MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Die Tagung fand unter Leitung von W. Felscher (Tübingen), H. Schwichtenberg (München) und A. S. Troelstra (Amsterdam) statt. Im Mittelpunkt des Interesses standen Fragen aus dem Gebiet der Mathematischen Logik mit den Schwerpunkten Beweistheorie, intuitionistische Logik, kombinatorische und lineare Logik, Logikprogrammierung, Rekursionstheorie und Graphentheorie mit Anwendungen in der Komplexitätstheorie, der konstruktiven Mathematik und der Informatik.

VORTRAGSAUSZÜGE

The weak truth table degrees of the recursively enumerable sets
K. Ambos-Spies

A weak truth table (wtt) reduction is a Turing reduction with a recursively bounded use function. The investigation of the weak r. e. wtt degrees was initiated by Ladner and Sasso in the 70s. It turned out that this structure is much more homogeneous than the corresponding structure for Turing reducibility: The partial ordering of the r. e. wtt degrees is a dense, distributive upper semi lattice in which all elements are join and meet reducible.
In our talk we discuss two recent results on the (first order) theory of this structure: First the undecidability of the theory (proven by Ambos-Spies, Nies and Shore); second a characterization of the finite lattices embeddable into the r. e. wtt degrees by maps which preserve the least and greatest elements (obtained by Ambos-Spies, Fejer, Lempp and Lerman) which leads to a decision procedure for the two quantifier theory of the partial ordering of the r. e. wtt degrees.

Extracting a Program from the Tait/Troelstra proof of strong Normalization
U. Berger

Using a standard proof-theoretic technique (modified realizability interpretation) we extract an efficient normalization program from a modification of the Tait/Troelstra proof of strong normalization. This

algorithm was discoverd by H. Schwichtenberg and is used for normalizing natural deduction proofs (represented as typed λ-terms).
In the normalization proof "Strong Computability Predicates" are used, which are formulas of unbounded logical complexity. This fact is reflected in the extracted normalization program by the use of arbitrary high types. The extracted algorithm may be described roughly as follows:

1. Evaluate a given term r, of higher type ρ say, and get a functional $|r|$ of type ρ (this part of the algorithm comes from the proof of "r is strongly computable under substitution").

2. Collapse the functional $|r|$ to ground type and get the long normal form of r, assuming that the ground type contains syntactic material like λ-terms (this comes from the proof of "r strongly computably $\rightarrow$ r strongly normalizable").

For step 1. we may use the (hopefully efficiently implemented) evaluation procedure of a functional language, and the collapsing functional $\rho \rightarrow$ ground type used in step 2. may be implemented by a very simple functional program. By a slight modification we may also get the "short" i.e. $\beta\eta$–normal form.
The division of the algorithm into these two part shows that the normal form of the term r in fact does not depend on r itself but only on its value $|r|$. From this fact we may deduce the following strong completeness theorem for the $\beta\eta$–calculus:

Theorem. If $r = s$ holds in some λ–model containing (representations of) the natural numbers and the primitive recursive functions (of type 1, no requirements for higher types)) then already $r =_{\beta\eta} s$.

Logical Tools for Specification of Programming Languages

E. Börger

We present a formal semantics of the full language PROLOG as emerging from the ISO WG17 standardisation effort. Our description uses the notion of evolving algebras as recently proposed by Y. Gurevich which allow to directly reflect the dynamic (and resource-bounded) aspects of computation. The proposed formal semantics for PROLOG, far from being hopelessly complicated, unnatural or machine-dependent, is simple, natural, abstract and in particular supports the process oriented understanding of programs by programmers. Our specific aim is to provide a mathematical precise but simple framework in which standards can be defined rigorously and in which different implementations may be compared and judged.

Our method provides a rigorous mathematical basis for Prolog compilation technology. Starting from our abstract specification, in joint work with D. Rosenzweig we have derived an evolving algebra specification of Warren's Abstract Machine together with a proof that the latter is correct wrt to the former. Our result holds for a large class of compilers which satisfy a small set of rigorously defined conditions. The power of our method is also demonstrated by a uniform treatment of diverging views on dynamic code in current implementations of Prolog, developed in joint work with D. Rosenzweig. Joint work with C. Beierle shows that our WAM description can naturally be extended to get a formal specification of the PAM, a virtual machine model for PROTOS-L (a logic programming language with types).
Adapting ideas from the evolving algebra description of OCCAM by Y. Gurevich and L. Moss, in joint work with E. Riccobene we have extended our standard sequential Prolog algebras to Parlog algebras thus obtaining a complete formal semantics of the parallel logic programming language PARLOG.
Joint work with P. Schmitt shows that our abstract Prolog specification can naturally be extended to constraint logic programming systems like PROLOG III.

Combinators and Reflective Truth

A. Cantini

We propose an extension STW of the first order theory of combinators by means of the Kripke-Feferman axioms of reflective truth and certain approximation principles. STW has a simple fixed point semantics and is a natural place to develop recursion for predicates and operations. In the spirit of Fefermans explicit mathematics, STW can be regarded as a rich type theory, which also involves partial types.

The main results are: 1) a proof-theoretic reduction of STW to first order arithmetic; 2) an inner model construction in STW for the Aczel-Feferman theory of abstraction; 3) an inner model construction in STW for the theory of Frege structures, extended by levels of implication (Flagg-Myhill 1987).
From a broader perspective, the investigation was suggested by the old problem of finding a nice blend between functional abstraction and class abstraction.

Second Order Systems for Polytime Reasoning
Stephen Cook
(Joint work with Stephen Bellantoni)

Let $L_2(QF^+)$ be second order logic with comprehension for positive quantifier-free formulas. Leivant (LICS '91) gave the following remarkable result: $f: \mathbf{N}^k \to \mathbf{N}$ is polytime iff some "program" (i.e. set of equations) for f provably (in $L_2(QF^+)$) takes numbers to numbers. Inspired by this result we give a new (STOC '92) recursion-theoretic characterization of the polytime functions, and use it to give an alternative proof of the $\Rightarrow$ direction in Leivant's theorem, and also to interpret the first order theory QPV into a second order theory based on Leivant's $L_2(QF^+)$.

On the theorem of I. Kříž
L. Gordeev

There are known two natural extensions of Kruskal-Friedman well-quasiordering theorems: one dealing with finite trees with vertices labeled by ordinals (see [1]), the other dealing with finite trees with edges labeled by ordinals (see [2]) – both under homeomorphic embeddability with symetrical gap-condition. I proved in [1] that the former variant has proof-theoretical strength of the subsystem of Analysis ITR_0 that extends the elementary analysis (ACA_0) by the axiom of Π^1_1-transfinite recursion. I. Kříž proved in [2] that the latter variant is provable in the subsystem of Analysis with Π^1_2-comprehension axiom, which is much stronger than ITR_0. Moreover, the edge-labeled variant seems stronger at he first glance. I prove that both extensions are proof-theoretically equivalent. Hence the theorem of I. Kříž actually has proof-theoretical strength of ITR_0.

[1] L. Gordeev: *Generalizations of the Kruskal-Friedman theorems*, Journal of Symbolic Logic 55(1):157–181, 1990 (Received March 28, 1988)
[2] I. Kříž: *Well-quasiordering finite trees with gap-conditions. Proof of Harvey Friedman's conjecture*, Annals of Mathematics 130: 215–226, 1989 (Received July 13, 1988)

Uncertainty and beliefs (Dempster-Shafer theory)
P. Hájek

Various approaches to inference under uncertainty are surveyed. Basic notions of Dempster-Shafer theory are presented (Dempster spaces, basic belief assignment, belief, plausibility, Dempster's rule of continuation). Axiomatic derivation of Dempster's rule from algebraic axioms is presented; it generalizes a result of Smets and uses the notion of a meet epimorphism among boolean algebras. Finally, possibilistic logic, which is known to be a particular case of belief functions (with coherent focal elements) is relied to a certain tense logic with linearly quasiordered time.

Partial fixed points in logic programming
G. Jäger

The notion of "partial fixed point model" of (general) logic programs was introduced, and the connections between this new approach and the more traditional three-valued models of the completions of logic programs were established. In addition, a rule based calculus for partial fixed points was presented. It was shown how general logic programs (with negation) can be transformed into systems of positive inductive definitions of partial fixed point and how to represent them in this calculus. By doing this, some interesting results about the relationship between logic programming and the theory of inductive definitions could be obtained.

Recursive Inseperability in Linear Logic

H. R. Jervell

Work done with Stål Aanderaa (Oslo).

Theorem For formulas F in propositional linear logic the following is recursively inseperable

- F is provable
- F has as a countermodel a finite lattice with products.

We start with the proof of Lincoln, Mitchell, Scedrov and Shankar.
They showed that propositional linear logic is undecidable by simulating executions on a certain type of register machines. We work further with the structure of computations of such machines and show that any computation can be represented in a nice way in a finite lattice with products.

Remarks on Herbrand normal forms

U. Kohlenbach

Let A^{H} be the Herbrand normal form of A. We show:

(1) There are theories $\mathcal{T}^+$ with function parameters such that for some A not containing function parameters $\mathcal{T}^+ \vdash A^{\mathrm{H}}, A^{\mathrm{H,D}}$ but $\mathcal{T}^+ \not\vdash A$ ($A^{\mathrm{H,D}}$ a suitable Herbrand realization of A^{H}).

(2) Similair for first order theories $\mathcal{T}$ if the index functions used in defining A^{H} are permitted to occur in instances of non-logical axiom schemata, i. e. for suitable $\mathcal{T}, A$ then $\mathcal{T}(f_1,\ldots,f_n) \vdash A^{\mathrm{H}}$ but $\mathcal{T} \not\vdash A$.

Examples for (1) and (2) are the fragments $(\Sigma^0_1\text{-IA})^+$ and $(\Sigma^{0,b}_1\text{-IA})$ of second order resp. first order arithmetic. ($\Sigma^{0,b}_1$ is the class of formulas $\exists x A(x)$ where $A(x)$ contains only bounded quantifiers).

(3) On the other hand

$$E-\mathrm{PA}^{\omega} \vdash A^{\mathrm{H}} \Rightarrow \mathrm{PA} \vdash A$$

for $A \in \mathcal{L}(\mathrm{PA})$, where $E-\mathrm{PA}^{\omega}$ is extensional arithmetic in the language of all finite types.

(3) does not generalize to sentences A containing positive existential quantifiers for functions.

Search problems and bounded arithmetic

J. Krajíček

Theorem (with S. Buss)
(Multivalued) functions Σ^b_1-definable in T^1_2 are exactly projections of PLS-problems.
Explanation: T^1_2 is a bounded arithmetic fragment based on induction for Σ^b_1-formulas; these formulas define precisely NP-predicates.
PLS is a class of polynomial search problems defined by Johnson et. al. (FOCS '85).

On algorithmic randomness

Antonín Kučera

Algorithmic randomness was studied from the computational point of view by many people, e.g. by Kolmogorov, Chaitin, Martin–Löf among others.
We use a standard notion of 1–randomness and 1–randomness relative to a given oracle.

Definition A set B is 1–RRA ("random relative and above") to a set A if B is 1–random relative to A and A is recursive in B.
Theorem 1. There is a nonrecursive r.e. set A such that there is a set B 1–RRA to A.
Theorem 2. If there is a set B 1–RRA to a set A then there is a degree $\underline{e}$ such that the class of degrees of sets 1–RRA to A contains the upper cone $\{\underline{d} : \underline{d} \geq \underline{e}\}$.
Theorem 3. If there is a set B 1–RRA to a set A then A belongs to GL_1 (i.e. $A' \equiv_T A \oplus \emptyset'$).

Logical aspects of the design language COLD
G. R. Renardel de Lavalette

COLD (Common Object-oriented Language for Design) is a formal wide-spectrum design language, to be used in the software development process for specification, design and implementation. COLD is based on ideas by Hans Jonkers and has been developed at Philips Research Laboratories Eindhoven in several ESPRIT projects.
The main feature of COLD is the class concept: an abstract machine model where the states are described as algebras (using functions and predicates), and transitions between states are modeled by (nondeterministic) procedures. Moreover, COLD has modularisation (with import, export and renaming) and parametrisation (based on lambda-abstraction).
In the formal definition of COLD-K, the kernel language of COLD, a logical semantics has been provided using the logic MPL_ω (Many-sorted Partial Infinitary Logic). The main properties of MPL_ω for this purpose are:
- it allows for the explicit rendering of inductive definitions (provided they are the fixpoint of continuous operators) using infinite disjunctions;
- it satisfies the Interpolation property.
As an illustration, I will discuss two of the theoretical themes that emerged from the work on the definition of COLD and its semantics. These are:
how to translate inductive definitions of predicastes, formulated as *the least predicate satisfying* ... to continuous predicate operators;
how to obtain a nice normal form for module expressions in the context of a theory semantics for modules: this normal form can be paraphrased by *first renaming, then import, finally export* and requires the Interpolation property.

Logische Probleme beim Entwurf und der Verifikation von Schaltungen
H. Leiß

An einer Fallstudie wird gezeigt, welche logischen Fragen beim Entwurf und bei der Verifikation von Schaltungen auftreten. Dabei wird insbesondere gezeigt, daß eine rekursiv auf abstrakten Datentypen definierte Funktion nicht ohne weiteres im Top-Down-Stil in ein Funktional übersetzt werden kann, das diese Funktion auf der Ebene zeitabhängiger Signale repräsentiert.
Außerdem wird erläutert, welche logischen Ausdrucksmittl zur Beschreibung und Verifikation von Hardwarekomponenten erforderlich sind. Die Fallstudie wurde mit einem interaktiven Theorembeweiser durchgeführt.

Gentzen-type systems and Hilbert's epsilon substitution method
G. Mints

The substituiton method was suggested by Hilbert in the framework of his program in the foundation of mathematics. It is a successive approximation method for finding finite function solution of a system of equations derived from a proof in a formal system. The problem of convergence, i.e. termination of the process after finite number of steps was treted by von Neumann (1928) for quantifier free induction, by Ackermann (1940) for the first order arithmetic, and by other authors including Kreisel and Tait. We present here new proof of the Ackermann's result allowing extensions to analysis (second order arithmetic). This settles one of three problems stated by Hilbert in (1930). The proof consists of the following parts: (1) Formalizing non-effective proof of the existence of solution in the infinitary sequent calculus. (2) Standard normalization proof. (3) Proof that normal form after this normalization is simply convergence protocol for the epsilon substitution process.

On an Intuitionistic Theory of Lawless Sequences

J. R. Moschovakis

In "Relative lawlessness in intuitionistic analysis", Jour. Symb. Logic, and "An intuitionistic theory of lawlike choice and lawless sequences", to appear in Proc., Finland 1990, we developed a notion of lawlessness relative to a countable information base. Now we simplify the predictability property characterizing relatively lawless sequences and derive it from the axiom of closed data (classically equivalent to open data) together with a natural principle of invariance under finite translation. We characterize relative lawlessness in terms of a notion of forcing. We study relative lawlessness on an arbitrary homogeneous spread and show that the collection of lawless binary sequences (which is comeager in the sense of Baire) has probability measure zero. The reasoning is predominantly constructive.

The Logic of Recursion

Y. N. Moschovakis

The expressions (terms) of the language FLR_0 are defined by the recursion

$$E :\equiv p \mid f(E_1, \ldots, E_n) \mid \perp \mid E_0 \text{ where } \{p_1 = E_1, \ldots, E_n\}$$

where p is any variable, f is any n-ary symbol in a fixed signature and the (key) last construct is meant to be understood as a *recursive* (mutual, simultaneous) definition.

The usual models of FLR_0 are of the form $(P, \leq, \mathcal{I})$, where $(P, \leq)$ is a *complete poset* and $\mathcal{I}$ interprets the function symbols by monotone operators on P. We consider many specialized modelings and establish the

Basic Theorem. The class of valid FLR_0 identities is the same for the following (among others) classes of models: (1) Arbitrary monotone, (2) Continuous, (3) Streams, (4) Procedure posets, (5) Behaviour posets, (6) Process models. In addition, this class of identities is decidable and can be defined by a simple axiomatization, which "codifies" the laws of valid inference under recursion.

(4) – (6) are special models of interest in Mathematical Computer Science, the last one not a least-fixed-point model.

This is a report of joint work with A. D. J. Hurkens, M. Mc Arthur and L. Moss.

Proof-theory and term rewriting theory for the categorical uniqueness conditions in the typed functional languages and the logical languages *M. Okada*

The normalization theorems (and the cut-elimination theorems) have been a central theme in the traditional proof-theory. However, the proof-reductions considered in the traditional proof theory seem insufficient from the point of view of category-theoretic semantics. The traditional proof-reductions can be derived from the existence conditions of certain operators defined in category theory. Then the uniqueness conditions for those operators provide another kind of proof reductions. In the typed λ–calculus (under the Curry–Howard isomorphism), the existence conditions correspond to eg. the β–rule and the pairing rule, while the uniqueness conditions correspond to the η–rule and the surjective pairing rule. In this talk, we generalize this distinction (between the existence and the uniqueness) and show how to provide a rewrite rule from a categorical uniqueness condition *in general*. In particular, we consider the uniqueness (so called Malcev–Lambek) condition for Gödels recursor/iterator, and show the strong normalizability result on it, among others.

On the length of proofs in propositional calculus

P. Pudlak

We shall show that the method introduced by M. Ajtai for bounded depth Frege systems can be applied to obtain a lower bound $2^{\Omega(\mu/\sqrt{m})}$ to the size of resolution proofs of the weak Pigeon Hole Principle PHP^m_μ.

An Elementary Proof of Kruskal's Theorem
M. Rathjen
Work done with A. Weiermann

We calibrate the exact proof-theoretic strength of Kruskal's Theorem in terms of provability of well-orderdness of ordinal notation systems as well as reflection principles over a weak base theory.

Theorem. $RCA_0 \vdash KT \leftrightarrow WO(\theta\Omega^\omega 0) \leftrightarrow WO(\Psi\Omega^{\Omega^\omega})$

There KT stands for Kruskal's theorem, and $WO(\theta\Omega^\omega 0)$ means the well-orderdness of the standard notation system for the ordinal $\theta\Omega^\omega 0$. The ordinal $\theta\Omega^\omega 0$ is also known as the Ackermann ordinal.
KT lends itself to natural restrictions which arise by considering the class of trees with at most n branchings. Let KT_n signify the corresponding Kruskal Theorem for this restricted class of trees.

Theorem. $RCA_0 \vdash \forall n KT_n \leftrightarrow KT$

In Simpson: *Nichtbeweisbarkeit gewisser kombinatorischer Eigenschaften von endlichen Bäumen, Archiv für Math. Logik und Grundlagenforschung, (1985)*, it is stated that KT is provable in $ACA_0 + \Pi^1_2 - BI$ ($\Pi^1_2 - BI :=$ bar induction for Π^1_2 formulas). It turns out that this is not quite true.

Theorem. The proof-theoretic ordinal of $T := ACA_0 + \Pi^1_2 - BI$ is $\theta\Omega^\omega 0$, hence KT is not provable in T. On the other hand $RCA_0 \vdash KT_n$ for every $n < \omega$ (given from the outside).

Theorem. $RCA_0 \vdash KT \leftrightarrow RFN_{\Pi^1_1}$

Here $RFN_{\Pi^1_1}$ stands for the uniform reflection principle of T restricted to Π^1_1 formulas with numerical parameters.

Non–Monotonic Logic, Preferences and Herbrand Models
M. M. Richter

We describe a unified approach to a wide class of non–monotonic logics which was originated and developed by Ulrich Junker. We present here the Logical basis of this theory which was used by U. Junker for his procedural techniques. The unifying element is the notion of a *preferential structure* $(\mathcal{C}, \leftarrow)$, $\mathcal{C} \subseteq S$, where S is a set and $\leftarrow \subseteq S \times S$. $s \leftarrow s'$ means "s is preferred to s'" and $s \in \mathcal{C}$ means "s is consistent". Underlying is a first order language with as set T of constants denoting assumptions and three predicates $a(t)$, $b(t)$ and $c(t)$ denoting applicability, blockade and choice of assumptions. If $\mathcal{A} = \{c(t) | t \in T\}$ then one example for S is $2^{\mathcal{A}}$. The preference relation can be defined by $a(t)$, $b(t)$ and $c(t)$ or by using priorities on the assumptions. Another example for S is the set of Herbrand models. The selection between these areas is discussed. The approach covers among others defaults logic, circumscription and autoepistemic logic.

Computational Expressiveness of Linear Logic
A. Scedrov

In previous joint work with P. Lincoln, J. Mitchell and N. Shankar we have shown that MALL, the multiplicative additive fragment of propositional (i.e. quantifier-free) linear logic, is PSPACE-hard. MALL is in PSPACE because the depth of a cut-free proof tree is linear in its conclusion. Here we consider an alternative proof of PSPACE hardness of MALL based on an embedding of the implicational intuitionistic propositional logic (IIL) into MALL. The embedding preserves cut-free proofs in a proof system that is a variant of IIL. The embedding is efficient and it exploits several proof-theoretic properties of intuitionistic implication that analyze the use of resources in IIL proofs. This is joint work with P. Lincoln and N. Shankar.
We also consider MALL with first order quantifiers, MALL1. Without function symbols, MALL1 is still PSPACE-complete, for the same reason as MALL. However, in the presence of function symbols the complexity is different in spite of the same bound on the depth. We show that MALL1 with function symbols is NEXPTIME-hard. On the other hand, it is still decidable. This is joint work with P. Lincoln.

Simple Models for the Untyped $\lambda\beta\eta$–calculus
Harold Schellinx

We describe a class of models for the untyped λ–calculus that includes the class of *graph models* (Engelers D_A and Scotts P_ω are examples). Whereas graph models never are extensional, our generalized construction enables us to formulate necessary and sufficient conditions for the construction of *extensional* combinatory algebras. From a more abstract point of view this can be seen as an application of the Karonbi–envelope construction to a *weak* cartesian closed category in order to obtain a cartesian closed category.

Two Remarks on the Resolution Calculus
U. Schmerl

The first remark concerns a technique to obtain definite answers using the classical resolution calculus. The second remark describes a procedure that allows to reduce a given set of clauses via Herbrand substitutions of depht one. These reduction steps preserve the unsatisfiability of the given set of clauses.

A Proof-theoretic Analysis of Negation as Failure
R. Stärk

What is the semantics of 'Negation as Failure' in logic programming? We try to answer this question by proof-theoretic methods. We have developed a rule based sequent calculus for negation as failure. Given any program P, a sequent Γ is provable in the calculus if and only if it is true in all three-valued models of the completion of P. The calculus is exactly the sequent calculus for the classical completion of a program but without axioms of the form $\Gamma, A, \neg A$. The reason that we do not use axioms of the form $\Gamma, A, \neg A$ is that they imply that a formula has to be true or false; in terms of logic programming this means that an atom A must succeed or fail which, in general, is not true. It is easy to transform SLDNF-computations into sequent proofs. For certain classes the converese is also possible. We will give a sufficient and necessary condition on a program such that it is possible. Via cut-elimination the complexity of a sequent proof is bounded to sequents constructed from equations and literals only. Such proofs can then be converted into SLDNF-computations. We obtain the main theorem that a normal program is *negation complete* if and only if it has the *cut-property*. From this theorem we can derive a very strong completeness result for SLDNF-resolution.

First Order Bounded Arithmetic for AC^k, NC^{k+1} and L
G. Takeuti

We present first order systems TAC^k, TNC^k and TLS and prove

Theorem 1. A function is in AC^k iff it is esb-definable in TAC^k.

Theorem 2. A function is in NC^{k+1} iff it is esb-definable in TNC^k.

Theorem 3. A function is in L iff it is esb-definable in TLS.

Definition. A formula A is *essentially sharply bounded (esb)* in a theory $\mathcal{T}$ iff it belongs to the smallest family $\mathcal{F}$ which includes all atomic formulas and is closed under Boolean combinations, sharply bounded quantifications and under the following operations:
If $A(\vec{a},x), B(\vec{a},x) \in \mathcal{F}$ and
$\mathcal{T} \vdash \exists x \leq s(\vec{a})A(\vec{a},x)$ and $\mathcal{T} \vdash b \leq s(\vec{a}), b \leq s(\vec{a}), A(\vec{a},b)A(\vec{a},c) \rightarrow b = c$
then $\exists x \leq s(\vec{a})(A(\vec{a},x) \wedge B(\vec{a},x))$ and $\forall x \leq s(\vec{a})(A(\vec{a},x) \supset B(\vec{a},x))$ belong to $\mathcal{F}$.
Definition. A function f is *esb-definable* in $\mathcal{T}$ iff there exists $s(\vec{a})$ and an esb-formula $A(\vec{a},x)$ satisfying the following conditions.

1. $\mathcal{T} \vdash \exists y \leq s(\vec{a})A(\vec{a},y)$,
2. $\mathcal{T} \vdash A(\vec{a},b), A(\vec{a},c) \rightarrow b = c$ and $\forall \vec{x} A(\vec{x}, f(\vec{x}))$ is satisfied.

TAC^0 consists of the definition axioms of basic functions O, 1, $+$, $2^{|y|} \cdot x$, $\dot{-}$, $|x|$, $x \# y$, $\lfloor x/2 \rfloor$, MSP, $x \cdot |y|$ and $\leq$ together with the following axioms

1) Bit-Extensionality Axiom: $|a| = |b|, \forall i < |a|(\mathrm{Bit}(i.a) = \mathrm{Bit}(i.b)) \rightarrow a = b$

2) Bit-Comprehension Axiom: $\exists y < 2^{|s|} \forall i < |s|(\mathrm{Bit}(i.y) = 1 \leftrightarrow A(i))$ where $A(i)$ is esb.

3) esb–PIND: $\frac{A(\lfloor a/2 \rfloor), \Gamma \rightarrow \Delta, A(a)}{A(0), \Gamma \rightarrow \Delta, A(t)}$ where $A(a)$ is esb.

All the systems TAC^k, TNC^k and TLS are obtained by introducing some axioms to TAC^0.

The Complexity of the Hajós Construction

T. Pitassi and A. Urquhart

The Hajós Construction is a simple, nondeterministic procedure for generating the class of graphs that are not k-colorable. Mansfield and Welch have posed the problem of proving whether or not there exists a polynomial-size Hajós construction for every non-k-colorable graph. The main result is a proof that the Hajós calculus is polynomially bounded if and only if extended Frege proof systems are polynomially bounded. This result links an open problem in graph theory to an important open problem in the complexity of propositional proof systems. In addition, we establish an exponential lower bound for a strong subsystem of the Hajós calculus. Lastly, we discuss an interesting graph theoretical consequence of this result.

Dynamic Contexts & Incremental Processing

A. Visser

In my talk I describe some first steps towards a semantics for certain formal languages and certain fragments of natural language(s), that satisfies the following constraints:

1) It is compositional

2) It is incremental

3) It unifies current relational, update & "database" (DRT) versions of semantics

4) It allows a reasonable repertoire of elementary actions (like *create*, *rename*)

5) It respects the distinction between *file* and *file name*

6) It allows modular construction of meanings.

In the talk I give some theory and I describe a simple example.

Unique Normal Forms for Combinatory Logic with Parallel Conditional, a case study in conditional rewriting

R.C. de Vrijer

A Term Rewriting System (TRS) has the unicity of normal forms property (UN), if convertible normal forms are identical. We present a simple proof of UN for Combinatory Logic, extended with 'Parallel Conditional'; that is, augmented with constants C, T, and F (conditional, true, false) and with the extra reduction rules: $\mathrm{CT}xy \rightarrow x$, $\mathrm{CF}xy \rightarrow y$ and $\mathrm{C}zxx \rightarrow x$. This TRS, we call it CL-pc, is known to fail the Church-Rosser property. So the usual route of establishing UN, via CR, is not available here.
The proof is based on a more general method for proving UN for certain non-leftlinear TRSs. This method proceeds by proving confluence for an associated left-linear conditional term rewriting system, that originates from the non-leftlinear original one by 'linearizing' the rewriting rules. Apart from the the application to CL-pc, we give a cogent presentation of the linearization method, and show another consequence: all TRSs that are non-ambiguous after linearization have unique normal forms.

The application of the method to CL-pc involves two specific features, that may be worth remarking. First the use of negative conditions, in order to disambiguate the rewrite rules. Secondly, although the method is essentially proof-theoretic, we use a lemma that depends on a model-thoretic argument, using the graph-model $P\omega$.

Proof versus Truth in Arithmetic

S. S. Wainer

The truth definition for first-order arithmetic recasts immediately as the cut-free (Tait-style) infinitary rules, but with "number declaration": $n \in \mathrm{N} \models A$. Adding cuts gives the usual proof theory: $n \in \mathbf{N} \vdash A$. We compare $\models$ with $\vdash$ by means of (Buchholz-style) assignments of "ordinal structures" measuring height thus: $n \in \mathrm{N} \models^{\alpha} A$, $n \in \mathrm{N} \vdash^{\alpha} A$.

Bounding Functions: $G_\alpha(n)/B_\alpha(n) := \max m\,(n \in \mathrm{N} \models^{\alpha} / \vdash^{\alpha} m \in \mathrm{N})$.

Bounding Lemma: For $A \in \Sigma^0_1$ and α satisfying mild conditions:
$n \in \mathrm{N} \models^{\alpha} / \vdash^{\alpha} A$ iff A true in finite model $G_\alpha(n)/B_\alpha(n)$.
(Hence this extends in an obvious way to $A \in \Pi^0_2$ also).

Theorem (Girard '81, Wainer '89, ...): There is a "natural" lifting
$+ : \Omega_1 \rightarrow \Omega_2$ such that $B_\alpha = G_{\phi(\alpha^+)}$.

Complete Cut-Elimination for $\Pi^0_2 \cap (\Pi^1_1 - \mathrm{CA})_0$:
$\vdash^{\alpha} = \models^{\phi(\alpha^+)}$. (E.g. $|\mathrm{ID}_n|^+ = |\mathrm{ID}_{n+1}|$).

Tecnalities concerning $+$-operation were discussed.

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