_2 s.t. R_i is conformally equivalent to A_i/Γ for $i = 1, 2$.*

Theorem 2. *Let $\alpha : S' \longrightarrow S'$ be an inner function.*
(i) If α ergodic and $\Phi : S' \longrightarrow S'$ absolutely continuous with $\beta = \Phi^{-1} \circ \alpha \circ \Phi$ inner then Φ is Möbius or anti – Möbius.
(ii) If α is not ergodic then there exists a nontrivial absolutely continuous $\Phi : S' \longrightarrow S'$ s.t. $\beta = \Phi^{-1} \circ \alpha \circ \Phi$ is inner.

F. Herrlich

A PARTIAL COMPACTIFICATION OF TEICHMÜLLER SPACE

We construct a locally ringed space $\overline{T}_g$ containing T_g as an open dense subspace s.t. the Teichmüller modular group Γ_g acts "weakly discontinuous" (i.e. allowing infinite stabilizers at the boundary) on $\overline{T}_g$ and the quotient is isomorphic to $\overline{\mathcal{M}}_g$, the Deligne – Mumford moduli space of stable curves of genus g, as a locally ringed space.
As a point set, $\overline{T}_g := \{(X,\tau) \,|\, X$ stable Riemann surface of genus g, $\tau : \pi_1(X_0) \to \pi_1(X)$ a homomorphism induced by a contraction $X_0 \to X\}/\sim$, where X_0 is a fixed closed surface of genus g and "$\sim$" is the obvious equivalence relation. A topology can be put on $\overline{T}_g$ by considering lengths of certain geodesics as in Abikoff's paper in Ann. of Math. 105.
To put a structure as locally ringed space on $\overline{T}_g$, we consider for each standard homomorphism $j : \pi_1(X_0) \to F_g$ (free group of rank g) the open subspace $\overline{T}_g(j) = \{(X,\tau) \in \overline{T}_g \,|\, \ker\tau \subset \ker j\}$ and on $\overline{T}_g(j)$ the map φ_j onto the extended Schottky space $\overline{S}_g$ which sends (X,τ) to (X,σ) s.t. $\sigma \circ \tau = j$. With the help of cross ratios of fixed poits as coordinates, one shows that $\overline{S}_g$ is a complex manifold that maps onto $\overline{\mathcal{M}}_g$. Using the analytic structure of $\overline{S}_g$ and the maps φ_j we finally define a structure sheaf on $\overline{T}_g$.

J. Jorgenson

STIRLINGS FORMULAE, WEIERSTRASS PRODUCTS, AND ARTIN FORMALISM

Let Y denote a compact Riemannian manifold and $\mathcal{E}$ a metrized vector sheaf on Y. Assume Y admits a fixed point finite group action by the group G, and that the metric on Y is G – invariant, and $\mathcal{E}$ is a G – sheaf. Assoziated to this data is a (de Rham) Laplacian and heat kernel.
In joint work with S. Lang it is shown that the trace of the heat kernel assoziated to $(Y,\mathcal{E})$ satisfies an Artin formalism identical to that satisfied by Artin $\mathcal{L}$ – functions from algebraic number theory. When applying these results to abelian varieties, the Artin formalism yields proofs of the Jacobi derivative formula, Kronecker's second limit formula, and the Riemann theta identities. Other applications to Selberg's zeta function, regularized products, and generalized Stirling formulae are studied.

L. Keen (joint work with C. Series)

PLEATING COORDINATES FOR DEFORMATION SPACES OF KLEINIAN GROUPS I

Our aim is to define new geometric coordinates for deformation spaces of Kleinian groups that have intrinsic geometric meaning. We consider the special case where the quotient Σ of the regular set Ω by the Kleinian group G is a four times punctured sphere with the punctures identified in pairs. We embed the deformation space Def(G) into $\mathbf{C}$ as follows. Consider the groups G_μ of the form $G_\mu = < \left(\begin{smallmatrix}1&1\\0&1\end{smallmatrix}\right), \left(\begin{smallmatrix}1&0\\\mu&1\end{smallmatrix}\right) >$ where $\mu \in \mathbf{C}$. The image of Def(G) is the space

$$\mathcal{R} = \{\mu \in \mathbf{C} \mid G_\mu \text{ is free, discrete and } \Omega(G_\mu)/G_\mu \cong \Sigma\}$$

The free homotopy classes of simple closed curves on Σ can be enumerated by the rational numbers and to each rational p/q there is a unique conjugacy class $\{g_{p/q}(\mu)\} \in G_\mu$. The traces $\{\mathrm{tr} g_{p/q}(\mu)\}$ form a family of analytic functions on $\mathcal{R}$.

Theorem. *We can choose branches of $\{\mu \mid trg_{p/q}(\mu) < -2\}$ s.t. these analytic curves form a dense set of nonsingular curves in $\mathcal{R}$ that are:*

(i) asymptotic to the ray $\exp(\pi i(p-q)/q)$ as $|\mu| \to \infty$

(ii) and end in unique points on $\partial\mathcal{R}$ s.t. $trg_{p/q}(\mu) = -2$. These points are cusps s.t. the curves in the p/q homotopy class are pinched.

Theorem. *There is a natural interpolation of the curves in $\mathcal{R}$ defined above by non – singular real analytic curves assoziated to the irrationals.*

S. Kerckhoff

DEFORMATIONS OF HYPERBOLIC CONE MANIFOLDS AND HODGE THEORY

We study the deformation theory of hyperbolic cone manifolds in dimension 2 and 3. These are closed manifolds which are locally modelled on hyperbolic space except at the singular locus, where they are modelled on a conical neighbourhood of the fixed point set of an elliptic element divided out by that element.

In dimension 2 the space of such structures on a surface of genus g with prescribed curve angles can be identified with the Teichmüller space of conformal structures (pointed). The tangent space can be identified with holomorphic quadratic differentials with at most simple poles.

In dimension 3 we expect cone structures with prescribed singular link and prescribed angles to be rigid. With C. Hodgson we prove local rigidity if the curve angles are at most 2π. The proofs involve analysis of the Hodge representations of the Zariski tangent space.

S. Krushkal

PLURISUBHARMONIC FUNCTIONS, INVARIANT METRICS ON TEICHMÜLLER SPACES, AND QUASICONFORMAL MAPS

It is interesting to investigate Teichmüller spaces from the point of view how the certain properties of complex manifolds are realized here in sharpened form caused by specific features of these spaces.

In this talk I consider the complex Green function $g_T(x,y)$ of the Teichmüller spaces $T(\Gamma)$ and its applications. The main theorem gives an explizit representation of $g_T(x,y)$ by the Kobayashi – Teichmüller metric of these spaces. The green function is an extremal plurisubharmonic function on $T(\Gamma)$ and has some remarkable properties. We prove that the invariant differential metric induced on $T(\Gamma)$ by $g_T(x,y)$ (Azukawa metric) coincides with the Finsler structure on $T(\Gamma)$ generating the Kobayashi – Teichmüller metric.

The representation of the Green function g_T leads also to other interesting applications, particulary it implies a complete solution of Gromov's question on the complex hyperconvexity of Teichmüller spaces and it gives a new characterisation of extremal quasiconformal maps.

M. Lustig (joint work with M.M. Cohen)

THE INDUCED ACTION OF GENERALIZED DEHN TWISTS ON TEICHMÜLLER SPACE AND ITS BOUNDARY

For any closed surface M^2 with negative Euler characteristic the fundamental group $\pi_1(M^2)$ is word – hyperbolic in the sense of Gromov, and every hyperbolic structure on M^2 gives rise to a δ – hyperbolic metric on $\pi_1(M^2)$. The mapping class group $\Gamma = \text{Out}(\pi_1(M^2))$ acts on Teichmüller space and on the space of δ – hyperbolic metrics on $\pi_1(M^2)$. This gives rise to the following generalisation: For any word – hyperbolic group G, we consider a "generalized Teichmüller space" $\mathcal{T}(G)$, which consists of certain δ – hyperbolic metrics on G. Just as in the case $G = \pi_1(M^2)$ there exists a Thurston boundary $\partial\mathcal{T}(G)$ of $\mathcal{T}(G)$, which consists of actions of G on metric trees. The action of $\text{Out}(G)$ on $\mathcal{T}(G)$ extends to $\partial\mathcal{T}(G)$.
There exists a particular class of automorphisms of G which we call "Dehn twist automorphisms", which coincide for the case $G = \pi_1(M^2)$ with the automorphisms induced by multiple Dehn twists. For such an automorphism D we study the induced action on $\overline{\mathcal{T}}(G)$. In particular we describe a simplex $\sigma(D) \subset \partial\mathcal{T}(G)$ of fixed points and prove that most other points lie on parabolic orbits with limit point in $\sigma(D)$. For G a free group this leads to a solution of the conjugacy problem for Dehn twist automorphisms (work in progress).

B. Maskit

FUNDAMENTAL DOMAINS FOR MAPPING CLASS GROUPS

Let S be a closed hyperbolic surface. A chain of geodesics $(A_1, \ldots, A_{2g+1})$ consists of simple non – dividing geodesics, where $A_{i+1} \cdot A_i = 1$ for $1 \leq i \leq 2g$ and $A_{i+1} \cdot A_j = 0$ for $j < i$. A short chain is defined as follows: A_1 is the shortest non – dividing geodesic and A_i is the shortest geodesic satisfying the above chain – condition. There is a unique short chain on a generic S; if one interprets "shortest" as "$\leq$", then there is a short chain on every surface.

Theorem: *Short chains on closed surfaces can be defined by finitely many inequalities.*

It is clear that choosing a short chain is equivalent to finding a homotopy basis; hence choosing a short chain on each surfaces is equivalent to finding a fundamental domain for the mapping class group.
In genus 2, we explicitly write down 32 inequalities which define this fundamental domain; it is not now known which, if any, of these inequalities are redundant.

H. Masur

PROJECTIONS OF TEICHMÜLLER DISKS TO MODULI SPACE

Let D be a Teichmüller disk in T_g, the Teichmüller space of genus g and let $\pi : T_g \to \mathcal{M}_g$ be the projection to moduli space. Let $\Gamma = \{\gamma \in \text{Mod}_g \mid \gamma(D) = D\}$.

Theorem. *(J. Smillie) $\pi(D)$ is closed in $\mathcal{M}_g$ if and only if Γ is a lattice; namely vol$(D/\Gamma) < \infty$.*

D determines a 1 – parameter family of Teichmüller geodesics, parametrized by an angle θ. For each geodesic $v_\theta(t)$, parametrize by arclength t. Let X_0 be the point in $\mathcal{M}_g$ and d_T denote the Teichmüller metric.

Theorem *For Lebesgue a.e. θ:* $\overline{\lim}_{t\to\infty} d_T(v_\theta(t), X_0)/\log(t) = 1$

J. Parker

COMPLEX HYPERBOLIC IDEAL TRIANGLE GROUPS

In the hyperbolic plane, reflections in three asymptotic geodesics freely generate a discrete group having the triangle bounded by these geodesics as its fundamental domain. Any such pair of triangle groups are conjugate. We are concerned with the analogous question for groups generated by inversions in three mutually asymptotic complex geodesics in complex hyperbolic space. Such an "ideal triangle group" is determined by a triple of points on the boundary of complex hyperbolic space; such triples are parametrized by a single real number, the angular invariant $\mathbb{A}$ of E. Cartan lying in the interval $[-\pi/2, \pi/2]$. We ask for which values of $\mathbb{A}$ the map

$$[-\frac{\pi}{2}, \frac{\pi}{2}] \longrightarrow \mathrm{Hom}(\mathbf{Z}_2 \star \mathbf{Z}_2 \star \mathbf{Z}_2, PU(2,1))/PU(2,1)$$

given by this correspondence yields a discrete and faithful representation.

Theorem. *(J.R.P., W.M. Goldman) Let u_0, u_1, u_2 be a triple of points with angular invariant $\mathbb{A}$. Let $\Psi_{\mathbb{A}}$ be the representation of the group generated by inversions as above. Then:*
$|\mathbb{A}| \leq \tan^{-1}\sqrt{35}\ (\approx 80.4°)\ \Rightarrow\ \Psi_{\mathbb{A}}$ *is discrete and faithful* $\Rightarrow\ |\mathbb{A}| \leq \tan^{-1}\sqrt{125/3}\ (\approx 81.2°)$

Conjecture. $\Psi_{\mathbb{A}}$ is discrete and faithful $\Longleftrightarrow\ |\mathbb{A}| \leq \tan^{-1}\sqrt{125/3}$

R. Penner

UNIVERSAL CONSTRUCTIONS IN TEICHMÜLLER THEORY

We study a new model $\mathcal{T}ess$ of a universal Teichmüller space which is defined to be the collection of all ideal tessalations of the Poincaré disc (together with a distinguished oriented edge) modulo the natural action of the Möbius group $\mathbf{P}SL_2(\mathbf{R})$. Applying techniques from hyperbolic geometry, we find: $\mathcal{T}ess$ is an infinite – dimensional Frechet manifold with canonical coordinates, there is a formal two – form ω (which generalizes the Weil – Petersson forms) on $\mathcal{T}ess$ invariant under the action of $\overline{\Gamma}$, the generalized mapping class group. $\mathcal{T}ess$ is homeomorphic to the right coset space of $\mathbf{P}SL_2(\mathbf{R})$ in the group of all orientation – preserving homeomorphisms of S^1, and one sees that $\mathcal{T}ess$ generalizes other models of a universal Teichmüller space, namely Bers' model and the right coset space of $\mathbf{P}SL_2(\mathbf{R})$ in the group of all smooth orientation – preserving diffeomorphisms of S^1. The latter model supports the Kirillov – Kostant symplectic form, and ω pulls back to this form. Furthermore, there is an assoziated *universal decorated Teichmüller space* $\widetilde{\mathcal{T}}ess$, which is again a Frechet manifold supporting a $\overline{\Gamma}$ action and a $\overline{\Gamma}$ – invariant two form. There is finally a $\overline{\Gamma}$ – invariant cell – decomposition of $\widetilde{\mathcal{T}}ess$.

C. Series (joint work with L. Keen)

PLEATING COORDINATES FOR DEFORMATION SPACES OF KLEINIAN GROUPS II

To prove the results in L. Keen's lecture (see above), we study, for a general finitely generated Kleinian group G, the boundary K of the convex hull of the limit set of G in $\mathbf{H}^3$. This is a pleated surface which carries a natural hyperbolic metric. K/G is homeomorphic to $\omega(G)/G$ and the bending lines are a geodesic lamination β on K/G.

Theorem. *If $G = G(\mu)$ for $\mu \in \mathbf{C}^n$ varies in some deformation space Def(G), then the hyperbolic structure and bending lamination of K/G depend continously on μ.*

Consider the case in which β is supported on a finite number of disjoined simple closed geodsics $\gamma_1, \ldots, \gamma_k$. The element $g_i \in G$ representig γ_i must be a pure translation and hence has real trace. We consider the Fuchsian subgroups F_i which stabilize the flat planes in K. These have the property that no limit point of G lies inside the invariant discs defined by the F_i and "facing" K. Such subgroups we call F-peripheral. This property is preserved under deformations of G which keep all relevant traces real. One deduces that $\{\mu \in \mathrm{Def}(G) \mid \beta(\mu) = \{\gamma_1, \ldots, \gamma_k\}\}$ is a union of connected components of the locus $\{\mathrm{Tr}\mu_i \in \mathbf{R}, i = 1, \ldots, k\}$. In the special case of the space $\mathcal{R}$ discussed in L. Keen's lecture this is enough to characterise the special branches of the locus $\mathrm{Tr} g_{p/q}(\mu) \in \mathbf{R}$ as those points in $\mathcal{R}$ along which $\gamma_{p/q}$ is the bending lamination.

K. Strebel

THE MAPPING BY HEIGHTS FOR QUADRATIC DIFFERENTIALS IN THE DISK

Let $\varphi \neq 0$ be a holomorphic quadratic differential of finite norm in the unit disk D. Set $w = u + iv = \Phi(z) = \int_z \sqrt{\varphi(z)}dz$. The vertical distance or height of a pair of boundary points r, s of D with respect to φ is $h_\varphi[r,s] = \inf \int_\gamma |dv|$, the infimum taken over all locally rectifiable arcs γ in D connecting r and s. Given a quasisymmetric selfmapping f of ∂D. Then, to every φ there exists a uniquely determined $\psi = H_f(\varphi)$ such that $h_\varphi[r,s] = h_\psi[f(r), f(s)]$, for all pairs of boundary points. H_f is called the mapping by heights assoziated with f. It satisfies the norm inequality $K^{-1}\,||\varphi|| \leq ||H_f(\varphi)|| \leq K\,||\varphi||$, with K the maximal dilatation of an extremal qc continuation of f into D.
The proof is based on the notion of a totally regular trajectory. Two boundary points p and q are connected by a totally regular trajectory of φ if $f(p)$ and $f(q)$ are connected by a totally regular trajectory of ψ. Vertical distances of corresponding pairs of totally regular trajectories are equal.

S.P. Tan (joint work with M.L. Lang and C.H. Lin)

INDEPENDANT GENERATORS FOR CONGRUENCE SUBGROUPS OF THE HECKE GROUPS

Let G_q, $q \geq 3$ be the Hecke groups, i.e. the subgroups of $\mathbf{P}SL_2(\mathbf{R})$ generated by the transformations $\binom{0\;1}{-1\;0}$ and $\binom{1\;\lambda_q}{0\;1}$ where $\lambda_q = 2\cos(\pi/q)$. The congruence subgroups for G_q are defined as follows: Let $I \subset \mathbf{Z}[\lambda_q]$ be an ideal.

$$G_0(q,I) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G_q \mid c \in I\}$$

$$G^1(q,I) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G_q \mid a-1, d-1, c \in I\}$$

$$G(q,I) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G_q \mid a-1, d-1, b, c \in I\}$$

where the matrices are defined up to ± 1. We give an inductive construction of special fundamental domains for the congruence subgroups so that the side – pairings which generate the group are independent in the sense of Rademacher.
There are many applications for our construction, for example we can use it to construct the subgroup of Γ leaving a modular function invariant. We also use this to give closed formulae for the number of subgroups of G_q of index n where $q \leq n < 2q$ when q is prime.

S. Trapani

CHERN CLASSES AND PLURISUBHARMONIC FUNCTIONS

Let X be a complex projective manifold, let L be an ample line bundle over X and let D be an effective divisor on X. If $L - D$ has top Kodaira dimension then $L^n > L^{n-1}D$. We use some new results by J.P. Demailly to show that if $L^n >$const$(X, D) \cdot L^{n-1}D$ then $L - D$ has top Kodaira dimension (const$(X, D) > 0$).
Using the above result we reduce the computation of the Kodaira dimension of $\overline{\mathcal{M}}_g$, the Deligne – Mumford compactification of moduli space, to the computation of the top intersection number of determinant of the bundle of quadratic holomorphic differentials on $\overline{\mathcal{M}}_g$.

M. Wolf (joint work with B. Zwiebach)

THE PLUMBING OF MINIMAL AREA METRICS

We study the following extremal length problem on a Riemann surface: a conformal metric is called admissible if it gives each non – trivial closed curve a length of at least 2π; we seek a metric of least area. In the case of a punctured Riemann surface we use reduced area. We show that if such a minimal metric exists and, in some neighborhood of a puncture, is smooth, non – vanishing and complete, then there is some neighborhood of the puncture in which the metric is isometric to a flat semi – infinite cylinder. The formal proof involves a partial converse to Beurling's criterion. We next show that conformal plumbing respects the minimality of the metrics. This ties with Zwiebach's objective of showing that minimal area string diagramms define a quantum closed string field theory. Finally, we exhibit an extremal metric which is not a Jenkins – Strebel differential.

P. Zograf

ON WEIL – PETERSSON VOLUMES OF MODULI SPACES

Let $\mathcal{M}_{g,n}$ be the moduli space of Riemann surfaces of genus g with n ordered punctures. Denote by ω_{WP} the Weil – Petersson symplectic form on $\mathcal{M}_{g,n}$. By definition,

$$\mathrm{Vol}_{WP}(\mathcal{M}_{g,n}) = \frac{1}{(3g-3+n)!}\int_{\mathcal{M}_{g,n}} \omega_{WP}^{3g-3+n} \quad \text{and set} \quad V_{g,n} = \frac{(3g-3+n)!}{\pi^{2(3g-3+n)}}\,\mathrm{Vol}_{WP}(\mathcal{M}_{g,n})$$

Then for $g = 0, 1$ we have the following recursion relations (where $V_{0,3} = 1$):

$$V_{0,n} = \frac{1}{2}\sum_{i=1}^{n-3} \frac{i(n-i-2)}{n-1}\binom{n-4}{i-1}\binom{n}{i+1} V_{0,i+2}V_{0,n-i}$$

$$V_{1,n} = \frac{n}{24}V_{0,n+2} + \sum_{i=1}^{n-1}(n-i)\binom{n-1}{i}\binom{n}{i-1} V_{1,i}V_{0,n-i}$$

Berichterstatter: R. Blache

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft

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